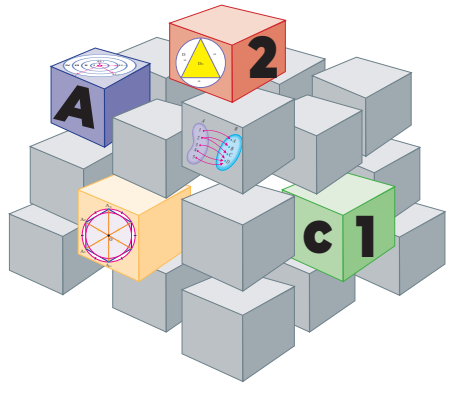
ALGEBRA
VA ANALIZ ASOSLARI
10



Umumiy o‘rta ta’lim maktablarining
10-sinfi uchun darslik

O‘zbekiston Respublikasi Xalq ta’limi vazirligi
nashrga tavsiya etgan

Yangi nashr

TOSHKENT – 2022

www.idum.uz saytidan yuklab olindi

UO‘K 512(075.3)
KBK 22.144ya72
A 45

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10-sinf Algebra va analiz asoslari [Matn]: darslik / A. Zaitov [va boshq.] – Toshkent: Respublika ta’lim markazi, 2022. – 192 b.

UNICEFning O‘zbekistondagi vakolatxonasi
bilan hamkorlikda tayyorlandi.

O‘zbekiston Respublikasi Fanlar akademiyasi V. I. Romanovskiy nomidagi
matematika instituti xulosasi asosida takomillashtirildi.

Original maket va dizayn konsepsiyasi
Respublika ta’lim markazi tomonidan ishlandi.

Respublika maqsadli kitob jamg‘armasi mablag‘lari hisobidan chop etildi.

SHARTLI BELGILAR:

– oson topshiriqlar.

– murakkabroq topshiriqlar.

– murakkab topshiriqlar.

– kichik mavzular.

ISBN 978-9943-8453-1-2

© Respublika ta’lim markazi, 2022

www.idum.uz saytidan yuklab olindi

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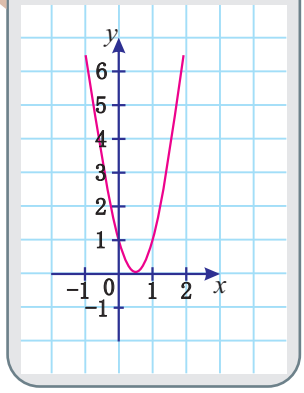


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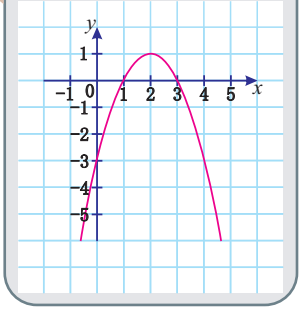
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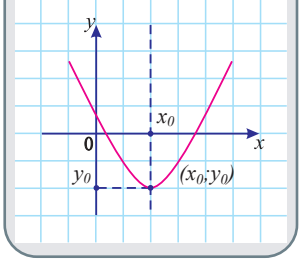
1-rasm



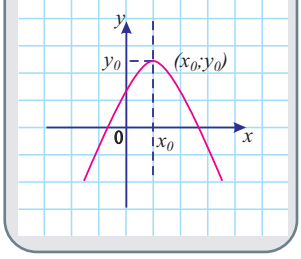
2-rasm



3-rasm



4-rasm



Kvadrat funksiyaning ta'rifi

Ta'rif

$y = ax^2 + bx + c$ ko'rinishidagi funksiya ***kvadrat funksiya*** deyiladi, bunda *a, b, c* berilgan haqiqiy sonlar, $a \neq 0$, x – haqiqiy o'zgaruvchi.

Masalan, quyidagi funksiyalar kvadrat funksiyalardir:
$y = 3x^2 + 2x - 1$, $y = -4x^2 - 5x$, $y = 6x^2 - 3$, $y = 4x^2$, $y = 2 - x^2$.

Kvadrat funksiyaning grafigi

1. $y = ax^2 + bx + c$ kvadrat funksiyaning grafigi *parabola* deb ataladigan egri chiziqdan iborat bo'ladi. 1-rasmda $y = 4x^2 - 4x + 1$ va 2-rasmda $y = -x^2 + 4x - 3$ funksiyalar grafiklari tasvirlangan.
2. $y = ax^2 + bx + c$ parabola tarmoqlari $a > 0$ bo'lganda (3-*rasm*) ordinata o'qi bo'yicha yuqoriga yo'nalgan, $a < 0$ bo'lganda (4-*rasm*) esa pastga yo'nalgan bo'ladi.
3. $y = ax^2 + bx + c$ parabola uchining $(x_0; y_0)$ koordinatalari $x_0 = -\frac{b}{2a}$, $y_0 = ax_0^2 + bx_0 + c$ yoki $y_0 = -\frac{b^2 - 4ac}{4a}$ formulalar bilan hisoblanadi.
4. $y = ax^2 + bx + c$ parabola o'zining uchi orqali o'tuvchi va ordinata o'qiga parallel to'g'ri chiziqqa nisbatan simmetrik bo'ladi.
5. $y = ax^2 + bx + c$ parabolaning *Ox* o'qi bilan kesishish nuqtalarining abssissalari kvadrat funksiyaning nollari bo'ladi. Kvadrat funksiya nollarini topish uchun $ax^2 + bx + c = 0$ tenglamani yechish kerak.
6. Kvadrat funksiyaning *Oy* o'qi bilan kesishish nuqtasining ordinatasi funksiyaning $x = 0$ nuqtadagi qiymatidan iborat.

$y = ax^2 + bx + c$ kvadrat funksiyaning grafigini yasash uchun:

1. Parabola tarmoqlari yo'nalishi aniqlanadi.
2. Parabola uchining koordinatalari $x_0 = -\frac{b}{2a}$, $y_0 = ax_0^2 + bx_0 + c$ formulalar yordamida topiladi va koordinata tekisligida belgilanadi.
3. Parabolaning abssissa o'qi bilan kesishish nuqtalari (nollari) topiladi. Agar funksiya nollari mavjud bo'lmasa, u holda odatda parabolaning simmetriya o'qiga nisbatan simmetrik bo'lgan ikkita nuqta topiladi. Masalan: parabolaning *Oy* o'qi bilan kesishish nuqtasi $(0; c)$ va unga simmetrik bo'lgan $(2x_0; c)$.
4. Yasalgan nuqtalarni uzluksiz silliq egri chiziq bilan tutashtiriladi (parabolaning grafigini aniqroq yasash uchun yana bir nechta nuqtasini yasash maqsadga muvofiq bo'ladi).

Kvadrat funksiyaning xossalari

1. Aniqlanish sohasi:

$D(y)=(-\infty;\infty)$.

2. Qiymatlar to‘plami:

a) $a>0$ bo‘lsa, $E(y)=[y_0;\infty)$.

b) $a<0$ bo‘lsa, $E(y)=(-\infty;y_0]$.

3. Eng katta va eng kichik qiymatlari:

a) $a>0$ bo‘lsa, $x=x_0$ nuqtada eng kichik qiymatga erishadi va bu qiymat $y_0=ax_0^2+bx_0+c$ ga teng bo‘ladi, eng katta qiymatga esa erishmaydi.

b) $a<0$ bo‘lsa, $x=x_0$ nuqtada eng katta qiymatga erishadi va bu qiymat $y_0=ax_0^2+bx_0+c$ ga teng bo‘ladi, eng kichik qiymatga esa erishmaydi.

4. Funksiya nollari:

a) $D=b^2-4ac>0$ bo‘lsa, funksiya ikkita nollarga ega: $x_1=\dfrac{-b+\sqrt{D}}{2a}$ va $x_2=\dfrac{-b-\sqrt{D}}{2a}$.

b) $D=b^2-4ac=0$ bo‘lsa, funksiya bitta (o‘zaro teng ikkita) nolga ega: $x=\dfrac{-b}{2a}$.

c) $D=b^2-4ac<0$ bo‘lsa, funksiya nollarga ega emas.

5. Monotonlik oraliqlari:

a) $a>0$ bo‘lsa, $y=ax^2+bx+c$ funksiya $(-\infty;x_0]$ da kamayuvchi, $[x_0;\infty)$ da o‘suvchi bo‘ladi.

b) $a<0$ da $y=ax^2+bx+c$ funksiya $(-\infty;x_0]$ da o‘suvchi, $[x_0;\infty)$ da kamayuvchi bo‘ladi (bu yerda x_0 – parabola uchining abssissasi).

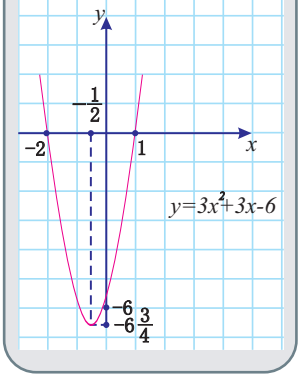
1-misol. $y=3x^2+3x-6$ kvadrat funksiya berilgan bo‘lsin. Uning xossalarini yozing va grafigini yasab ko‘rsating.

Yechish

1. Aniqlanish sohasi: $D(y)=(-\infty;\infty)$.
2. $a=3>0$ va $x_0=-\dfrac{1}{2}$, $y_0=-6,75$, $E(y)=[-6,75;\infty)$.
3. $x=-\dfrac{1}{2}$ bo‘lganda eng kichik qiymati $y=-6,75$ ga teng, eng katta qiymatga erishmaydi.
4. $D=81>0$, demak, nollari ikkita: $x_1=1, x_2=-2$.
5. $x\in(-\infty;-2)\cup(1;\infty)$ da $y>0$ va $x\in(-2;1)$ da $y<0$ bo‘ladi.
6. Funksiya juft ham, toq ham emas.
7. Funksiya $\left(-\infty;-\dfrac{1}{2}\right]$ oraliqda kamayuvchi, $\left[-\dfrac{1}{2};\infty\right)$ oraliqda o‘suvchi bo‘ladi.

Funksiya grafigi 5-rasmda ko‘rsatilgan.

5-rasm



MISOLLAR

1. Qaysi funksiyalar kvadrat funksiya bo‘ladi?

a) $y=\frac{1}{3}x+2$ b) $y=-x^2+5x+1$ c) $y=x^2-x^3$ d) $y=x^2$

2. $x=-3$ bo‘lganda, $y=4x^2+7x-5$ funksiyaning qiymati nechaga teng bo‘ladi?

3. x ning qanday qiymatlarida $y=-3x^2+x+1$ funksiyaning qiymati -1 ga teng bo‘ladi?

4. $y=-5x^2+x+\sqrt{7}$ funksiya x ning qanday qiymatlarida aniqlangan?

5. -5 soni $y=x^2-5x$ funksiyaning noli bo‘ladimi?

6. Funksiya grafigini yasang.

a) $y=x^2$ b) $y=-x^2$ c) $y=3x^2$

d) $y=-3x^2-5$ e) $y=x^2-2x$ f) $y=-2x^2+5x$

7. Funksiya nollarini toping.

a) $y=2x^2+5x+2$ b) $y=3x^2+10x+3$ c) $y=-2x^2+x-5$

8. Funksiyaning qiymatlar to‘plamini toping.

a) $y=x^2+2$ b) $y=(x-4)^2-1$ c) $y=(x-5)^2+3$ d) $y=3-4x^2$

e) $y=3x-x^2$ f) $y=3x^2+2x$ g) $y=2x^2-8x+19$ h) $y=-3x^2-12x+1$

9. x ning qanday qiymatlarida funksiya eng katta (yoki eng kichik) qiymat qabul qilishini aniqlang va uni toping.

a) $y=x^2+9x+34$ b) $y=-9x^2-3x+7$ c) $y=-2x^2-5x+1$

10. t ning qanday qiymatlarida $y=2x^2-tx+8$ funksiyaning nollari mavjud emas?

11. x ning qanday qiymatlarida $y=5x^2-4x-1$ funksiyaning qiymatlari manfiy bo‘ladi?

12. $y=x^2+6x+13$ funksiya manfiy qiymatlarni qabul qila oladimi?

13. $y=-x^2-4x-5$ funksiya musbat qiymatlarni qabul qila oladimi?

14. $y=6x^2+7x+1$ funksiya grafigini yasang va grafik bo‘yicha funksiyaning qiymatlari musbat, manfiy bo‘ladigan x ning qiymatlarini toping.

15. $y=-x^2+4x-3$ funksiya grafigini yasang. Grafik yordamida funksiyaning o‘sish va kamayish oraliqlarini toping.

16. x ning qanday qiymatlarida $y=x^2-22x+27$ va $y=2x^2-20x+3$ funksiyalarning qiymatlari teng bo‘ladi?

17. Agar parabolaning $(-1;\ 6)$ nuqta orqali o‘tishi va uning uchi $(1;\ 2)$ nuqtada ekani ma’lum bo‘lsa, parabolaning tenglamasini toping.

18. $y=x^2+px+q$ parabolaning uchi $A(1;\ -2)$ bo‘lsa, p va q koeffitsiyentlarni toping.

19. Agar $y=ax^2+bx+c$ parabolaning uchi $M(-1;\ -7)$ va parabola ordinatalar o‘qi bilan $N(0;\ -4)$ nuqtada kesishsa, $a,\ b,\ c$ koeffitsiyentlarni toping.

20. $A(1; 4)$, $B(-1; 10)$, $C(2; 7)$ nuqtalardan o‘tuvchi $y=ax^2+bx+c$ funksiyani toping.

KVADRAT TENGSIZLIK

Ta'rif

Agar tengsizlikning chap qismida kvadrat uchhad, o'ng qismida esa nol turgan bo'lsa, bunday tengsizlik ***kvadrat* (bir noma'lumli ikkinchi darajali) *tengsizlik*** deyiladi.

$ax^2+bx+c>0$, $ax^2+bx+c<0$, $ax^2+bx+c\geq 0$, $ax^2+bx+c\leq 0$ $(a\neq 0)$ tengsizliklar kvadrat tengsizliklardir, bunda a, b, c – berilgan sonlar, x esa noma'lum son.

Tengsizlikning yechimi deb noma'lumning shu tengsizlikni to'g'ri sonli tengsizlikka aylantiruvchi barcha qiymatlari to'plamiga aytiladi.

Tengsizlikni yechish uning yechimini topish yoki yechimi yo'qligini ko'rsatish demakdir.

Kvadrat tengsizlikni quyidagi usullar bilan yechish mumkin:

1-usul. Chiziqli tengsizliklar sistemasiga keltirib yechish

Agar $ax^2+bx+c=0$ kvadrat tenglama ikkita turli ildizga ega bo'lsa, u holda kvadrat tengsizlikni yechishni birinchi darajali tengsizliklar sistemasini yechishga keltirish mumkin.

1-misol. $x^2-5x+6<0$ tengsizlikni yeching.

Yechish

Tengsizlikning chap tomonini ko'paytuvchilarga ajratamiz:

$$(x-2)(x-3)<0.$$

1-hol: $\begin{cases} x-2>0 \\ x-3<0 \end{cases} \Rightarrow \begin{cases} x>2 \\ x<3 \end{cases} \Rightarrow x\in(2;3).$ 2-hol: $\begin{cases} x-2<0 \\ x-3>0 \end{cases} \Rightarrow \begin{cases} x<2 \\ x>3 \end{cases} \Rightarrow x\in\varnothing.$

Javob: (2; 3).

2-usul. Kvadrat tengsizlikni kvadrat funksiya grafigi yordamida yechish

Kvadrat tengsizliklarni kvadrat funksiya grafigini yasab, grafik bo'yicha bu funksiya musbat yoki manfiy qiymatlarni qabul qiladigan oraliqlarni topib, yechish mumkin.

Kvadrat tengsizlikni grafik usulda yechish uchun:

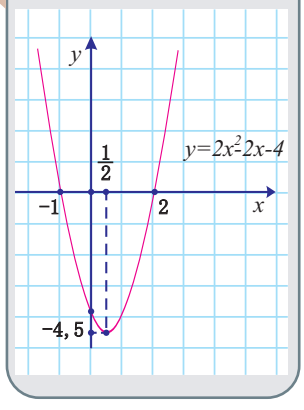
1) parabola tarmoqlari yo'nalishi aniqlanadi;

2) funksiya nollari (agar ular mavjud bo'lsa) topiladi yoki ularning yo'qligi aniqlanadi;

3) $y=ax^2+bx+c$ funksiya grafigining eskizi chiziladi;

4) grafik bo'yicha funksiya musbat yoki manfiy qiymatlar qabul qiladigan oraliqlar ko'rsatiladi.

1-rasm



2-misol. $2x^2-2x-4\ge 0$ tengsizlikni kvadrat funksiya grafigi yordamida yeching.

Yechish. $y=2x^2-2x-4$ funksiya grafigini yasaymiz (1-*rasm*).

Avval parabola uchini topamiz:

$$x_0=-\frac{b}{2a}=-\frac{-2}{4}=\frac{1}{2};\quad y_0=2\left(\frac{1}{2}\right)^2-2\cdot\frac{1}{2}-4=-4{,}5.$$

Keyin diskriminantni hisoblab: $D=b^2-4ac=4+32=36$, parabola nollarini topamiz:

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{2\pm\sqrt{36}}{4}=\frac{2\pm 6}{4},$$

$$x_1=-1,\ x_2=2.$$

Javob: $(-\infty;-1]\cup[2;\infty)$.

Tengsizlikni bu usulda yechishda parabola uchining koordinatalarini topish shart emas, shuningdek, parabolaning *Oy* o‘qi bilan kesishish nuqtalarining grafikda ko‘rsatilishi muhim emas. Eng muhimi, parabola tarmoqlarining yo‘nalishini va funksiya nollari bor yoki yo‘qligini bilishdir.

3-usul. Kvadrat tengsizlikni oraliqlar (intervallar) usuli bilan yechish

Agar biror $(a;\ b)$ oraliqda $y=f(x)$ funksiya grafigini qalamni qog‘ozdan uzmasdan chizish mumkin bo‘lsa, bu funksiya $(a;\ b)$ oraliqda ***uzluksiz*** deyiladi.

Masalan, $y=kx+b$, $y=ax^2+bx+c$ funksiyalar o‘z aniqlanish sohasida uzluksiz funksiyalardir.

Uzluksiz funksiyalarning bir muhim xossasini isbotsiz qabul qilamiz.

Agar $f(x)$ funksiya $(a;\ b)$ oraliqda uzluksiz bo‘lsa va nolga aylanmasa, u holda bu oraliqda funksiyaning qiymatlari bir xil ishoraga ega bo‘ladi, ya’ni shu oraliqda funksiya o‘z ishorasini saqlaydi.

Kvadrat funksiyaning aniqlanish sohasini uning x_1 va x_2 nollari yordamida uchta $(-\infty;\ x_1)$, $(x_1;\ x_2)$, $(x_2;\ \infty)$ oraliqlarga ajratish mumkin (bunda $x_1<x_2$). Bu oraliqlarning har birida kvadrat funksiya uzluksiz va nolga aylanmaydi, ya’ni o‘z ishorasini saqlaydi. Bir noma’lumli tengsizliklarni yechishning ***oraliqlar usuli*** aynan shu xossaga asoslangan.

Kvadrat tengsizliklarni yechishda oraliqlar usulining qo‘llanishini ko‘rib chiqamiz.

1-hol. $D>0$. Bu holatda kvadrat funksiyaning nollari hisoblangan ikkita haqiqiy x_1 va x_2 $(x_1<x_2)$ sonlar mavjud bo‘ladi. Ular kvadrat funksiyaning aniqlanish sohasini: $(-\infty;\ x_1)$, $(x_1;\ x_2)$, $(x_2;\ \infty)$ oraliqlarga ajratadi va bu oraliqlarning har birida funksiyaning qiymatlari doimiy ishoraga (“+” yoki “–”) ega bo‘ladi.

Kvadrat funksiya qiymatlarining hosil qilingan oraliqlardagi ishorasini turlicha usullar bilan topish mumkin:

1) $y=ax^2+bx+c$ funksiya qiymatining $(-\infty;\ x_1)$, $(x_2;\ \infty)$ oraliqlarning har biridagi ishorasi a koeffitsiyentning ishorasi bilan bir xil; $(x_1;\ x_2)$ oraliqdagi ishorasi esa a koeffitsiyent ishorasiga qarama-qarshi bo‘ladi;

2) funksiya qiymatlarining ishorasini har bir oraliqdagi “qulay” nuqtada aniqlash mumkin;

3) $y = ax^2 + bx + c$ funksiyani $y = a(x - x_1)(x - x_2)$ ko‘rinishda yozib, har bir oraliqda chiziqli ko‘paytuvchilarning ishoralarini topish orqali aniqlash mumkin.

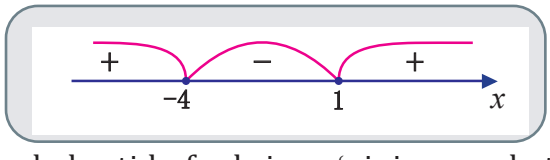
3-misol. $x^2 + 3x - 4 \le 0$ tengsizlikni oraliqlar usuli bilan yeching.

Yechish. Tengsizlikning chap qismini ko‘paytuvchilarga ajratamiz:

$$(x + 4)(x - 1) \le 0.$$

Uning nollari: – 4 va 1.

Topilgan nuqtalarni son o‘qida belgilaymiz va o‘qni oraliqlarga ajratamiz. Har bir oraliqda $y = x^2 + 3x - 4$ funksiyaning ishorasini aniqlaymiz:



Berilgan misol shartida funksiya o‘zining musbat bo‘lmagan qiymatlariga qaysi oraliqda erishishi so‘ralgani uchun yechim [–4; 1] bo‘ladi.

Javob: [–4; 1].

2-hol. $D = 0$ bo‘lsin. U holda $y = ax^2 + bx + c$ funksiya faqat bitta x_0 nuqtada nolga aylanadi. x_0 nuqta koordinata o‘qini ikkita: $(-\infty; x_0)$ va $(x_0; \infty)$ oraliqlarga ajratadi. Har bir $x \ne x_0$ uchun $y = ax^2 + bx + c$ kvadrat funksiya qiymatlarining ishorasi a koeffitsiyentning ishorasi bilan bir xil bo‘ladi (2, 3-*rasmlar*).

3-hol. $D < 0$. U holda $y = ax^2 + bx + c$ kvadrat funksiya nollarga ega bo‘lmaydi.

Bu holatda x ning ixtiyoriy qiymatlarida funksiya a koeffitsiyentning ishorasi bilan ustma-ust tushadigan bir xil ishorali qiymatlarni qabul qiladi:

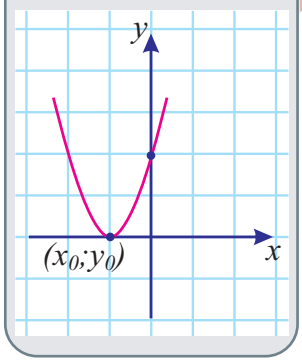
1) agar $a > 0$ bo‘lsa, x ning ixtiyoriy qiymatida $ax^2 + bx + c > 0$;

2) agar $a < 0$ bo‘lsa, x ning ixtiyoriy qiymatida $ax^2 + bx + c < 0$.

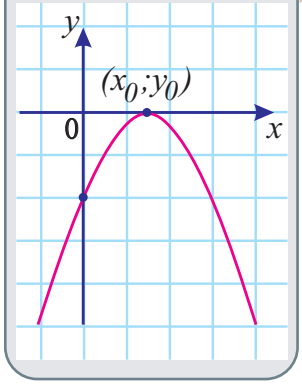
Quyidagilarni bilish va qo‘llay olish muhim:

1) $a > 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ tengsizliklarning yechimlari barcha haqiqiy sonlar to‘plamidan iborat (4-*rasm*);
2) $a > 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$ tengsizliklarning yechimlari bo‘sh to‘plamdan iborat (4-*rasm*);
3) $a < 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c > 0$, $ax^2 + bx + c \ge 0$ tengsizliklarning yechimlari bo‘sh to‘plamdan iborat (5-*rasm*);
4) $a < 0$ va $D < 0$ bo‘lganda $ax^2 + bx + c < 0$, $ax^2 + bx + c \le 0$ tengsizliklarning yechimlari barcha haqiqiy sonlar to‘plamidan iborat bo‘ladi (5-*rasm*).

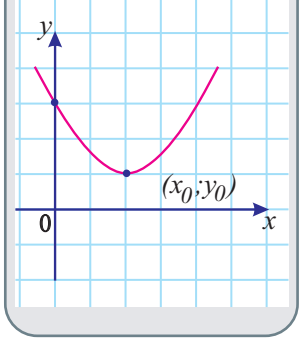
2-rasm



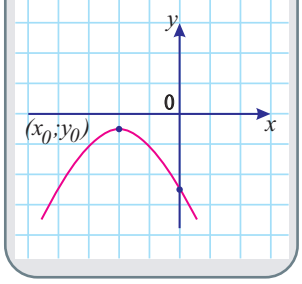
3-rasm



4-rasm



5-rasm



MISOLLAR

1. 0; –2; 3 sonlaridan qaysilari $-4x^2+5x-5>0$ tengsizlikni qanoatlantiradi?

2. Quyidagi tengsizliklarni chiziqli tengsizliklar sistemasiga keltirib yeching.

a) $(x+4)(2x-3)>0$ b) $x^2+10x-11<0$

c) $(5x-2)(4x+3)\le 0$ d) $2x^2-5x+2\ge 0$

3. Tengsizliklar teng kuchlimi?

a) $5x^2 > 2x$ va $5x > 2$ b) $3x^3 < 7x^2$ va $3x < 7$ c) $\dfrac{x^2-1}{x}>0$ va $(x^2-x)(x+1)>0$

4. Tengsizliklarni yeching.

a) $x^2>0$ b) $4x^2\ge 0$ c) $x^2<0$

d) $-x^2\le 0$ e) $x^2+7>0$ f) $5x^2+11\le 0$

g) $-x^2-5>0$ h) $3x^2-2x<0$ i) $-4x^2+11x<0$

j) $x^2-9x+20<0$ k) $x^2-10x+25>0$ l) $-x^2+6x-8>0$

m) $3x^2-x+2\ge 0$ n) $-9x^2+24x+20>0$ o) $-7\cdot(3-x)^2>0$

5. Yechimi berilgan oraliqlardan iborat bo‘lgan biror kvadrat tengsizlik tuzing.

a) $(-\infty;-3)\cup(6;\infty)$ b)$(-\infty;\infty)$

6. Abssissa o‘qida $x^2+9x\le -14$ tengsizlikning yechimi bo‘lgan kesma uzunligini toping.

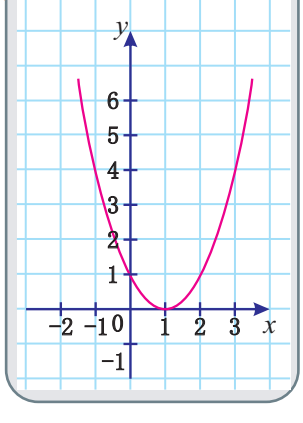
7. Nechta butun son $2x^2+7x-15<0$ tengsizlikni qanoatlantiradi?

8. Tengsizlikni oraliqlar usuli bilan yeching.

a) $x^2+5x-6>0$ b)$-x^2+x+2<0$ c) $x^2+3x+7>0$ d) $x^2+3x+7\le 0$

e) $-2x^2+5x+3>0$ f) $6x^2-x-2<0$ g) $2x^2+5x+9\le 0$ h) $49x^2-28x+4\le 0$

6-rasm



9. Tengsizliklarni yeching.

a) $8x^2+3x-5\ge 0$ b) $5x^2-12x+8\le 0$

c) $49x^2-70x+25>0$ d) $(2x^2+3x+4)(x+3)\ge 0$

e) $(7+6x-x^2)(3x-5)<0$

10. Tengsizlikni kvadrat funksiya grafigi yordamida yeching.

a) $2x^2+5x-3>0$ b) $4x^2-9x-90>0$

11. 6-rasmda $y = ax^2 + bx + c$ funksiya grafigi tasvirlangan. Quyidagi tengsizliklarning yechimini toping.

a) $ax^2+bx+c>0$ b) $ax^2+bx+c\le 0$

12. Tengsizlikning barcha butun yechimlari yig‘indisini toping.

a) $2x^2-9x+4<0$ b) $\frac{x-1}{4}+\frac{3-2x}{2}>\frac{3x+x^2}{8}$

c) $(5x+7)(x-2)\le 21x^2-11x+3$

13. $3x(x-2)-2x(x+4)-(x-16)\le 0$ tengsizlikning $[0;9]$ kesmaga tegishli bo‘lgan nechta butun yechimi bor?

14. $y=-x^2+4x-3$ funksiya grafigi yordamida quyidagi tengsizliklarning yechimini toping.

a) $-x^2+4x-3>0$ b) $-x^2+4x-3\ge 0$ c) $-x^2+4x-3<0$ d) $-x^2+4x-3\le 0$

15. a ning qanday qiymatlarida $ax^2+2ax+4=0$ tenglama ildizlarga ega bo‘lmaydi?

16. Tengsizlikni yeching: $(x-1)^2(x^2-2)<(x-1)^2(6-2x)$

17. $f(x)=(x-1)^4(x+1)^3x^2$ funksiya berilgan.

a) $f(x)<0$ b) $f(x)\le 0$ c) $f(x)>0$ d) $f(x)\ge 0$

bo‘ladigan x ning barcha qiymatlarini toping.

18. Tengsizliklarni yeching:

a) $x^2-2(b-c)x+a^2>0$, bunda a, b, c lar uchburchakning tomonlari;

b) $x^2+(a^2+b^2-c^2)x+a^2b^2>0$, bunda a, b, c lar uchburchakning tomonlari.

19. Agar $a^2+12b<0$ bo‘lsa, $3x^2-b\le ax$ ni yeching.

20. Agar $b>0{,}05a^2$ bo‘lsa, $5x^2-ax+b>0$ ni yeching.

21. Agar $b^2\le 4ac$ va $a+c>b$ bo‘lsa, $ax^2+bx+c\le 0$ ni yeching.

22. c ning qanday qiymatlarida $y=cx^2+x+c$ va $y=cx+1-c$ funksiyalar grafiklari umumiy nuqtaga ega bo‘lmaydi?

23. p ning qanday qiymatlarida $y=px^2-24x+1$ va $y=12x^2-2px-1$ funksiyalar grafigi kesishmaydi?

24. a ning qanday qiymatlarida $x^2+3x+a=0$ tenglamaning ildizlari $\frac{x_1}{x_2}+\frac{x_2}{x_1}+1>0$ shartni qanoatlantiradi?

25. b ning qanday qiymatlarida $x^2-2bx+b+6=0$ tenglamaning:

a) ildizlari manfiy; b) ildizlari musbat; c) ildizlari turli ishorali bo‘ladi?

26. a ning qanday qiymatlarida barcha haqiqiy sonlar tengsizlikni qanoatlantiradi?

a) $x^2-(a+2)x+8a+1>0$ b) $\frac{1}{24}x^2+ax-a+1>0$

27. b ning qanday qiymatlarida tengsizlik yechimga ega emas?

a) $x^2+2bx+1<0$ b) $bx^2+4bx+5<0$ c) $bx^2+(2b+3)x+b-1\ge 0$

TRIGONOMETRIK AYNIYATLAR

Asosiy trigonometrik ayniyatlar

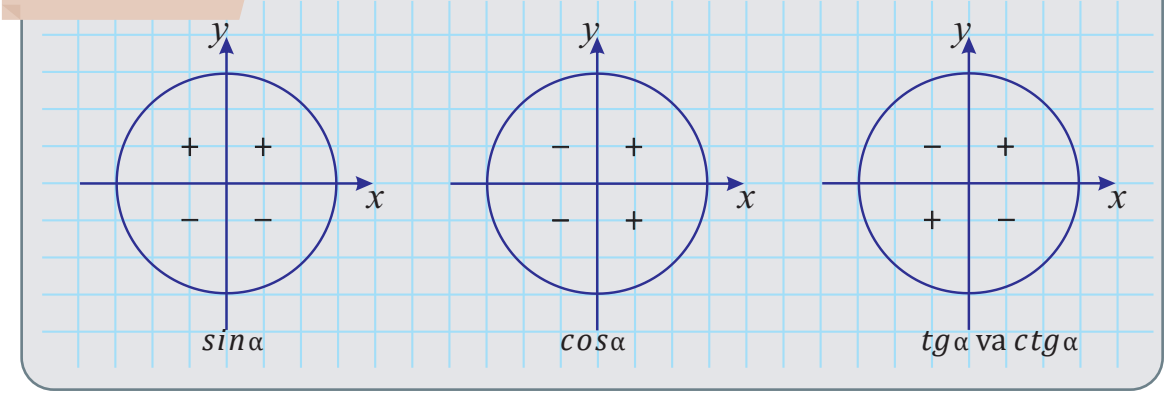
1. $\sin^2\alpha+\cos^2\alpha=1$
2. $tg\alpha=\frac{\sin\alpha}{\cos\alpha},\ \cos\alpha\neq 0$
3. $ctg\alpha=\frac{\cos\alpha}{\sin\alpha},\ \sin\alpha\neq 0$
4. $tg\alpha\cdot ctg\alpha=1$
5. $1+tg^2\alpha=\frac{1}{\cos^2\alpha},\ \cos\alpha\neq 0$
6. $1+ctg^2\alpha=\frac{1}{\sin^2\alpha},\ \sin\alpha\neq 0$

Ba'zi burchaklarning sinusi, kosinusi, tangensi va kotangensining qiymatlari

α	$0^0\,(0)$	$30°\left(\frac{\pi}{6}\right)$	$45°\left(\frac{\pi}{4}\right)$	$60°\left(\frac{\pi}{3}\right)$	$90°\left(\frac{\pi}{2}\right)$	$180°(\pi)$
$\sin\alpha$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\alpha$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	–1
$tg\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Mavjud emas	0
$ctg\alpha$	Mavjud emas	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	Mavjud emas

Sinus, kosinus, tangens va kotangensning ishoralari

1-rasm



α va $(-\alpha)$ burchaklarning sinusi, kosinusi, tangensi va kotangensi

1. $\sin(-\alpha)=-\sin\alpha$
2. $\cos(-\alpha)=\cos\alpha$
3. $tg(-\alpha)=-tg\alpha$
4. $ctg(-\alpha)=-ctg\alpha$

Keltirish formulalari

	$\frac{\pi}{2}-\alpha$	$\frac{\pi}{2}+\alpha$	$\pi-\alpha$	$\pi+\alpha$	$\frac{3\pi}{2}-\alpha$	$\frac{3\pi}{2}+\alpha$	$2\pi-\alpha$	$2\pi+\alpha$
$\sin\alpha$	$\cos\alpha$	$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$
$\cos\alpha$	$\sin\alpha$	$-\sin\alpha$	$-\cos\alpha$	$-\cos\alpha$	$-\sin\alpha$	$\sin\alpha$	$\cos\alpha$	$\cos\alpha$
$tg\,\alpha$	$ctg\,\alpha$	$-ctg\,\alpha$	$-tg\,\alpha$	$tg\,\alpha$	$ctg\,\alpha$	$-ctg\,\alpha$	$-tg\,\alpha$	$tg\,\alpha$
$ctg\,\alpha$	$tg\,\alpha$	$-tg\,\alpha$	$-ctg\,\alpha$	$ctg\,\alpha$	$tg\,\alpha$	$-tg\,\alpha$	$-ctg\,\alpha$	$ctg\,\alpha$

Keltirish formulalaridagi ushbu qonuniyatga e'tibor qarating: agar α ni I chorakka tegishli deb olsak, $\pi\pm\alpha$, $2\pi\pm\alpha$ burchaklar uchun keltirish formulalarida funksiya nomi almashmaydi, $\frac{\pi}{2}\pm\alpha$, $\frac{3\pi}{2}\pm\alpha$ burchaklar uchun esa sinus kosinusga, kosinus sinusga, tangens kotangensga, kotangens tangensga almashadi.

Masalan, $\sin\left(n\cdot\frac{\pi}{2}\pm\alpha\right)$ ni qarasak, $\frac{\pi}{2}$ lar sonini ifodalaydigan n – natural son juft bo'lsa, funksiya nomi almashmaydi; toq bo'lsa, funksiya nomi almashadi. Ishorani aniqlash esa $n\cdot\frac{\pi}{2}\pm\alpha$ burchak qaysi chorakka tegishli ekani va bu chorakda sinusning ishorasi qandayligiga qarab aniqlanadi.

1-misol. Hisoblang.

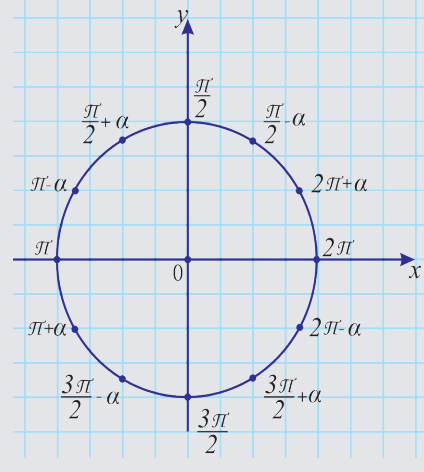
a) $\sin 855^\circ = \sin(9\cdot 90^\circ+45^\circ) = \cos 45^\circ = \frac{\sqrt{2}}{2}$

b) $\cos 2025^\circ = \cos(22\cdot 90^\circ+45^\circ) = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

c) $tg\,1680^\circ = tg(18\cdot 90^\circ+60^\circ) = tg\,60^\circ = \sqrt{3}$

d) $ctg\,1200^\circ = ctg(13\cdot 90^\circ+30^\circ) = -tg\,30^\circ = -\frac{\sqrt{3}}{3}$

2-rasm



Qo‘shish formulalari

1. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$
2. $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$
3. $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$
4. $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$
5. $tg(\alpha+\beta)=\dfrac{tg\alpha+tg\beta}{1-tg\alpha tg\beta}$
6. $tg(\alpha-\beta)=\dfrac{tg\alpha-tg\beta}{1+tg\alpha tg\beta}$
7. $ctg(\alpha+\beta)=\dfrac{ctg\alpha ctg\beta-1}{ctg\alpha+ctg\beta}$
8. $ctg(\alpha-\beta)=\dfrac{ctg\alpha ctg\beta+1}{ctg\beta-ctg\alpha}$

Ikkilangan burchak formulalari

1. $\sin 2\alpha=2\sin\alpha\cdot\cos\alpha$
2. $\cos 2\alpha=\cos^2\alpha-\sin^2\alpha=2\cos^2\alpha-1=1-2\sin^2\alpha$
3. $tg2\alpha=\dfrac{2tg\alpha}{1-tg^2\alpha}$
4. $ctg2\alpha=\dfrac{ctg^2\alpha-1}{2ctg\alpha}$

Yig‘indi va ayirmani ko‘paytmaga almashtirish formulalari

1. $\sin\alpha+\sin\beta=2\sin\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2}$
2. $\sin\alpha-\sin\beta=2\sin\dfrac{\alpha-\beta}{2}\cdot\cos\dfrac{\alpha+\beta}{2}$
3. $\cos\alpha+\cos\beta=2\cos\dfrac{\alpha+\beta}{2}\cdot\cos\dfrac{\alpha-\beta}{2}$
4. $\cos\alpha-\cos\beta=-2\sin\dfrac{\alpha+\beta}{2}\cdot\sin\dfrac{\alpha-\beta}{2}$
5. $tg\alpha+tg\beta=\dfrac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}$
6. $tg\alpha-tg\beta=\dfrac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$
7. $ctg\alpha+ctg\beta=\dfrac{\sin(\alpha+\beta)}{\sin\alpha\cdot\sin\beta}$
8. $ctg\alpha-ctg\beta=\dfrac{-\sin(\alpha-\beta)}{\sin\alpha\cdot\sin\beta}$

Ko‘paytmani yig‘indiga almashtirish formulalari

1. $\sin\alpha\cdot\cos\beta=\dfrac{1}{2}(\sin(\alpha+\beta)+\sin(\alpha-\beta))$
2. $\cos\alpha\cdot\cos\beta=\dfrac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$
3. $\sin\alpha\cdot\sin\beta=\dfrac{1}{2}(\cos(\alpha-\beta)-\cos(\alpha+\beta))$

Daraja pasaytirish formulalari

1. $\sin^2\alpha=\dfrac{1-\cos 2\alpha}{2}$
2. $\cos^2\alpha=\dfrac{1+\cos 2\alpha}{2}$
3. $tg^2\alpha=\dfrac{1-\cos 2\alpha}{1+\cos 2\alpha}$
4. $ctg^2\alpha=\dfrac{1+\cos 2\alpha}{1-\cos 2\alpha}$

Yarim burchak formulalari

1. $cos^2\frac{\alpha}{2}=\frac{1+cos\alpha}{2}$
2. $sin^2\frac{\alpha}{2}=\frac{1-cos\alpha}{2}$
3. $tg^2\frac{\alpha}{2}=\frac{1-cos\alpha}{1+cos\alpha}$
4. $ctg^2\frac{\alpha}{2}=\frac{1+cos\alpha}{1-cos\alpha}$
5. $tg\frac{\alpha}{2}=\frac{sin\alpha}{1+cos\alpha}=\frac{1-cos\alpha}{sin\alpha}$
6. $ctg\frac{\alpha}{2}=\frac{sin\alpha}{1-cos\alpha}=\frac{1+cos\alpha}{sin\alpha}$

$sin\alpha$, $cos\alpha$, $tg\alpha$ larni $tg\frac{\alpha}{2}$ orqali ifodalash formulalari

1. $sin\alpha=\frac{2tg\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$
2. $cos\alpha=\frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}}$
3. $tg\alpha=\frac{2tg\frac{\alpha}{2}}{1-tg^2\frac{\alpha}{2}}$

MISOLLAR

1. Agar $ctg\alpha=-\frac{3}{4}$ va $\frac{3\pi}{2}<\alpha<2\pi$ bo‘lsa, $cos\alpha$ ni toping.

2. Agar $tg\alpha=-\sqrt{5}$ va $\frac{\pi}{2}<\alpha<\pi$ bo‘lsa, $sin\alpha$ ni toping.

3. Agar $tg\alpha=\frac{3}{2}$ bo‘lsa, $\frac{2sin\alpha+5cos\alpha}{3sin\alpha-4cos\alpha}$ ni toping.

4. Soddalashtiring.

a) $\frac{2sin^2\alpha-1}{2cos^2\alpha-1}$ b) $\frac{ctg\left(2sin^2\alpha+tg\alpha\right)}{\left(sin\alpha+cos\alpha\right)^2}$ c) $\frac{1-cos^2\alpha}{sin^4\alpha}-ctg^2\alpha$ d) $2-\frac{1-sin^4\alpha}{cos^2\alpha}$

5. Hisoblang.

a) $4cos150°-sin240°-3tg210°$ b) $2cos135°+tg60°-ctg240°$

c) $sin300°-3cos135°+2cos210°$ d) $tg150°-ctg315°+5sin135°$

6. Hisoblang.

a) $tg\frac{7\pi}{6}-2tg\frac{5\pi}{3}+3tg\frac{11\pi}{6}$ b) $2cos\frac{4\pi}{3}+sin\frac{5\pi}{4}-cos\frac{4\pi}{3}$

c) $20ctg\frac{3\pi}{2}-sin\frac{2\pi}{3}+cos\frac{4\pi}{3}$ d) $ctg\frac{7\pi}{4}+2ctg\frac{4\pi}{3}-2cos\frac{5\pi}{6}$

7. Soddalashtiring.

a) $\frac{1-tg(360°-\alpha)tg\alpha}{tg(270°-\alpha)+tg\alpha}$ b) $\frac{cos(90°+\alpha)+sin(90°-\alpha)}{cos(270°-\alpha)+cos(180°+\alpha)}$

8. Soddalashtiring.

a) $\dfrac{\sin\left(\frac{\pi}{2}+\beta\right)}{tg^2\left(\frac{\pi}{2}-\frac{\beta}{2}\right)-ctg^2\left(\frac{\pi}{2}+\frac{\beta}{2}\right)}$

b) $\dfrac{\cos\left(\frac{3\pi}{2}-2a\right)+\sin\left(\frac{3\pi}{2}+2a\right)}{\cos\left(\frac{3\pi}{2}+2a\right)+\sin\left(\frac{3\pi}{2}-2a\right)}$

9. Ayniyatni isbotlang.

a) $\dfrac{\sin(\pi-2\alpha)-2\sin\left(\frac{\pi}{2}-\alpha\right)}{\sin^2(\pi+\alpha)-\cos\left(\frac{3\pi}{2}+\alpha\right)}=2ctg\alpha$

b) $\dfrac{\sin^4\left(\alpha-\frac{\pi}{2}\right)-\cos^2(2\alpha+\pi)}{1-3\cos(2\alpha+\pi)}=\dfrac{\sin^2\alpha}{2}$

10. Hisoblang.

a) $\sin(-43^\circ)\cos 88^\circ+\cos(-43^\circ)\sin 88^\circ$

b) $\cos 11^\circ\cos 19^\circ-\sin 19^\circ\sin 11^\circ$

11. Hisoblang.

a) $\sin\dfrac{2\pi}{7}\cos\dfrac{3\pi}{14}+\cos\dfrac{2\pi}{7}\sin\dfrac{3\pi}{14}$

b) $\dfrac{1+tg33^\circ tg78^\circ}{tg78^\circ-tg33^\circ}$

12. Hisoblang.

a) $\cos\left(-\dfrac{19\pi}{36}\right)\cos\dfrac{7\pi}{9}-\sin\dfrac{7\pi}{9}\sin\left(-\dfrac{19\pi}{36}\right)$

b) $\dfrac{1-tg\frac{\pi}{11}tg\frac{5\pi}{66}}{tg\frac{5\pi}{66}+tg\frac{\pi}{11}}$

13. Soddalashtiring.

a) $\cos(\alpha-\beta)-\sin\alpha\sin\beta$

b) $\sin(\alpha-\beta)+\sin(\alpha+\beta)$

c) $\sin 4\alpha\cos\alpha-\cos 4\alpha\sin\alpha$

d) $\cos\alpha\cos 2\alpha+\sin 2\alpha\sin\alpha$

e) $\dfrac{1}{2}\cos\alpha+\dfrac{\sqrt{3}}{2}\sin\alpha$

f) $\dfrac{1}{\sqrt{2}}\cos\alpha-\dfrac{\sqrt{2}}{2}\sin\alpha$

14. a) $\cos\alpha=\dfrac{2\sqrt{2}}{3}$ va $\dfrac{3\pi}{2}<\alpha<2\pi$ bo‘lsa, $\cos\left(\alpha+\dfrac{\pi}{4}\right)$ ni toping.

b) $\cos\alpha=-\dfrac{1}{\sqrt{3}}$ va $\pi<\alpha<\dfrac{3\pi}{2}$ bo‘lsa, $\sin\left(\alpha+\dfrac{\pi}{6}\right)$ ni toping.

15. Hisoblang.

a) $\dfrac{6\sin 10^\circ\cos 10^\circ}{\sin 20^\circ}$

b) $\dfrac{\sin 88^\circ}{\sin 22^\circ\cos 22^\circ\cos 44^\circ}$

c) $\sin\dfrac{\pi}{12}\left(2\sin^2\dfrac{\pi}{24}-1\right)$

16. a) Agar $\cos\alpha=0{,}4$ bo‘lsa, $\cos 2\alpha$ ni toping.

b) Agar $\sin\alpha=-0{,}7$ bo‘lsa, $\cos 2\alpha$ ni toping.

17. a) Agar $cos\alpha = \frac{2\sqrt{2}}{3}$ va $\frac{3\pi}{2} < \alpha < 2\pi$ bo‘lsa, $sin2\alpha$ ni toping.

b) Agar $sin\alpha = \frac{1}{5}$ va $\frac{\pi}{2} < \alpha < \pi$ bo‘lsa, $sin2\alpha$ ni toping.

18. Soddalashtiring:

a) $2cos^2\frac{\alpha}{2}(cos\alpha - 1)$ b) $\frac{sin2\alpha}{1+cos2\alpha}$

19. Agar $tg\alpha = -2$ bo‘lsa, $sin2\alpha$, $cos2\alpha$, $tg2\alpha$, $ctg2\alpha$ larni toping.

20. a) $cos123°tg231°sin312°$ ifodaning ishorasini aniqlang.

b) $sin\frac{1}{3}cos\frac{7}{8}tg4ctg5,7$ ifodaning ishorasini aniqlang.

21. Sonlarni taqqoslang: $sin200°$ va $sin(-200°)$.

22. $sin\alpha = \frac{12}{13}$ va $cos\alpha = \frac{5}{13}$ tengliklarning ikkalasi bir vaqtda o‘rinli bo‘lishi mumkinmi?

23. Ayniyatni isbotlang.

a) $\left(sin\alpha + \frac{1}{sin\alpha}\right)^2 + \left(cos\alpha + \frac{1}{cos\alpha}\right)^2 - \left(tg^2\alpha + ctg^2\alpha\right) = 7$ b) $\frac{tg3\alpha}{tg^2 3\alpha - 1} \cdot \frac{1 - ctg^2 3\alpha}{ctg3\alpha} = 1$

24. Ifodani soddalashtiring.

a) $\frac{1 - ctg^2(-\alpha)}{cos\alpha + sin(-\alpha)} \cdot sin(-\alpha) + ctg(-\alpha)$ b) $\frac{sin(\alpha-\beta) - sin(\beta-\alpha)}{cos(\alpha-\beta) + cos(\beta-\alpha)}$

25. Ayniyatni isbotlang:

a) $sin\alpha + cos\alpha = \sqrt{2}\,cos\left(\alpha - \frac{\pi}{4}\right)$ b) $2sin2\alpha cos5\alpha = sin7\alpha - sin3\alpha.$

26. $cos\alpha = \frac{2}{3}$; $sin\beta = -\frac{\sqrt{7}}{4}$; $\frac{3\pi}{2} < \alpha < 2\pi$; $\pi < \beta < \frac{3\pi}{2}$ bo‘lsa, $cos(\alpha+\beta)$ ni toping.

27. Ayniyatni isbotlang: $\frac{tg\alpha - tg\beta}{tg\alpha + tg\beta} = \frac{sin(\alpha-\beta)}{sin(\alpha+\beta)}$.

28. Hisoblang: $sin(-300^0)cos(-135^0)tg(-210^0)ctg(-120^0)$.

29. Agar $sin\alpha + cos\alpha = \frac{2}{3}$ bo‘lsa, $sin\alpha cos\alpha$ ni toping.

30. Agar $sin\alpha\, cos\alpha = \frac{1}{3}$ va $\pi < a < \frac{3\pi}{2}$ bo‘lsa, $sin\alpha + cos\alpha$ ni toping.

ARIFMETIK VA GEOMETRIK PROGRESSIYA

Arifmetik progressiya

1. $a_{n+1} = a_n + d$, $n \in N$;
2. $a_n = a_1 + (n-1)d$, $n \in N$;
3. $a_n = a_k + (n-k)d$, $n,k \in N$, $n>k$;
4. $a_n = \frac{a_{n-1} + a_{n+1}}{2}$, $n \in N$;
5. $a_n = \frac{a_{n-k} + a_{n+k}}{2}$, $n,k \in N$, $n>k$;
6. $\{a_n\}$ arifmetik progressiya hadlari uchun $a_n + a_m = a_k + a_l$ tenglik o‘rinli, bunda $n + m = k + l$;
7. $S_n = \frac{(a_1 + a_n)n}{2}$;
8. $S_n = \frac{(2a_1 + (n-1)d)n}{2}$.

Geometrik progressiya

1. $b_{n+1} = b_n \cdot q$, $n \in N$;
2. $b_n = b_1 \cdot q^{n-1}$, $n \in N$;
3. $b_n = b_k \cdot q^{n-k}$, $n,k \in N$ va $n>k$;
4. $b_n^2 = b_{n-k} \cdot b_{n+k}$, $n,k \in N$, $n>k$;
5. $\{b_n\}$ arifmetik progressiya hadlari uchun $b_n \cdot b_m = b_k \cdot b_l$ tenglik o‘rinli, bunda $n + m = k + l$;
6. $S_n = \frac{b_1(1-q^n)}{1-q}$, $q \neq 1$, agar $q = 1$ bo‘lsa, $S_n = b_1 \cdot n$;
7. $S_n = \frac{b_n q - b_1}{q-1}$, $q \neq 1$;
8. Cheksiz kamayuvchi geometrik progressiya (barcha hadlari) yig‘indisi:

$$S = \frac{b_1}{1-q}, \quad |q| < 1, q \neq 0.$$

MISOLLAR

1. Agar $a_1 = -3$ va $d = 6$ bo‘lsa, arifmetik progressiyaning saksoninchi hadini toping.

2. 2, 6, 10, 14, 18, ... ketma-ketlik arifmetik progressiya tashkil qiladi. Uning n-hadi formulasini yozing.

3. Arifmetik progressiyada:

a) $a_7 = -5$, $a_{32} = 70$ bo‘lsa, a_1 va d ni;

b) $a_5 = 2$, $a_{40} = 142$ bo‘lsa, a_7 ni;

c) $a_{14} = 5$, $a_{12} = 1$ bo‘lsa, a_{13} ni;

d) $a_{25} - a_{20} = 10$, $a_{16} = 13$ bo‘lsa, a_{10} ni toping.

4. Agar geometrik progressiyada $b_2 = 4$ va $b_3 = 6$ bo‘lsa, b_7 ni toping.

5. Agar geometrik progressiyada $b_1 = 3$ va $q = -2$ bo‘lsa, b_8 ni toping.

6. Geometrik progressiyada:

a) $b_1 = 18$, $q = \frac{1}{9}$ bo‘lsa, b_2 ni;

b) $b_1 = \frac{1}{2}$, $q = \frac{1}{2}$ bo‘lsa, b_7 ni;

c) $b_4 = 8$, $b_8 = 128$ bo‘lsa, b_1 va q ni;

d) $b_9 = -1$, $q = -1$ bo‘lsa, b_1 va b_{17} ni toping.

7. Geometrik progressiyada $b_1 = 3$, $q = 2$ bo‘lsa, S_6 ni toping.

8. Geometrik progressiyada $b_2 = 6$, $q = 3$ bo‘lsa, S_8 ni toping.

9. Geometrik progressiyada $b_1 = 4$, $q = \frac{1}{2}$ bo‘lsa, dastlabki 10 ta hadi yig‘indisini toping.

10. Geometrik progressiyaning birinchi hadi 5, oltinchi hadi 1215 ga teng. Shu progressiya maxrajini toping.

11. Cheksiz kamayuvchi geometrik progressiyada $b_1 = 8$, $q = \frac{1}{2}$ bo‘lsa, uning yig‘indisini toping.

12. 12, 4, $\frac{4}{3}$, ... cheksiz kamayuvchi geometrik progressiya yig‘indisini toping.

13. Geometrik progressiyada:

a) $b_1 = 24$, $b_2 = 36$ bo‘lsa, q ni;

b) $b_5 = 36$, $b_7 = 144$ bo‘lsa, b_6 ni;

c) $b_6 = \frac{1}{486}$, $b_8 = \frac{1}{4374}$ bo‘lsa, b_7 ni toping.

14. Cheksiz kamayuvchi geometrik progressiya yig‘indisi 150 ga teng. Agar $q = \frac{1}{3}$ bo‘lsa, b_1 ni toping.

15. Cheksiz kamayuvchi geometrik progressiyada $b_1 = \frac{1}{4}$, $S = 16$ bo‘lsa, q ni toping.

16. Geometrik progressiyada:

a) $b_1 = 3$, $q = 5$ bo‘lsa, S_4 ni;

b) $b_2 = 8$, $b_3 = 4$ bo‘lsa, S_6 ni;

c) $b_1 = -2$, $b_6 = -486$ bo‘lsa, S_6 ni toping.

17. Geometrik progressiyada $q=-\frac{1}{2}$, $S_8=\frac{85}{64}$ bo‘lsa, b_1 ni toping.

18. Arifmetik progressiyaning n-hadining formulasini yozing.

a) $a_1=5,\ a_2=-5$ b) $a_1=-3,\ a_6=12$ c) $a_1=6,\ a_{10}=33$

19. Arifmetik progressiyaning to‘rtinchi va oltinchi hadlari mos ravishda 16 va 19 ga teng bo‘lsa, birinchi hadini toping.

20. Dastlabki 25 ta natural son yig‘indisini toping.

21. (a_n) arifmetik progressiyada $a_3+a_7=5$ va $a_4=1$ bo‘lsa, uning dastlabki o‘nta hadi yig‘indisini toping.

22. Arifmetik progressiyada $a_1=-20{,}7$, $d=1{,}8$ bo‘lsa, nechanchi hadidan boshlab progressiyaning barcha hadlari musbat bo‘ladi?

23. Arifmetik progressiyada $a_{12}+a_{15}=20$ bo‘lsa, S_{26} ni toping.

24. Arifmetik progressiyada $a_2+a_6=44$, $a_5-a_1=20$ bo‘lsa, a_{100} ni toping.

25. Arifmetik progressiyaning uchinchi va to‘qqizinchi hadlari yig‘indisi 8 ga teng. Shu progressiyaning dastlabki o‘n bitta hadi yig‘indisini toping.

26. Arifmetik progressiyada $S_n=3n^2+n$ bo‘lsa, a_1 va d ni toping.

27. O‘suvchi geometrik progressiyada $b_{12}=4\cdot b_{10}$ va $b_3=6$ bo‘lsa, b_7 ni toping.

28. Geometrik progressiyada $b_5=\sqrt[3]{2}$. Shu progressiya dastlabki to‘qqizta hadi ko‘paytmasini toping.

29. Geometrik progressiyada $S_4=10\frac{5}{8}$, $S_5=42\frac{5}{8}$, $b_1=\frac{1}{8}$ bo‘lsa, q ni toping.

30. Geometrik progressiyada $b_1=1$ va $b_3+b_5=90$ bo‘lsa, q ni toping.

31. Uchta x, y va 12 soni kamayuvchi geometrik progressiyani tashkil qiladi. Agar 12 ning o‘rniga 9 qo‘yilsa, arifmetik progressiya hosil bo‘ladi. $x+y$ ni toping.

32. Geometrik progressiyada $b_2\cdot b_4\cdot b_6=216$ va $b_3=12$. Shu progressiyaning dastlabki oltita hadi yig‘indisini toping.

33. Cheksiz kamayuvchi geometrik progressiyaning toq o‘rinlarda turgan hamma hadlarining yig‘indisi 36 ga teng. Juft o‘rinlarda turgan hamma hadlarining yig‘indisi 12 ga teng. Shu progressiya maxrajini va ikkinchi hadini toping.

34. Cheksiz kamayuvchi geometrik progressiyaning birinchi va to‘rtinchi hadlarining yig‘indisi 54 ga, ikkinchi va uchinchi hadlarining yig‘indisi 36 ga teng. Shu progressiyaning yig‘indisini toping.

35. Arifmetik progressiyada:

a) $a_1=-3$, $a_3\cdot a_7=24$ bo‘lsa, S_{12} ni;

b) $a_2+a_9=20$ bo‘lsa, S_{10} ni;

c) $a_3+a_6=19$ bo‘lsa, S_8 ni;

d) $S_4=-28$, $S_6=58$ bo‘lsa, S_{16} ni toping.



1-BOB. FUNKSIYALAR

- FUNKSIYA. FUNKSIYANING BERILISH USULLARI
- FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TO‘PLAMI
- FUNKSIYALAR USTIDA ARIFMETIK AMALLAR
- MURAKKAB, TESKARI, DAVRIY FUNKSIYALAR
- FUNKSIYA XOSSALARI
- FUNKSIYA GRAFIGI USTIDA SODDA ALMASHTIRISHLAR
- CHIZIQLI VA KVADRATIK MODELLASHTIRISHLAR
- LOYIHA ISHI

FUNKSIYA. FUNKSIYANING BERILISH USULLARI

Funksiya

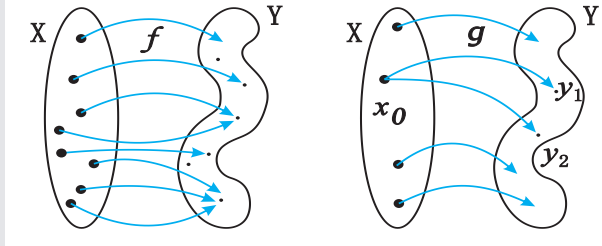
Tabiat, ishlab chiqarish, iqtisodiyot va boshqa sohalarda ayrim miqdorlar orasidagi bog‘lanishlarni o‘rganishda ***funksiya*** tushunchasi muhim ahamiyatga ega.

X va Y sonli to‘plamlar bo‘lsin. Har bir $x \in X$ nuqtaga yagona $y \in Y$ nuqtani mos qo‘yuvchi qonuniyat ***funksiya*** deyiladi.

Funksiyani aniqlovchi qonuniyatlar f, g, ... harflari orqali belgilanadi. $y = f(x)$ yozuv f qonuniyat $x \in X$ nuqtaga $y \in Y$ nuqtani mos qo‘yganini anglatadi va bu holda X to‘plamning nuqtalarini Y to‘plamning nuqtalariga mos qo‘yuvchi f funksiya berilgan deyiladi. Bunda, x ***erkli o‘zgaruvchi*** yoki ***argument***, y esa ***erksiz o‘zgaruvchi*** yoki ***funksiya*** deb yuritiladi. f funksiya odatda $y = f(x)$ yoki $f(x)$ ko‘rinishlarda ifodalanadi.

Quyida ayrim funksiyalar keltirilgan:

1-rasm



f qonuniyat funksiya bo‘ladi: X ning har bir x elementiga Y dan yagona y element mos qo‘yilgan.

g qonuniyat funksiya emas: $x_0 \in X$ elementga ikkita $y_1, y_2 \in Y$ elementlar mos qo‘yilgan.

Funksiya bo‘ladigan (f) va funksiya bo‘lmaydigan (g) qonuniyatlar

1) *chiziqli funksiya*: $y = kx + b$

2) *kvadrat funksiya*: $y = ax^2 + bx + c$

3) *darajali funksiya*: $y = x^n$

4) *kasr darajali funksiya*: $y = \sqrt[n]{x^m}$

5) *teskari proporsionallik funksiyasi*: $y = \frac{k}{x}$ (bu yerda $k \neq 0$)

6) *modulli funksiya*: $y = |x|$

Funksiyaning berilish usullari

Funksiyalar quyidagi usullarda berilishi mumkin:

1. Funksiya berilishining ***analitik usuli.*** Agar funksiya bitta yoki bir nechta formula yoki tenglamalar bilan berilgan bo‘lsa, u holda bu funksiya analitik usulda berilgan deyiladi. Masalan, moddiy nuqtaning harakat tenglamasi $s = 20 - 5t + \frac{1}{4}t^2$ analitik usulda berilgan funksiyaga misol bo‘la oladi.

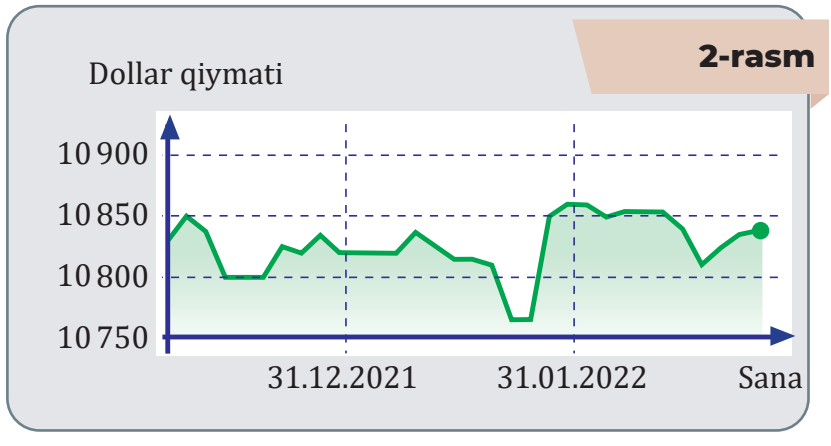
2. Funksiya berilishining ***jadval usuli*** odatda amaliy tajribalarda o‘zgaruvchilar orasidagi o‘zaro bog‘liqlikni ifodalaydi. Masalan, haroratning kunlik o‘zgarishi jad-

val usulida berilishi mumkin. Bu yerda kunlar erkli o‘zgaruvchi (ya’ni argument), harorat esa erksiz o‘zgaruvchi (ya’ni funksiya) bo‘ladi. Toshkent shahrida 2022-yil 20–26-yanvar kunlari havo haroratining haftalik o‘zgarishi quyidagi jadvalda keltirilgan:

Sana		20.01	21.01	22.01	23.01	24.01	25.01	26.01
Harorat, t °C	Kunduzi	13	9	3	4	6	7	8
	Kechasi	–2	–3	–1	–2	–3	–4	–3

3. Ayrim amaliy ishlarda o‘zgaruvchilarning bog‘liqligi ***grafik usulda*** beriladi. Masalan, dollarning so‘mga nisbatan qiymatining oylik, yillik o‘zgarishi grafik usulda ifodalanishi mumkin. Bu yerda sanalar *argument*, dollarning so‘mga nisbatan qiymati esa *funksiya* bo‘ladi.

2-rasm



4. Funksiya ***matn usulida*** berilishi ham mumkin. Masalan: 4 nafar a’zosi bor oila osh pishirish uchun 1 kg guruch sarflaydi. Uyga 2 nafar mehmon kelganda osh pishirish uchun necha kg guruch kerak bo‘ladi? Bu masalada oshdagi guruch miqdori uydagi kishilar soniga bog‘liq bo‘lib, kishilar soni argument, guruch miqdori funksiya bo‘ladi.

MISOLLAR

1. Matn bilan berilgan funksiyaning analitik ko‘rinishini yozing. Masalan, “argumentning kvadratidan 5 ni ayiring” deyilishi quyidagi funksiyani beradi: $f(x) = x^2 - 5$.

a) argumentni 3 ga ko‘paytirib, undan 5 ni ayiring.
b) argumentning kvadratiga 2 ni qo‘shing.
c) argumentdan 1 ni ayirib, keyin kvadratga ko‘taring.
d) argumentga 1 ni qo‘shing, keyin kvadrat ildizini topib, 6 ga bo‘ling.

2. Matn usulida berilgan funksiyaning ***analitik***, ***jadval*** va ***grafik*** ko‘rinishini toping:

a) $f(x)$ ni topish uchun argumentni 3 ga bo‘ling, keyin $\frac{2}{3}$ ni qo‘shing.

b) $g(x)$ni topish uchun argumentdan 4 ni ayiring, keyin $\frac{3}{4}$ ga ko‘paytiring.

c) $T(x)$ funksiya x so‘mga sotib olingan mahsulotning soliq miqdori funksiyasi bo‘lsin, soliq miqdorini topish uchun mahsulot narxining 8% ini hisoblang.

d) $V(d)$ funksiya d diametrli sharning hajmini topish funksiyasi bo‘lsin, hajmni topish uchun diametrning 3-darajasini π ga ko‘paytirib 6 ga bo‘ling.

3. Berilgan funksiyalar uchun qiymatlar jadvalini to‘ldiring:

a) $f(x) = 2(x-1)^2$

x	$f(x)$
−1	
0	
1	
2	
3	

b) $g(x) = |2x+3|$.

x	$g(x)$
-3	
-2	
0	
1	
3	

4. Funksiyaning berilgan argumentdagi qiymatlarini toping.

a) $f(x) = x^2-6$ $\quad f(-3), f(3), f(0), f\left(\frac{1}{2}\right)$

b) $f(x) = x^3+2x$ $\quad f(-2), f(-1), f(0), f\left(\frac{1}{2}\right)$

c) $f(x)=\frac{|x|}{x}$ $\quad f(-2), f(-1), f(0), f(5), f(x^2), f\left(\frac{1}{x}\right)$

d) $f(x)=\frac{1-2x}{3}$ $\quad f(2), f(-2), f\left(\frac{1}{2}\right), f(a), f(-a), f(a-1)$

e) $h(x)=\frac{x^2+4}{5}$ $\quad h(2), h(-2), h(a), h(-x), h(a-2), h\left(\sqrt{x}\right)$

f) $f(x)=x^2+2x$ $\quad f(0), f(3), f(-3), f(a), f(-x), f\left(\frac{1}{a}\right)$

g) $h(t)=t+\frac{1}{t}$ $\quad h(-1),\ h(2),\ h\left(\frac{1}{2}\right),\ h(x-1),\ h\left(\frac{1}{x}\right)$

5. Berilgan tengliklardan qaysilari x o‘zgaruvchili funksiya bo‘la oladi?

a) $3x-5y=7$ b) $3x^2-y=5$ c) $x=y^2$ d) $x^2+(y-1)^2=4$

e) $2x-4y^2=3$ f) $2x^2-4y^2=3$ g) $2xy-5y^2=4$ h) $\sqrt{y}-x=5$

i) $2|x|+y=0$ j) $2x+|y|=0$ k) $x=y^3$ l) $x=y^4$

6. Quyidagi jadvallardan qaysi biri x o‘zgaruvchili funksiya bo‘la oladi?

a)

x	y
−5	−12
9	2
11	2

b)

x	y
−10	−9
$3\frac{1}{2}$	−6
−10	−1

c)

x	y
2	0
−5	−3
−17	7
6	17
11	7

d)

x	y
−4	$3\frac{1}{2}$
$3\frac{1}{2}$	$-3\frac{1}{2}$
$9\frac{3}{5}$	−10

FUNKSIYANING ANIQLANISH SOHASI VA QIYMATLAR TO‘PLAMI

Funksiyaning aniqlanish sohasi va qiymatlar to‘plami

$y = f(x)$ funksiyada x argument qabul qilishi mumkin bo‘lgan sonlar to‘plami berilgan funksiyaning ***aniqlanish sohasi,*** y funksiya qabul qilishi mumkin bo‘lgan sonlar to‘plami berilgan funksiyaning ***qiymatlar to‘plami*** deb ataladi va ular mos ravishda $D(f)$ va $E(f)$ yoki $D(y)$ va $E(y)$ kabi belgilanadi.

Ba’zi funksiyalar uchun aniqlanish sohasi va qiymatlar to‘plami jadvali:

Funksiya	Aniqlanish sohasi	Qiymatlar to‘plami
1) $y = kx + b$	$D(y) = (-\infty; +\infty)$	$E(y) = (-\infty;\ +\infty)$
2) $y = x^2$	$D(y) = (-\infty; +\infty)$	$E(y) = [0;\ +\infty)$
3) $y = \lvert x \rvert$	$D(y) = (-\infty; +\infty)$	$E(y) = [0;\ +\infty)$
4) $y = \frac{k}{x}$	$D(y) = (-\infty; 0) \cup (0; +\infty)$	$E(y) = (-\infty; 0) \cup (0; +\infty)$
5) $y = \sqrt{x}$	$D(y) = [0; +\infty)$	$E(y) = [0;\ +\infty)$
6) $y = \sqrt[2n]{x}$	$D(y) = [0; +\infty)$	$E(y) = [0;\ +\infty)$
7) $y = \sqrt[3]{x}$	$D(y) = (-\infty; +\infty)$	$E(y) = (-\infty;\ +\infty)$
8) $y = \sqrt[2n+1]{x}$	$D(y) = (-\infty; +\infty)$	$E(y) = (-\infty;\ +\infty)$

x argumentning $y = f(x)$ funksiyaning aniqlanish sohasiga tegishli bo‘lmagan har qanday qiymatida $y = f(x)$ funksiya aniqlanmagan bo‘ladi, boshqacha aytganda, $f(x)$ funksiya ma’noga ega bo‘lmaydi. Masalan, $y = \sqrt{x}$ funksiya $x = -1$ bo‘lganda; $y = \frac{k}{x}$ funksiya $x = 0$ bo‘lganda ma’noga ega emas.

1-misol. $y = \frac{1}{x^2 - x}$ funksiyaning aniqlanish sohasini toping.

Yechish. Ratsional ifodaning maxraji nolga teng bo‘lishi mumkin emas, ya’ni:

$$x^2 - x \neq 0$$
$$x(x-1) \neq 0$$
$$x \neq 0 \text{ va } x \neq 1.$$

Demak, x argument 0 va 1 qiymatlarni qabul qila olmaydi. Shuning uchun berilgan funksiyaning aniqlanish sohasi $D(y) = (-\infty; 0) \cup (0; 1) \cup (1; +\infty)$.

Javob: $D(y) = (-\infty; 0) \cup (0; 1) \cup (1; +\infty)$.

2-misol. $y=\sqrt{9-x^2}$ funksiyaning aniqlanish sohasini toping.

Yechish. Kvadrat ildiz ostidagi ifoda manfiy bo‘la olmaydi. Ya’ni:

$$9-x^2\ge 0$$
$$(3-x)(3+x)\ge 0$$
$$-3\le x\le 3.$$

Demak, x argument faqat [–3; 3] kesmadan qiymat qabul qila oladi. Shuning uchun funksiyaning aniqlanish sohasi: $D(y)=[-3;3]$.

Javob: $D(y)=[-3;3]$.

3-misol. $y=\dfrac{1}{\sqrt{x+1}}$ funksiyaning aniqlanish sohasini toping.

Yechish. Berilgan funksiya maxrajida kvadrat ildiz ostidagi ifoda berilgan, bu ifoda nolga teng bo‘lishi mumkin emas hamda manfiy bo‘lmasligi kerak. Shuning uchun

$$x+1>0$$
$$x>-1.$$

Demak, funksiyaning aniqlanish sohasi $D(y)=(-1;\infty)$.

Javob: $D(y)=(-1;\infty)$.

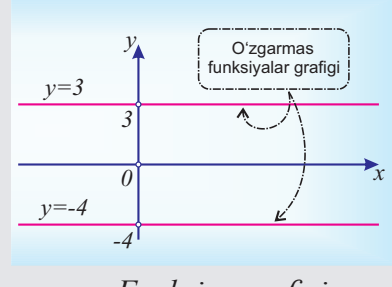
Funksiya grafigi

$y=f(x)$ funksiya o‘zining $D(f)$ aniqlanish sohasidan olingan har bir x elementga $E(f)$ qiymatlar to‘plamidan yagona $f(x)$ qiymatni mos qo‘yadi. Natijada har bir $x\in D(f)$ element Oxy koordinatalar tekisligida yagona $(x,f(x))$ nuqtani aniqlaydi.

Oxy koordinatalar tekisligida hosil qilingan barcha $(x,f(x))$ nuqtalar to‘plami $y=f(x)$ ***funksiyaning grafigi*** deyiladi.

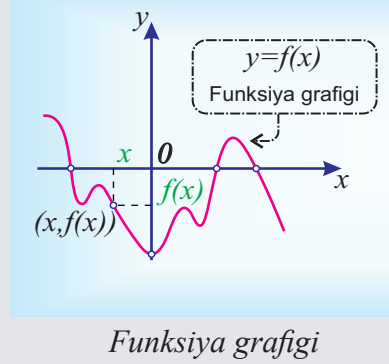
1-, 2-rasmlarda funksiya grafiklari tasvirlangan.

1-rasm



Funksiya grafigi

2-rasm



Funksiya grafigi

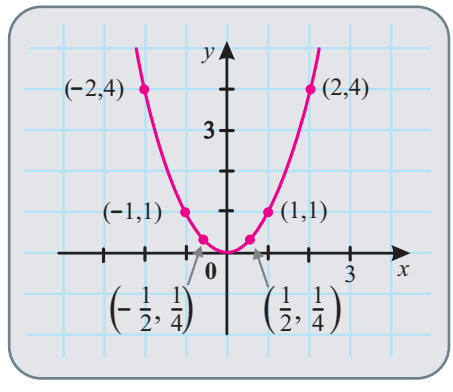
4-misol. Quyidagi funksiyalarning grafiklarini yasang.

a) $y=x^2$ b) $y=x^3$ d) $y=\sqrt{x}$

Yechish. Bu funksiyalarning grafiklarini yasash uchun dastlab ayrim qiymatlar uchun jadvalini tuzib olamiz. Keyin bu nuqtalarni koordinata tekisligida belgilaymiz va ularni egri chiziq bilan tutashtiramiz.

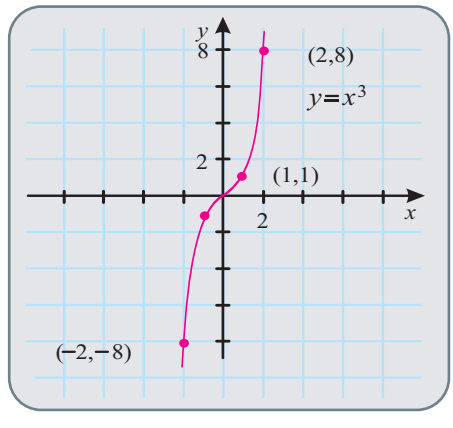
a)

x	–3	–2	–1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9



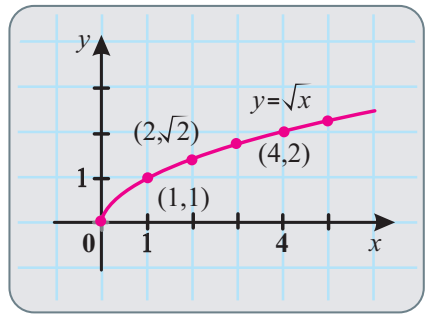
b)

x	–2	–1	–1/2	0	1/2	1	2
$y = x^3$	–8	–1	–1/8	0	1/8	1	8



c)

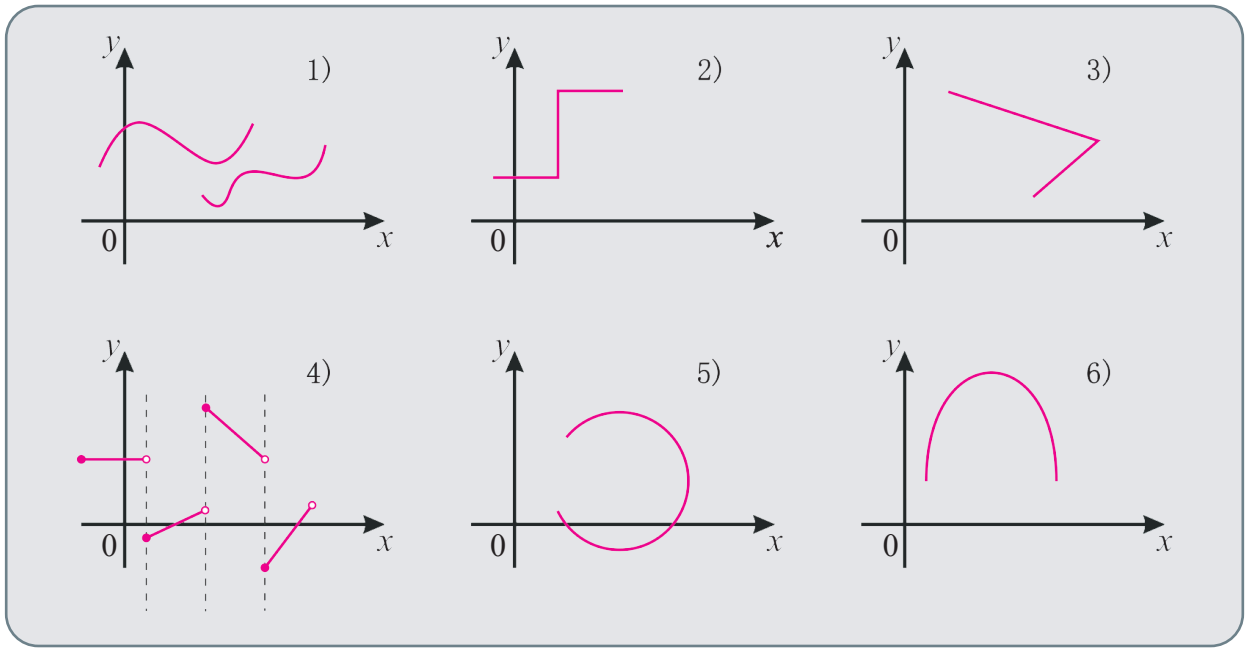
x	0	1/4	1	2	4	9
$y = \sqrt{x}$	0	1/2	1	$\sqrt{2}$	2	3



Oy o‘qiga parallel ixtiyoriy to‘g‘ri chiziq berilgan grafikni bittadan ortiq bo‘lmagan nuqtada kesib o‘tsa *Oxy* tekisligida tasvirlangan shakl $y = f(x)$ funksiyaning grafigi bo‘ladi.

Agar Oy o‘qiga parallel qandaydir to‘g‘ri chiziq berilgan chiziqni bittadan ortiq nuqtada kesib o‘tgan taqdirda Oxy tekisligida tasvirlangan chiziq funksiya grafigi bo‘la olmaydi.

Quyidagi rasmda keltirilgan 4- va 6-chiziqlar biror funksiya ning grafigi bo‘ladi. 1-, 2-, 3- va 5-chiziqlar esa funksiya grafigi bo‘lmaydi.



MISOLLAR

1. Funksiyaning aniqlanish sohasini va qiymatlar to‘plamini toping.

a) $f(x)=3x$ b) $f(x)=3x,\ 2\le x\le 6$

c) $f(x)=5x^2+2$ d) $f(x)=5x^2+2,\ 0\le x\le 2$

2. Funksiyaning aniqlanish sohasini toping.

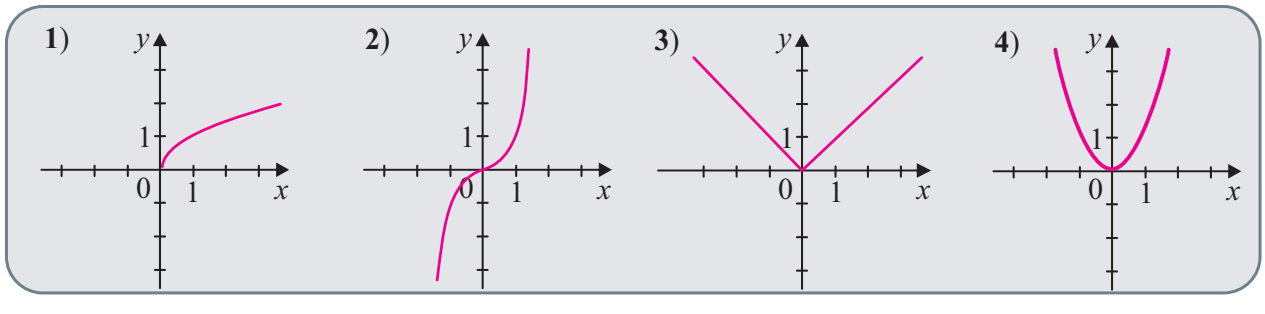
a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=\dfrac{1}{3x-6}$ c) $f(x)=\dfrac{x+2}{x^2-1}$

d) $f(x)=\dfrac{x^4}{x^2+x-6}$ e) $f(t)=\sqrt{t+1}$ f) $g(t)=\sqrt{t^2+9}$

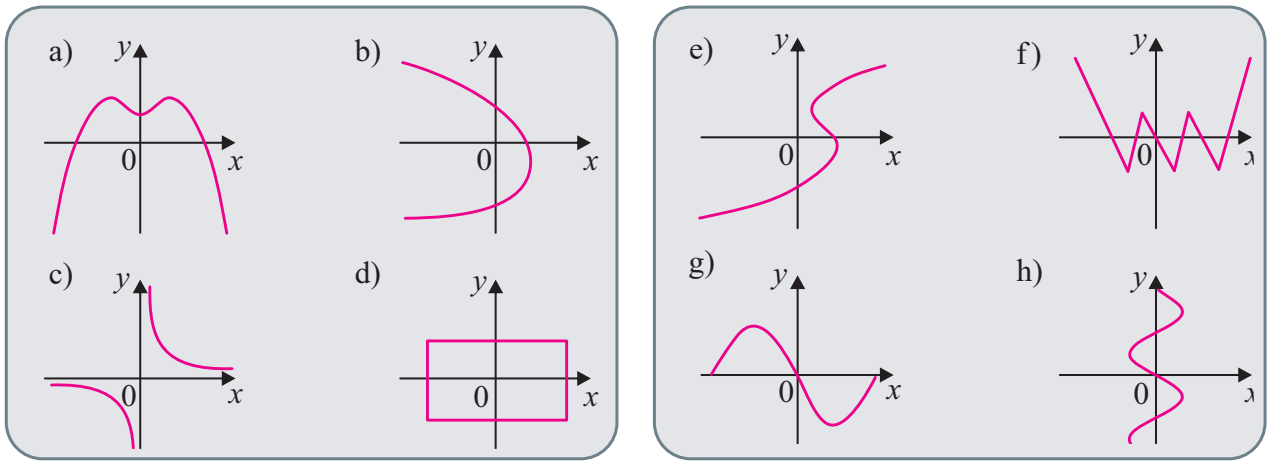
g) $f(t)=\sqrt[3]{t-1}$ h) $g(x)=\sqrt{7-3x}$ i) $f(x)=\sqrt{1-2x}$

3. Funksiyaga mos grafikni aniqlang.

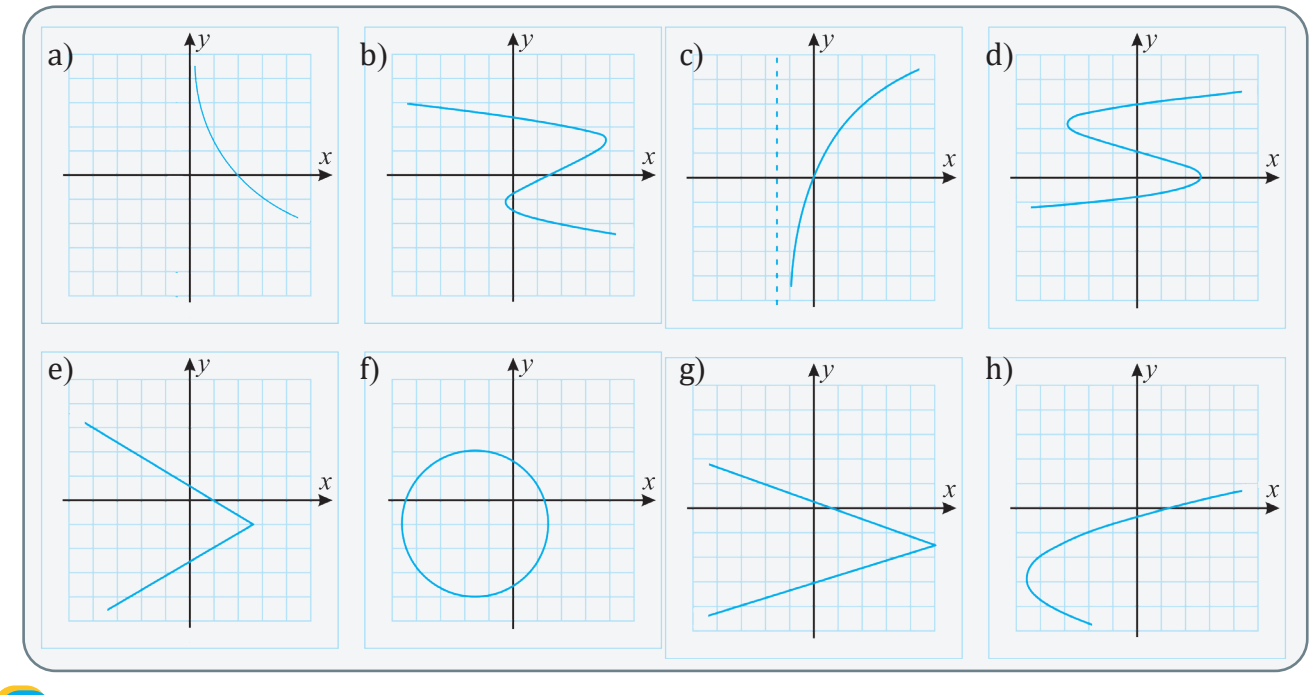
a) $f(x)=x^2$ b) $f(x)=x^3$ c) $f(x)=\sqrt{x}$ d) $f(x)=|x|$



4. Berilganlardan qaysilari funksiya grafigi bo‘la oladi?



5. Berilganlardan qaysilari funksiya grafigi bo‘la olmaydi?



6. Berilgan funksiyalarning grafiklarini yasang.

a) $f(x)=8x-x^2$ b) $g(x)=x^2-x-20$ c) $h(x)=x^3-5x-4$

7. Berilgan funksiyalarning qiymatlar jadvalini tuzing va grafigini yasang.

a) $f(x)=-x^2$ b) $f(x)=x^2-4$ c) $g(x)=-(x+1)^2$

d) $r(x)=3x^4$ e) $r(x)=1-x^4$ f) $g(x)=x^3-8$

g) $k(x)=\sqrt[3]{-x}$ h) $k(x)=-\sqrt[3]{x}$ i) $f(x)=1+\sqrt{x}$

j) $C(t)=\frac{1}{t^2}$ k) $C(t)=-\frac{1}{t+1}$ l) $H(x)=|2x|$

m) $G(x)=|x|+x$ n) $G(x)=|x|-x$ o) $f(x)=|2x-2|$

FUNKSIYALAR USTIDA ARIFMETIK AMALLAR

Funksiyalar ustida arifmetik amallar

Funksiyalar ustida qo‘shish (+), ayirish (–), ko‘paytirish (×), bo‘lish (÷) arifmetik amallarini bajarish mumkin.

$f(x)$ va $g(x)$ funksiyalarning aniqlanish sohalari mos ravishda A va B to‘plamlar bo‘lsin. Bu funksiyalarning $A \cap B$ to‘plamdagi ***yig‘indisi*** deb har bir $x \in A \cap B$ elementda $f(x) + g(x)$ qiymatni qabul qiladigan funksiyaga aytiladi. $f(x)$ va $g(x)$ funksiyalarning yig‘indisi $(f + g)(x)$ kabi belgilanadi. Demak:

$$(f + g)(x) = f(x) + g(x).$$

Xuddi shuningdek, $f(x)$ va $g(x)$ funksiyalarning ***ayirmasi, ko‘paytmasi, bo‘linmasini*** aniqlash mumkin:

$$(f - g)(x) = f(x) - g(x)$$
$$(f \cdot g)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$

Diqqat qiling!

1. $A \cap B = \varnothing$ bo‘lsa, bu amallar aniqlanmaydi.

2. Ikkita $f(x)$ va $g(x)$ funksiyalarning bo‘linmasini aniqlashda X to‘plamdan olingan har bir x element uchun $g(x) \neq 0$ bo‘lishi talab etiladi.

MISOLLAR

1-misol. $f(x) = \frac{1}{x-2}$ va $g(x) = \sqrt{x}$ funksiyalar berilgan.

a) $(f+g)(x), (f-g)(x), (f \cdot g)(x)$ va $\left(\frac{f}{g}\right)(x)$ funksiyalarni va ularning aniqlanish sohasini toping.

b) $(f+g)(4), (f-g)(4), (fg)(4)$ va $\left(\frac{f}{g}\right)(4)$ ni toping.

Yechish. a) f funksiyaning aniqlanish sohasi $x \neq 2$, g funksiyaniki esa $x \geq 0$. f va g funksiyalarning aniqlanish sohalari kesishmasi $[0; 2) \cup (2; \infty)$ bo‘ladi.

U holda ular ustida amallar quyidagicha bajariladi:

$$(f+g)(x) = f(x) + g(x) = \frac{1}{x-2} + \sqrt{x}$$

$$(f-g)(x) = f(x) - g(x) = \frac{1}{x-2} - \sqrt{x}$$

$$(f\cdot g)(x) = f(x)g(x) = \frac{\sqrt{x}}{x-2}$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{1}{(x-2)\sqrt{x}}.$$

b) $x = 4$ qiymat har bir yangi funksiyaning aniqlanish sohasiga tegishli ekanidan quyidagi qiymatlar aniqlangan:

$$(f+g)(4) = f(4)+g(4) = \frac{1}{4-2}+\sqrt{4} = \frac{5}{2}$$

$$(f-g)(4) = f(4)-g(4) = \frac{1}{4-2}-\sqrt{4} = -\frac{3}{2}$$

$$(fg)(4) = f(4)g(4) = \left(\frac{1}{4-2}\right)\sqrt{4} = 1$$

$$\left(\frac{f}{g}\right)(4) = \frac{f(4)}{g(4)} = \frac{1}{(4-2)\sqrt{4}} = \frac{1}{4}.$$

2-misol. Funksiyalarni grafik usulda qo‘shish

f va g funksiyalarning grafigi 1-rasmda berilgan bo‘lsin. Grafik usuldagi qo‘shish yordamida $f + g$ funksiyaning grafigini chizing.

Yechish. Ma’lumki, f funksiya grafigi Oxy tekislikdagi

$$\{(x, f(x)) : x \in D(f)\}$$

to‘plamdan iborat. Xuddi shuningdek,

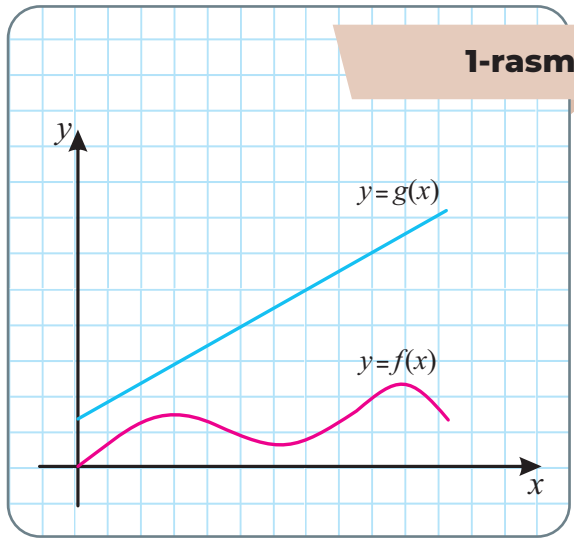
$$\{(x, g(x)) : x \in D(g)\}$$

to‘plam g funksiyaning grafigi bo‘ladi. f va g funksiyalarni qo‘shishning grafik usuli deganda ushbu to‘plam tushuniladi:

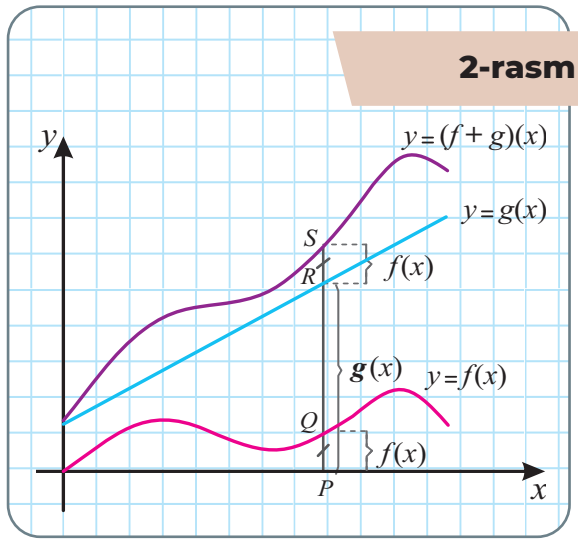
$$\{(x, f(x) + g(x)) : x \in D(f) \cup D(g)\}.$$

PQ kesma PR kesmaning yuqorisiga $f + g$ funksiyaning S nuqtasini hosil qilish uchun nusxalab ko‘chirilgan ($PQ = RS$).

1-rasm



2-rasm



MISOLLAR

1. Funksiyalarni qo‘shing va ayiring.

a) $f(x) = 5x+1$, $g(x) = -2x$

b) $f(x) = -3x+3$, $g(x) = -5x+4$

c) $f(x) = 2x+1$, $g(x) = -5x+3$

d) $f(x) = -3x^2+7x$, $g(x) = 2x+4$

2. Funksiyalarni ko‘paytiring.

a) $f(x)=-x^2,\ g(x)=-3x+1$

b) $f(x)=-3x^2+3,\ g(x)=-x$

c) $f(x)=-x+3,\ g(x)=5x+6$

d) $f(x)=-4x+5,\ g(x)=-3x+1$

3. $(f+g)(x),(f-g)(x),(f\cdot g)(x)$ va $\left(\frac{f}{g}\right)(x)$ funksiyalarni va ularning aniqlanish sohasini toping.

a) $f(x)=x,\ g(x)=2x$

b) $f(x)=x,\ g(x)=\sqrt{x}$

c) $f(x)=x^2+x,\ g(x)=x^2$

d) $f(x)=3-x^2,\ g(x)=x^2-4$

e) $f(x)=5-x,\ g(x)=x^2-3x$

f) $f(x)=x^2+2x,\ g(x)=3x^2-1$

g) $f(x)=\sqrt{25-x^2},\ g(x)=\sqrt{x+3}$

h) $f(x)=\sqrt{16-x^2},\ g(x)=\sqrt{x^2-1}$

i) $f(x)=\frac{2}{x},\ g(x)=\frac{4}{x+4}$

j) $f(x)=\frac{2}{x+1},\ g(x)=\frac{x}{x+1}$

4. Funksiyalarning aniqlanish sohasini toping.

a) $f(x)=\sqrt{x}+\sqrt{3-x}$

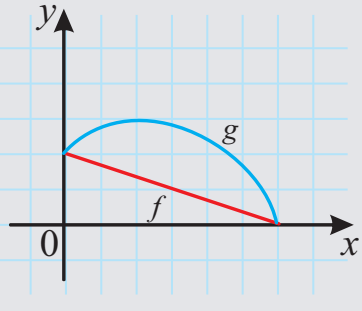
b) $f(x)=\sqrt{x+4}-\frac{\sqrt{1-x}}{x}$

c) $h(x)=(x-3)^{-\frac{1}{4}}$

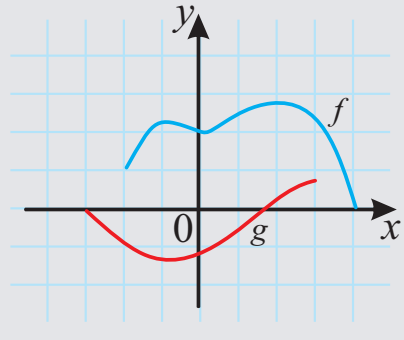
d) $k(x)=\frac{\sqrt{x+3}}{x-1}$

5. Grafikdan foydalanib qo‘shish yordamida $f+g$ funksiyaning grafigini yasang (3–4-*rasmlar*).

3-rasm



4-rasm



MURAKKAB, TESKARI, DAVRIY FUNKSIYALAR

Murakkab funksiya

Funksiyalarni ketma-ket qo‘llash natijasida o‘zgaruvchilarning yangi bog‘lanishlari hosil bo‘ladi. Agar X to‘plamda $y = f(x)$ funksiya berilgan bo‘lib, x argument T to‘plamda aniqlangan biror $x = g(t)$ funksiya bo‘lsa, u holda T to‘plamda $y = f(g(t))$ ***murakkab funksiya*** aniqlangan deyiladi.

Masalan, $y = 2x^2 - 3x$ funksiya $X = (-\infty; +\infty)$ to‘plamda, $x = \sqrt{t}$ funksiya esa $T = [0; +\infty)$ to‘plamda berilgan bo‘lsin. U holda $y = 2t - 3\sqrt{t}$ funksiya $T = [0; +\infty)$ to‘plamda $y = 2x^2 - 3x$ va $x = \sqrt{t}$ funksiyalarning murakkab funksiyasi bo‘ladi.

1-misol. $f(x) = x^2$ va $g(x) = x - 3$ funksiyalar berilgan:

a) $f(g(x))$ va $g(f(x))$ murakkab funksiyalarni va ularning aniqlanish sohasini toping;

b) $f(g(5))$ va $g(f(7))$ ni toping.

Yechish. a) Quyidagi tengliklar o‘rinli:

g funksiyaning berilishiga ko‘ra, $f(g(x)) = f(x-3)$,

f funksiyaning berilishiga ko‘ra, $f(g(x)) = (x-3)^2$ bo‘ladi.

f funksiyaning berilishiga ko‘ra, $g(f(x)) = g(x^2)$,

g funksiyaning berilishiga ko‘ra, $g(f(x)) = x^2 - 3$ bo‘ladi.

$f(g(x))$ va $g(f(x))$ funksiyaning aniqlanish sohasi haqiqiy sonlar to‘plami.

b) Topilgan murakkab funksiyalarda x ning o‘rniga berilgan qiymatni qo‘yamiz:

$f(g(5)) = (5-3)^2 = 2^2 = 4$, $\quad g(f(7)) = 7^2 - 3 = 49 - 3 = 46$.

2-misol. Agar $f(x) = \sqrt{x}$ va $g(x) = \sqrt{2-x}$ berilgan bo‘lsa, quyidagi funksiyalarni va ularning aniqlanish sohasini toping (1-*rasm*).

a) $f(g(x))$ b) $g(f(x))$

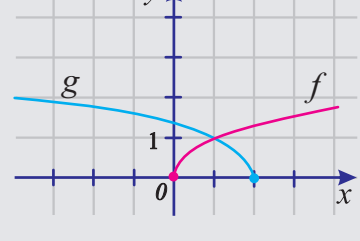
c) $f(f(x))$ d) $g(g(x))$

Yechish. a) Murakkab funksiya ta’rifi hamda f va g funksiyalarning berilishiga ko‘ra,

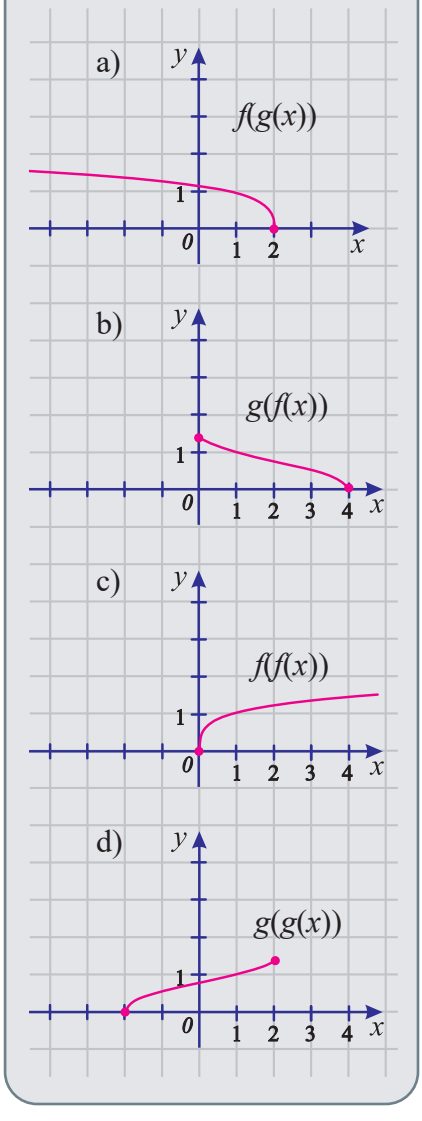
$$f(g(x)) = f\left(\sqrt{2-x}\right) = \sqrt{\sqrt{2-x}} = \sqrt[4]{2-x} \text{ bo‘ladi.}$$

$\sqrt[4]{2-x}$ ifodaning aniqlanish sohasi $2 - x \geq 0$. Bundan $x \leq 2$.

1-rasm



2-rasm



Demak, $f(g(x))$ ning aniqlanish sohasi $(-\infty; 2]$ oraliqdan iborat (2a-*rasm*).

b) f ning berilishiga ko‘ra, $g(f(x)) = g\left(\sqrt{x}\right)$ bo‘lib,

g ning berilishiga ko‘ra, $g(f(x)) = \sqrt{2-\sqrt{x}}$ bo‘ladi.

$\sqrt{x}$ ning aniqlanish sohasi: $x \geq 0$.

$\sqrt{2-\sqrt{x}}$ ning aniqlanish sohasi: $2-\sqrt{x} \geq 0$, bundan

$\sqrt{x} \leq 2$ yoki $x \leq 4$. Demak, $0 \leq x \leq 4$ (2b-*rasm*).

c) f ning berilishiga ko‘ra, $f(f(x)) = f\left(\sqrt{x}\right)$ bo‘lib,

f ning berilishiga ko‘ra, $f(f(x)) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$ bo‘ladi.

$\sqrt[4]{x}$ ning aniqlanish sohasi: $[0; \infty)$ (2c-*rasm*).

d) g ning berilishiga ko‘ra, $g(g(x)) = g\left(\sqrt{2-x}\right)$ bo‘lib,

g ning berilishiga ko‘ra, $g(g(x)) = \sqrt{2-\sqrt{2-x}}$ bo‘ladi.

$\sqrt{2-\sqrt{2-x}}$ ning aniqlanish sohasi: $2-x \geq 0$ va $\sqrt{2-x} \leq 2$

Bundan $x \leq 2$ va $x \geq -2 \Rightarrow -2 \leq x \leq 2$, demak, $g(g(x))$ ning aniqlanish sohasi: $[-2; 2]$ (2d-*rasm*).

3-misol. $f(x) = \dfrac{x}{x+1}, g(x) = x^{10}$ va $h(x) = x+3$ bo‘lsa, $f(g(h(x)))$ ni toping.

Yechish. h ning berilishiga ko‘ra,

$f(g(h(x))) = f\left(g\left(x+3\right)\right)$ bo‘lib, g funksiyaning berilishiga ko‘ra, $f(g(h(x))) = f\left(\left(x+3\right)^{10}\right)$,

f funksiyaning berilishiga ko‘ra, $f(g(h(x))) = \dfrac{\left(x+3\right)^{10}}{\left(x+3\right)^{10}+1}$ bo‘ladi.

4-misol. $F(x) = \sqrt[4]{x+9}$ funksiya berilgan. $F(x) = f(g(x))$ bo‘ladigan f va g funksiyalarga misol keltiring.

Yechish. f va g funksiyalarni quyidagicha olishimiz mumkin: $g(x) = x+9$ va $f(x) = \sqrt[4]{x}$.

Bunda, g ning berilishiga ko‘ra, $f(g(x)) = f\left(x+9\right)$ bo‘lib, f ning berilishiga ko‘ra, $f(g(x)) = \sqrt[4]{x+9}$ bo‘ladi.

Ushbu topshiriq shartini qanoatlantiruvchi f va g funksiyalarni bir nechta holatda tanlab olish mumkin. Shulardan yana biri $f(x)=\sqrt{x}$ va $g(x)=\sqrt{x+9}$.

5-misol. Murakkab funksiyaning qo‘llanishi

Kema 20 km/h o‘zgarmas tezlikda qirg‘oqqa parallel ravishda harakat qilmoqda. Kema mayoqning ro‘parasidan soat 12:00 da, qirg‘oqdan 5 km uzoqlikda o‘tadi.

a) Mayoq va kema orasidagi s masofani kemaning soat 12:00 dan keyin yurgan masofasi d ga bog‘liq funksiyasi ko‘rinishida yozing:

$$s=f(d).$$

b) d ni soat 12:00 dan keyin o‘tgan vaqt t (soat) ga bog‘liq funksiyasi ko‘rinishida yozing:

$$d=g(t).$$

c) $f(g(t))$ murakkab funksiyani toping. Bu funksiya nimani anglatadi?

3-rasm



Yechish. 3-rasmga qaraymiz.

a) s va d masofalarning bog‘liqligini Pifagor teoremasi yordamida ko‘rsatamiz. Boshqacha aytganda, s ning d ga bog‘liq funksiya ekanini quyidagicha ifodalaymiz:

$$s=f(d)=\sqrt{25+d^2}.$$

b) kema 20 km/h o‘zgarmas tezlikda harakatlanayotgani uchun d masofaning t vaqtga bog‘liqligi

$$d=g(t)=20t$$

ko‘rinishdagi funksiya orqali ifodalanishi mumkin.

c) shunday qilib:

g ning berilishiga ko‘ra, $f(g(t))=f(20t)$ bo‘lib,

f funksiyaning berilishiga ko‘ra, $f(g(t))=\sqrt{25+(20t)^2}$ bo‘ladi. Bu yerda $f(g(t))$ funksiya kema va mayoq orasidagi masofaning vaqtga nisbatan funksiyasini anglatadi.

Teskari funksiya

Agar $f(x)=y$ tenglama har bir y uchun x ga nisbatan yagona $g(y)$ ildizga ega bo‘lsa, u holda $x=g(y)$ funksiya $y=f(x)$ funksiyaga ***teskari funksiya*** deyiladi. $x=g(y)$ funksiya o‘rniga odatdagi belgilashlarga ko‘ra, $y=g(x)$ yozuvi ishlatiladi. $y=f(x)$ funksiyaga teskari funksiya $y=f^{-1}(x)$ kabi yoziladi.

1-misol. $y=3x-5$ funksiyani ko‘rib chiqaylik. Bu yerdan x ni y orqali ifodalaylik:

$$3x-5=y \Rightarrow 3x=y+5 \Rightarrow x=\frac{y+5}{3}.$$

Oxirgi tenglikda x va y larning o‘rinlarini almashtirib:

$$y=\frac{x+5}{3}$$

funksiyaga ega bo‘lamiz. Demak, $f^{-1}(x)=\frac{x+5}{3}$ funksiya $y=3x-5$ funksiyaga teskari funksiya bo‘ladi.

Eslatma. Berilgan $y=f(x)$ funksiya va unga teskari $y=f^{-1}(x)$ funksiya uchun $D\left(f^{-1}\right)=E(f)$ hamda $E\left(f^{-1}\right)=D(f)$ bo‘ladi.

Diqqat qiling! $\left(f(x)\right)^{-1}=\frac{1}{f(x)}$ bo‘lib, bu tenglikdagi (-1) daraja ko‘rsatkichini anglatadi. $f^{-1}(x)$ yozuvdagi (-1) esa teskari funksiyani bildiradi. Umuman olganda, $\left(f(x)\right)^{-1}\neq f^{-1}(x)$. Masalan:

$f(x)=3x-5$ funksiya uchun $f^{-1}(x)=\frac{x+5}{3}$ hamda $\left(f(x)\right)^{-1}=\frac{1}{3x-5}$ bo‘ladi.

2-misol. Berilgan funksiyaning teskari funksiyasini toping: $f(x)=\frac{x^5-3}{2}$.

Yechish. Funksiyani $y=\frac{x^5-3}{2}$ kabi yozib olamiz va x ni y orqali ifodalaymiz:

$$y=\frac{x^5-3}{2}$$
$$2y=x^5-3$$
$$x^5=2y+3$$
$$x=\sqrt[5]{2y+3}.$$

Endi x va y larning o‘rnini almashtiramiz: $y=\sqrt[5]{2x+3}$. Demak, teskari funksiya quyidagicha: $f^{-1}(x)=\sqrt[5]{2x+3}$.

3-misol. Berilgan funksiyaning teskari funksiyasini toping: $f(x)=\frac{2x+3}{x-1}$.

Yechish. Funksiyani quyidagicha yozib olamiz: $y=\frac{2x+3}{x-1}$, x ni esa y orqali ifodalaymiz:

$$y=\frac{2x+3}{x-1}$$
$$y\cdot(x-1)=2x+3$$
$$yx-y=2x+3$$
$$yx-2x=y+3$$

$$x\cdot(y-2)=y+3$$

$$x=\frac{y+3}{y-2}$$

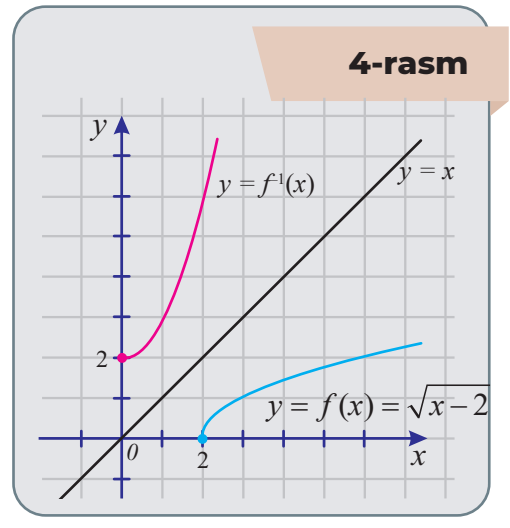
Demak, $f^{-1}(x)=\frac{x+3}{x-2}$ teskari funksiya bo‘ladi.

4-misol. Teskari funksiyaning grafigini yasash.

$f(x)=\sqrt{x-2}$ funksiyaning grafigidan foydalanib f^{-1} funksiyaning grafigini yasang va uning analitik ko‘rinishini yozing.

Yechish.

1. $y=\sqrt{x-2}$ funksiyaning grafigi 4-rasmda keltirilgan.



4-rasm

2. f^{-1} funksiyaning grafigi f funksiyaning grafigini $y=x$ to‘g‘ri chiziqqa nisbatan simmetrik akslantirish yordamida yasaladi (4-*rasm*).

3. $y=\sqrt{x-2}$ funksiyada x ni y orqali ifodalanadi, bunda $y\geq 0$ ekani inobatga olinadi.

$$\sqrt{x-2}=y$$

$$x-2=y^2$$

$$x=y^2+2,\quad y\geq 0.$$

Endi x va y larning o‘rnini almashtiramiz: $y=x^2+2$, $x\geq 0$.

Demak, teskari funksiya $f^{-1}(x)=x^2+2$ bo‘lar ekan, $x\geq 0$.

Bu topilgan $f^{-1}(x)$ teskari funksiya $y=x^2+2$ parabolaning o‘ng tarmog‘idan iborat. Buni grafikdan ham ko‘rsa bo‘ladi.

Davriy funksiyalar

$D(f)$ berilgan $y=f(x)$ funksiyaning aniqlanish sohasi bo‘lsin. Shunday $T\neq 0$ topilib, har bir $x\in D(f)$ uchun:

1. $x-T$ va $x+T$ lar $D(f)$ ga tegishli,

2. $f(x-T)=f(x)=f(x+T)$ munosabatlar bajarilsa, u holda $y=f(x)$ ***davriy funksiya*** deyiladi.

Agar T soni $y=f(x)$ funksiyaning davri bo‘lsa, u holda har bir n butun son uchun nT soni ham $y=f(x)$ funksiyaning davri bo‘ladi:

$$f(x+nT)=f(x),\quad n\in\mathbb{Z}.$$

Eng kichik musbat T davr $f(x)$ funksiyaning ***asosiy davri*** deb yuritiladi.

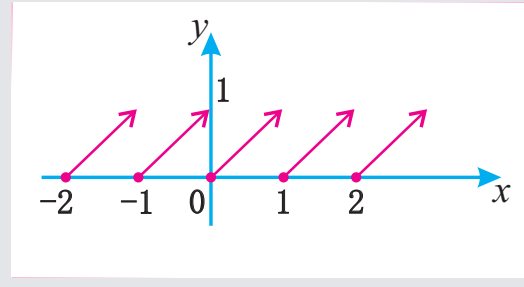
Davriy funksiyaning grafigini bitta davr oralig‘ida chizish kifoya qiladi, boshqa davr oraliqlarida

shu grafik takrorlanadi.

Masalan, sonning kasr qismi $\{x\}$ – berilgan x songa uning kasr qismini mos qo‘yuvchi funksiya (5-*rasm*) davriy funksiya bo‘ladi. Uning asosiy davri $T_0 = 1$, ya’ni ixtiyoriy $x \in (-\infty; +\infty)$ son uchun $(x + 1) \in (-\infty; +\infty)$ hamda $\{x+1\} = \{x\}$ munosabatlar o‘rinli bo‘ladi.

Agar $y = f(x)$ funksiyaning asosiy davri T_0 bo‘lsa, u holda $y = kf(ax+b)+c$ funksiyaning asosiy davri $T_1 = \frac{T_0}{|a|}$ bo‘ladi $(a \neq 0)$.

5-rasm



$y = \{x\}$ funksiya grafigi

MISOLLAR

1. $f(x) = 2x - 3$ va $g(x) = 4 - x^2$ dan foydalanib murakkab funksiyalarning qiymatini toping.

a) $f(g(0))$ b) $g(f(0))$ c) $f(f(2))$ d) $g(g(3))$

e) $f(g(-2))$ f) $g(f(-2))$ g) $f(f(-1))$ h) $g(g(-1))$

2. $f(g(x))$, $g(f(x))$, $f(f(x))$ va $g(g(x))$ funksiyalarni va ularning aniqlanish sohasini toping.

a) $f(x) = 2x + 3$, $g(x) = 4x - 1$

b) $f(x) = 6x - 5$, $g(x) = \frac{x}{2}$

c) $f(x) = x^2$, $g(x) = x + 1$

d) $f(x) = x^3 + 2$, $g(x) = \sqrt[3]{x}$

e) $f(x) = \frac{1}{x}$, $g(x) = 2x + 4$

f) $f(x) = x^2$, $g(x) = \sqrt{x-3}$

g) $f(x) = |x|$, $g(x) = 2x + 3$

h) $f(x) = 4 - x$, $g(x) = |x + 4|$

i) $f(x) = \frac{x}{x+1}$, $g(x) = 2x - 1$

j) $f(x) = \frac{1}{\sqrt{x}}$, $g(x) = x^2 - 4x$

k) $f(x) = \frac{x}{x+1}$, $g(x) = \frac{1}{x}$

l) $f(x) = \frac{2}{x}$, $g(x) = \frac{x}{x+2}$

3. $f(x) = 3 - x$ va $g(x) = x^2 + 1$ dan foydalanib funksiyalarni toping.

a) $f(g(x))$ b) $g(f(x))$ c) $f(f(x))$ d) $g(g(x))$

4. $f(g(h(x)))$ murakkab funksiyani toping.

a) $f(x) = x - 1$, $g(x) = \sqrt{x}$, $h(x) = x - 1$

b) $f(x) = \frac{1}{x}$, $g(x) = x^3$, $h(x) = x^2 + 2$

c) $f(x)=x^4+1,\ g(x)=x-5,\ h(x)=\sqrt{x}$

d) $f(x)=\sqrt{x},\ g(x)=\dfrac{x}{x-1},\ h(x)=\sqrt[3]{x}$

5. F(x) = $f(g(x))$ tenglikni qanoatlantiradigan f va g sodda funksiyalarga misollar keltiring.

a) $F(x)=(x-9)^5$ b) $F(x)=\sqrt{x}+1$ c) $F(x)=\dfrac{x^2}{x^2+4}$

d) $F(x)=\dfrac{1}{x+3}$ e) $F(x)=|1-x^3|$ f) $F(x)=\sqrt{1+\sqrt{x}}$

6. Berilgan f funksiyaga teskari funksiyani toping.

a) $f(x)=3x+5$ b) $f(x)=7-5x$ c) $f(x)=5-4x^3$

d) $f(x)=3x^3+8$ e) $f(x)=\dfrac{1}{x+2}$ f) $f(x)=\dfrac{5}{x-6}$

g) $f(x)=\dfrac{3-4x}{8x-1}$ h) $f(x)=\dfrac{3x}{x-2}$ i) $f(x)=\dfrac{2x+5}{x-7}$

j) $f(x)=\sqrt{5+8x}$ k) $f(x)=2+\sqrt[3]{x}$ l) $f(x)=x^6,\ x\ge 0$

m) $f(x)=\dfrac{1}{x^2},\ x>0$ n) $f(x)=4-x^2,\ x\ge 0$ o) $f(x)=x^2+x,\ x\ge -\dfrac{1}{2}$

7. Berilgan funksiyaga teskari funksiyani toping. f funksiyaning grafigidan foydalanib teskari funksiya grafigini yasang.

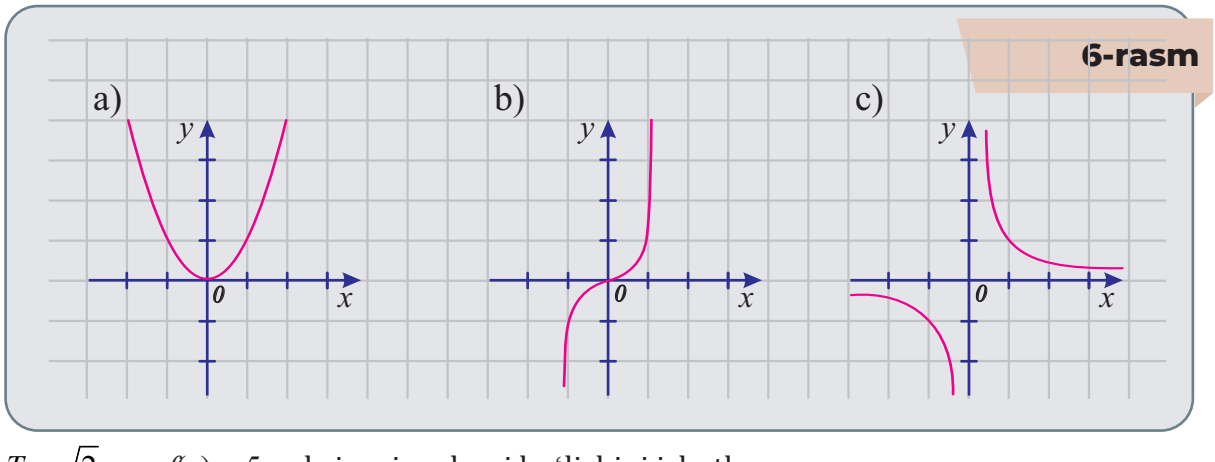
a) $f(x)=3x-6$ b) $f(x)=16-x^2,\ x\ge 0$

c) $f(x)=\sqrt{x+1}$ d) $f(x)=x^3$

8. 6-rasmda berilgan grafiklarga mos funksiyalarni tanlang va ularga teskari funksiya grafigini yasang:

1) $f(x)=x^3$; 2) $f(x)=\dfrac{1}{x}$; 3) $f(x)=x^2$

6-rasm



9. $T=\sqrt{2}$ son $f(x)=5$ unksiyaning davri bo‘lishini isbotlang.

10. Berilgan funksiyalar davriy emasligini ko‘rsating.

a) $f(x)=\dfrac{1}{x-3}$ b) $f(x)=-\dfrac{2}{x-2}$ c) $f(x)=\dfrac{x}{x}$

d) $f(x)=x^2-4$ e) $f(x)=\dfrac{x^2+2}{x^2+5x+8}$ f) $f(x)=\sqrt[3]{x}+3x-1$

FUNKSIYA XOSSALARI

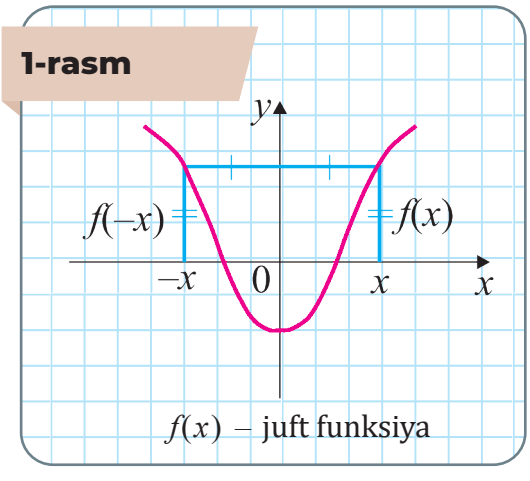
Juft va toq funksiyalar

Ixtiyoriy $x \in D(f)$ uchun $f(-x) = f(x)$ tenglik bajarilsa, u holda $f(x)$ ***juft funksiya*** deyiladi. Juft funksiyaning grafigi *Oy* o‘qiga nisbatan simmetrik bo‘ladi (1-*rasm*).

Ixtiyoriy $x \in D(f)$ uchun $f(-x) = -f(x)$ tenglik bajarilsa, u holda $f(x)$ ***toq funksiya*** deyiladi. Toq funksiyaning grafigi koordinata boshiga nisbatan simmetrik bo‘ladi (2-*rasm*).

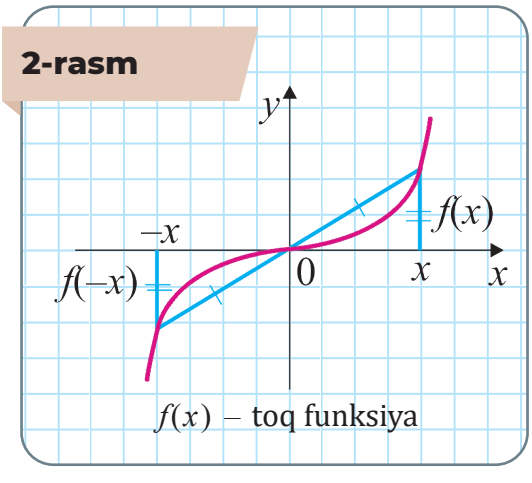
Yuqoridagi ikkita tenglikdan birortasi ham bajarilmasa, u holda $f(x)$ ***juft ham, toq ham emas funksiya*** deyiladi.

1-rasm



$f(x)$ – juft funksiya

2-rasm



$f(x)$ – toq funksiya

1-misol. $f(x) = 2x^2 + 5$ funksiyani juft yoki toqligini tekshiring.

Yechish

$f(x) = 2x^2 + 5$ funksiya uchun:

$$f(-x) = 2(-x)^2 + 5 = 2x^2 + 5 = f(x)$$

ekanidan $f(x)$ funksiya juft funksiya bo‘ladi.

2-misol. $f(x) = 2x^3 + 5x$ funksiyani juft yoki toqligini tekshiring.

Yechish

$f(x) = 2x^3 + 5x$ funksiya uchun:

$$f(-x) = 2(-x)^3 + 5(-x) = -(2x^3 + 5x) = -f(x)$$

ekanligidan $f(x)$ funksiya toq funksiya bo‘ladi.

3-misol. $f(x) = 2x^3 + 5x^2 - 3x + 1$ funksiyani juft yoki toqligini tekshiring.

Yechish

$$f(-x) = 2(-x)^3 + 5(-x)^2 - 3(-x) + 1 =$$
$$= -(2x^3 - 5x^2 - 3x - 1)$$

Demak, $f(-x) \neq f(x)$, $f(-x) \neq -f(x)$ bo‘lib, bu funksiya juft ham, toq ham emas ekan.

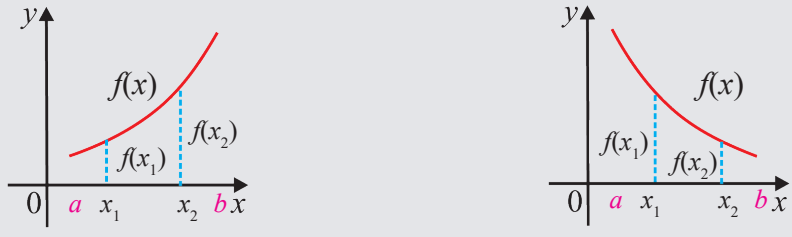
Funksiyalarning o‘sishi va kamayishi

$f(x)$ funksiya $(a; b)$ oraliqda aniqlangan bo‘lib, $x_1 < x_2$ shartni qanoatlantiruvchi barcha $x_1, x_2 \in (a; b)$ lar uchun:

- $f(x_1) < f(x_2)$ bo‘lsa, $f(x)$ funksiya $(a; b)$ oraliqda o‘suvchi;
- $f(x_1) > f(x_2)$ bo‘lsa, $f(x)$ funksiya $(a; b)$ oraliqda kamayuvchi;
- $f(x_1) \geq f(x_2)$ bo‘lsa, $f(x)$ funksiya $(a; b)$ oraliqda o‘smaydigan;
- $f(x_1) \leq f(x_2)$ bo‘lsa, $f(x)$ funksiya $(a; b)$ oraliqda kamaymaydigan funksiya deyiladi.

O‘suvchi, kamayuvchi, o‘smaydigan va kamaymaydigan funksiyalar umumiy nom bilan ***monoton funksiyalar*** deyiladi.

3-rasm



O‘suvchi funksiya **Kamayuvchi funksiya**

Funksiya ekstremum nuqtalari va ekstremumlari

- Agar:

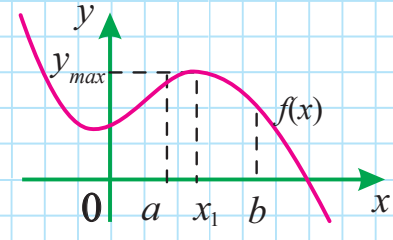
1) $f(x)$ funksiya x_1 nuqta tegishli bo‘lgan biror $(a;\ b)$ intervalda aniqlangan bo‘lib;

2) $(a;\ b)$ intervalning x_1 dan farqli barcha x nuqtalarida $f(x) < f(x_1)$ shart bajarilsa, u holda x_1 nuqta $f(x)$ ***funksiyaning maksimum nuqtasi*** deyiladi (4-*rasm*).

Agar $x_1 \in D(f)$ nuqta $f(x)$ funksiya uchun maksimum nuqta bo‘lsa, u holda $f(x)$ funksiyaning x_1 nuqtadagi $f(x_1)$ qiymati ***funksiyaning maksimumi*** deyiladi va y_{max} kabi belgilanadi. Demak,

$$y_{max} = f(x_1).$$

4-rasm



$f(x)$ uchun $(a;\ b)$ intervalda
x_1 – funksiyaning maksimum nuqtasi;
$y_{max} = f(x_1)$ – funksiyaning maksimumi.

- Agar:

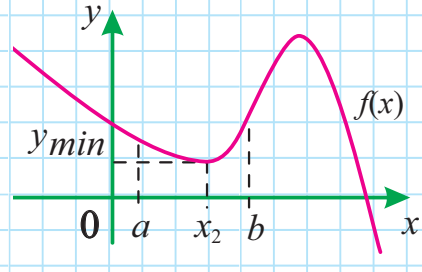
1) $f(x)$ funksiya x_2 tegishli bo‘lgan biror $(a;\ b)$ intervalda aniqlangan bo‘lib;

2) $(a;\ b)$ intervalning x_2 dan farqli barcha x nuqtalarida $f(x) > f(x_2)$ shart bajarilsa, u holda x_2 nuqta $f(x)$ ***funksiyaning minimum nuqtasi*** deyiladi (5-*rasm*).

Agar $x_2 \in X$ nuqta $f(x)$ funksiya uchun minimum nuqta bo‘lsa, u holda $f(x)$ funksiyaning x_2 nuqtadagi $f(x_2)$ qiymati $f(x)$ ***funksiyaning minimumi*** deyiladi va y_{min} kabi belgilanadi. Demak,

$$y_{min} = f(x_2).$$

5-rasm



$f(x)$ uchun $(a;\ b)$ intervalda
x_2 – funksiyaning minimum nuqtasi;
$y_{min} = f(x_2)$ – funksiyaning minimumi.

Funksiyaning maksimum va minimum nuqtalari ***ekstremum nuqtalari*** deyiladi.

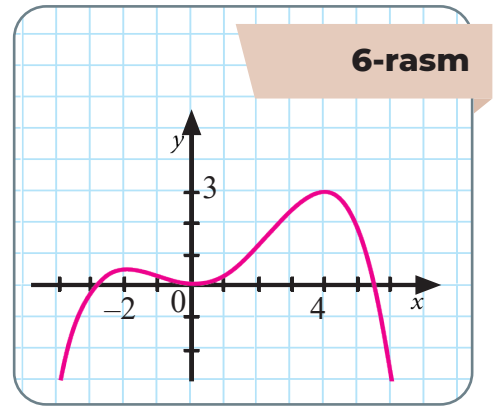
Funksiyaning ***ekstremum nuqtalardagi*** qiymatlari funksiya ***ekstremumlari*** deyiladi.

4-misol. $f(x)$ funksiyaning grafigi 6-rasmda keltirilgan. Funksiyaning o‘sish va kamayish oraliqlarini toping.

Yechish

$f(x)$ funksiyaning grafigidan funksiya $(-\infty;-2]$ va $[0;\ 4]$ oraliqlarda o‘sishini hamda $[-2;\ 0]$ va $[4;\infty)$ oraliqlarda kamayishini aniqlaymiz.

6-rasm



MISOLLAR

1. Berilgan funksiyalarning juft yoki toqligini tekshiring.

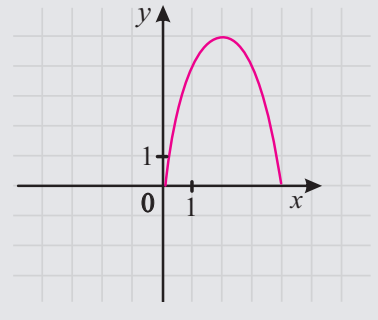
a) $f(x)=x^4$ b) $f(x)=x^3$

c) $f(x)=x^2+x$ d) $f(x)=x^4-4x^2$

e) $f(x)=x^3-x$ f) $f(x)=3x^3+2x^2+1$

g) $f(x)=1-\sqrt[3]{x}$ h) $f(x)=x+\dfrac{1}{x}$

7-rasm



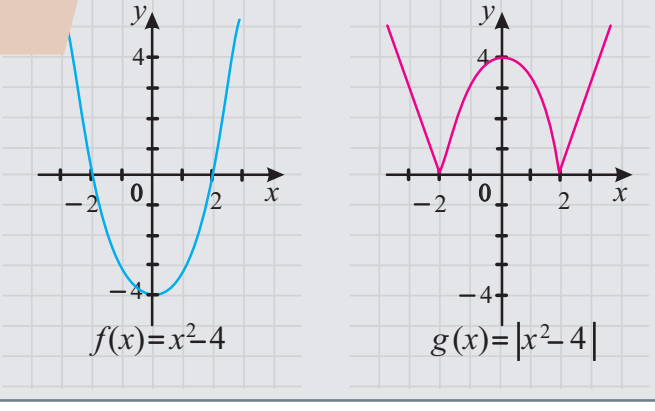
2. 7-rasmda $x\geq 0$ soha uchun funksiyaning grafigi berilgan. $x<0$ sohada grafikni shunday quringki:

1) juft funksiya;

2) toq funksiya grafigi hosil bo‘lsin.

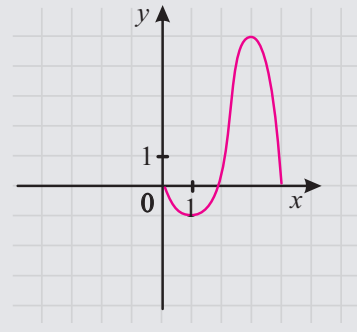
3. 8-rasmda $f(x)=x^2-4$ va $g(x)=|x^2-4|$ funksiya grafiklari berilgan. $g(x)$ funksiyaning grafigi $f(x)$ funksiyaning grafigidan qanday hosil qilinganini tushuntirib bering.

8-rasm



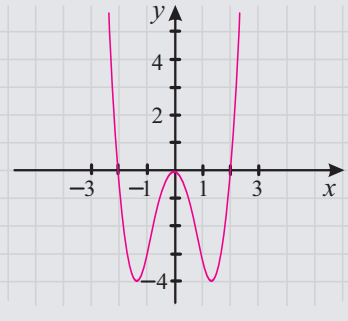
4. 9-rasmda $x \geq 0$ soha uchun funksiyaning grafigi berilgan. $x < 0$ sohada grafikni shunday quringki:
1) juft funksiya;
2) toq funksiya grafigi hosil bo‘lsin.

9-rasm



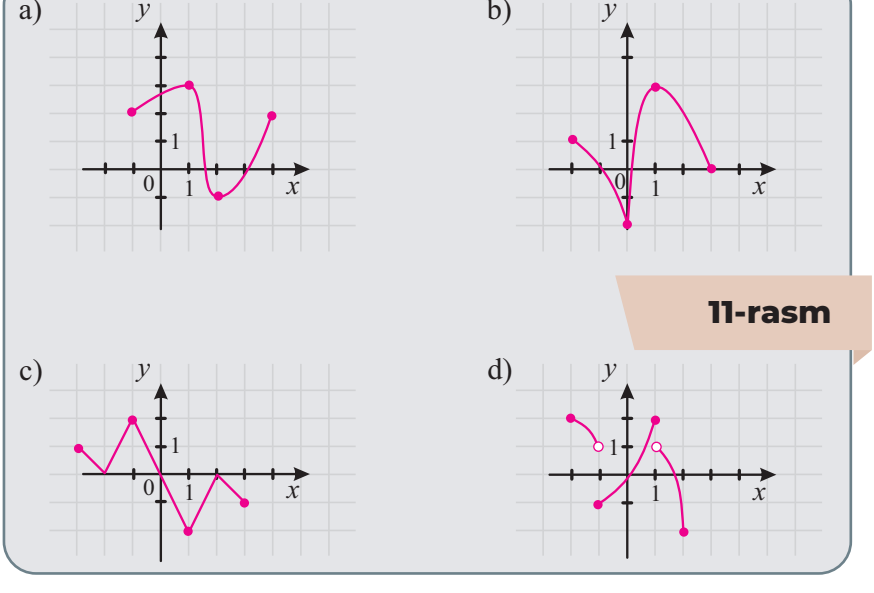
5. $f(x) = x^4 - 4x^2$ funksiyaning grafigi berilgan (10-*rasm*). Undan foydalanib $g(x) = |x^4 - 4x^2|$ funksiyaning grafigini yasang.

10-rasm



6. 11-rasmda f funksiyaning grafigi berilgan. Bu grafikdan foydalanib quyidagilarni aniqlang:
1) f funksiyaning aniqlanish sohasini va qiymatlar to‘plamini.
2) f funksiyaning o‘sish va kamayish oraliqlarini.

11-rasm



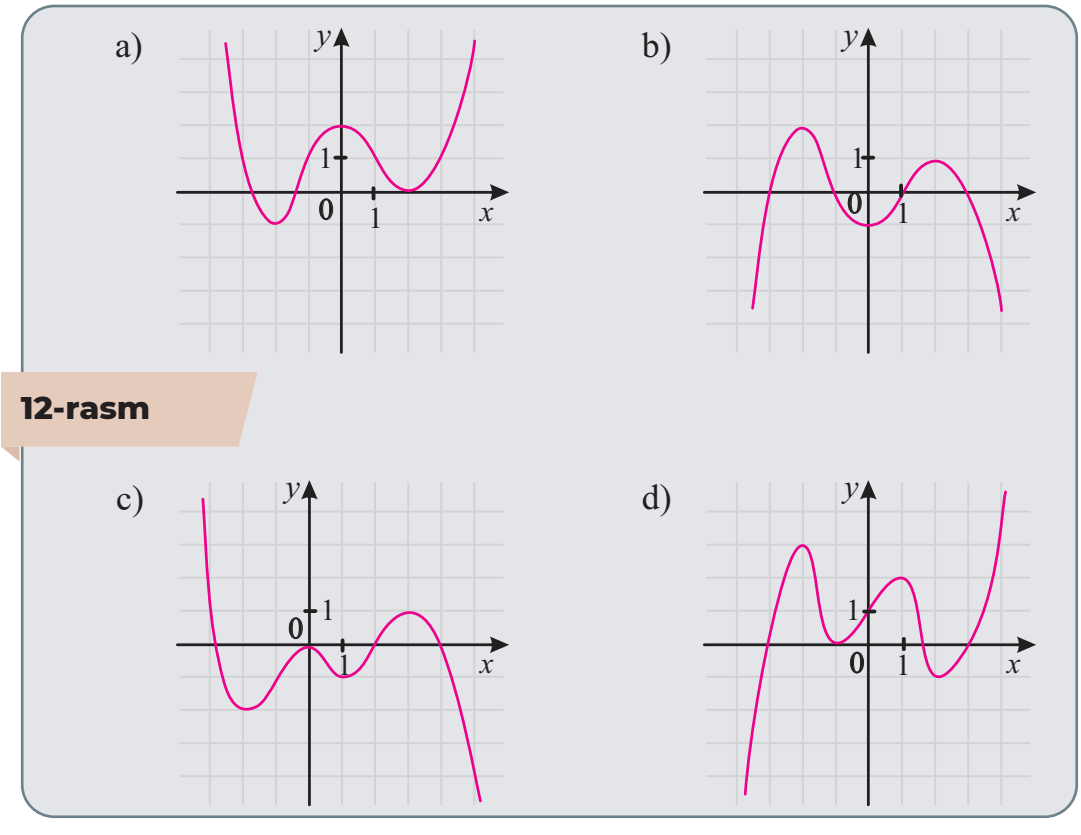
7. Berilgan funksiyalarning grafigini yasang, aniqlanish sohasini va qiymatlar to‘plamini aniqlang, o‘sish va kamayish oraliqlarini taxminiy toping.

a) $f(x)=x^2-5x$ b) $f(x)=x^3-4x$ c) $f(x)=x^4-16x^2$

8. f funksiyaning grafigi 12-rasmda berilgan. Bu grafikdan foydalanib quyidagilarni taxminiy aniqlang:

1) funksiyaning barcha ekstremum nuqtalarini va ekstremumlarini;

2) funksiyaning o‘sish va kamayish oraliqlarini.



12-rasm

9. Quyidagi ma’lumotlar asosida funksiya grafigining eskizini yasang.

a) $(-\infty;3]$ kamayadi, $[3;+\infty)$ o‘sadi;

b) $(-\infty;0]\cup[1;+\infty)$ kamayadi, $[0;1]$ o‘zgarmaydi;

c) $(-\infty;-6]$ kamayadi, $[-6;0]$ o‘sadi va $[0;+\infty)$ o‘zgarmaydi;

d) $[-5;10]$ o‘sadi, $[10;+\infty)$ o‘zgarmaydi va $x=-5$ da eng kichik qiymatni qabul qiladi.

FUNKSIYA GRAFIGI USTIDA SODDA ALMASHTIRISHLAR

Funksiya grafigini siljitish

Berilgan $f(x)$ funksiyaning grafigini Oxy tekisligida siljitish mumkin. Funksiya grafigining quyida keltiriladigan siljitishlarini ko‘rib o‘tamiz.

1. Funksiya grafigini Ox o‘qi bo‘yicha siljitish.
2. Funksiya grafigini Oy o‘qi bo‘yicha siljitish.
3. Funksiya grafigini biror vektor yo‘nalishida siljitish.

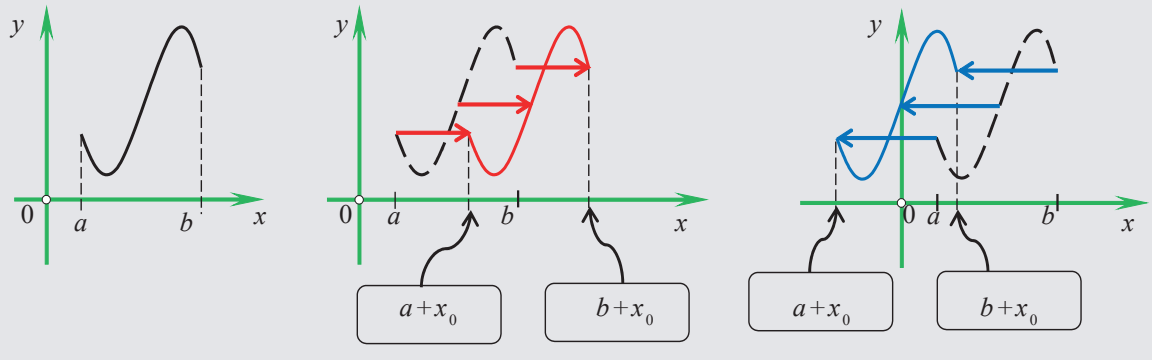
1. Funksiya grafigini Ox o‘qi bo‘yicha x_0 birlikka siljitish (1-*rasm*)

a) agar $x_0 > 0$ bo‘lsa, grafik Ox o‘qi yo‘nalishida x_0 birlik siljiydi.

b) agar $x_0 < 0$ bo‘lsa, grafik Ox o‘qi yo‘nalishiga qarshi $|x_0|$ birlik siljiydi.

1-rasm

$y = f(x)$ funksiya grafigini Ox o‘qi bo‘yicha siljitish



a) berilgan $y = f(x)$ funksiya grafigi.

b) $x_0 > 0$ bo‘lganda $y = f(x - x_0)$ funksiya grafigi; $y = f(x)$ funksiya grafigi o‘ngga x_0 birlik siljigan.

c) $x_0 < 0$ bo‘lganda $y = f(x - x_0)$ funksiya grafigi; $y = f(x)$ funksiya grafigi chapga $|x_0|$ birlik siljigan.

1-misol. $f(x) = x^2$ funksiya grafigidan foydalanib quyidagi funksiyalar grafigini yasang.

a) $g(x) = (x+4)^2$ b) $h(x) = (x-2)^2$

Yechish. 2-rasmda ko‘rsatilgandek,

a) g funksiyaning grafigini yasash uchun f funksiya grafigini chapga 4 birlikka siljitamiz.

b) h funksiyaning grafigini yasash uchun f funksiya grafigini o‘ngga 2 birlikka siljitamiz.

2-rasm



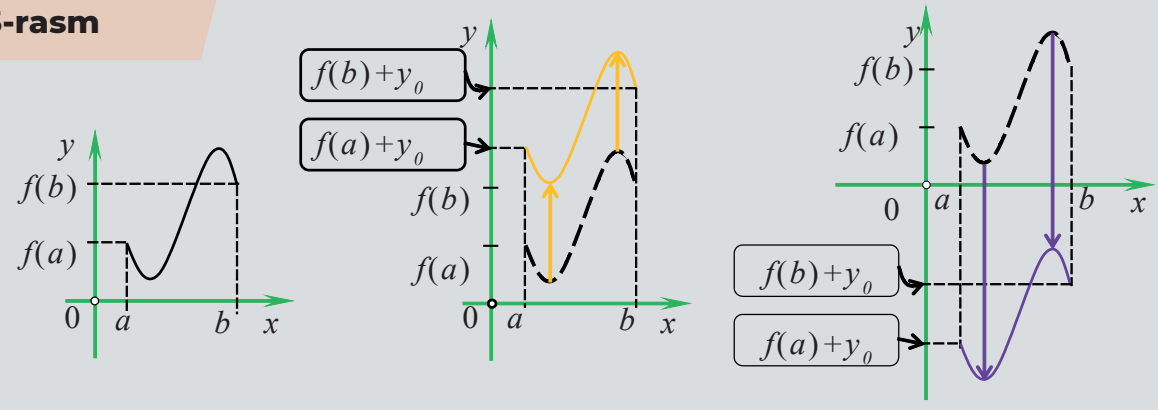
2. Funksiya grafigini *Oy* o‘qi bo‘yicha y_0 birlikka siljitish (3-*rasm*)

a) agar $y_0 > 0$ bo‘lsa, grafik *Oy* o‘qi yo‘nalishida y_0 birlik siljiydi;

b) agar $y_0 < 0$ bo‘lsa, grafik *Oy* o‘qi yo‘nalishiga qarshi $|y_0|$ birlik siljiydi (3-*rasm*).

3-rasm

$y = f(x)$ funksiya grafigini *Oy* o‘qi bo‘yicha siljitish.



a) Berilgan $y = f(x)$ funksiya grafigi.

b) $y_0 > 0$ bo‘lganda $y = f(x)+y_0$ funksiya grafigi. $y = f(x)$ funksiya grafigi yuqoriga y_0 birlik siljigan.

c) $y_0 < 0$ bo‘lganda $y = f(x)+y_0$ funksiya grafigi. $y = f(x)$ funksiya grafigi pastga $|y_0|$ birlik siljigan.

2-misol. $f(x) = x^2$ funksiyadan foydalanib quyidagi funksiyalarning grafigini yasang.

a) $g(x) = x^2 + 3$ b) $h(x) = x^2 - 2$

Yechish

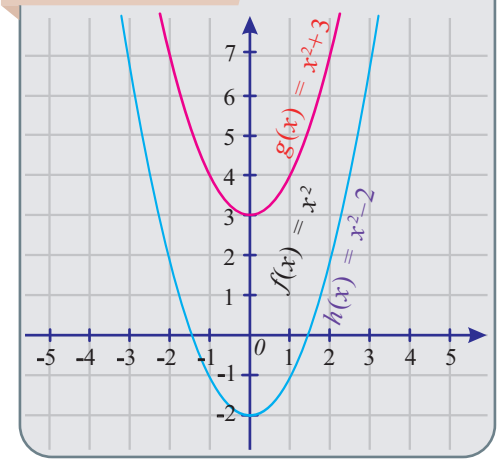
a) Quyidagiga e’tibor bering:

$$g(x) = x^2 + 3 = f(x) + 3$$

Demak, 4-rasmda ko‘rsatilgandek *g* funksiya grafigini chizish uchun *f* funksiyaning grafigini yuqoriga 3 birlikka siljitamiz (ko‘taramiz).

b) Xuddi shunday, *h* funksiyaning grafigini chizish uchun *f* funksiyaning grafigini pastga 2 birlikka siljitamiz (tushiramiz).

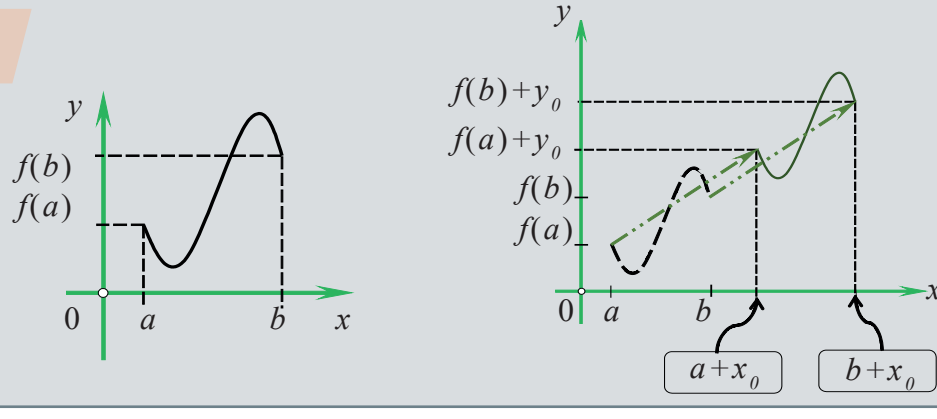
4-rasm



3. Funksiya grafigini ham *Ox*, ham *Oy* o‘qlari bo‘yicha siljitish (5-*rasm*)

$y = f(x - x_0) + y_0$ funksiya grafigini yasash uchun $y = f(x)$ funksiya grafigini *Ox* o‘qi bo‘yicha x_0 birlikka, *Oy* o‘qi bo‘yicha esa y_0 birlikka siljitish yetarli.

5-rasm

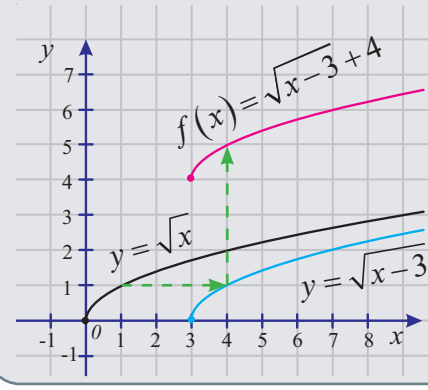


3-misol. $f(x)=\sqrt{x-3}+4$ funksiyaning grafigini yasang.

Yechish

Dastlab $y=\sqrt{x}$ funksiya grafigini yasaymiz. Hosil bo‘lgan funksiya grafigini 3 birlik o‘ngga siljitamiz va $y=\sqrt{x-3}$ funksiyaning grafigini hosil qilamiz. So‘ng bu grafikni 4 birlik yuqoriga siljitamiz va $f(x)=\sqrt{x-3}+4$ funksiya grafigini hosil qilamiz (6-*rasm*).

6-rasm



Funksiya grafiklarini siqish va cho‘zish

Berilgan $f(x)$ funksiyaning grafigini Oxy tekisligida deformatsiyalash (siqish yoki cho‘zish) mumkin.

1-hol. Berilgan $y=f(x)$ funksiya grafigidan foydalanib $y=kf(x)$ funksiya grafigi quyidagicha hosil qilinadi (7-*rasm*):

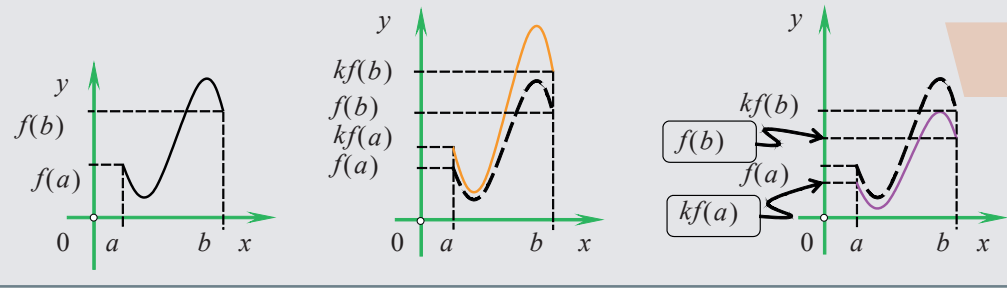
a) agar $k>1$ bo‘lsa, grafik Ox o‘qidan Oy o‘qi bo‘yicha k baravar cho‘ziladi.

b) agar $0<k<1$ bo‘lsa, grafik Ox o‘qiga Oy o‘qi bo‘ylab $\frac{1}{k}$ baravar siqiladi.

c) agar $k<0$ bo‘lsa, u holda $y=kf(x)$ funksiya grafigi $y=|k|f(x)$ funksiya grafigining Ox o‘qqa nisbatan simmetrik aksi bo‘ladi.

7-rasm

$y=kf(x)$ funksiya grafigini hosil qilish



2-hol. Berilgan $y=f(x)$ funksiya grafigidan foydalanib $y=f(mx)$ funksiya grafigi quyidagicha hosil qilinadi (8-*rasm*):

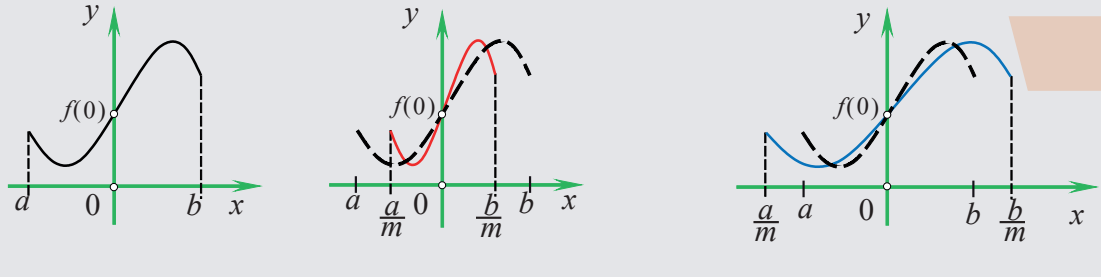
a) agar $m>1$ bo‘lsa, grafik Oy o‘qiga Ox o‘qi bo‘ylab m baravar siqiladi;

b) agar $0<m<1$ bo‘lsa, grafik Oy o‘qidan Ox o‘qi bo‘ylab $\frac{1}{m}$ baravar cho‘ziladi.

c) agar $m<0$ bo‘lsa, u holda $y=f(mx)$ funksiyaning grafigi $y=f(|m|x)$ funksiya grafigining Oy o‘qqa nisbatan simmetrik aksi bo‘ladi.

8-rasm

$y=f(mx)$ funksiya grafigini hosil qilish



4-misol. Quyidagi funksiyaning grafigini yasang.

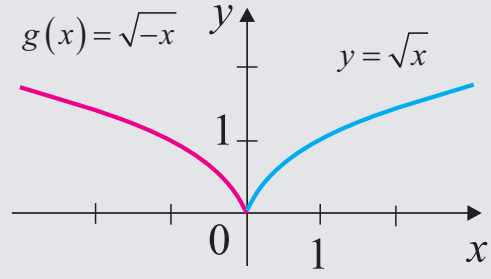
$$g(x)=\sqrt{-x}$$

Yechish

9-rasmda $y=\sqrt{x}$ funksiyaning grafigini chizamiz. Bu grafikni y o‘qiga nisbatan simmetrik akslantirish orqali $g(x)=\sqrt{-x}$ funksiyaning grafigini hosil qilamiz.

E’tibor qiling: $g(x)=\sqrt{-x}$ funksiyaning aniqlanish sohasi: $x\le 0$ dan iborat.

9-rasm



5-misol. 10-rasmda $f(x)=x^2$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini yasang.

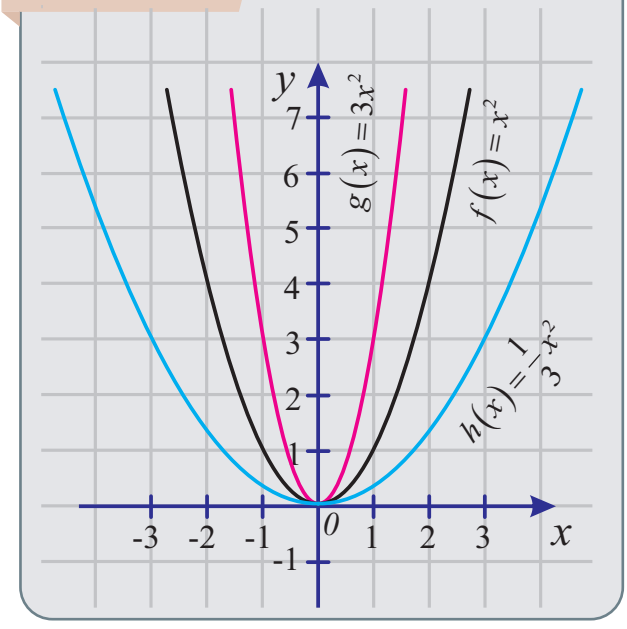
a) $g(x)=3x^2$ b) $h(x)=\frac{1}{3}x^2$

Yechish

a) g funksiyaning grafigi f funksiyaning har bir nuqtasining y koordinatasini 3 ga ko‘paytirishdan hosil bo‘ladi. Ya’ni g funksiyaning grafigini hosil qilish uchun f funksiyaning grafigini vertikal 3 baravar cho‘zish kerak.

b) h funksiyaning grafigi f funksiyaning har bir nuqtasining y koordinatasini $\frac{1}{3}$ ga ko‘paytirishdan hosil bo‘ladi. Ya’ni h funksiyaning grafigini hosil qilish uchun f funksiyaning grafigini vertikal yo‘nalishda x o‘qqa 3 baravar siqish kerak.

10-rasm

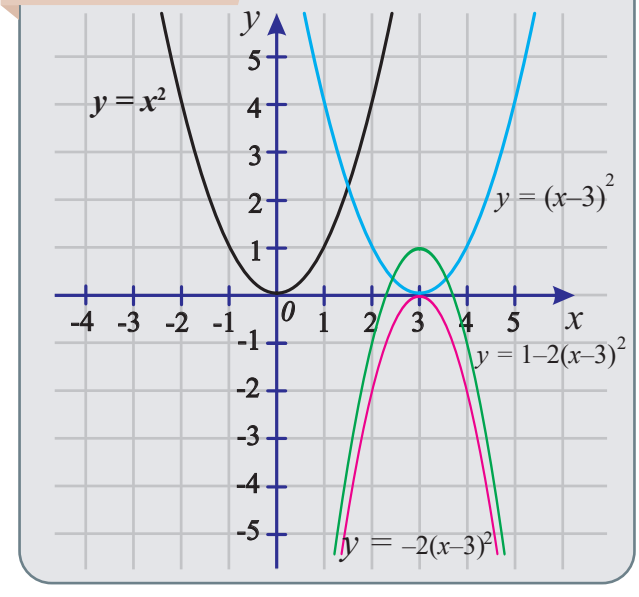


6-misol. $f(x)=1-2(x-3)^2$ funksiyaning grafigini yasang.

Yechish

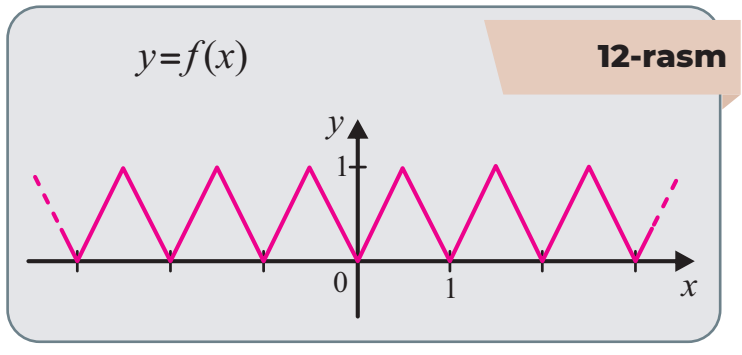
Dastlab $y=x^2$ funksiyaning grafigini o‘ngga 3 birlikka gorizontal siljitamiz va $y=(x-3)^2$ funksiyaning grafigini hosil qilamiz. Keyin bu grafikni Ox o‘qiga nisbatan simmetrik akslantiramiz. Oy o‘qi bo‘yicha 2 baravar cho‘zishni bajaramiz va $y=-2(x-3)^2$ funksiyaning grafigini hosil qilamiz. Nihoyat bu grafikni yuqoriga 1 birlikka siljitamiz va $f(x)=1-2(x-3)^2$ funksiyaning grafigini hosil qilamiz. (11-*rasm*).

11-rasm



7-misol. 12-rasmda $y=f(x)$ funksiyaning grafigi berilgan. Bu grafikdan foydalanib quyidagi funksiyalarning grafigini yasang.

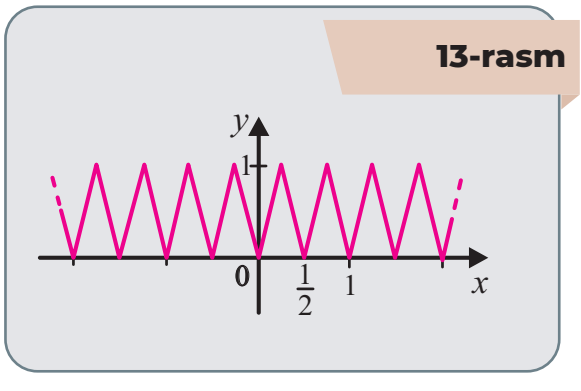
a) $y=f(2x)$; b) $y=f\left(\frac{1}{2}x\right)$.



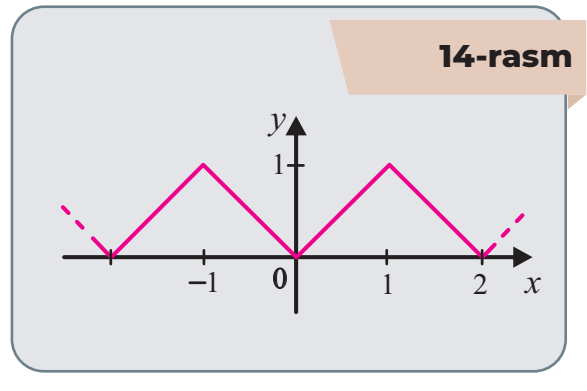
Yechish

a) $y=f(2x)$ funksiya grafigini yasash uchun $f(x)$ funksiya grafigini Oy o‘qiga Ox o‘qi bo‘yicha 2 baravar siqamiz (13-*rasm*).

b) $y=f\left(\frac{1}{2}x\right)$ funksiya grafigini yasash uchun $f(x)$ funksiya grafigini Oy o‘qidan Ox o‘qi bo‘yicha 2 baravar cho‘zamiz (14-*rasm*).



$y=f(2x)$



$y=f\left(\frac{1}{2}x\right)$

MISOLLAR

1. $f(x)$ funksiya grafigi berilgan bo‘lsa, quyidagi funksiyalarning grafigi qanday yasalishini tushuntiring.

a) $y=f(x)-1$ b) $y=f(x-2)$ c) $y=f\left(\frac{1}{4}x\right)$

d) $y=f(x)+4$ e) $y=f(-x)$ f) $y=3f(x)$

g) $y=-f(x)$ h) $y=\frac{1}{3}f(x)$ i) $y=f(x-5)+2$

j) $y=f(x+1)-1$ k) $y=4f(x+1)+3$ l) $y=f(4x)$

m) $y=-f(x)+5$ n) $y=3f(x)-5$ o) $y=1-f(-x)$

2. g funksiyaning grafigi f funksiyaning grafigidan qanday almashtirishlar yordamida hosil qilinganini tushuntiring.

a) $f(x)=x^2,\ g(x)=(x+2)^2$
b) $f(x)=x^2,\ g(x)=x^2+2$
c) $f(x)=x^3,\ g(x)=(x-4)^3$
d) $f(x)=x^3,\ g(x)=x^3-4$
e) $f(x)=|x|,\ g(x)=|x+2|-2$
f) $f(x)=|x|,\ g(x)=|x-2|+2$
g) $f(x)=\sqrt{x},\ g(x)=-\sqrt{x}+1$
h) $f(x)=\sqrt{x},\ g(x)=\sqrt{-x}+1$

3. $y=x^2$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini chizing.

a) $g(x)=x^2+1$
b) $g(x)=(x-1)^2$
c) $g(x)=-x^2$
d) $g(x)=(x-1)^2+3$

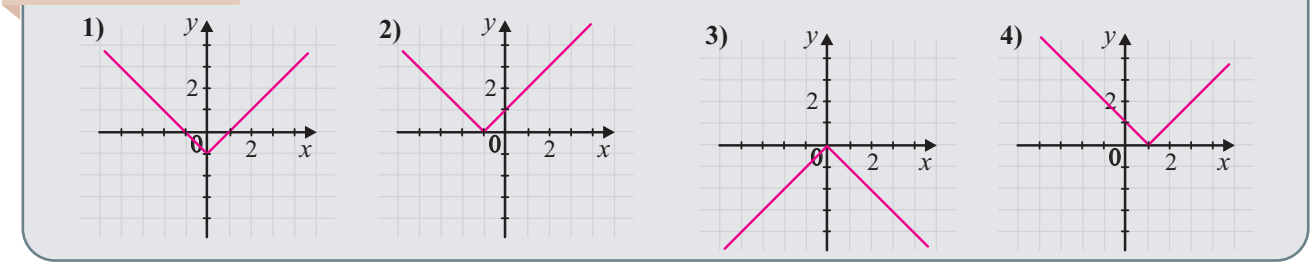
4. $y=\sqrt{x}$ funksiyaning grafigidan foydalanib quyidagi funksiyalarning grafigini chizing.

a) $g(x)=\sqrt{x-2}$
b) $g(x)=\sqrt{x}+1$
c) $g(x)=\sqrt{x+2}+2$
d) $g(x)=-\sqrt{x}+1$

5. Berilgan funksiyalarga 15-rasmda berilgan grafiklardan mosini toping.

a) $y=|x+1|$
b) $y=|x|-1$
c) $y=|x-1|$
d) $y=-|x|$

15-rasm



6. Quyidagi funksiyalarning grafiklarini standart funksiyaning grafigi ustida mos almashtirishlarni bajarib chizing.

a) $f(x)=x^2+3$
b) $f(x)=\sqrt{x}+1$
c) $f(x)=|x|-1$
d) $f(x)=\sqrt{x}+1$
e) $f(x)=(x-5)^2$
f) $f(x)=(x+1)^2$
g) $f(x)=|x+2|$
h) $f(x)=\sqrt{x-4}$
i) $f(x)=-x^3$
j) $f(x)=-|x|$
k) $y=\sqrt[4]{-x}$
l) $y=\sqrt[3]{-x}$
m) $y=\frac{1}{4}x^2$
n) $y=-5\sqrt{x}$
o) $y=3|x|$
p) $y=\frac{1}{2}|x|$
q) $y=(x-3)^2+5$
r) $y=\sqrt{x+4}-3$
s) $y=3-\frac{1}{2}(x-1)^2$
t) $y=2-\sqrt{x+1}$
u) $y=|x+2|+2$
v) $y=2-|x|$
w) $y=\frac{1}{2}\sqrt{x+4}-3$
x) $y=3-2(x-1)^2$.

7. Berilgan f funksiyaning grafigiga ko‘rsatilgan almashtirishlar qo‘llangan. Yakuniy funksiyaning formulasini yozing.

a) $f(x)=x^2$, 3 birlik pastga siljiting.

b) $f(x)=x^3$, 5 birlik yuqoriga siljiting.

c) $f(x)=\sqrt{x}$, 2 birlik chapga siljiting.

d) $f(x)=\sqrt[3]{x}$, 1 birlik o‘ngga siljiting.

e) $f(x)=|x|$, 2 birlik chapga va 5 birlik pastga siljiting.

f) $f(x)=|x|$, x o‘qiga nisbatan akslantirib, 4 birlik o‘ngga va 3 birlik yuqoriga siljiting.

g) $f(x)=\sqrt[4]{x}$, y o‘qiga nisbatan simmetrik akslantiring va 1 birlik yuqoriga siljiting.

h) $f(x)=x^2$, 2 birlik chapga siljiting va x o‘qiga nisbatan simmetrik akslantiring.

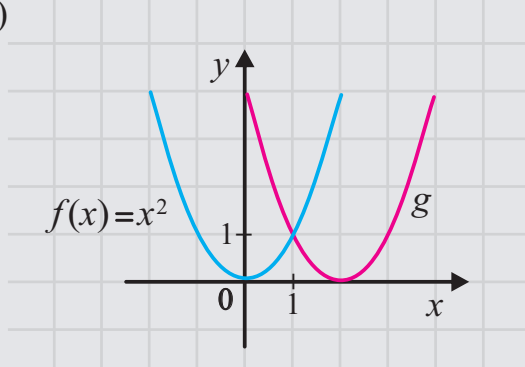
i) $f(x)=x^2$, 2 baravar vertikal cho‘zib, 2 birlik pastga va 3 birlik o‘ngga siljiting.

j) $f(x)=|x|$, $\frac{1}{2}$ baravar vertikal yo‘nalishda siqishni bajarib, 1 birlik chapga va 3 birlik yuqoriga siljiting.

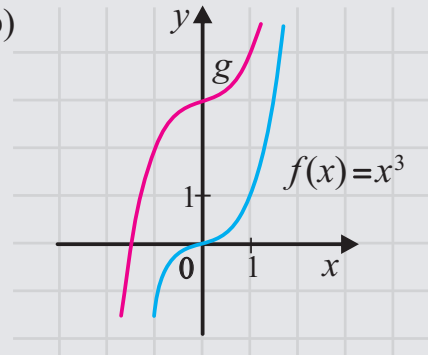
8. f va g funksiyalarning grafigi berilgan (16-*rasm*). f funksiyadan foydalanib g funksiyaning formulasini toping.

16-rasm

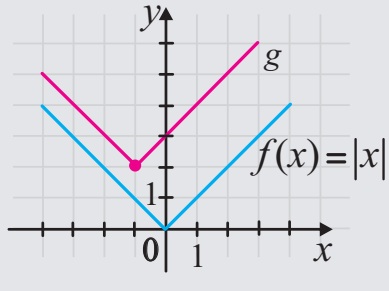
a)



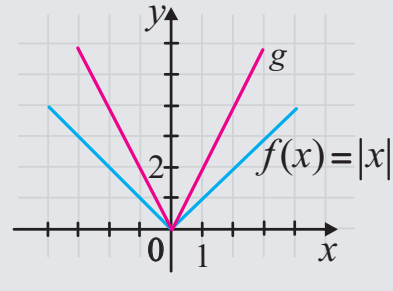
b)



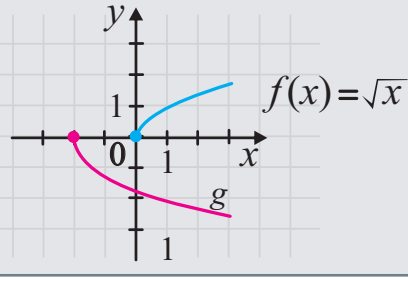
c)



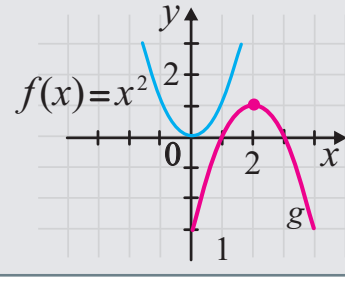
d)



e)



f)



9. $y = f(x)$ funksiya berilgan, 17-rasmda quyidagilarga mos grafikni toping.

a) $y = f(x-4)$

b) $y = f(x)+3$

c) $y = 2f(x+6)$

d) $y = -f(2x)$

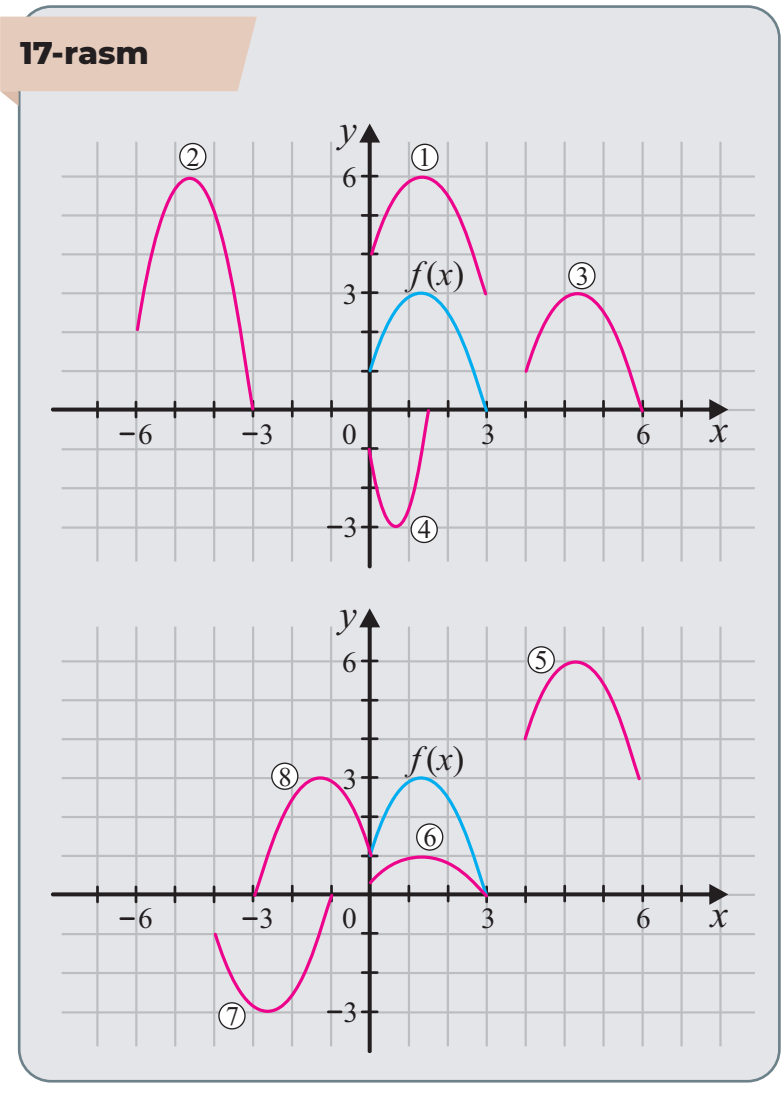
e) $y = \frac{1}{3}f(x)$

f) $y = -f(x+4)$

g) $y = f(x-4)+3$

h) $y = f(-x)$

17-rasm



CHIZIQLI VA KVADRATIK MODELLASHTIRISHLAR

Matematik modellashtirish kundalik hayotimizdagi turli jarayonlarni o‘rganishning asosiy analitik vositasi hisoblanadi.

Quyidagi masalalarni ko‘rib chiqamiz.

1-masala. Porshenli nasos eng ko‘pi bilan qanday chuqurlikdan suv chiqara olishini toping (1-*rasm*).

Yechish

Ma’lumki, porshenli nasos trubasidagi suv ustunining bosimi

$$p = \rho g h$$

formula bilan hisoblanadi.

Bu yerda $\rho = 1000\ \text{kg/m}^3$ – suv zichligi, $g = 10\ \text{m/s}^2$ – erkin tushish tezlanishi, h – suv ustunining balandligi.

Nasos yer sathida joylashgani uchun suv ustuni balandligi *suv chuqurligi* deb yuritiladi. Demak, suv chuqurligini

$$h = \frac{p}{\rho g}$$

tenglikdan topish mumkin.

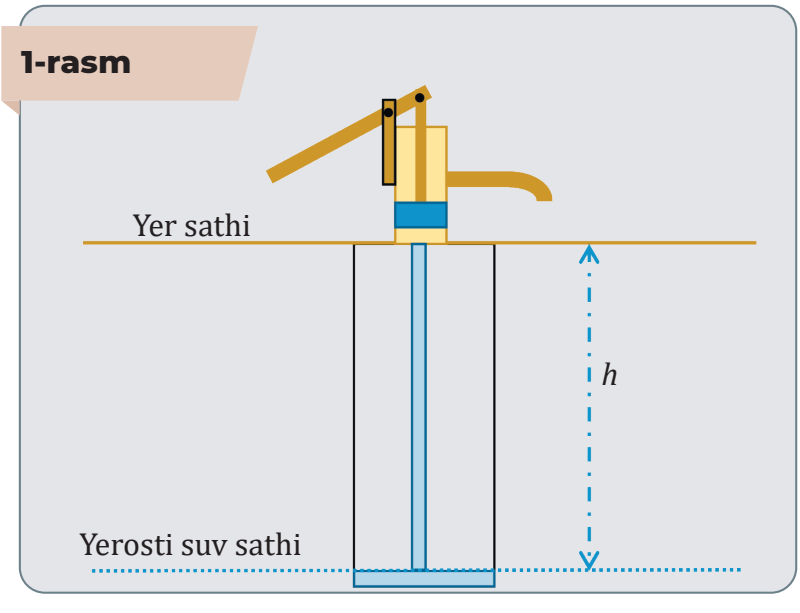
1643-yilda italyan fizigi Evangelista Torrichelli tajribalarida suv ustuni bosimi atmosfera bosimi $p_0 = 100\ 000$ Pa dan oshib ketmasligi isbotlangan, ya’ni $p \le p_0$. Shuning uchun porshenli nasosda suv chuqurligi

$$h = \frac{p}{\rho g} \le \frac{p_0}{\rho g} = \frac{100000}{1000 \cdot 10} = 10\ \text{m}$$

dan oshib keta olmas ekan.

Javob: Porshenli nasos eng ko‘pi bilan 10 m chuqurlikdan suv chiqara oladi.

1-rasm



Bu modeldagi o‘zgaruvchilar birinchi darajali bo‘lib, o‘zaro chiziqli amallar (qo‘shish va songa ko‘paytirish) orqali bog‘langan. Shuning uchun bu tipdagi matematik modellar ***chiziqli modellar*** deyiladi. Qo‘yilgan masalani chiziqli model shakliga olib kelish jarayoni ***chiziqli modellashtirish*** deyiladi.

2-masala. O‘quvchi Oxy koordinatalar tekisligini shunday tanladiki, bunda o‘z uyini koordinata boshi $O(0; 0)$ deb oldi. Keyin o‘zi o‘qiydigan maktab $C(4; 3)$ nuqtada joylashganini aniqladi. Yo‘lning uyi va maktab orasidan o‘tadigan to‘g‘ri chiziqli qismi Ox o‘qini $(6; 0)$ nuqtada, Oy o‘qini $(0; 4)$ nuqtada kesib o‘tishini hisoblab chiqdi.

Maktabga uyali aloqa kompaniyasining antennasi o‘rnatilganligi ma’lum. O‘quvchi yo‘lda harakatlanayotgan avtomobildagi yo‘lovchining uyali aloqa vositasi antennadan tarqalayotgan to‘lqinni eng yaxshi tutadigan nuqtani topishga qiziqib qoldi.

Topshiriq. Siz bu masalani qanday yechgan bo‘lar edingiz?

Yechish. Ravshanki, yo‘lning maktabga eng yaqin nuqtasida uyali aloqa vositasi to‘lqinni eng yaxshi tutadi. Bu masalani yechishda yo‘lni tavsiflovchi (AB) to‘g‘ri chiziq tenglamasini tuzish va uning maktabga eng yaqin nuqtasining koordinatalarini topish kerak. Buning uchun avvalo bayon etilganlar asosida vaziyatning chizmasi chiziladi (2-*rasmga qarang*).

Keyin $A(6; 0)$ va $B(0; 4)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasi tuziladi. Buning uchun to‘g‘ri chiziqning

$$y = kx+b$$

tenglamasiga $A(6; 0)$ va $B(0; 4)$ nuqtalarning koordinatalarini qo‘yib, ushbu

$$0 = k\cdot 6+b$$
$$4 = k\cdot 0+b$$

tengliklar hosil qilinadi. Ulardan

$$b = 4,\ k = -\frac{2}{3}$$

koeffitsiyentlar topiladi. Demak, (AB) to‘g‘ri chiziq tenglamasi

$$y = -\frac{2}{3}x+4$$

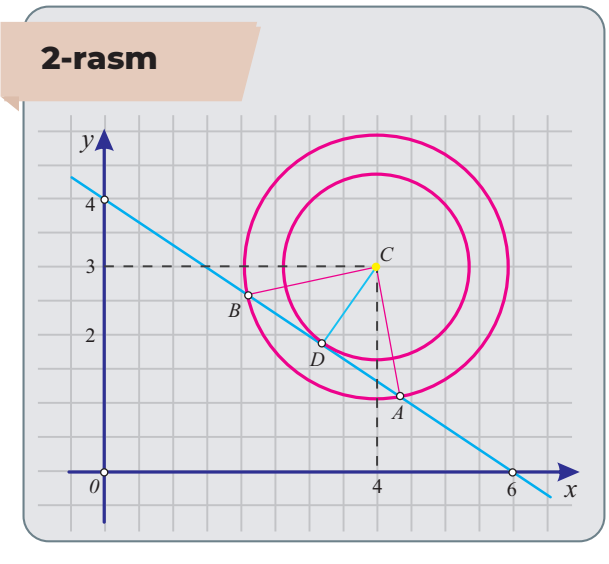
bo‘ladi.

Masalaning yechimi (AB) to‘g‘ri chiziqning $C(4; 3)$ nuqtaga eng yaqin $D(x; y)$ nuqtasini topishdan iborat. Bu vaziyatning matematik modeli quyidagicha yoziladi:

$$F = \sqrt{(x-4)^2+(y-3)^2} \to min,$$

$$y = -\frac{2}{3}x+4$$

2-rasm



Bu modeldagi o‘zgaruvchilar birinchi va ikkinchi darajali bo‘lgani uchun bu tipdagi matematik modellar ***kvadratik modellar*** deyiladi. Qo‘yilgan masalani kvadrat model shakliga olib kelish jarayoni ***kvadratik modellashtirish*** deyiladi.

MISOLLAR

1. Har bir berilgan topshiriqning chiziqli modelini yozing:

a) Siz velosipedni 10 000 so‘m boshlang‘ich to‘lov va soatiga 5000 so‘m tarif bo‘yicha ijaraga oldingiz.

b) Avtomobillarni ta’mirlash ustaxonasi 50 000 so‘m bazaviy to‘lov hamda soatiga 15 000 so‘mdan to‘lov belgiladi.

c) Shamning uzunligi 30 cm va u soatiga 1,4 cm tezlikda yonadi.

d) Dasturlash bo‘yicha mutaxassis maslahat uchun alohida $75 va undan so‘ng soatiga $35 oladi.

e) Hozirgi harorat 25 °C va u kechasi har soatda 2 °C ga tushishi kutilmoqda.

f) Qishloq aholisi 6791 kishini tashkil etadi va yiliga 7 taga kamayib bormoqda.

2. Berilgan jadvaldagi funksiya chiziqli yoki kvadratik ekanini aniqlang.

x	0	1	3	4	6
y	5	10	20	25	35

3. To‘p tepaga va pastga sakraganda uning erishadigan balandligi doimiy ravishda kamayadi. Quyidagi jadvalda vaqt bo‘yicha sakrash balandligi ko‘rsatilgan.

a) Eng mos keladigan kvadrat funksiyani toping.

b) To‘pning maksimal balandligini toping.

c) 2,5 sekundda to‘pning qancha balandlikda bo‘lganini taxmin qiling.

t (s)	2	2,2	2,4	2,6	3
h (dyuym)	2	16	26	33	42

4. Agar tosh 70 metrli binoning tepasidan otilgan bo‘lsa, toshning vaqtga bog‘liq balandligi $h(t) = -5t^2 - 20t + 70$ kvadrat funksiya bilan berilgan, bu yerda t sekundda, balandligi esa metrda. Necha sekunddan keyin tosh yerga tegadi?

5. Malika xonasini tozalashga Umidadan ikki baravar ko‘p vaqt sarflaydi. Aziza xonasini tozalashi uchun Umidadan 10 minut ko‘proq vaqt ketkazadi. Ular xonalarini tozalash uchun jami 90 minut sarflaydi. Malika xonasini tozalashi uchun qancha vaqt sarflaydi?

6. Dilshod dengizga durni olish uchun sho‘ng‘idi. Uning t sekunddan keyingi sho‘ng‘ish chuqurligi $h(t) = -4t^2 + 4t + 3$ metr bo‘ldi, $t \geq 0$.

a) durlar qanday chuqurlikda joylashgan?

b) Dilshod durni olish uchun qancha vaqt sarflaydi?

c) Dilshod qanday balandlikdan suvga sho‘ng‘idi?

7. Jasmina ko‘ylak tikish uchun buyurtma oldi. U bir kunda x ta ko‘ylak tiksa, $P(x) = -x^2 + 20x$ ming so‘m miqdorida daromad oladi.

a) Eng katta daromad olish uchun u qancha ko‘ylak tikishi kerak?

b) Eng katta daromad necha so‘mga teng?

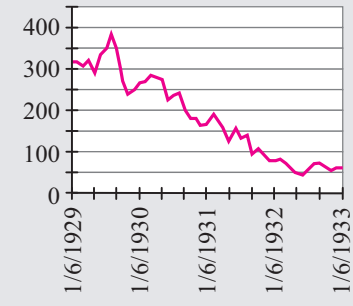
8. 2005-yilda Zarafshon shahri aholisi 55 000 ga yaqin edi. O‘sha paytda aholi soni yiliga 2000 ga yaqin sur’atlarda o‘sib borardi. Har qanday yil uchun Zarafshon aholisini topish zarur. Buning uchun uning chiziqli modelini tuzing. 2010-yili Zarafshon aholisi qancha bo‘lgan? Zarafshon aholisi soni 2025-yili qancha bo‘lishini hisoblang.

LOYIHA ISHI

Har bir grafik hikoya qiladi

Agar rasm ming so‘zga arzigulik bo‘lsa, unda grafik hech bo‘lmaganda bir necha qator jumlalariga arziydi. Darhaqiqat, grafik ba’zan hikoyani ko‘p so‘zlarga qaraganda tezroq va samaraliroq aytib berishi mumkin. 1929-yildagi fond bozori qulashining halokatli ta’siri Dou Jons indeksining (DJIA) grafigidan (1-*rasm*) darhol ko‘rinadi. O‘sha paytdagi gazetalarda bunday grafiklar halokatning naqadar kattaligini yetkazishning samarali usuli sifatida chop etilgan.

1-rasm



2-rasmdagi xabarni yetkazish uchun hech qanday so‘z kerak emas. Grafik oddiy bir voqeani aytib beradi: nimadir pastga tushdi – ehtimol, savdo, foyda yoki mahsuldorlik, mas’ul shaxs esa juda xavotirda.

2-rasm

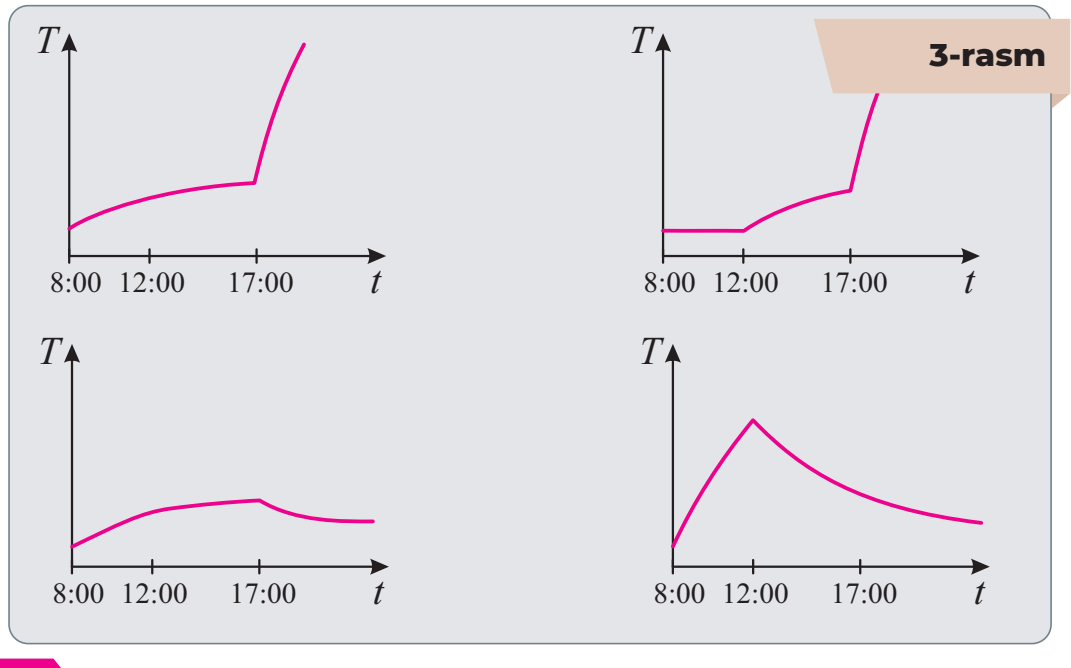


Ushbu loyiha tadqiqotida biz grafiklar aytib beradigan hikoyalarni o‘qiymiz va hikoya qiluvchi grafiklarni yaratamiz.

Hikoyani grafikdan o‘qish

1. Quyida haroratning vaqtga nisbatan to‘rtta grafigi (soat 8:00 dan boshlab) ko‘rsatilgan bo‘lib, ulardan keyin uchta hikoya berilgan. (3-*rasm*).

3-rasm



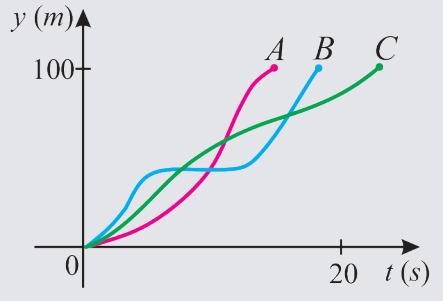
a) hikoyalarning har birini grafiklarning biri bilan moslang.

b) hech qanday hikoyaga mos kelmaydigan grafik uchun xuddi shunday hikoya yozing.

1-hikoya	Tushda muzlatkichdan go‘shtni olib, eritish uchun peshtaxtaga qo‘ydim va ishga ketdim. Ishdan uyga qaytganimdan keyin go‘shtni pechda pishirdim.
2-hikoya	Ertalab muzlatkichdan go‘shtni olib, eritish uchun peshtaxtaga qo‘ydim va ishga ketdim. Ishdan uyga qaytganimdan keyin go‘shtni pechda pishirdim.
3-hikoya	Ertalab muzlatkichdan go‘shtni olib, eritish uchun peshtaxtaga qo‘ydim va ishga ketdim. Men buni unutib, ishdan uyga qaytayotganimda kafeda tanovul qildim. Uyga kelganimdan so‘ng esa go‘shtni muzlatkichga qayta qo‘ydim.

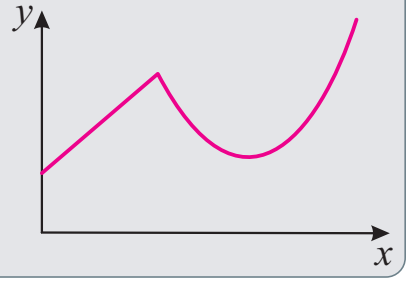
2. 100 metrga to‘siqlar osha yugurishda uchta yuguruvchi qatnashdi. Grafikda yugurish masofasi har bir yuguruvchi uchun vaqt funksiyasi sifatida ko‘rsatilgan (4-*rasm*). Grafik sizga ushbu poyga haqida nima deyishini tasvirlab bering. Poygada kim g‘olib chiqdi? Har bir yuguruvchi poygani tugatdimi? Sizningcha, B sportchi bilan nima sodir bo‘lgan?

4-rasm



3. Quyidagi chizmaga mos keladigan (har qanday vaziyatni o‘z ichiga olgan) hikoya tuzing. (5-*rasm*).

5-rasm





2-BOB. RATSIONAL TENGLAMALAR VA TENGSIZLIKLAR. IRRATSIONAL TENGLAMALAR

- RATSIONAL TENGLAMALAR
- RATSIONAL TENGLAMALAR SISTEMASI
- RATSIONAL TENGSIZLIKLAR
- RATSIONAL TENGSIZLIKLAR SISTEMASI
- IRRATSIONAL TENGLAMALAR
- IRRATSIONAL TENGLAMALAR SISTEMASI

RATSIONAL TENGLAMALAR

Asosiy ta'rif va tushunchalar

Ta'rif

$f(x)=g(x)$ ko'rinishidagi tenglik ***bir noma'lumli tenglama*** deyiladi (bu yerda $f(x)$ va $g(x)$ lar x noma'lumli ifodalar).

Tenglamaning ildizi deb noma'lumning berilgan tenglamani to'g'ri sonli tenglikka aylantiradigan qiymatiga aytiladi.

Tenglamani yechish deganda uning barcha ildizlarini topish yoki uning ildizi mavjud emasligini ko'rsatish tushuniladi.

Tenglamaning barcha ildizlari to'plami ***tenglamaning yechimi*** deyiladi.

Agar x noma'lumning hech bir qiymati tenglamani to'g'ri sonli tenglikka aylantirmasa, u holda "***tenglamaning ildizi yo'q***" yoki "***tenglamaning yechimi – bo'sh to'plam***" iborasi ishlatiladi, bu holatni $x \in \varnothing$ kabi ham yozish mumkin.

1-misol. $(x+3)(2x-1)(x-2)=0$ tenglamani yeching.

Yechish. Bu tenglamaning o'ng tarafi nolga teng, chap tarafi esa 3 ta ifodaning ko'paytmasidan iborat. Ko'paytuvchilaridan hech bo'lmaganda bittasi nolga teng bo'lgandagina ko'paytma nolga teng bo'lgani uchun har bir ko'paytuvchi nolga teng bo'lgan holni alohida ko'rib chiqamiz: $x+3=0,\ 2x-1=0,\ x-2=0$. Hosil bo'lgan ushbu tenglamalardan tenglamaning ildizlari

$$x_1=-3,\ x_2=\frac{1}{2},\ x_3=2 \text{ ekanini aniqlab olamiz.}$$

2-misol. Ildizlari 0, –1 va $\sqrt{2}$ ga teng bo'lgan tenglama tuzing.

Yechish. Turli ko'rinishdagi tenglamalar javob tariqasida berilishi mumkin. Eng sodda tenglama $x(x+1)\left(x-\sqrt{2}\right)=0$ ko'rinishida bo'lishini eslatib o'tamiz.

Bu sonlar yana quyidagi tenglamaning ham ildizi bo'la oladi:

$$(x^2+x^3)\left(x-\sqrt{2}\right)\left(x^2+3\right)=0$$

Ta'rif

Agar $f(x)=g(x)$ tenglamaning barcha ildizlari $f_1(x)=g_1(x)$ tenglamaning ildizlari bo'lsa va, aksincha, $f_1(x)=g_1(x)$ tenglamaning barcha ildizlari $f(x)=g(x)$ tenglamaning ildizlari bo'lsa, ya'ni ularning yechimlari ustma-ust tushsa, bunday tenglamalar ***teng kuchli tenglamalar*** deyiladi.

3-misol. $3x-6=0$ va $2x-1=3$ tenglamalarning teng kuchliligini tekshiring.

Yechish. $3x-6=0$ va $2x-1=3$ tenglamalar teng kuchli, chunki har birining ildizi $x=2$ dan iborat.

Yechimi bo'sh to'plam bo'lgan har qanday ikkita tenglama ham teng kuchli bo'ladi.

Teng kuchli tenglamalar quyidagicha belgilanadi: $3x-6=0 \Leftrightarrow 2x-1=3$.

Tenglama quyidagi holatlarda o‘ziga teng kuchli bo‘lgan tenglamaga o‘tadi:

a) Tenglamaning biror-bir hadi tenglikning bir qismidan ikkinchi qismiga qarama-qarshi ishora bilan o‘tkazilganda. Masalan: $f(x)=g(x)+t(x) \Leftrightarrow f(x)-g(x)=t(x)$.

b) Tenglamaning ikkala tarafi noldan farqli songa ko‘paytirilganda yoki bo‘linganda.

Butun ratsional tenglamalar

Agar $f(x)$ va $g(x)$ funksiyalar butun ratsional ifodalar bilan berilgan bo‘lsa,

$$f(x)=g(x)$$

tenglama ***butun ratsional tenglama*** deyiladi.

Bunday tenglamaning aniqlanish sohasi barcha haqiqiy sonlar to‘plami bo‘ladi.

Ta’rif

$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x+a_n=0$, $a_0\neq 0$ ko‘rinishidagi tenglama **standart ko‘rinishdagi *n*-darajali butun ratsional tenglama** deb ataladi. Bu yerda a_0, a_1, ..., a_{n-1} koeffitsiyentlar, a_n ozod had, $n \in N$.

Agar $a_0=1$ bo‘lsa, $x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n=0$ tenglama **keltirilgan *n*-darajali butun ratsional tenglama** deb ataladi.

Ma’lumki, *n*-darajali ko‘phad *n* tadan ko‘p bo‘lmagan ildizlarga ega bo‘lishi mumkin, demak, har bir standart ko‘rinishdagi *n*-darajali butun ratsional tenglama ham *n* tadan ko‘p bo‘lmagan ildizlarga ega bo‘ladi.

Teorema. Butun koeffitsiyentli keltirilgan butun ratsional tenglamaning ildizlari butun son bo‘lsa, ular ozod hadning bo‘luvchilari bo‘ladi.

4-misol. $x^4+2x^3=11x^2-4x-4$ tenglamani yeching.

Yechish. Avval uni standart ko‘rinishga keltiramiz: $x^4+2x^3-11x^2+4x+4=0$.

Bu tenglamaning butun ildizlari borligini tekshirish uchun ozod hadi 4 ning barcha butun bo‘luvchilarini yozib olamiz: $\pm1, \pm2, \pm4$. Bu sonlarni ketma-ket tenglamaga qo‘yib ko‘rib, $x_1=1$ va $x_2=2$ sonlar tenglamaning ildizlari bo‘lishini aniqlab olamiz. Demak, $x^4+2x^3-11x^2+4x+4$ ko‘phad $(x-1)(x-2)=x^2-3x+2$ ko‘phadga qoldiqsiz bo‘linadi.

$$\begin{array}{r|l}
x^4+2x^3-11x^2+4x+4 & x^2-3x+2 \\
\underline{-\,x^4-3x^3+2x^2} & \overline{x^2+5x+2} \\
5x^3-13x^2+4x+4 & \\
\underline{-\,5x^3-15x^2+10x} & \\
2x^2-6x+4 & \\
\underline{-\,2x^2-6x+4} & \\
0 &
\end{array}$$

Tenglamani $(x-1)(x-2)(x^2+5x+2)=0$ ko‘rinishida yozib olamiz.

Hosil bo‘lgan tenglama berilgan tenglamaga teng kuchli tenglamadir. Har bir ko‘paytuvchini nolga tenglashtirib, tenglamaning ildizlarini topamiz.

Javob: $x_1 = 1;\ x_2 = 2,\ x_{3,4} = \frac{-5 \pm \sqrt{17}}{2}$.

5-misol. Tenglamani yeching: $x^3 - 3x^2 - 13x + 15 = 0$.

Yechish. Ozod hadning bo‘luvchilari: ±1, ±3, ±5. Bulardan –3, 1, 5 sonlari tenglamaning chap tomonini 0 ga teng qilishini osonlikcha aniqlashimiz mumkin. Demak, $x^3 - 3x^2 - 13x + 15$ ko‘phadni quyidagicha ko‘paytuvchilarga ajratishimiz mumkin:

$$x^3 - x^2 - 2x^2 + 2x - 15x + 15 = 0$$

$$x^2(x-1) - 2x(x-1) - 15(x-1) = 0$$

$$(x-1)(x^2 - 2x - 15) = 0$$

$$(x-1)(x-5)(x+3) = 0$$

Bu ko‘paytuvchilarning har birini 0 ga tenglashtirib yechib, tenglamaning ildizlari $x_1 = 1,\ x_2 = 5,\ x_3 = -3$ ga teng ekaniga ishonch hosil qilamiz.

Javob: $x_1 = 1,\ x_2 = 5,\ x_3 = -3$.

6-misol. $x^4 - 5x^3 + 8x^2 - 5x + 1 = 0$ tenglamani yeching.

Yechish. Berilgan tenglama 4-darajali qaytma (simmetrik) tenglama. Uni yechish uchun tenglamaning ikkala tomonini $x^2 \neq 0$ ga bo‘lamiz va unga teng kuchli tenglamani hosil qilamiz.

$$x^2 - 5x + 8 - \frac{5}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 5\left(x + \frac{1}{x}\right) + 8 = 0$$

$x + \frac{1}{x} = t$ belgilash kiritamiz. U holda

$$t^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + 2x \cdot \frac{1}{x} + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2} \quad \Rightarrow \quad x^2 + \frac{1}{x^2} = t^2 - 2 \text{ bo‘ladi.}$$

Bulardan $t^2 - 5t + 6 = 0$ tenglamani hosil qilamiz. Bu tenglamaning yechimlari: $t_1 = 2$ va $t_2 = 3$. Bu qiymatlarni belgilashga qayta qo‘yib, berilgan tenglamaning yechimi $x + \frac{1}{x} = 2$ va $x + \frac{1}{x} = 3$ tenglamalarning yechimi birlashmasiga teng bo‘lishini ko‘ramiz.

Bu tenglamalarni yechib, $x_1 = 1, x_2 = \frac{3+\sqrt{5}}{2}$ va $x_3 = \frac{3-\sqrt{5}}{2}$ ekanini topamiz.

Javob: $x_1 = 1, x_2 = \frac{3+\sqrt{5}}{2}, x_3 = \frac{3-\sqrt{5}}{2}$.

7-misol. $3x^3 + 4x^2 + 4x + 3 = 0$ tenglamani yeching.

Yechish. Berilgan tenglama 3-darajali qaytma (simmetrik) tenglama. Uni yechish uchun avval ko‘paytuvchilarga ajratamiz va unga teng kuchli tenglamani hosil qilamiz.

$$3(x^3 + 1) + 4x(x + 1) = 0$$

$$(x + 1)(3x^2 - 3x + 3 + 4x) = 0$$

$$(x + 1)(3x^2 + x + 3) = 0.$$

Bu tenglamaning yechimi quyidagi 2 ta tenglamaning yechimi birlashmasiga teng.

$$x + 1 = 0 \quad \text{va} \quad 3x^2 + x + 3 = 0.$$

1-tenglamaning yechimi $x = -1$, 2-tenglama esa haqiqiy yechimga ega emas.

Javob: $x = -1$.

Kasr-ratsional tenglamalar

$\frac{f(x)}{g(x)} = 0$ ko‘rinishiga keltirish mumkin bo‘lgan tenglamalar ***kasr-ratsional tenglama*** deyiladi (bu yerda $f(x)$ va $g(x)$ lar x noma’lumli ko‘phadlar).

$\frac{f(x)}{g(x)} = 0$ ko‘rinishidagi ratsional tenglamaning **aniqlanish sohasi** $g(x) \neq 0$.

Ratsional tenglamalarni yechish qadamlari:

- tenglamadagi barcha ifodalar tenglikning chap tarafiga o‘tkaziladi;
- barcha ifodalar umumiy maxrajga keltiriladi;
- tenglama $\frac{f(x)}{g(x)} = 0$ ko‘rinishiga keltiriladi;
- suratining nollari topiladi;
- aniqlanish sohasi topiladi;
- suratning aniqlanish sohasiga tegishli bo‘lgan nollari tenglamaning ildizlari bo‘ladi.

Yoki $\frac{f(x)}{g(x)} = 0$ ratsional tenglamaning yechimini topish uchun u $\begin{cases} f(x) = 0 \\ g(x) \neq 0 \end{cases}$ ko‘rinishdagi teng kuchli sistema yozib olinadi va yechiladi.

Masalan, quyidagi tenglamani qaraylik: $\frac{x^2 + x - 2}{x - 1} = 0,$

Kasrning suratini nolga tenglashtiramiz:

$$x^2 + x - 2 = 0 \Rightarrow x_1 = -2,\ x_2 = 1.$$

Bu tenglamaning aniqlanish sohasi $x \neq 1$, ya’ni $x = 1$ qiymat berilgan tenglamaning yechimi bo‘la olmaydi, demak, $x = 1$ chet ildiz bo‘ladi.

8-misol. Tenglamaning ildizini toping: $\frac{2x+3}{x-1}=0$.

Yechish. $\begin{cases}2x+3=0\\ x-1\neq 0\end{cases}\Rightarrow\begin{cases}2x=-3\\ x\neq 1\end{cases}\Rightarrow\begin{cases}x=-1,5\\ x\neq 1\end{cases}$

Javob: $x=-1,5$.

9-misol. Tenglamani yeching: $\frac{4x+4}{3(x+2)-3}=0$.

Yechish. $\begin{cases}4x+4=0\\ 3(x+2)-3\neq 0\end{cases}\Rightarrow\begin{cases}4x=-4\\ 3x+6-3\neq 0\end{cases}\Rightarrow\begin{cases}x=-1\\ x\neq -1\end{cases}$

ko‘rinib turibdiki, x ning qiymati -1 ga teng bo‘lishi mumkin emas.

Javob: $x\in\varnothing$.

10-misol. Tenglamaning ildizini toping: $\frac{-2x-4}{x^2-4}=\frac{x+5}{x-2}$.

Yechish. Barcha ifodalarni tenglikdan chap tarafga o‘tkazamiz va umumiy maxrajga keltiramiz.

$$\frac{x+5}{x-2}+\frac{2x+4}{x^2-4}=0\Rightarrow\frac{(x+5)(x+2)+2x+4}{x^2-4}=0\Rightarrow$$
$$\Rightarrow\frac{x^2+7x+10+2x+4}{x^2-4}=\frac{x^2+9x+14}{x^2-4}=0$$

Kasr-ratsional ifodaning suratini nolga tenglashtiramiz va nollarini topamiz. Viyet teoremasidan foydalanamiz.

$$x^2+9x+14=0\Rightarrow x=-2;x=-7$$

Aniqlanish sohasi $x^2-4=(x-2)(x+2)\neq 0\Rightarrow x\neq -2;x\neq 2$

Ko‘rinib turibdiki, tenglamaning bitta ildizi bor: $x=-7$.

Javob: $x=-7$.

Diqqat qiling! Kasr-ratsional tenglamani yechishda har doim suratining nollari tenglama aniqlanish sohasiga tegishli ekanini tekshiring.

11-misol. Tenglamani yeching: $\frac{(x^2-x-56)(x-3)}{x^2+5x+6}=0$.

Yechish. Berilgan tenglama kasr-ratsional tenglamadir. Avval suratining nollarini topamiz.

$$(x^2-x-56)(x-3)=0\Rightarrow x=3;\ \ x^2-x-56=0$$
$$D=(-1)^2-4\cdot 1\cdot(-56)=225=15^2$$
$$x_{1,2}=\frac{1\pm 15}{2}\Rightarrow x_1=8;x_2=-7.$$

Suratning 3 ta nolini topdik: $x_1=8;x_2=-7;x_3=3$.

Bu nollarni berilgan tenglamaning maxrajidagi ifodaga qo‘yib tekshiramiz va ular maxrajning nollari bo‘lmasligiga ishonch hosil qilamiz.

Javob: $x_1=8;x_2=-7;x_3=3$.

12-misol. Tenglamaning ildizlarini toping.

$$\frac{2}{(x-2)(x+2)}-\frac{1}{x(x-2)}=\frac{4-x}{x(x+2)}$$

Yechish. Tenglikning o‘ng tarafidagi ifodani chap tarafga o‘tkazamiz:

$$\frac{2}{(x-2)(x+2)}-\frac{1}{x(x-2)}-\frac{4-x}{x(x+2)}=0$$

va umumiy maxrajga keltiramiz:

$$\frac{2x-(x+2)-(4-x)(x-2)}{x(x-2)(x+2)}=0$$

Suratidagi qavslarni ochib, kvadrat tenglamaga keltiramiz:

$$\frac{2x-x-2-4x+x^2+8-2x}{x(x-2)(x+2)}=0\Rightarrow\frac{x^2-5x+6}{x(x-2)(x+2)}=0$$

Suratining nollarini topamiz:

$$x^2-5x+6=0\Rightarrow D=(-5)^2-4\cdot 6=1,\ x_{1,2}=\frac{5\pm 1}{2}\Longrightarrow x_1=2;\ \ x_2=3.$$

x ning topilgan qiymatlarini berilgan tenglamaning maxrajidagi ifodaga qo‘yib, tekshiramiz. $x=2$ qiymat maxrajdagi ifodani nolga aylantirgani uchun chet ildiz bo‘ladi. Demak, tenglama bitta $x=3$ ildizga ega ekan.

Javob: $x=3$.

13-misol. Tenglamani yeching: $x^2+x+1=\dfrac{15}{x^2+x+3}$.

Yechish. $x^2+x+1=t$ belgilash kiritamiz. Tenglama quyidagi ko‘rinishga keladi:

$$t=\frac{15}{t+2}$$

$t\neq -2$ bo‘lishini inobatga olib, quyidagi tenglamani yechamiz:

$$t(t+2)=15$$

$$t^2+2t-15=0$$

$$t_1=-5;\ t_2=3$$

t ning o‘rniga qo‘yib, $x^2+x+1=-5$ va $x^2+x+1=3$ tenglamalarga ega bo‘ldik. Ularning har birini alohida yechamiz:

$$x^2+x+6=0\ \Rightarrow \text{haqiqiy ildizi yo‘q};\quad x^2+x-2=0\ \Rightarrow\ x_1=-2;\ x_2=1.$$

Javob: $x_1=-2;\ x_2=1$.

Matnli masalalar yechishda ratsional tenglamalar ishlatilishi mumkin. Quyida harakat va ishga doir masalalar ratsional tenglama ko‘rinishida modellashtirilib yechilgan.

Harakatga doir masala

Vertolyot avvaliga shamol yo‘nalishida 120 km masofani uchib o‘tdi, keyin ortga qaytdi. Bunga u 6 soat vaqt sarfladi. Agar vertolyotning shamolsiz havodagi tezligi 45 km/h ga teng bo‘lsa, shamolning tezligini toping.

Yechish. Shamolning tezligini x km/h bilan belgilaylik. Unda shamol yo‘nalishi bo‘yicha vertolyotning tezligi $(45 + x)$ km/h va shamolga qarshi yo‘nalishda esa $(45 - x)$ km/h ga teng bo‘ladi. Masalaning sharti bo‘yicha, vertolyot jami 6 soat vaqt sarflagan. Masofani tezlikka bo‘lib, qo‘shsak, jami vaqtga teng bo‘ladi.

$$\frac{120}{45+x}+\frac{120}{45-x}=6$$

Kasr-ratsional tenglama hosil bo‘ldi: $\frac{120}{45+x}+\frac{120}{45-x}-6=0$

$$\frac{120(45-x)+120(45+x)-6(45+x)(45-x)}{(45+x)(45-x)}=0$$

Suratini soddalashtiramiz va nolga tenglab yechamiz:

$$6x^2-1350=0$$
$$x^2=225$$
$$x_1=-15;\ x_2=15$$

Tezlik manfiy qiymat qabul qilmagani uchun $x=-15$ ildiz bo‘la olmaydi. Demak, shamolning tezligi 15 km/h.

Javob: shamolning tezligi 15 km/h.

Ishga doir masala

Ikkita traktorchi birgalikda dalani 4 kunda shudgor qildi. Agar 1-traktorchiga shudgorni alohida bajarishi uchun 2-traktorchiga nisbatan 6 kun kam vaqt kerak bo‘lsa, har bir traktorchi ishni necha kunda bajaradi?

Yechish. 1-traktorchi dalani x kunda shudgor qilsin. Unda 2-traktorchi shu dalani $(x + 6)$ kunda shudgor qiladi. Demak, 1-traktorchi 1 kunda dalaning $\frac{1}{x}$ qismini, 2-traktorchi esa $\frac{1}{x+6}$ qismini shudgor qiladi. Masalaning shartiga ko‘ra, shu dalani ular birgalikda 4 kunda shudgor qiladi. Ya’ni ikkalasi 1 kunda dalaning $\frac{1}{4}$ qismini shudgor qiladi.

Tenglamani tuzamiz va yechamiz: $\frac{1}{x}+\frac{1}{x+6}=\frac{1}{4}$

$$\frac{4(x+6)+4x-x(x+6)}{4x(x+6)}=0$$

$$\frac{-x^2+2x+24}{4x(x+6)}=0$$

Hosil bo‘lgan ratsional tenglama quyidagi sistemaga teng kuchli.

$$\begin{cases} x^2-2x-24=0; \\ 4x(x+6)\neq 0; \end{cases}$$

$$D=(-2)^2-4\cdot(-24)=100;\ x_{1,2}=\frac{2\pm 10}{2}\Rightarrow x_1=6;\ \ x_2=-4$$

Kunlar soni manfiy bo‘lmaydi, shuning uchun $x = -4$ javob bo‘la olmaydi. Demak, 1-traktorchi shudgorni 6 kunda, 2-traktorchi esa $x+6=6+6=12$ kunda bajaradi.

Javob: 1-traktorchi 6 (kun), 2-traktorchi 12 (kun).

MISOLLAR

Kasr-ratsional tenglamalarni yeching (1–10).

1. $\frac{1}{x}-\frac{2x}{x+1}=0$

2. $\frac{2y-5}{y+5}=\frac{3y+21}{2y-1}$

3. $\frac{5x-7}{x-3}=\frac{4x-3}{x}$

4. $\frac{x+1}{2(x-1)}=\frac{9}{2(x+4)}+\frac{1}{x-1}$

5. $\frac{2x}{x-1}-\frac{1}{x+1}=\frac{4x}{x^2-1}$

6. $\frac{x^2-2x}{x-2}=x^2-2$

7. $\frac{7}{2x+9}-6=5x$

8. $\frac{15}{x-2}=\frac{14}{x}+1$

9. $\frac{4}{x-2}+\frac{4}{x+2}=\frac{3}{2}$

10. $\frac{3x}{x^2-1}=2\left(\frac{2x-1}{x+1}\right)$

Kasr-ratsional tenglamalarni yeching (11–30).

11. $\frac{1}{x^2-12x+36}+\frac{12}{36-x^2}=\frac{1}{x+6}$

12. $\frac{8c-3}{4c^2-2c+1}+\frac{6}{8c^3+1}=\frac{2}{2c+1}$

13. $\frac{3x-2}{x-1}+\frac{x-4}{x+3}=\frac{3x^2+1}{(x-1)(x+3)}$

14. $\frac{2-3x}{x+1}-\frac{4}{3}\cdot\frac{x+1}{2-3x}=\frac{4}{3}$

15. $\frac{x-49}{x+6}+\frac{2x+50}{x+5}=2$

16. $\frac{(x+2)^2-9}{x-1}\cdot(x-5)=-24$

17. $(x+4)(x^2-1)=4x^2+24x-\frac{4x^2+20x}{5x+x^2}$

18. $\frac{25x-21}{2x^2+5x-12}=\frac{x-4}{2x-3}-\frac{2x-3}{x+4}$

19. $\frac{3}{x^2-2x+1}+\frac{2}{1-x^2}=\frac{1}{1+x}$

20. $\frac{6}{x-1}+\frac{6}{(x-1)(x-3)}+\frac{3}{3-x}=7$

21. $\frac{x^5-4x^3}{x-2}=16+2x^3$

22. $\frac{1}{(x-2)(x-3)}-\frac{9}{(x+2)(x-7)}=1$

23. $x^2+x+1=\frac{15}{x^2+x+3}$

24. $\frac{x^2+2}{3x-2}-\frac{3x-2}{x^2+2}=2\frac{2}{3}$

25. $x^2-5x+\frac{24}{x^2-5x}+10=0$

26. $\frac{x^2+1}{x}+\frac{x}{x^2+1}=2\frac{1}{2}$

27. $\frac{2}{x^2+3}+\frac{4}{x^2+7}=1$

28. $\frac{1}{x(x+2)}-\frac{1}{(x+1)^2}=\frac{1}{12}$

29. $\dfrac{x^2-3x-1}{x^2-2x+4}=\dfrac{x^2-2x}{x^2-x+1}$

30. $\dfrac{x^2-4x-1}{x^2-3x+5}=\dfrac{x^2-3x+1}{x^2-2x+2}$

31. Ikki shahar orasidagi masofa daryo yo‘li bilan 80 km. Anvar kemada shu shaharlarning biridan ikkinchisiga borib-kelishi uchun 8 soat 20 minut vaqt sarf qildi. Daryo oqimining tezligi 4 km/h bo‘lsa, kemaning turg‘un suvdagi tezligini toping.

32. Ikki ishchi ayni bir ishni birgalashib bajarsa, 12 kunda tugatadi. Agar oldin bittasi ishlab, ishning yarmini tugatgandan keyin uning o‘rniga ikkinchisi ishlasa, ish 25 kunda tugaydi. Shu ishni har qaysi ishchi yolg‘iz o‘zi bajarsa, necha kunda tugatadi?

33. “A” traktor 3 kunda 7 ha, “B” traktor esa 2 kunda 17 ha yerni shudgor qila oladi. Fermer xo‘jaligida “A” traktordan 2 ta va “B” traktordan 1 ta bor. Agar bu traktorlar birgalikda ishlatilsa, fermer xo‘jaligining 237 ha yerini necha kunda shudgor qiladi?

34. Avtomobil yo‘lning 80 kilometrlik qismida 120 km/h, keyingi 25 kilometrlik qismida 50 km/h hamda so‘ngi 35 kilometrlik qismida 70 km/h tezlik bilan harakatlandi. Uning butun yo‘l davomidagi o‘rtacha tezligini toping.

35. Bir ishni birinchi ishchining yolg‘iz o‘zi a kunda bajaradi, ikkinchi ishchi shu ishni bajarish uchun birinchi ishchiga qaraganda b kun ortiq vaqt sarf qiladi. Agar uchinchi ishchining yolg‘iz o‘zi birinchi ishchiga qaraganda b kun tezroq bajara olsa, shu ishni uchala ishchi birgalikda ishlab necha kunda tugatadi?

36. Daryo bo‘yida joylashgan A va B shaharlar orasidagi masofa 96 km. Katerda A shahardan B shaharga borib kelish uchun 10 soat sarflandi. Agar daryo oqimining tezligi 4 km/h bo‘lsa, katerning turg‘un suvdagi tezligini toping.

RATSIONAL TENGLAMALAR SISTEMASI

Ikki noma'lumli ikkita tenglama qatnashgan sistemalarni yechish bizga ma'lum bo'lgan algebraik qo'shish, o'rniga qo'yish, o'zgaruvchini almashtirish usullariga tayanadi. Bunda ishtirok etgan kasr-ratsional ifodalarning maxrajlari nolga teng bo'lmasligini qayd qilamiz.

O'rniga qo'yish usuli

1-misol. Quyidagi tenglamalar sistemasini o'rniga qo'yish usulidan foydalanib yeching.

$$\begin{cases} 3xy = 21 \\ x - 8y = -1 \end{cases}$$

Yechish. 2-tenglamadan $x - 8y = -1 \Rightarrow x = 8y - 1$.

x ning hosil bo'lgan bu qiymatini 1-tenglamaga qo'yib, $3(8y-1)y = 21$ tenglamaga kelamiz. Bu tenglamani yechib,

$$(8y-1)y = 7$$

$8y^2 - y - 7 = 0 \Rightarrow y_1 = -\frac{7}{8}; y_2 = 1$ qiymatlarni topamiz va ularni $x = 8y - 1$ ga qo'yib $\Rightarrow x_1 = -8;$ $x_2 = 7$ ekanini aniqlaymiz.

Javob: $\left(-8; -\frac{7}{8}\right), (7; 1)$.

2-misol. O'rniga qo'yish usulidan foydalanib $\begin{cases} 2x^2 + y = 4 \\ x^4 + y^2 = 16 \end{cases}$ tenglamalar sistemasini yeching.

Yechish.

$$y = 4 - 2x^2.$$

$$x^4 + \left(4 - 2x^2\right)^2 = 16$$

$$x^4 + 16 - 16x^2 + 4x^4 = 16$$

$$5x^4 - 16x^2 = 0$$

$$x^2\left(5x^2 - 16\right) = 0, \quad \Rightarrow x_1 = 0,\ x_2 = \frac{4}{\sqrt{5}},\ x_3 = -\frac{4}{\sqrt{5}}$$

$$\Rightarrow\ y_1 = 4;\ y_2 = -\frac{12}{5};\ y_3 = -\frac{12}{5}$$

Javob: $(0;4)$, $\left(\frac{4}{\sqrt{5}}; -2\frac{2}{5}\right), \left(-\frac{4}{\sqrt{5}}; -2\frac{2}{5}\right)$.

Algebraik qo'shish usuli

3-misol. Ushbu tenglamalar sistemasini yeching: $\begin{cases} x^2 + y = 27 \\ x - y = 3 \end{cases}$

Yechish. Ikkala tenglamada y noma'lum qarama-qarshi ishorali koeffitsiyent bilan qatnashgan, shuning uchun bu tenglamalarni hadma-had qo'shamiz.

$$+\begin{cases} x^2+y=27 \\ x-y=3 \end{cases}$$
$$x^2+x=30$$

$x^2+x-30=0$ bir noma'lumli kvadrat tenglamaga keltirib oldik.

$$x_1=\frac{-1-11}{2}=-6 \Rightarrow y_1=-9$$
$$x_2=\frac{-1+11}{2}=5 \Rightarrow y_2=2$$

Javob: (–6; –9), (5; 2).

4-misol. Tenglamalar sistemasini algebraik qo'shish usuli yordamida yeching.

$$\begin{cases} x^3-y^3-3x^2+3y^2x=-2 \\ x^2-x^2y=1. \end{cases}$$

Yechish. 2-tenglamani 3 ga ko'paytirib, 1-tenglamaga qo'shsak:

$$+\begin{cases} x^3-y^3-3x^2+3y^2x=-2 \\ 3x^2-3x^2y=3 \end{cases}$$

1-tenglama ayirmaning kubi formulasiga keladi: $x^3-y^3-3x^2y+3y^2x=1$.

Bundan:
$$\begin{cases} (x-y)^3=1 \\ x^2-x^2y=1 \end{cases}$$
$$\begin{cases} x-y=1 \\ x^2-x^2y=1 \end{cases}$$

Endi esa o'rniga qo'yish usulidan foydalanamiz va sistemani yechamiz.

$$\begin{cases} y=x-1 \\ x^2-x^2(x-1)=1 \end{cases} \Rightarrow \begin{cases} y=x-1 \\ x^2-x^3+x^2=1 \end{cases} \Rightarrow \begin{cases} y=x-1 \\ x^3-2x^2+1=0 \end{cases}$$

$$\Rightarrow \begin{cases} y=x-1 \\ (x-1)(x^2-x-1)=0 \end{cases} \Rightarrow x_1=1;\quad x_2=\frac{1+\sqrt{5}}{2};\quad x_3=\frac{1-\sqrt{5}}{2},$$ bu qiymatlarni $y=x-1$

tenglamaga qo'yib, $y_1=0$, $y_2=\frac{-1+\sqrt{5}}{2}$, $y_3=\frac{-1-\sqrt{5}}{2}$ ekanini topamiz.

Javob: $(1;0),\left(\frac{1+\sqrt{5}}{2};\frac{-1+\sqrt{5}}{2}\right),\left(\frac{1-\sqrt{5}}{2};\frac{-1-\sqrt{5}}{2}\right)$.

O'zgaruvchilarni almashtirish usuli

5-misol. Tenglamalar sistemasini yeching:

$$\begin{cases} x+xy+y=11 \\ x^2y+xy^2=30. \end{cases}$$

Yechish. Quyidagicha belgilash kiritamiz.

$$x+y=a \quad \text{va} \quad xy=b$$

Shunda sistema quyidagi ko‘rinishga keladi:

$$\begin{cases} a+b=11 \\ ab=30 \end{cases}$$

Bu sistemani yechib, $a_1=6,\ b_1=5$ va $a_2=5,\ b_2=6$ larni aniqlaymiz. Endi quyidagi sistemalarni yechamiz:

$$\begin{cases} x+y=6 \\ xy=5 \end{cases} \text{ va } \begin{cases} x+y=5 \\ xy=6 \end{cases}$$

Ularning ildizlaridan tuzilgan to‘plam tenglamalar sistemasining yechimi bo‘ladi.

Javob: $(5;1),\ (1;5),\ (2;3),\ (3;2)$.

MISOLLAR

Tenglamalar sistemasini yeching.

1. $\begin{cases} y-x^2+x=1 \\ x=y-4 \end{cases}$

2. $\begin{cases} 4x^2-y=2 \\ 3x-2y=-1 \end{cases}$

3. $\begin{cases} 4x+3y=-1 \\ 2x^2=y+11 \end{cases}$

4. $\begin{cases} xy=20 \\ x-4y=2 \end{cases}$

5. $\begin{cases} x^2+y^2-2xy=1 \\ x+y=3 \end{cases}$

6. $\begin{cases} 3x-y=10 \\ x^2-y^2=20-xy \end{cases}$

7. $\begin{cases} x+y=8 \\ x^2+y^2=36 \end{cases}$

8. $\begin{cases} x\cdot y=300 \\ x+y=35 \end{cases}$

9. $\begin{cases} x^2+y^2=74 \\ x+y=12 \end{cases}$

10. $\begin{cases} x+y=8 \\ xy=15 \end{cases}$

11. $\begin{cases} x+y=1 \\ x^3+y^3=19 \end{cases}$

12. $\begin{cases} x^3+8y^3=35 \\ x^2-2xy+4y^2=7 \end{cases}$

13. $\begin{cases} \dfrac{xy}{x+2y}+\dfrac{x+2y}{xy}=2 \\ \dfrac{xy}{x-2y}+\dfrac{x-2y}{xy}=4 \end{cases}$

14. $\begin{cases} x^2-xy+\dfrac{1}{4}y^2+x-\dfrac{1}{2}y=2 \\ \dfrac{1}{4}x^2+xy+y^2+2y+x=3 \end{cases}$

15. $\begin{cases} x^2-xy+y^2=19 \\ x^2+xy+y^2=49 \end{cases}$

16. $\begin{cases} x^2+y^2=x+y \\ x^4+y^4=\dfrac{1}{2}(x+y)^2 \end{cases}$

17. $\begin{cases} \dfrac{x-y}{x+y}+6\dfrac{x+y}{x-y}=5 \\ xy=-2 \end{cases}$

18. $\begin{cases} \dfrac{2x}{y}+\dfrac{3y}{x}+6=\dfrac{3}{xy} \\ \dfrac{6y}{x}+\dfrac{4x}{y}-1=\dfrac{45}{xy} \end{cases}$

19. $\begin{cases} \dfrac{1}{x}+\dfrac{1}{y}=5 \\ \dfrac{1}{x^2}+\dfrac{1}{y^2}=13 \end{cases}$

20. $\begin{cases} \dfrac{x+y}{xy}+\dfrac{xy}{x+y}=2 \\ \dfrac{x-y}{xy}+\dfrac{xy}{x-y}=\dfrac{5}{2} \end{cases}$

21. $\begin{cases} x^3-y^3=61(x-y) \\ (x+1)(y+1)=12 \end{cases}$

22. $\begin{cases} \dfrac{x^2}{y}+\dfrac{y^2}{x}=\dfrac{9}{2} \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{3}{2} \end{cases}$

23. $\begin{cases} x^4+y^4=17(x+y)^2 \\ xy=2(x+y) \end{cases}$

24. $\begin{cases} x^2+y^2=x-y \\ x^4+y^4=\dfrac{1}{2}(x-y)^2 \end{cases}$

25. $\begin{cases} x^2\left(1+y+y^2+y^3\right)=160 \\ x^2\left(1-y+y^2-y^3\right)=-80 \end{cases}$

26. $\begin{cases} 2x^2y^2-3y^2+5xy-6=0 \\ 3x^2y^2-4y^2+3xy-2=0 \end{cases}$

27. $\begin{cases} x^3y+xy^3=\dfrac{10}{9}(x+y)^2 \\ x^4y+xy^4=\dfrac{2}{3}(x+y)^3 \end{cases}$

28. $\begin{cases} \dfrac{x\left(y^2+1\right)}{x^2+y^2}=\dfrac{3}{5} \\ \dfrac{y\left(x^2-1\right)}{x^2+y^2}=\dfrac{4}{5} \end{cases}$

29. $\begin{cases} \dfrac{y^2}{x}+\dfrac{x^2}{y}=12 \\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{1}{3} \end{cases}$

30. $\begin{cases} xy=6 \\ yz=15 \\ zx=10 \end{cases}$

RATSIONAL TENGSIZLIKLAR

Ratsional tengsizliklarni yechish xuddi ratsional tenglamalarni yechish kabi, avval tengsizlikni sodda teng kuchli tengsizlikka keltirish orqali bajariladi. Bunda quyidagi qoidalarga rioya etiladi:

1-qoida. Tengsizlikning ixtiyoriy hadini tengsizlikning bir qismidan ikkinchi qismiga qarama-qarshi ishora bilan o‘tkazish mumkin.

2-qoida. Tengsizlikning ikkala qismini bir xil musbat songa ko‘paytirish yoki bo‘lish mumkin, bunda tengsizlik belgisi o‘zgarmaydi.

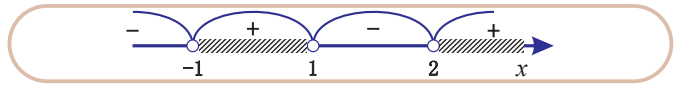
3-qoida. Tengsizlikning ikkala qismini bir xil manfiy songa ko‘paytirish yoki bo‘lish mumkin, bunda tengsizlik belgisi qarama-qarshisiga o‘zgaradi.

Ratsional tengsizlikni yechishda ***oraliqlar usulidan*** foydalaniladi.

1-misol. Tengsizlikni yeching: $(x-1)(x+1)(x-2)>0$.

Yechish. 1. Tengsizlikning o‘ng tarafi nolga teng, demak, chap tarafdagi ifodaning nollarini topamiz: $x=1, x=-1, x=2$.

2. x ning bu qiymatlarini son o‘qida belgilaymiz va hosil bo‘lgan intervallarda chap tarafining ishorasini aniqlaymiz.



3. Tengsizlik belgisi noldan katta bo‘lgani uchun musbat ishorali oraliqlar berilgan tengsizlikning yechimi bo‘ladi.

Javob: $x\in(-1;1)\cup(2;\infty)$

2-misol. Tengsizlikni yeching: $x^4-3<2x\left(2x^2-x-2\right)$.

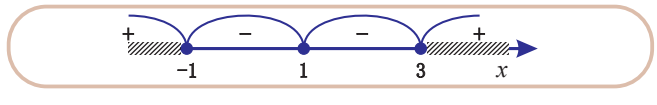
Yechish. 1. Butun ratsional tengsizlik berilgan. Uni yechish uchun dastlab barcha ifodalarni tengsizlikning chap tarafiga o‘tkazamiz.

$$x^4-4x^3+2x^2+4x-3<0$$

2. Chap tarafda hosil bo‘lgan ifodani ko‘paytuvchilarga ajratamiz. Buning uchun uning nollarini topamiz: $x_1=-1,\ x_2=1$ va $x_3=3$.

$$(x-1)^2(x+1)(x-3)\ge 0$$

3. Nollarni son o‘qida belgilaymiz va intervallarda chap tarafdagi ifodaning ishoralarini belgilaymiz.



4. Chap tarafdagi ifodada $(x-1)$ ikkihad ikkinchi (juft) darajada, shuning uchun son o‘qida 1 dan o‘tishda ishora o‘zgarmaydi.

5. Tengsizlikning belgisi noldan katta yoki teng bo‘lgani uchun musbat ishorali oraliqlar va 1 soni tengsizlikning yechimi bo‘ladi.

Javob: $x\in(-\infty;-1]\cup[3;+\infty)\cup\{1\}$

Kasr-ratsional tengsizliklar

$\frac{f(x)}{g(x)}>0$, $\frac{f(x)}{g(x)}<0$, $\frac{f(x)}{g(x)}\geq 0$, $\frac{f(x)}{g(x)}\leq 0$ ko‘rinishga keltirish mumkin bo‘lgan tengsizliklar ***kasr-ratsional tengsizliklar*** deyiladi (bu yerda $f(x)$ va $g(x)$ lar x noma’lumli ko‘phadlar).

Kasr-ratsional tengsizliklarni yechish qadamlari:

- suratining nollari topiladi;
- maxrajining nollari topiladi;
- surat va maxrajning nollari son o‘qida belgilanadi;
- hosil bo‘lgan intervallarda $\frac{f(x)}{g(x)}$ ning ishoralari topiladi;
- tengsizlikni qanoatlantiradigan oraliq(lar) tengsizlikning yechimi bo‘ladi.

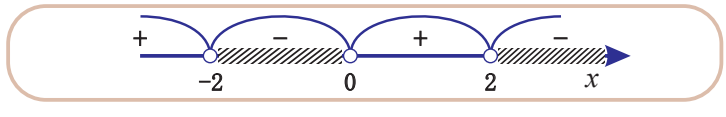
3-misol. Tengsizlikni yeching: $\frac{4}{x}-x<0$.

Yechish

1. Umumiy maxrajga keltiramiz: $\frac{4-x^2}{x}<0$.

2. Suratning nollari $x=2$, $x=-2$, maxrajning noli $x=0$.

3. Nollarni son o‘qida belgilaymiz va intervallarda ishoralarni aniqlaymiz.



Tengsizlik belgisi noldan kichik bo‘lgani uchun manfiy ishorali oraliqlar berilgan tengsizlikning yechimi bo‘ladi.

Javob: $x\in(-2;0)\cup(2;\infty)$.

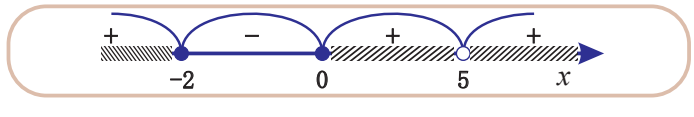
4-misol. Tengsizlikni yeching: $\frac{x(x+2)^3}{(x-5)^2}\geq 0$.

Yechish

1. Suratining nollari $x=0$ va $x=-2$.

2. Maxrajining noli $x=5$.

3. Son o‘qida bu qiymatlarni belgilab chiqamiz va intervallarda ishoralarni aniqlaymiz, bunda chap tarafdagi ifodada $(x-5)$ ifoda ikkinchi (juft) darajada qatnashgan, shuning uchun son o‘qida 5 sonidan ikki tarafda joylashgan intervallar bir xil ishoraga ega.



4. Tengsizlik belgisi noldan katta yoki teng bo‘lgani uchun musbat ishorali oraliqlar berilgan tengsizlikning yechimi bo‘ladi.

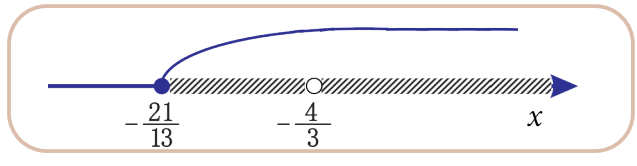
Javob: $x\in(-\infty;-2]\cup[0;5)\cup(5;\infty)$.

Diqqat qiling! $\frac{f(x)}{g(x)} < a$ ko‘rinishdagi tengsizliklarni yechishda tengsizlikning ikkala tarafini $g(x) \neq 0$ deb hisoblab, $g(x)$ ga ko‘paytirish noto‘g‘ri javobga olib kelishi mumkin. Chunki $g(x)$ ning musbat yoki manfiy ekanligi aniq emas.

Masalan, $\frac{2x-1}{3x+4} \le 5 \quad \Big| \cdot (3x+4) \neq 0$

$$\begin{cases} \frac{2x-1}{3x+4} \cdot (3x+4) \le 5 \cdot (3x+4) \\ 3x \neq -4 \end{cases} \qquad \begin{cases} 2x-1 \le 15x+20 \\ x \neq -\frac{4}{3} \end{cases} \qquad \begin{cases} x \ge -\frac{21}{13} \\ x \neq -\frac{4}{3} \end{cases}$$

Bu sistemani son o‘qida tasvirlaymiz:



Ko‘rinib turibdiki, tengsizlikning hosil bo‘lgan yechimi $\left[-\frac{21}{13};-\frac{4}{3}\right) \cup \left(-\frac{4}{3};\infty\right)$ bo‘ladi degan noto‘g‘ri xulosaga kelamiz.

To‘g‘ri javobni aniqlash uchun bu tengsizlikni qadamma-qadam mustaqil ravishda ishlab chiqing va nima uchun bunday usul to‘g‘ri javob bermagani haqida mulohaza yuriting.

MISOLLAR

Tengsizliklarni yeching.

1. $\frac{x+4}{(x+5)x} < 0$

2. $\frac{x-4}{(x-3)x} < 0$

3. $\frac{5+4x}{(x-2)(x+1)} \ge 0$

4. $\frac{4-3x}{(x+2)(x-1)} \ge 0$

5. $\frac{4x+3}{x+2} > 5$

6. $\frac{4x-3}{x-5} > 5$

7. $\frac{25-16x^2}{x^2+4x+4} > 0$

8. $\frac{16-25x^2}{x^2-4x+4} > 0$

9. $\frac{2x-7}{6} + \frac{7x-2}{3} < 3 - \frac{1-x}{2}$ tengsizlikning butun sonlardan iborat yechimlaridan eng kattasini ko‘rsating.

10. $\frac{x-4}{2x+6} \le 0$ tengsizlikning barcha butun sonlardagi yechimlari yig‘indisini toping.

11. $\frac{1}{x} < 1$ tengsizlikning $(-3; 3)$ oraliqdagi butun yechimlari sonini toping.

12. $\frac{(x+3)(x-5)}{x+1} \geq 0$ tengsizlikning manfiy butun sonlardan iborat yechimlaridan eng kattasidan eng kichigining ayirmasini toping.

13. $\frac{(x+4)^2 - 8x - 25}{(x-6)^2} \geq 0$ tengsizlikning butun sonlardan iborat yechimlaridan nechtasi $[-5; 6]$ kesmada joylashgan?

14. $\frac{6x-1}{4x+3} \leq \frac{3x-2}{2x-1}$

15. $\frac{5}{-6x+3} + \frac{6x}{1-2x} \geq 0$

16. $\frac{x^2+3x}{49x^2+70x+25} \leq 0$

17. $\frac{6x+1}{4x-3} \leq \frac{3x+2}{2x+1}$

18. $\frac{6}{-4x+2} - \frac{5x}{1-2x} \leq 0$

19. $\frac{49x^2-70x+25}{x^2-3x} \leq 0$

20. $\frac{x^2+3x-2}{(x-1)^2-9} - \frac{3x+1}{3x-12} \leq 0$

21. $\frac{x^2+7x+8}{(x+1)^2-9} - \frac{3x+7}{3x-6} \leq 0$

22. $\frac{1}{2x^2-5x} - \frac{2}{25+10x} + \frac{4}{25-4x^2} \geq 0$

23. $\frac{6}{-4x-x^2} - \frac{2}{x^2-4x} + \frac{x}{x^2-16} \geq 0$

24. $\left(\frac{4}{x^2+4x} + \frac{32-3x}{x^3+64}\right) : \frac{x+8}{x^3-4x^2+16x} \geq \frac{4}{4+x}$

25. $\left(\frac{x^2+2x+4}{4x^2-1} \cdot \frac{2x^2-x}{-x^3+8} - \frac{2-x}{2x^2+x}\right) : \frac{4}{x^2-2x} \geq \frac{4-x}{x+2x^2}$

RATSIONAL TENGSIZLIKLAR SISTEMASI

Tengsizliklar sistemasini yechish qadamlari:

- har bir tengsizlikning yechimi alohida topiladi;
- ikkala tengsizlik uchun umumiy yechim topiladi (bu qadam son o‘qida tasvirlash orqali bajarilishi mumkin).

1-misol. Tengsizliklar sistemasini yeching: $\begin{cases} x^2-9\ge 0 \\ 2x-8<0 \end{cases}$

Yechish

$$\begin{cases} x^2-9\ge 0 \\ 2x-8<0 \end{cases} \Rightarrow \begin{cases} (x+3)(x-3)\ge 0 \\ x<4 \end{cases} \Rightarrow \begin{cases} x\in(-\infty;-3]\cup[3;\infty) \\ x<4 \end{cases} \Rightarrow x\in(-\infty;-3]\cup[3;4)$$

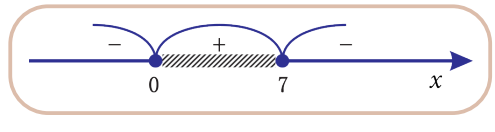
Javob: $x\in(-\infty;-3]\cup[3;4)$.

2-misol. Tengsizliklar sistemasini yeching: $\begin{cases} 7x-x^2\ge 0 \\ x^2-6x+5<0 \end{cases}$

Yechish

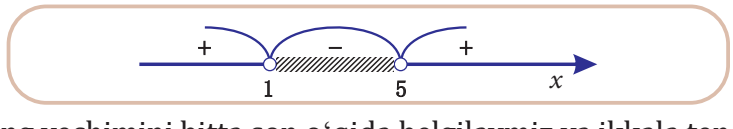
1-tengsizlikni yechamiz: $x(7-x)\ge 0$.

$x=0$ va $x=7$ nollarini son o‘qida belgilaymiz va hosil bo‘lgan intervallarda ishoralarni aniqlaymiz.

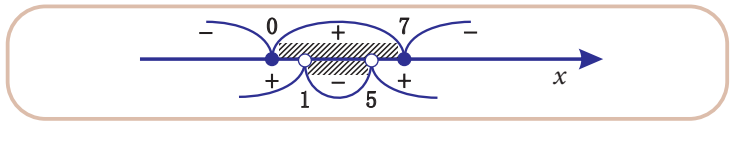


2-tengsizlikni yechamiz: $x^2-6x+5<0$.

Nollari $x=1$ va $x=5$ ga teng. Ularni son o‘qida belgilaymiz va hosil bo‘lgan intervallarda ishoralarni aniqlaymiz.



Ikkala tengsizlikning yechimini bitta son o‘qida belgilaymiz va ikkala tengsizlikni ham qanoatlantiradigan oraliq sistemaning yechimi bo‘ladi.



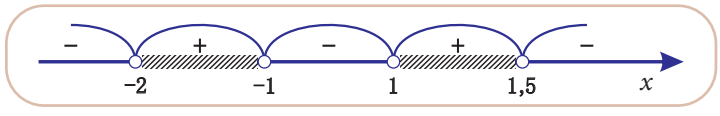
Javob: $x\in(1;5)$.

3-misol. Tengsizliklar sistemasini yeching. $\begin{cases} \dfrac{(3-2x)(x+2)}{x^2-1}>0 \\ 1+2x\le\dfrac{3}{x} \end{cases}$

1-tengsizlikni yechamiz:

$$\frac{(3-2x)(x+2)}{x^2-1}>0$$

Surat va maxrajning $x=-2, x=-1, x=1,\ x=1{,}5$ nollarini son o‘qida belgilaymiz. Hosil bo‘lgan intervallarda ishoralarni aniqlaymiz.

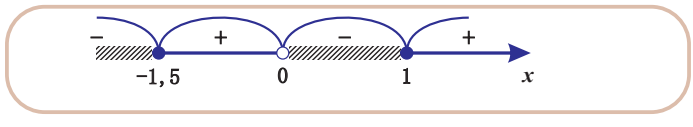


2-tengsizlikni yechamiz: $1+2x\le\frac{3}{x}$.

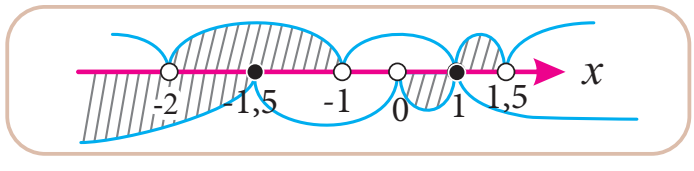
Barcha ifodalarni tengsizlikning chap tarafiga o‘tkazib olamiz va umumiy maxrajga keltiramiz.

$$\frac{2x^2+x-3}{x}\le 0$$

Kasrning nollari $x=1$ va $x=-1{,}5$ ga teng, aniqlanish sohasi $x\neq 0$ bo‘ladigan qiymatlardan iborat. Ularni son o‘qida belgilaymiz va hosil bo‘lgan intervallarda ishoralarni aniqlaymiz.



Ikkala tengsizlikning yechimini bitta son o‘qida belgilaymiz va ikkala tengsizlikni ham qanoatlantiradigan oraliq sistemaning yechimi bo‘ladi.



Javob: $x\in(-2;-1{,}5]$.

MISOLLAR

1. Tengsizliklar sistemasining yechimi bo‘lgan barcha butun sonlarni toping.

a) $\begin{cases}0{,}2x>-1\\ -\frac{x}{3}\ge 1\end{cases}$ b) $\begin{cases}\frac{x-1}{2}<\frac{x}{3}\\ \frac{x+1}{2}\ge\frac{x}{5}\end{cases}$ c) $\begin{cases}1-\frac{x}{4}>x\\ x-\frac{x-4}{5}>1\end{cases}$ d) $\begin{cases}x-\frac{x}{4}\ge 2\\ \frac{x-1}{2}+\frac{x-2}{3}>1\end{cases}$

2. Tengsizliklar sistemasini yeching.

a) $\begin{cases}\frac{x-1}{2}-\frac{x-2}{3}\ge\frac{x-3}{4}-x\\ 1-x>0{,}5x-4\end{cases}$ b) $\begin{cases}\frac{5x+7}{6}-\frac{3x}{4}<\frac{11x-7}{12}\\ \frac{1-3x}{2}-\frac{1-4x}{3}\ge\frac{x}{6}-1\end{cases}$

c) $\begin{cases}\frac{2x-1}{6}+\frac{x+2}{3}-\frac{x-8}{2}>x-1\\ 2-2x>0{,}5+0{,}5\end{cases}$ d) $\begin{cases}\frac{8x+1}{3}>\frac{4x+9}{2}-\frac{x-1}{3}\\ \frac{5x-2}{3}<\frac{2x+13}{2}-\frac{x+2}{3}\end{cases}$

3. Ifodaning aniqlanish sohasini toping.

a) $\sqrt{(x-3)(x-5)}+\sqrt{(1-x)(7-x)}$ b) $\sqrt{\frac{3x+2}{5-x}}+\sqrt{\frac{4-x}{7-2x}}$

c) $\sqrt{(x-2)(x-3)}+\sqrt{(5-x)(6-x)}$ d) $\sqrt{\frac{4x+1}{x+2}}+\sqrt{\frac{2x+1}{x-7}}$

4. Funksiyaning aniqlanish sohasini toping.

a) $y=\sqrt{12-3x}+\sqrt{x+2}$ b) $y=\frac{\sqrt{3-5x-2x^2}}{10x}$ c) $y=\sqrt{15-3x}+\sqrt{4+x}$ d) $y=\frac{\sqrt{-3x^2+12}}{1-5x}$

5. Tengsizliklar sistemasini yeching.

a) $\begin{cases}\frac{2x+1}{x-2}<1\\ \frac{3x+2}{2x-3}>2\end{cases}$ b) $\begin{cases}\frac{7-3x}{2-5x}\le 2\\ \frac{2x+1}{3x-3}>4\end{cases}$ c) $\begin{cases}\frac{3x-2}{x-2}<2\\ \frac{5x+1}{4x-5}\ge 3\end{cases}$ d) $\begin{cases}\frac{x+3}{3x-1}\le 1\\ \frac{2x+5}{x-4}\ge 2\end{cases}$

6. Tengsizliklar sistemasini yeching.

a) $\begin{cases}x^2\le 9\\ x>0\end{cases}$ b) $\begin{cases}3x^2+7x-6\le 0\\ 6(x+4)-3(4-3x)<2\end{cases}$ c) $\begin{cases}5x^2-2x+1\le 0\\ 2(x+3)-(x-8)<4\end{cases}$

d) $\begin{cases}-2x^2+3x-2<0\\ -3(6x-1)-2x<x\end{cases}$ e) $\begin{cases}12(x+2)-5(5-4x)<2\\ 9x^2-6x-8\le 0\end{cases}$ f) $\begin{cases}3x-1<0\\ x^2-3x+2\ge 0\end{cases}$

7. Tengsizliklar sistemasini qanoatlantiruvchi butun sonlar yig‘indisini toping.

a) $\begin{cases}\frac{9-x^2}{x}\ge 0\\ 2x-1\ge 0\end{cases}$ b) $\begin{cases}\frac{(x+5)(x-1)}{x}\ge 0\\ 10x-1<0\end{cases}$ c) $\begin{cases}\frac{(x-2)(x+3)}{x(x+7)}<0\\ 20x\ge 20\end{cases}$ d) $\begin{cases}\frac{25-x^2}{x}\le 0\\ 5x-10\le 35\end{cases}$

8. Tengsizliklar sistemasini yeching.

a) $\begin{cases}3x^2-5x-2<0\\ 4-x^2>0\end{cases}$ b) $\begin{cases}3x^2+x+2>0\\ x^2<9\end{cases}$ c) $\begin{cases}2x^2+5x+10>0\\ x^2\ge 16\end{cases}$ d) $\begin{cases}-7x^2+5x-2>0\\ x^2\le 25\end{cases}$

e) $\begin{cases}\frac{x+7}{x-5}+\frac{3x+1}{2}\ge 0\\ (5-x)^2\le 4\end{cases}$ f) $\begin{cases}-5x^2+x-1>0\\ x^2>81\end{cases}$ g) $\begin{cases}x^2-6x+8<0\\ x^2-36\ge 0\end{cases}$ h) $\begin{cases}x^2-16\ge 0\\ x^2-7x+12\ge 0\end{cases}$

i) $\begin{cases}x^2-5x+4\ge 0\\ 2x^2-5x+2\le 0\end{cases}$ j) $\begin{cases}x^2-9x+14<0\\ x^2-7x-8\le 0\end{cases}$ k) $\begin{cases}x^2+4x+3\le 0\\ 2x^2+5x<0\end{cases}$ l) $\begin{cases}\frac{(x-3)^2}{(x-3)(x+1)}\ge 0\\ (x-4)(x+4)\le 0\end{cases}$

IRRATSIONAL TENGLAMALAR

$\sqrt{2x-5}=7$, $2\sqrt{x}+5=8$, $\sqrt[3]{x+3}=-1-x$ tenglamalarda noma'lum son ildiz belgisi ostida qatnashgan. Bu kabi tenglamalar ***irratsional tenglamalar*** deyiladi.

Ushbu $\sqrt{2+\sqrt{x-5}}=\sqrt{13-x}$, $\sqrt[5]{(x+1)^2}-\sqrt[5]{(x-1)^2}=\sqrt[5]{x^2-1}$ tenglamalar ham irratsional tenglamalardir.

Ko'p hollarda irratsional tenglamalar o'zining natijasi bo'lgan ratsional tenglamalarga keltirib yechiladi. Bunda **quyidagi qadamlar bajariladi:**

- irratsional tenglamani ratsional tenglamaga keltirish uchun berilgan tenglamaning ikki tarafini bir yoki bir necha marta biror natural darajaga ko'tariladi;
- hosil bo'lgan ratsional tenglamaning ildizlari topiladi va berilgan tenglamani qanoatlantirishi tekshiriladi.

Buni quyidagi teorema tasdiqlaydi.

Teorema. $f_1(x)=f_2(x)$ tenglamaning ikki tomonini kvadratga ko'tarishdan hosil bo'lgan $f_1^2(x)=f_2^2(x)$ tenglamaning ildizlari $f_1(x)=f_2(x)$ va $f_1(x)=-f_2(x)$ tenglamalarning ildizlaridan iborat bo'ladi.

Bu teorema $f_1(x)=f_2(x)$ tenglamadan $f_1^2(x)=f_2^2(x)$ tenglamaga o'tishda ildizlar yo'qolishi ro'y bermasdan, balki chet ildizlar hosil bo'lishi mumkinligini ko'rsatadi.

Irratsional tenglamada birgina ildiz belgisi qatnashsa, bu ildiz belgisini tenglamaning bir tomonida qoldirib, tenglamaning qolgan hadlarini ikkinchi tomonga o'tkazamiz. Keyin esa tenglamaning ikki tomonini tenglama ildiz belgisidan qutuladigan qilib biror darajaga ko'taramiz. Natijada ratsional tenglama hosil bo'ladi. Bu hosil bo'lgan tenglamani yechib, uning ildizlarini berilgan irratsional tenglamaga qo'yib, tekshirib ko'rish kerak. Agar topilgan ildizlardan birortasi berilgan tenglamani qanoatlantirmasa, u chet ildiz hisoblanadi.

1-misol. $\sqrt{2x-1}=5$ tenglamani yeching.

Yechish. Tenglamaning aniqlanish sohasi $2x-1\ge 0 \Rightarrow x\ge\frac{1}{2}$

Tenglamaning har ikki tarafini kvadratga oshiramiz. $\left(\sqrt{2x-1}\right)^2=5^2$

$2x-1=25$ tenglama hosil bo'ladi. Bundan $x=13$ ekani kelib chiqadi.

Tekshirish: $\sqrt{2\cdot 13-1}=\sqrt{25}=5$.

Javob: $x=13$.

2-misol. $\sqrt{x^2-x-2}=x-3$ tenglamani yeching.

Yechish. $\sqrt{x^2-x-2}=x-3$ tenglamaning ikki tomonini kvadratga ko'taramiz: $x^2-x-2=x^2-6x+9 \Rightarrow 5x=11 \Rightarrow x=2,2$.

Tekshirish: $\sqrt{2,2^2-2,2-2}=2,2-3$, $\sqrt{0,64}=-0,8$; $0.8\ne -0,8$. Demak, $x=2,2$ chet ildiz, tenglama yechimga ega emas.

Javob: $\varnothing$.

Irratsional tenglamalarni yechish:

I. $\sqrt{f(x)} = g(x)$ ko‘rinishidagi tenglamani unga teng kuchli bo‘lgan $\begin{cases} g(x) \geq 0 \\ f(x) = g^2(x) \end{cases}$ sistemaga keltirib yechish mumkin.

3-misol. $\sqrt{3x^2 - 6x + 16} = 2x - 1$ tenglamani yeching.

Yechish

$$\sqrt{3x^2 - 6x + 16} = 2x - 1 \Rightarrow \begin{cases} 3x^2 - 6x + 16 = (2x-1)^2 \\ 2x - 1 \geq 0 \end{cases} \Rightarrow$$

$$\begin{cases} 3x^2 - 6x + 16 = 4x^2 - 4x + 1 \\ 2x \geq 1 \end{cases} \Rightarrow \begin{cases} x^2 + 2x - 15 = 0 \\ x \geq \frac{1}{2} \end{cases}$$

$x^2 + 2x - 15 = 0$ tenglamani yechamiz. $x_{1,2} = \frac{-2 \pm \sqrt{4+60}}{2} = \frac{-2 \pm 8}{2}$, $\Rightarrow x_1 = 3,\ x_2 = -5$.

$x \geq \frac{1}{2}$ bo‘lgani sababli tenglamaning yechimi $x = 3$.

Javob: $x = 3$.

II. $\sqrt{f(x)} \cdot g(x) = 0$ ko‘rinishidagi tenglama quyidagicha yechiladi.

1-qadam: $g(x) = 0$ va $f(x) \geq 0$ 2-qadam: $f(x) = 0$

4-misol. Tenglamani yeching: $(x^2 - 25)\sqrt{6 - 2x} = 0$.

Yechish

1-qadam: $\begin{cases} x^2 - 25 = 0 \\ 6 - 2x > 0 \end{cases} \Rightarrow \begin{cases} x_{1,2} = \pm 5 \\ x < 3 \end{cases} \Rightarrow x = -5$

2-qadam: $6 - 2x = 0 \Rightarrow x = 3$

Javob: $x_1 = -5;\ x_2 = 3$.

III. $\sqrt{f(x)} = \sqrt{g(x)}$ ko‘rinishidagi tenglama quyidagicha yechiladi:

$\begin{cases} f(x) = g(x), \\ f(x) \geq 0 \end{cases}$ yoki $\begin{cases} f(x) = g(x), \\ g(x) \geq 0 \end{cases}$

5-misol. Tenglamani yeching: $\sqrt{x+1} = \sqrt{2x-3}$.

Yechish

$$\begin{cases} x + 1 = 2x - 3 \\ 2x - 3 \geq 0 \end{cases} \Rightarrow \begin{cases} x = 4 \\ x \geq 1,5 \end{cases} \Rightarrow x = 4$$

Javob: $x = 4$.

6-misol. Tenglamani yeching: $\sqrt{x^2+4x}=\sqrt{14-x}$.

Yechish

Tenglamaning ikkala tarafini kvadratga oshiramiz. $\left(\sqrt{x^2+4x}\right)^2=\left(\sqrt{14-x}\right)^2$

$x^2+4x=14-x$ bundan $x_1=2;\ \ x_2=-7$ ekanini topamiz.

Tekshirish bu sonlarning ildiz bo‘lishini ko‘rsatadi.

Javob: $x_1=2;\ \ x_2=-7$.

IV. $\sqrt{f(x)}\cdot\sqrt{g(x)}=0$ ko‘rinishidagi tenglamani o‘ziga teng kuchli ikkita sistemalarga keltirib yechish mumkin:

$$\begin{cases} f(x)=0 \\ g(x)>0 \end{cases} \quad \text{va} \quad \begin{cases} g(x)=0 \\ f(x)>0 \end{cases}$$

Ba’zi hollarda tenglamaning aniqlanish sohasini bilish tenglamaning yechimi mavjud yoki mavjud emasligini aniqlashga yoki yechimini topishga yordam beradi.

7-misol. Tenglamani yeching: $\sqrt{x^2-4}\cdot\sqrt{x+5}=0$.

1) $\begin{cases} x^2-4=0 \\ x+5\ge 0 \end{cases} \Rightarrow \begin{cases} x_{1,2}=\pm 2 \\ x\ge -5 \end{cases} \Rightarrow x_1=-2;\ x_2=2$

2) $\begin{cases} x+5=0 \\ x^2-4\ge 0 \end{cases} \Rightarrow x=-5$

Javob: $x_1=2; x_2=-2; x_3=-5$.

8-misol. Tenglamani yeching: $\sqrt{1-x^2}\cdot\sqrt{x^2-9}=0$.

Yechish

Tenglamaning aniqlanish sohasini topamiz.

$\begin{cases} 1-x^2\ge 0, \\ x^2-9\ge 0 \end{cases} \Rightarrow \begin{cases} x^2\le 1, \\ x^2\ge 9 \end{cases} \Rightarrow \begin{cases} -1\le x\le 1, \\ x\le -3,\ x\ge 3 \end{cases} \Rightarrow \varnothing$

Tenglamaning aniqlanish sohasi bo‘sh to‘plam bo‘lgani sababli tenglama yechimga ega emas.
Javob: $\varnothing$.

9-misol. $\sqrt{3x+7}-\sqrt{x+1}=2$ tenglamani yeching.

Yechish

Tenglamaning ikki tarafini kvadratga ko‘taramiz.

$$\left(\sqrt{3x+7}-\sqrt{x+1}\right)^2=2^2$$

$$3x+7-2\sqrt{(3x+7)(x+1)}+x+1=4,$$

$$\sqrt{(3x+7)(x+1)} = 2x+2$$

$\sqrt{(3x+7)(x+1)} = 2x+2$ tenglamani ikkala tarafini kvadratga ko‘tarsak,

$(3x+7)(x+1) = 4x^2+8x+4$ tenglama hosil bo‘ladi. Bundan $x^2-2x-3=0$ kelib chiqadi.

Bu tenglamaning ildizlari $x_1=-1,\ x_2=3$.

Tekshirish: $x=-1$ da $\sqrt{3(-1)+7} - \sqrt{-1+1} = 2-0=2$.

$x=3$ da $\sqrt{3\cdot 3+7} - \sqrt{3+1} = 4-2=2$.

Ikkala ildiz ham berilgan tenglamani qanoatlantiradi.

Javob: $x_1=-1; x_2=3$.

10-misol. Tenglamani yeching: $\sqrt{3-2x}+\sqrt{x-7}=5$.

Yechish

Tenglamaning aniqlanish sohasini topamiz.

$$\begin{cases} 3-2x\ge 0 \\ x-7\ge 0 \end{cases} \Rightarrow \begin{cases} x\le 1,5 \\ x\ge 7 \end{cases} \Rightarrow x\in\varnothing$$

Aniqlanish sohasi bo‘sh to‘plamdan iborat bo‘lgani uchun tenglamaning ildizi yo‘q.

Javob: Ø.

11-misol. $\sqrt[5]{25+\sqrt{x+13}}-2=0$ tenglamani yeching.

Yechish

$\sqrt[5]{25+\sqrt{x+13}}=2 \Rightarrow 25+\sqrt{x+13}=2^5 \Rightarrow \sqrt{x+13}=7$

$\sqrt{x+13}=7,\ x+13=7^2,\ x=49-13=36$

Tekshirish: $\sqrt[5]{25+\sqrt{36+13}}=\sqrt[5]{25+\sqrt{49}}=\sqrt[5]{25+7}=\sqrt[5]{32}=2$

Javob: $x=36$.

12-misol. $\sqrt{\dfrac{3x+2}{x}}+\sqrt{\dfrac{x}{3x+2}}=\dfrac{5}{2}$ tenglamani yeching.

Yechish.

1. $\sqrt{\dfrac{3x+2}{x}}=a$ belgilash kiritsak, $\sqrt{\dfrac{x}{3x+2}}=\dfrac{1}{a}$ bo‘lib, tenglama $a+\dfrac{1}{a}=\dfrac{5}{2}$ ko‘rinishga keladi. Bu tenglamani yechib, $a_1=2$ va $a_2=\dfrac{1}{2}$ larni topamiz.

2. $\sqrt{\dfrac{3x+2}{x}}=a$ almashtirishdan foydalansak $x_1=2$ va $x_2=-\dfrac{8}{11}$ kelib chiqadi. Demak, tenglamaning ildizlari $x_1=2$ va $x_2=-\dfrac{8}{11}$.

Javob: $x_1=2$ va $x_2=-\dfrac{8}{11}$.

13-misol. $\sqrt[3]{x^3+4x^2+3x-3}=x+1$ tenglamani yeching.

Yechish

$\sqrt[3]{x^3+4x^2+3x-3}=x+1 \Rightarrow x^3+4x^2+3x-3=(x+1)^3 \Rightarrow$

$\Rightarrow x^3+4x^2+3x-3=x^3+3x^2+3x+1 \Rightarrow x^2-4=0.$

$x^2=4,\ x_{1,2}=\pm 2.$

Tekshirish. $x=2$ da $\sqrt[3]{2^3+4\cdot 2^2+3\cdot 2-3}=2+1,\ \sqrt[3]{27}=3.$

$x=-2$ da $\sqrt[3]{(-2)^3+4\cdot(-2)^2+3\cdot(-2)-3}=-2+1,\ \sqrt[3]{-1}=-1.$

Javob: $x=\pm 2.$

14-misol. $\sqrt{x^2-3x+5}+x^2=3x+7$ tenglamani yeching.

Yechish

$\sqrt{x^2-3x+5}+x^2=3x+7 \Rightarrow \sqrt{x^2-3x+5}+x^2-3x+5-12=0$

$\sqrt{x^2-3x+5}=a$ belgilash kiritsak, $a^2+a-12=0$ kvadrat tenglama hosil bo‘ladi.

$a^2+a-12=0$ tenglamani yechsak, $a_1=3;\ a_2=-4.$

$a=3$ da $\sqrt{x^2-3x+5}=3,\ x^2-3x+5=9,\ x^2-3x-4=0$ tenglamani yechamiz: $x_1=4; x_1=-1.$

$a=-4\notin[0;\infty)$ bo‘lgani uchun $\sqrt{x^2-3x+5}=-4$ tenglama yechimga ega emas.

$x_1=4; x_1=-1$ tenglamaning yechimlari.

Javob: $x_1=4; x_2=-1.$

Tenglamaning aniqlanish sohasi deb noma’lumning shunday qiymatlari to‘plamiga aytiladiki, bu qiymatlarda tenglamaning chap va o‘ng taraflari ma’noga ega bo‘ladi. Irratsional tenglamani aniqlanish sohasini topmasdan ham to‘g‘ri yechish mumkin. Buning uchun tekshirish kifoya. Ba’zi tenglamalarda aniqlanish sohasini topish foydali.

Masalan:

1) $\sqrt{x^3+4x-1-8\sqrt{x^4-x}}=\sqrt{x^3-1}+2\sqrt{x}$ tenglamaning aniqlanish sohasini topish yetarlicha murakkab va foydasiz (*ko‘rsatma: tenglamaning o‘ng va chap tarafini kvadratga oshiring*).

2) $\sqrt{x^2-x}+\sqrt{2-x-x^2}=\sqrt{x}-1$ tenglamani yechish uchun uning aniqlanish sohasini topish kifoya qilishini mustaqil tekshiring.

V. $\sqrt{f^2(x)}=f(x)$ tenglama $f(x)\geq 0$ tengsizlikka teng kuchli,

$\sqrt{f^2(x)}=-f(x)$ tenglama esa $f(x)\leq 0$ tengsizlikka teng kuchli.

MISOLLAR

Tenglamalarni yeching.

1. $\sqrt{5x+2}=10$
2. $\sqrt{4x-6}=12$
3. $\sqrt{10-2x}=4$
4. $\sqrt{-12+7x}=x$
5. $\sqrt{x+12}+x=0$
6. $\sqrt{4+3x}=-x$
7. $x-3=\sqrt{9-x}$
8. $-x=\sqrt{15-2x}$
9. $x-6=\sqrt{8-x}$
10. $\sqrt{\dfrac{3x-17}{7}}=4$
11. $\sqrt{\dfrac{11}{6-4x}}=\dfrac{1}{2}$
12. $\sqrt{\dfrac{4}{5x-2}}=1$
13. $\sqrt{5x-3}=\sqrt{2x}$
14. $\sqrt{4-2x}=2\sqrt{x-1}$
15. $\sqrt{x^2-3x+1}=\sqrt{2x-5}$
16. $3x+2\sqrt{2x^2+3x-5}=12$
17. $3+\sqrt{3x^2-8x+14}=2x$
18. $\sqrt{15x^2-7x+8}=4x$
19. $\sqrt{x^2+x}=2-x$
20. $\left(x^2-25\right)\sqrt{6-2x}=0$
21. $\left(4-x^2\right)\sqrt{-1-3x}=0$
22. $\left(x^2-16\right)(x-3)(x-6)\sqrt{5-x}=0$
23. $\left(x^2-9x+14\right)\sqrt{x^2-9}=0$
24. $(x-4)\cdot\sqrt{3+2x-x^2}=0$
25. $\sqrt{5x+4}-\sqrt{x+3}=1$
26. $\sqrt{x-2}+\sqrt{1-x}=2$
27. $\sqrt{x-13}+\sqrt{10-x}=4$
28. $\sqrt{(2x-3)^2}=2x-3$
29. $\sqrt{3x-2}+\sqrt{x-1}=3$
30. $2\sqrt{x-2}+2=\sqrt{3x+1}$
31. $\sqrt{x^2+77}-2\sqrt[4]{x^2+77}-3=0$
32. $\sqrt{x}+\sqrt[4]{x}=12$
33. $\sqrt{x}+\sqrt{x-2}=1-x$
34. $\sqrt{x^2+32}=2\sqrt[4]{x^2+32}+3$
35. $x^2+5x+4-5\sqrt{x^2+5x+28}=0$
36. $x^2+\sqrt{x^2+2x+8}=12-2x$
37. $\sqrt{x-2}+\sqrt{x-1}=\sqrt{3x-5}$
38. $\sqrt{x+2}+\sqrt{x+5}=\sqrt{2x+11}$
39. $\sqrt[3]{2-x}=1-\sqrt{x-1}$
40. $\sqrt[3]{7-x}=\sqrt{3-x}$

IRRATSIONAL TENGLAMALAR SISTEMASI

Irratsional tenglamalar sistemasini yechishda turli usullar qo‘llanadi: ko‘paytuvchilarga ajratish, belgilash, algebraik qo‘shish, o‘zgaruvchilarni almashtirish va boshqalar.

1-misol. $\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{xy}=7\end{cases}$ tenglamalar sistemasini yeching.

Yechish

Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\ge 0,\ y\ge 0.$

$\sqrt{x}=a, \sqrt{y}=b$ belgilash kiritamiz.

$$\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{xy}=7\end{cases}\Leftrightarrow\begin{cases}a+b=8\\ ab=7\end{cases}\Rightarrow\begin{cases}a=8-b\\ (8-b)b=7\end{cases}\Rightarrow\begin{cases}a=8-b\\ b^2-8b+7=0\end{cases}$$

$b^2-8b+7=0$ tenglamani yechamiz, $b_{1,2}=\dfrac{8\pm\sqrt{64-28}}{2}=\dfrac{8\pm 6}{2},\Rightarrow b_1=7,\ b_2=1.$

$a_1=8-b_1=8-7=1,\ \Rightarrow\ a_1=1.$

$a_2=8-b_2=8-1=7,\ \Rightarrow\ a_2=7.$

$a_1=1,\ b_1=7$ da $\sqrt{x}=1,\ \sqrt{y}=7.\Rightarrow x=1,\ y=49.$

$a_2=7,\ b_2=1$ da $\sqrt{x}=7,\ \sqrt{y}=1.\Rightarrow x=49,\ y=1.$

Tekshirish: $x=1,\ y=49$ da

$$\begin{cases}\sqrt{1}+\sqrt{49}=8\\ \sqrt{49}=7\end{cases}\Rightarrow\begin{cases}1+7=8\\ 7=7\end{cases}$$

$x=49,\ y=1$ da

$$\begin{cases}\sqrt{49}+\sqrt{1}=8\\ \sqrt{49}=7\end{cases}\Rightarrow\begin{cases}7+1=8\\ 7=7\end{cases}$$

Javob: $(1;\ 49),\ (49;\ 1).$

2-misol. $\begin{cases}x-y=21\\ \sqrt{x}-\sqrt{y}=3\end{cases}$ tenglamalar sistemasini yeching.

Yechish

Tenglamalar sistemasining aniqlanish sohasini topamiz: $x\ge 0,\ y\ge 0.$

$x-y=(\sqrt{x}-\sqrt{y})\left(\sqrt{x}+\sqrt{y}\right)$ qisqa ko‘paytirish formuladan foydalanib yechamiz.

$$\begin{cases} x-y=21 \\ \sqrt{x}-\sqrt{y}=3 \end{cases} \Rightarrow \begin{cases} \left(\sqrt{x}-\sqrt{y}\right)\left(\sqrt{x}+\sqrt{y}\right)=21 \\ \sqrt{x}-\sqrt{y}=3 \end{cases} \Rightarrow \begin{cases} 3\left(\sqrt{x}+\sqrt{y}\right)=21 \\ \sqrt{x}-\sqrt{y}=3 \end{cases} \Rightarrow$$

$$+\begin{cases} \sqrt{x}+\sqrt{y}=7 \\ \sqrt{x}-\sqrt{y}=3 \end{cases} \Rightarrow \begin{cases} \sqrt{x}=5 \\ \sqrt{y}=2 \end{cases} \Rightarrow \begin{cases} x=25 \\ y=4. \end{cases}$$

Tekshirish: $x=25,\ y=4$ da $\begin{cases} 25-4=21 \\ \sqrt{25}-\sqrt{4}=3 \end{cases} \Rightarrow \begin{cases} 21=21 \\ 5-2=3 \end{cases}$

Javob: (25; 4).

3-misol. $\begin{cases} x+y+\sqrt{x+y}=20 \\ x^2+y^2=136 \end{cases}$ tenglamalar sistemasini yeching.

Yechish

Dastlab sistemadagi birinchi tenglamani yechib olamiz.

$x+y+\sqrt{x+y}=20$, $\sqrt{x+y}=a$ belgilash kiritsak, $a^2+a-20=0$ kvadrat tenglama hosil bo‘ladi. $\sqrt{x+y}\geq 0$ bo‘lgani uchun $a\geq 0$ bo‘ladi. $a\in[0;\infty)$.

$a^2+a-20=0$ tenglamani yechamiz, $a_{1,2}=\dfrac{-1\pm\sqrt{1+80}}{2}=\dfrac{-1\pm 9}{2}$, $a_1=4,\ a_2=-5$.

$4\in[0;\infty)$, $-5\notin[0;\infty)$. Demak, $\sqrt{x+y}=4$. $x+y=16$.

$$\begin{cases} x+y+\sqrt{x+y}=20 \\ x^2+y^2=136 \end{cases} \Rightarrow \begin{cases} x+y=16 \\ x^2+y^2=136 \end{cases} \Rightarrow \begin{cases} x=16-y \\ (16-y)^2+y^2=136 \end{cases}$$

$(16-y)^2+y^2=136 \Rightarrow y^2-16y+60=0$ tenglamani yechamiz.

$y_{1,2}=\dfrac{16\pm\sqrt{256-240}}{2}=\dfrac{16\pm 4}{2}$, $\Rightarrow y_1=10,\ y_2=6$.

$y_1=10,\ y_2=6$ bo‘lsa, $x+y=16$ dan $x_1=6,\ x_2=10$ kelib chiqadi. (10; 6) va (6; 10) sistemaning yechimi.

Tekshirish: $\begin{cases} 10+6+\sqrt{10+6}=16+4=20 \\ 10^2+6^2=100+36=136 \end{cases}$

Javob: (10; 6), (6; 10).

4-misol. $\begin{cases} x+y=28 \\ \sqrt[3]{x}+\sqrt[3]{y}=4 \end{cases}$ tenglamalar sistemasini yeching.

Yechish

Tenglamalar sistemasining aniqlanish sohasini topamiz: $x \in R, y \in R$.

$\sqrt[3]{x} = a,\ \sqrt[3]{y} = b$ belgilash kiritamiz: $x = a^3,\ y = b^3$.

$$\begin{cases} x + y = 28 \\ \sqrt[3]{x} + \sqrt[3]{y} = 4 \end{cases} \Leftrightarrow \begin{cases} a^3 + b^3 = 28 \\ a + b = 4 \end{cases}$$

$$\Rightarrow \begin{cases} (a+b)\left(a^2 - ab + b^2\right) = 28 \\ a + b = 4 \end{cases} \Rightarrow \begin{cases} 4\left(a^2 - ab + b^2\right) = 28 \\ a + b = 4 \end{cases}$$

$$\Rightarrow \begin{cases} a^2 - ab + b^2 = 7 \\ a + b = 4 \end{cases} \Rightarrow \begin{cases} (a+b)^2 - 3ab = 7 \\ a + b = 4 \end{cases}$$

$$\Rightarrow \begin{cases} 4^2 - 3ab = 7 \\ a + b = 4 \end{cases} \Rightarrow \begin{cases} 3ab = 9 \\ a + b = 4 \end{cases} \Rightarrow \begin{cases} ab = 3 \\ a + b = 4 \end{cases}$$

$\begin{cases} ab = 3 \\ a + b = 4 \end{cases}$ tenglamalar sistemasidan $a_1 = 1,\ b_1 = 3$ va $a_2 = 3,\ b_2 = 1$ kelib chiqadi.

$\sqrt[3]{x} = a,\ \sqrt[3]{y} = b,\ a_1 = 1,\ b_1 = 3$ da $\sqrt[3]{x} = 1,\ \sqrt[3]{y} = 3 \Rightarrow x = 1,\ y = 27.$

$a_2 = 3,\ b_2 = 1$ da $\sqrt[3]{x} = 3,\ \sqrt[3]{y} = 1 \Rightarrow x = 27, y = 1.$

Tekshirish: $x = 1, y = 27$ yoki $x = 27, y = 1$ da $\begin{cases} 1 + 27 = 28 \\ \sqrt[3]{1} + \sqrt[3]{27} = 1 + 3 = 4 \end{cases}$

Javob: (1; 27), (27; 1).

5-misol. $\begin{cases} 3x - \sqrt{y + 2x} = 1 \\ y + 3x = 5 \end{cases}$ tenglamalar sistemasini yeching.

Yechish. 1) $\begin{cases} 3x - \sqrt{y + 2x} = 1 \\ y + 3x = 5 \end{cases} \Rightarrow \begin{cases} 3x - \sqrt{y + 2x} = 1 \\ y = 5 - 3x \end{cases}$

$$\Rightarrow \begin{cases} 3x - \sqrt{5 - 3x + 2x} = 1 \\ y = 5 - 3x \end{cases} \Rightarrow \begin{cases} \sqrt{5 - x} = 3x - 1 \\ y = 5 - 3x \end{cases}$$

2) $\sqrt{5 - x} = 3x - 1$ tenglamani yechamiz. $\sqrt{5 - x} \geq 0$ bo‘lgani uchun

$$3x - 1 \geq 0,\ \Rightarrow\ 3x \geq 1,\ \Rightarrow\ x \geq \frac{1}{3},\ \Rightarrow\ x \in \left[\frac{1}{3}; \infty\right).$$

$\sqrt{5 - x} = 3x - 1$ tenglikning ikki tomonini kvadratga ko‘tarsak,

$5-x=(3x-1)^2$, $\Rightarrow$ $5-x=9x^2-6x+1$, $\Rightarrow$ $9x^2-5x-4=0$ kvadrat tenglama hosil bo‘ladi. Tenglamaning ildizlarini topamiz:

$$x_{1,2}=\frac{5\pm\sqrt{25+144}}{18}=\frac{5\pm13}{18},\Rightarrow x_1=1,\ x_2=-\frac{4}{9}.$$

$x\in\left[\frac{1}{3};\infty\right)$ bo‘lgani uchun $1\in\left[\frac{1}{3};\infty\right)$, $-\frac{4}{9}\notin\left[\frac{1}{3};\infty\right)$.

$x=1$ da $y=5-3x=5-3=2$, $y=2$. (1; 2) sistemaning yechimi.

Tekshirish: (1; 2) da $\begin{cases}3\cdot1-\sqrt{2+2\cdot1}=3-\sqrt{4}=3-2=1\\2+3\cdot1=5\end{cases}$

Javob: (1; 2).

6-misol. $\begin{cases}\sqrt{x-2y+2}=2,\\\sqrt{y-2x+11}=x-5\end{cases}$ tenglamalar sistemasini yeching.

Yechish

$\sqrt{y-2x+11}\ge0$ bo‘lgani uchun $x-5\ge0$, $x\ge5$. $x\in[5;\infty)$.

$$\begin{cases}\sqrt{x-2y+2}=2,\\\sqrt{y-2x+11}=x-5\end{cases}\Rightarrow\begin{cases}\left(\sqrt{x-2y+2}\right)^2=4\\\left(\sqrt{y-2x+11}\right)^2=(x-5)^2\end{cases}$$

$$\Rightarrow\begin{cases}x-2y+2=4,\\y-2x+11=x^2-10x+25\end{cases}\Rightarrow\begin{cases}x-2y=2,\\y=x^2-8x+14\end{cases}$$

$$\Rightarrow\begin{cases}x-2(x^2-8x+14)=2,\\y=x^2-8x+14\end{cases}\Rightarrow\begin{cases}2x^2-17x+30=0,\\y=x^2-8x+14\end{cases}$$

$2x^2-17x+30=0$ tenglamani yechamiz, $x_{1,2}=\frac{17\pm\sqrt{289-240}}{4}=\frac{17\pm7}{4}$,

$x_1=6,\ x_2=\frac{5}{2}$.

$x\in[5;\infty)$ shartga asosan, $6\in[5;\infty)$, $\frac{5}{2}\notin[5;\infty)$.

$x_1=6$ da $y_1=6^2-8\cdot6+14=36-48+14=2$. $\Rightarrow y_1=2$

Tekshirish: (6; 2) da $\begin{cases}\sqrt{6-2\cdot2+2}=\sqrt{4}=2,\\\sqrt{2-2\cdot6+11}=6-5=1\end{cases}$

Javob: (6; 2).

7-misol. $\begin{cases}\sqrt{x+y}+\sqrt{2x+y+2}=7\\3x+2y=23\end{cases}$ tenglamalar sistemasini yeching.

Yechish

$\sqrt{x+y}=a$ va $\sqrt{2x+y+2}=b$ belgilash kiritsak, $a\geq 0$, $b\geq 0$ bo‘ladi.

$x+y=a^2$, $2x+y+2=b^2$

$$+\begin{cases}x+y=a^2\\2x+y+2=b^2\end{cases}\qquad\Rightarrow 3x+2y+2=a^2+b^2.$$

$3x+2y=23$ ekanini hisobga olsak, $3x+2y+2=a^2+b^2\Rightarrow 25=a^2+b^2$.

$$\begin{cases}\sqrt{x+y}+\sqrt{2x+y+2}=7\\3x+2y=23\end{cases}\Leftrightarrow\begin{cases}a+b=7\\a^2+b^2=25\end{cases}\Rightarrow\ a_1=3, b_1=4.\ a_2=4, b_2=3.$$

$a_1=3, b_1=4$ da $\begin{cases}\sqrt{x+y}=3,\\\sqrt{2x+y+2}=4\end{cases}\Rightarrow\begin{cases}x+y=9,\\2x+y+2=16\end{cases}$

$$\Rightarrow\begin{cases}x+y=9\\2x+y=14\end{cases}\qquad\Rightarrow x=5, y=4.$$

$a_2=4, b_2=3$ da $\begin{cases}\sqrt{x+y}=4,\\\sqrt{2x+y+2}=3\end{cases}\Rightarrow\begin{cases}x+y=16,\\2x+y+2=9\end{cases}$

$$\Rightarrow\begin{cases}x+y=16\\2x+y=7\end{cases}\qquad\Rightarrow x=-9,\ y=25.$$

Tekshirish: $(5;\ 4)$ da $\begin{cases}\sqrt{5+4}+\sqrt{2\cdot 5+4+2}=\sqrt{9}+\sqrt{16}=3+4=7,\\3\cdot 5+2\cdot 4=15+8=23\end{cases}$

$(-9;\ 25)$ da $\begin{cases}\sqrt{(-9)+25}+\sqrt{2\cdot(-9)+25+2}=\sqrt{16}+\sqrt{9}=4+3=7,\\3\cdot(-9)+2\cdot 25=-27+50=23\end{cases}$

Javob: $(5;\ 4), (-9;\ 25)$.

MISOLLAR

Tenglamalar sistemasini yeching.

1. a) $\begin{cases}\sqrt{x}-\sqrt{y}=4\\2\sqrt{x}+3\sqrt{y}=18\end{cases}$ b) $\begin{cases}3\sqrt{x}-\sqrt{y}=8\\\sqrt{x}+2\sqrt{y}=19\end{cases}$

2. a) $\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=3\\\sqrt[3]{x}+\sqrt[3]{y}=5\end{cases}$ b) $\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=1\\\sqrt[3]{x}+\sqrt[3]{y}=3\end{cases}$

3. a) $\begin{cases} 2\sqrt[3]{x}+3\sqrt[3]{y}=-1 \\ 2\sqrt[3]{x}-3\sqrt[3]{y}=-7 \end{cases}$ b) $\begin{cases} 3\sqrt[3]{x}+2\sqrt[3]{y}=3 \\ 3\sqrt[3]{x}-2\sqrt[3]{y}=-9 \end{cases}$

4. a) $\begin{cases} \sqrt{x}+\sqrt{y}=26 \\ \sqrt[4]{x}+\sqrt[4]{y}=6 \end{cases}$ b) $\begin{cases} \sqrt{x}-\sqrt{y}=5 \\ \sqrt[4]{x}-\sqrt[4]{y}=1 \end{cases}$

5. a) $\begin{cases} \sqrt{x}+\sqrt{y}=9 \\ \sqrt[6]{x}+\sqrt[6]{y}=3 \end{cases}$ b) $\begin{cases} \sqrt{x}-\sqrt{y}=7 \\ \sqrt[6]{x}-\sqrt[6]{y}=1 \end{cases}$

6. a) $\begin{cases} \sqrt{x}+\sqrt{y}=8 \\ \sqrt{x}\sqrt{y}=15 \end{cases}$ b) $\begin{cases} \sqrt{x}+\sqrt{y}=7 \\ \sqrt{x}\sqrt{y}=12 \end{cases}$

7. a) $\begin{cases} \sqrt{x}+3\sqrt{y}=10 \\ \sqrt{x}\sqrt{y}=8 \end{cases}$ b) $\begin{cases} 2\sqrt{x}-\sqrt{y}=5 \\ \sqrt{x}\sqrt{y}=3 \end{cases}$

8. a) $\begin{cases} \sqrt{x}-\sqrt{y}=4 \\ x-y=32 \end{cases}$ b) $\begin{cases} \sqrt{x}+\sqrt{y}=8 \\ x-y=16 \end{cases}$

9. a) $\begin{cases} \sqrt{6+x}-3\sqrt{3y+4}=-10 \\ 4\sqrt{3y+4}-\sqrt{6+x}=14 \end{cases}$ b) $\begin{cases} 2\sqrt{x-2}+\sqrt{5y+1}=8 \\ 3\sqrt{x-2}-2\sqrt{5y+1}=-2 \end{cases}$

10. a) $\begin{cases} \sqrt[4]{x+y}-\sqrt[4]{x-y}=2 \\ \sqrt{x+y}-\sqrt{x-y}=8 \end{cases}$ b) $\begin{cases} \sqrt[4]{x+y}+\sqrt[4]{x-y}=4 \\ \sqrt{x+y}+\sqrt{x-y}=10 \end{cases}$

11. a) $\begin{cases} \sqrt[3]{x}+\sqrt[3]{y}=5 \\ x\cdot y=216 \end{cases}$ b) $\begin{cases} \sqrt[3]{x}-\sqrt[3]{y}=3\frac{3}{4} \\ x\cdot y=1 \end{cases}$

c) $\begin{cases} \sqrt[3]{x}+\sqrt[3]{y}=-3 \\ x\cdot y=8 \end{cases}$ d) $\begin{cases} \sqrt[3]{x}-\sqrt[3]{y}=2 \\ x\cdot y=27 \end{cases}$

12. a) $\begin{cases} y\sqrt{x}+x\sqrt{y}=30 \\ \sqrt{x}+\sqrt{y}=5 \end{cases}$ b) $\begin{cases} y\sqrt{x}-x\sqrt{y}=-12 \\ \sqrt{x}-\sqrt{y}=1 \end{cases}$

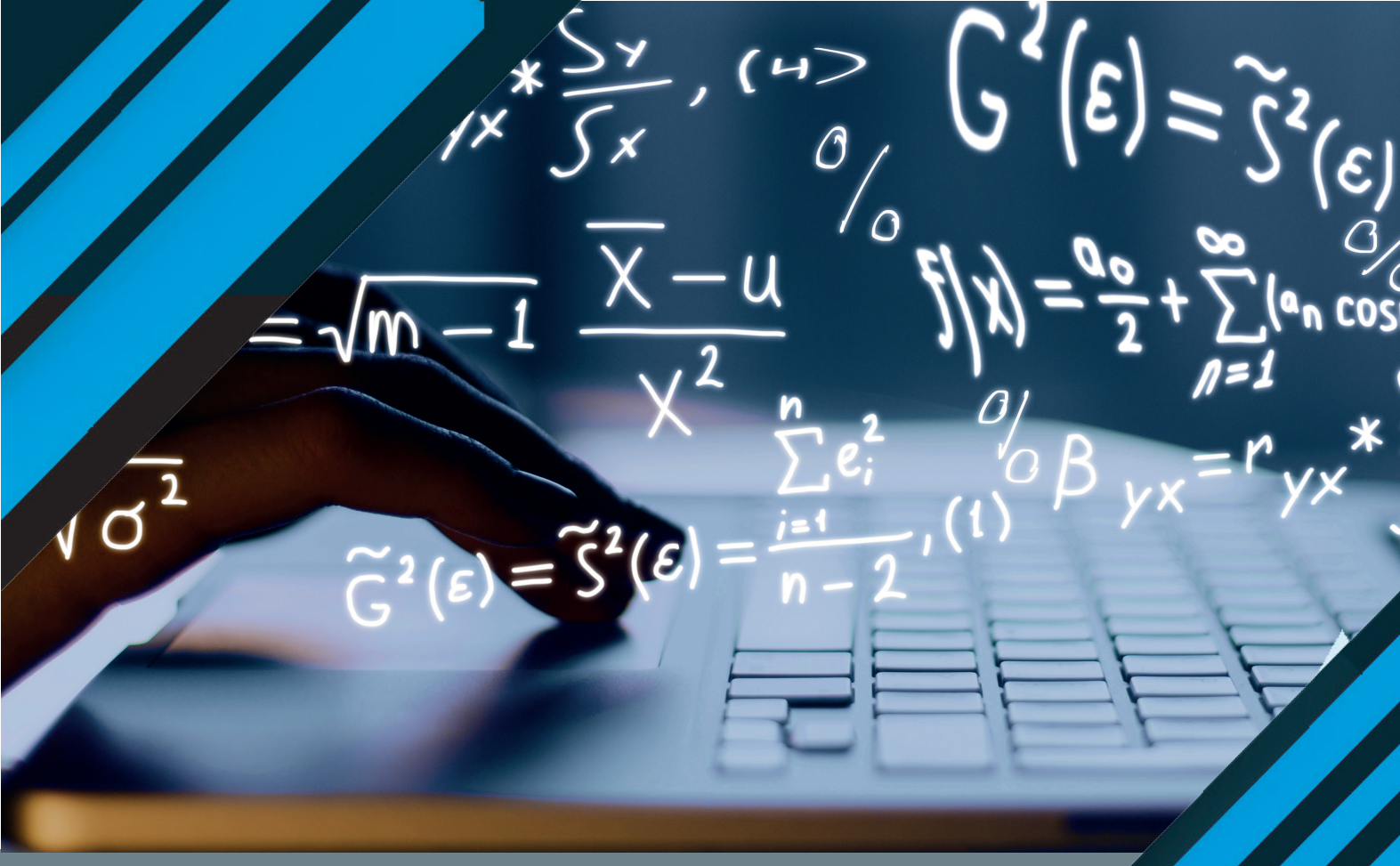
13. a) $\begin{cases} y+x-\sqrt{xy}=7 \\ xy=9 \end{cases}$ b) $\begin{cases} x-y+\sqrt{xy}=20 \\ xy=64 \end{cases}$

14. a) $\begin{cases} \sqrt[3]{x}-\sqrt[3]{y}=3 \\ \sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}=3 \end{cases}$ b) $\begin{cases} \sqrt[3]{x}-\sqrt[3]{y}=-1 \\ \sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2}=7 \end{cases}$

15. a) $\begin{cases} 3\sqrt{\frac{x}{y}}+2\sqrt{\frac{y}{x}}=5 \\ 4\sqrt{x}+\sqrt{y}=10 \end{cases}$ b) $\begin{cases} 4\sqrt{\frac{x}{y}}+2\sqrt{\frac{y}{x}}=9 \\ 7\sqrt{x}+2\sqrt{y}=48 \end{cases}$

16. a) $\begin{cases} \sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{10}{3} \\ y^2+x^2=82 \end{cases}$ b) $\begin{cases} \sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=\frac{5}{2} \\ y^2-x^2=15 \end{cases}$

17. a) $\begin{cases} 4y+5x-\sqrt{xy}=79 \\ 5x-4y+\sqrt{xy}=81 \end{cases}$ b) $\begin{cases} 9y+2x-\sqrt{xy}=71 \\ 2x-9y+\sqrt{xy}=73 \end{cases}$



3-BOB. KO‘RSATKICHLI VA LOGARIFMIK FUNKSIYALAR

- KO‘RSATKICHLI FUNKSIYA
- KO‘RSATKICHLI TENGLAMALAR
- KO‘RSATKICHLI TENGSIZLIKLAR
- LOGARIFM TUSHUNCHASI. LOGARIFMIK FUNKSIYA
- LOGARIFMIK IFODALARNI AYNIY ALMASHTIRISH
- LOGARIFMIIK TENGLAMALAR
- KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI
- LOGARIFMIK TENGSIZLIKLAR
- KO‘RSATKICHLI VA LOGARIFMIK FUNKSIYALARNING TATBIQI

KO‘RSATKICHLI FUNKSIYA

Keyingi vaqtlarda yer sathidan chang-to‘zon ko‘tarilishi tez-tez kuzatilmoqda. Bunda chang miqdori yuqoriga ko‘tarilgani sari kamayib borishi isbotlangan. Chang miqdorining balandlikka bog‘liqligi ko‘rsatkichli funksiya orqali ifodalanar ekan. Undan tashqari, viruslarning ko‘payishi, radioaktiv moddalarning yemirilishi kabi hodisalar ham ko‘rsatkichli funksiyalar orqali tavsiflanadi.

Masalan, chang miqdori y ning x balandlikka bog‘liqligi $y = p \cdot e^{-qx}$ ko‘rinishdagi funksiya orqali ifodalanadi. Bu yerda p, q sonlari ***parametrlar*** deb ataluvchi kattaliklar, e esa ***Eyler soni*** deb ataluvchi irratsional son. Uning taqribiy qiymati 2,71 ga teng.

Ko‘rsatkichli funksiyalarni o‘rganish uchun quyidagi xossalarni bilish talab etiladi:

1) $a^0 = 1, \quad a \neq 0$ 2) $a^1 = a$ 3) $a^n \cdot a^m = a^{n+m}$

4) $\dfrac{a^n}{a^m} = a^{n-m}$ 5) $\left(a^n\right)^m = a^{nm}$ 6) $(ab)^n = a^n \cdot b^n$

7) $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, \quad b \neq 0$ 8) $a^{\frac{m}{n}} = \sqrt[n]{a^m}, \quad a > 0, n \in N, m \in Z$

Ma’lumki, kasr ko‘rsatkichli $a^{\frac{m}{n}}$ yoki haqiqiy ko‘rsatkichli a^p ko‘rinishidagi darajalarni ham qarash mumkin. Bunda ko‘rsatkichning ayrim qiymatlarida a^p daraja ma’noga ega bo‘lmay qolishi mumkin. Masalan, $(-3)^{\frac{1}{2}} = \sqrt{-3}$ ifoda haqiqiy sonlar to‘plamida ma’noga ega bo‘lmaydi. Undan tashqari, $0^{-3} = \dfrac{1}{0^3} = \dfrac{1}{0}$ ifoda ham aniqlanmagan. Bunday holatlarning oldini olish maqsadida haqiqiy p ko‘rsatkichli a^p daraja uchun $a > 0$ tengsizlik bajarilishi talab etiladi. Har qanday p haqiqiy son uchun $1^p = 1$ ekani sababli asosi 1 bo‘lgan darajalarni o‘rganish orqali hech qanday yangi ma’lumotga erishilmaydi.

Demak, yuqorida bayon etilganlar asosida quyidagi xulosaga kelish mumkin.

Xulosa. Ixtiyoriy p haqiqiy ko‘rsatkichli a^p daraja aniq qiymat qabul qilishi uchun a asos $a > 0$ va $a \neq 1$ shartlarni bajarishi talab etiladi.

$a > 0$ va $a \neq 1$ shartlarni qanoatlantiradigan a haqiqiy son uchun ushbu $\boldsymbol{y = a^x}$ ko‘rinishdagi funksiya ***ko‘rsatkichli funksiya*** deyiladi (daraja ko‘rsatkichi – o‘zgaruvchi miqdor).

$y = a^x$ ko‘rsatkichli funksiya quyidagi xossalarga ega:

- $y = a^x$ ko‘rsatkichli funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to‘plamidan iborat:

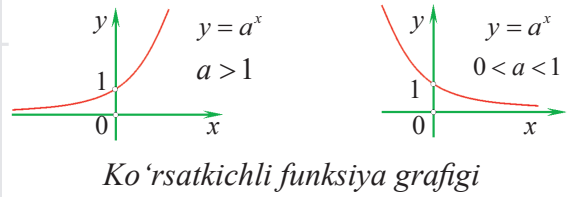
$$D(y) = (-\infty; +\infty)$$

- $y = a^x$ ko‘rsatkichli funksiyaning qiymatlar to‘plami barcha musbat haqiqiy sonlar to‘plamidan iborat:

$$E(y) = (0; +\infty)$$

- $y = a^x$ ko‘rsatkichli funksiya grafigi Ox o‘qi bilan kesishmaydi.

1-rasm



Ko‘rsatkichli funksiya grafigi

● $y = a^x$ ko‘rsatkichli funksiya grafigi Oy o‘qi bilan esa $(0,1)$ nuqtada kesishadi.

● Ko‘rsatkichli funksiya davriy bo‘lmaydi, juft ham emas, toq ham emas.

● a asosning $0 < a < 1$ tengsizliklarni qanoatlantiruvchi qiymatlarida $y = a^x$ funksiya kamayadi: Kamayish oralig‘i $(-\infty; +\infty)$ dan iborat.

● a asosning $a > 1$ tengsizlikni qanoatlantiruvchi qiymatlarida $y = a^x$ funksiya o‘sadi: O‘sish oralig‘i $(-\infty; +\infty)$ dan iborat.

1-misol. $(0,1)^{\sqrt{2}}$ ni 1 bilan taqqoslang.

Yechish

$1 = (0,1)^0$ va $y = (0,1)^x$ funksiya $x \in R$ da kamayuvchi bo‘lgani uchun

$\sqrt{2} > 0 \Rightarrow (0,1)^{\sqrt{2}} < (0,1)^0 \Rightarrow (0,1)^{\sqrt{2}} < 1$

Javob: $(0,1)^{\sqrt{2}} < 1$.

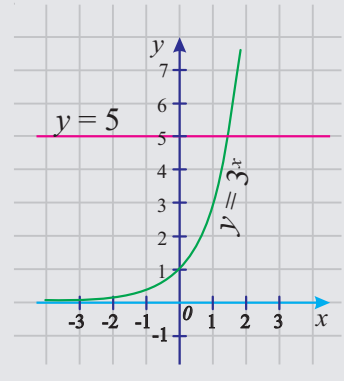
2-misol. Ushbu $y = \left(\frac{1}{2}\right)^x$, $y = e^x$, $y = 2,6^x$ funksiyalardan qaysilari kamayuvchi?

Yechish

Berilgan uchta funksiyadan faqat birinchi funksiyada qatnashuvchi ko‘rsatkichli ifodaning asosi 0 va 1 oralig‘iga tegishli, shuning uchun birinchi funksiya kamayuvchi funksiya bo‘ladi.

Javob: $y = \left(\frac{1}{2}\right)^x$.

2-rasm



3-misol. $3^x = 5$ tenglama bitta ildizga ega ekanini ko‘rsating.

Yechish

$y = 3^x$ va $y = 5$ funksiyalarning grafiklarini bitta koordinata tekisligida yasaymiz (2-*rasm*).

Chizmadan ko‘rinib turibdiki, grafiklar yagona nuqtada kesishadi. Demak, tenglama bitta ildizga ega ekan.

MISOLLAR

1. Funksiya xossalarini ayting va uning grafigini yasang.

a) $y = 3^x$ b) $y = 0,4^x$ c) $y = 0,8^x$ d) $y = 1,5^x$

2. Funksiyaning qiymatlar sohasini toping.

a) $y = 3^x$ b) $y = \left(\frac{1}{2}\right)^x - 1$

c) $y = -\left(\frac{1}{3}\right)^x$ d) $y = 4^x + 2$

3. Sonlarni taqqoslang.

a) $\left(\frac{3}{5}\right)^{-\frac{\sqrt{3}}{2}}$ va 1 b) $3,2^{-\sqrt{2}}$ va 1 c) $0,7^{\frac{\sqrt{5}}{9}}$ va $0,7^{\frac{1}{6}}$ d) $5^{-\sqrt{13}}$ va $\left(\frac{1}{5}\right)^{2,1}$

4. Hisoblang.

a) $((\sqrt{3})^{\sqrt{3}})^{\sqrt{3}}$ b) $3^{1-2\sqrt{3}}\cdot 9^{1+\sqrt{3}}$ c) $64^{\sqrt{2}}:64^{3\sqrt{2}}$ d) $(5^{\sqrt[5]{16}})^{\sqrt[5]{2}}$

5. Ifodani soddalashtiring.

a) $(c^{\sqrt{3}})^{\sqrt{3}}$ b) $b^{\sqrt{2}}\cdot(\frac{1}{b})^{\sqrt{2}-1}$ c) $x^{\pi}\cdot\sqrt[4]{x^2:6x^{4\pi}}$ d) $y^{\sqrt{2}}\cdot y^{1,5}:6\sqrt[3]{y^{3\sqrt{2}}}$

6. Ushbu $y=\left(\frac{5}{9}\right)^x$, $y=\pi^x$, $y=1,7^x$ funksiyalardan qaysilari o‘suvchi?

7. Quyidagi funksiyalar grafiklarini sxematik ko‘rinishda tasvirlang.

a) $y=2^{|x|}$ b) $y=-2^{|x|+1}$ c) $y=2^{-|x|}-1$

8. Ifodani soddalashtiring.

a) $\frac{a^{2\sqrt{2}}-b^{2\sqrt{3}}}{(a^{\sqrt{2}}-b^{\sqrt{3}})^2}+1$ b) $\frac{\left(a^{2\sqrt{3}}-1\right)\left(a^{2\sqrt{3}}+a^{\sqrt{3}}+a^{3\sqrt{3}}\right)}{a^{4\sqrt{3}}-a^{\sqrt{3}}}$

c) $\frac{a^{\sqrt{5}}-b^{\sqrt{7}}}{a^{\frac{2\sqrt{5}}{3}}+a^{\frac{\sqrt{5}}{3}}b^{\frac{\sqrt{7}}{3}}+b^{\frac{2\sqrt{7}}{3}}}$ d) $\sqrt{\left(x^{\pi}+y^{\pi}\right)^2-\left(4^{\frac{1}{\pi}}xy\right)^{\pi}}$

9. Ikkita funksiyadan qaysi biri o‘suvchi, qaysi biri kamayuvchi ekanini aniqlang.

a) $y=\left(\sqrt{2}\right)^x$, $y=\left(\frac{1}{\sqrt{2}}\right)^x$ b) $y=\left(\frac{\pi}{3}\right)^x$, $y=\left(\frac{3}{\pi}\right)^x$

c) $y=\left(\sqrt{5}-2\right)^x$, $y=\frac{1}{\left(\sqrt{5}-2\right)^x}$ d) $y=\left(3-\sqrt{7}\right)^x$, $y=\frac{1}{\left(3-\sqrt{7}\right)^x}$

10. Funksiyaning qiymatlar sohasini toping.

a) $y=3^{x+1}-3$ b) $y=\left(\frac{1}{2}\right)^{x-1}+2$ c) $y=\left|2^x-2\right|$ d) $y=4^{|x|}$

11. Funksiyaning eng katta va eng kichik qiymatini toping.

a) $y=\left(\frac{1}{2}\right)^{\sin x}$ b) $y=4^{\cos x}$ c) $y=5+3^{|\cos x|}$ d) $y=\left(\frac{1}{3}\right)^{|\sin x|}-2$

12. a ning ishorasini aniqlang.

a) $3^a=10$ b) $10^a=4$ c) $0,3^a=0,1$ d) $0,7^a=5$

13. Ifodaning qiymatini toping.

a) $6^{x-1}=12$ bo‘lsa, $6^x=?$ b) $5^{x-3}=4$ bo‘lsa, $5^{4-x}=?$ c) $12^{x+5}=6$ bo‘lsa, $12^{-3-x}=?$

14. Qaysi hollarda $3^{x_1} > 3^{x_2}$ tengsizlik bajariladi?

15. $y = \left(\frac{1}{3}\right)^x$ funksiyaning x natural son bo‘lgandagi qiymatlari ketma-ketligi geometrik progressiya tashkil etishini isbotlang.

Ko‘rsatkichli funksiyaning hayotda qo‘llanishi



Qaynayotgan choynak olovdan olinsa, u dastlab tez soviydi, so‘ng esa sovish tezligi pasayadi. Gap shundaki, sovish tezligi choynak temperaturasi va tashqi muhit temperaturasining ayirmasiga proporsional. Bu ayirma qancha kamaysa, choynak shuncha sekin soviydi. Choynakning dastlabki temperaturasi T_0, havo temperaturasi T_1 bo‘lsa, u holda t sekunddan keyin choynak temperaturasi $T = (T_1 - T_0)e^{-kt} + T_1$ formula bilan aniqlanadi.

Fizikada qo‘llanishi



Havosiz bo‘shliqda (vakuum) jismning erkin tushishida uning tezligi ortib boradi. Havoda ham jismlarning tushish tezligi ortib boradi, ammo ma’lum bir qiymatdan ortib ketmaydi. Agar havoning qarshilik kuchi parashyutchining tushish tezligiga to‘g‘ri proporsional bo‘lsa, ya’ni $F = kv$ bo‘lsa, u holda t sekunddan keyin uning tushish tezligi $v = \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$ ga teng bo‘ladi. Bu yerda m parashyutchining tezligi.

Aholining o‘sishi



Mamlakatda aholi sonining ma’lum vaqt oralig‘ida o‘zgarishi $N = N_0e^{kt}$ formula bilan ko‘rsatiladi. Bu yerda $t = 0$ vaqtdagi aholi soni N_0, t vaqtdagi aholi soni N, k esa o‘zgarmas.

Biologiyada qo‘llanishi



Organik olamning ko‘payish qonuni: organizm uchun qulay muhitda (yirtqichlar soni kamligi, oziq miqdori yetarli bo‘lishi) tirik organizmlar ko‘rsatkichli funksiya qonuni bo‘yicha ko‘payadi. Masalan, bitta uy pashshasi yoz davomida $8 \cdot 10^{14}$ miqdorida yangi avlod hosil qiladi. Shu miqdorda ko‘payib boraverganda, ularning og‘irligi bir necha million tonnani tashkil qilardi (ikkita uy pashshasining nasli esa sayyoramiz massasidan ortardi), ular juda katta maydonni egallab olardi. Agar ularni zanjir qilib joylashtirsa, u holda bu zanjir uzunligi Yerdan Quyoshgacha bo‘lgan masofadan ham katta bo‘lardi. Ammo tabiatda pashshaning tabiiy “dushmani” hisoblangan ko‘plab jonivor va o‘simliklar mavjudligi pashshalarning soni bu darajada ortishiga yo‘l qo‘ymaydi.

KO‘RSATKICHLI TENGLAMALAR

Ko‘rsatkichli tenglamalar

Daraja ko‘rsatkichida noma’lum qatnashgan tenglama ***ko‘rsatkichli tenglama*** deyiladi. $3^x = 9$, $4^x - 9 = 7$, $2^{x+1} = 2^{8-2x}$ tenglamalar ko‘rsatkichli tenglamaga misol bo‘la oladi.

Noma’lumning berilgan ko‘rsatkichli tenglamani to‘g‘ri sonli tenglikka aylantiradigan qiymati ko‘rsatkichli tenglamaning ***ildizi*** deyiladi.

Ko‘rsatkichli tenglamalar va ularni yechish

Ushbu x noma’lumli $a^x = a^p$ ko‘rsatkichli tenglamaning ildizi $x = p$ bo‘ladi.

Ko‘rsatkichli tenglamalarni yechishda ushbu qoida ishlatiladi:

$a > 0$, $a \neq 1$ bo‘lganda $a^{f(x)} = a^{g(x)}$ tenglamaning ildizlari $f(x) = g(x)$ tenglamaning ildizlaridan iborat bo‘ladi.

1-misol. Tenglamani yeching: $2^{x-1} = 16$.

Yechish

Tenglamani quyidagi ko‘rinishda yozib olamiz: $2^{x-1} = 2^4 \Rightarrow x - 1 = 4 \Rightarrow x = 5$

Javob: $x = 5$.

2-misol. Tenglamani yeching: $3^{2x} \cdot 3^{x^2} = 3^{15}$.

Yechish

Tenglamani quyidagi ko‘rinishda yozib olamiz: $3^{2x+x^2} = 3^{15} \Rightarrow 2x + x^2 = 15$.

$x^2 + 2x - 15 = 0$ kvadrat tenglamaning ildizlari $x_1 = -5; x_2 = 3$ bo‘ladi.

Javob: $x_1 = -5;\ x_2 = 3$.

3-misol. Tenglamani yeching: $\left(5^{x+1}\right)^x = \left(\dfrac{5^x}{5^{24}}\right)^{-1}$.

Yechish

Tenglamani quyidagi ko‘rinishda yozib olamiz: $5^{x^2+x} = 5^{24-x} \Rightarrow x^2 + x = 24 - x$.

$x^2 + 2x - 24 = 0$ kvadrat tenglama ildizlari $x_1 = -6; x_2 = 4$ bo‘ladi.

Javob: $x_1 = -6; x_2 = 4$.

4-misol. $6^{x^2} + 36 = 2^{1-x^2} \cdot 12^{x^2}$ tenglamaning ildizlari ko‘paytmasini toping.

Yechish

$12^{x^2} = (6 \cdot 2)^{x^2} = 6^{x^2} \cdot 2^{x^2}$ ekanidan foydalanib tenglamani quyidagi ko‘rinishda yozib olamiz:

$$6^{x^2}+36=2^{1-x^2}\cdot 6^{x^2}\cdot 2^{x^2} \Rightarrow 6^{x^2}+36=2^{1-x^2+x^2}\cdot 6^{x^2} \Rightarrow 6^{x^2}+36=2\cdot 6^{x^2};\Rightarrow 6^{x^2}=36 \Rightarrow 6^{x^2}=6^2$$

$$x^2=2 \Rightarrow x_{1,2}=\pm\sqrt{2}$$

Demak, $x_1\cdot x_2=-\sqrt{2}\cdot\sqrt{2}=-2$

Javob: –2.

5-misol. Tenglamani yeching: $3^{2x-1}=7^{2x-1}$.

Yechish

Berilgan tenglamada tenglikning ikkala tarafidagi ko‘rsatkichli ifodalarning daraja ko‘rsatkichlari bir xil bo‘lgani uchun tenglikning ikkala tarafini 7^{2x-1} ifodaga bo‘lamiz:

$$\frac{3^{2x-1}}{7^{2x-1}}=\frac{7^{2x-1}}{7^{2x-1}} \Rightarrow \left(\frac{3}{7}\right)^{2x-1}=1 \Rightarrow \left(\frac{3}{7}\right)^{2x-1}=\left(\frac{3}{7}\right)^{0} \Rightarrow 2x-1=0 \Rightarrow x=\frac{1}{2}$$

Javob: $x=\frac{1}{2}$.

6-misol. $9^{x^2-1}-36\cdot 3^{x^2-3}+3=0$ tenglamaning ildizlari yig‘indisini toping.

Yechish.

Tenglamani quyidagi ko‘rinishda yozib olamiz:

$$\frac{1}{9}\cdot 9^{x^2}-\frac{36}{27}\cdot 3^{x^2}+3=0$$

$3^{x^2}=t$ deb belgilaymiz, demak, $9^{x^2}=t^2$

$\frac{1}{9}\cdot t^2-\frac{4}{3}\cdot t+3=0$ kvadrat tenglamani yechib, $t_1=9;\ t_2=3$ ekanini topamiz.

$$3^{x^2}=9\Rightarrow x^2=2\Rightarrow x_{1,2}=\pm\sqrt{2} \quad \text{va} \quad 3^{x^2}=3\Rightarrow x^2=1\Rightarrow x_{3,4}=\pm 1$$

tenglama ildizlari yig‘indisi: $x_1+x_2+x_3+x_4=-\sqrt{2}+\sqrt{2}-1+1=0.$

Javob: 0.

MISOLLAR

1. Ko‘rsatkichli tenglamalarni yeching.

a) $3^x\cdot 3=81$ b) $4^{3x}\cdot 2^x=128$ c) $5^{x+1}-4\cdot 5^x=25$ d) $7^x\cdot 8^x=1$

e) $4^{x^2-3x-4}=1$ f) $0,3^{2x-1}=0,09$ g) $2^{2x}=4^{2\sqrt{3}}$ h) $\left(\frac{1}{3}\right)^{3x}=9$

i) $27^x=\frac{1}{3}$ j) $400^x=\frac{1}{20}$ k) $\left(\frac{1}{3}\right)^x=\frac{1}{81}$ l) $0,6^{x+3}=0,6^{2x-5}$

2. Tenglamani yeching.

a) $3^x = 81$ b) $\left(\frac{1}{2}\right)^x = \frac{1}{1024}$ c) $7^x = -49$ d) $13^x = -169$

e) $5^x = 0$ f) $8^{2x} = 0$ g) $3^{6-x} = 3^{3x-2}$ h) $\left(\frac{3}{7}\right)^{3x+1} = \left(\frac{7}{3}\right)^{5x-9}$

i) $2^{7x-15} = 2^{9-4x}$ j) $13^{5-2x} = 13^{6x+1}$ k) $2^{x^2+x-0,5} = 4\sqrt{2}$ l) $\left(\frac{4}{9}\right)^x \cdot \left(\frac{27}{8}\right)^{x-1} = \frac{2}{3}$

3. $\left(\frac{21}{6}\right)^{29x^2-8x} = \left(\frac{6}{21}\right)^{8x^2-29x}$

4. $\sqrt[3]{5^{2x-3}} = \frac{5}{\sqrt[4]{5}}$

5. $\left(\frac{37}{5}\right)^{71\sqrt{x}-3} = \left(\frac{5}{37}\right)^{3\sqrt{x}-293}$

6. $\left(\frac{1}{\sqrt{2}}\right)^{x^2-9x} = 1$

7. $2^{x^2-3} \cdot 5^{x^2-3} = 0,01\left(10^{x-1}\right)^3$

8. $2^{x+1} = 5^{x+1}$

9. $7^{x+2} + 4 \cdot 7^{x-1} = 347$

10. $2 \cdot 3^{x+1} - 4 \cdot 3^{x-2} = 150$

11. $5^{2x} + 5^{2x+2} + 5^{2x+4} = 651$

12. $4 \cdot 7^{x+3} - 7^{x+2} - 3 \cdot 7^{x+1} = 1302$

13. $6 \cdot 2^{x+4} - 4 \cdot 2^{x+3} + 3 \cdot 2^{x+2} = 152$

14. $7^{3x} - 7^{3x-1} = 6$

15. $4^x - 6 \cdot 2^x + 8 = 0$

16. $5 \cdot 25^x - 6 \cdot 5^x + 1 = 0$

17. $9^x + 3 \cdot 3^x - 18 = 0$

18. $3^{2x+3} - 4 \cdot 3^{x+1} + 1 = 0$

19. $(0,25)^{2-x} = \frac{256}{2^{x+3}}$

20. $3^{4\sqrt{x}} - 4 \cdot 3^{2\sqrt{x}} + 3 = 0$

21. $9 \cdot 16^x + 2 \cdot 12^x - 32 \cdot 9^x = 0$

22. $64 \cdot 9^x - 84 \cdot 12^x + 27 \cdot 16^x = 0$

23. $3 \cdot 16^x + 2 \cdot 81^x = 5 \cdot 36^x$

24. $4^{x^2} + 6^{x^2} = 2 \cdot 9^{x^2}$

25. $8^x - 6 \cdot 12^x + 11 \cdot 18^x = 2 \cdot 27^{x+\frac{1}{3}}$

26. $x \cdot 3^{x-1} + 3 \cdot 3^{\sqrt{3-x}} = 3^x + x \cdot 3^{\sqrt{3-x}}$

27. $x^2 \cdot 4^{\sqrt{6-x}} + 4^{2+x} = 16 \cdot 2^{2\sqrt{6-x}} + x^2 \cdot 2^{2x}$

28. $\left(\frac{1}{2}\right)^{2-x} + 2^{x-3} = 80 + \sqrt{4^{x-4}}$

KO‘RSATKICHLI TENGSIZLIKLAR

$4^x < 64$, $8^x + 11 > 75$, $2^{x-2} \le 2^{5+3x}$, $9^x < 7^x$ tengsizliklar ko‘rsatkichli tengsizlikka misol bo‘la oladi.

Quyidagi jadvalda bir xil asosli ko‘rsatkichli tengsizliklarni ratsional tengsizliklarga keltirish ko‘rsatilgan.

Ko‘rsatkichli tengsizliklar turlari	$a^{f(x)} \le a^{g(x)}$	$a^{f(x)} < a^{g(x)}$	$a^{f(x)} > a^{g(x)}$	$a^{f(x)} \ge a^{g(x)}$
$0 < a < 1$ bo‘lganda	$f(x) \ge g(x)$	$f(x) > g(x)$	$f(x) < g(x)$	$f(x) \le g(x)$
$a > 1$ bo‘lganda	$f(x) \le g(x)$	$f(x) < g(x)$	$f(x) > g(x)$	$f(x) \ge g(x)$

1-misol. Tengsizlikni yeching: $2^x > 32$.

Yechish

Tengsizlikni quyidagicha yozib olamiz: $2^x > 2^5$.

$2 > 1$ bo‘lgani uchun $x > 5$.

Javob: $(5;\infty)$.

2-misol. Tengsizlikni yeching: $\left(\frac{3}{4}\right)^x \ge \frac{16}{9}$.

Yechish

Tengsizlikni quyidagicha yozib olamiz: $\left(\frac{3}{4}\right)^x \ge \left(\frac{3}{4}\right)^{-2}$

$0 < \frac{3}{4} < 1$ bo‘lgani uchun $x \le -2$.

Javob: $(-\infty;-2]$

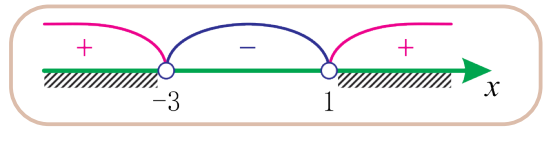
3-misol. $3^{x^2+2x} > 3^3$ tengsizlikni yeching.

Yechish

$3 > 1$ bo‘lgani uchun $x^2 + 2x > 3$ tengsizlikning yechimini topish kifoya.

$$x^2 + 2x - 3 > 0,$$

$$(x+3)(x-1) > 0,$$



Javob: $x \in (-\infty;-3) \cup (1;+\infty)$.

Har xil asosli ko‘rsatkichli tengsizliklarni yechish

$a > 0$, $a \ne 1$ va $b > 0$, $b \ne 1$ bo‘lganda ushbu $a^{f(x)} < b^{f(x)}$ ko‘rsatkichli tengsizlik $\left(\frac{a}{b}\right)^{f(x)} < 1$ tengsizlikka keltirilib, yechiladi.

MISOLLAR

Tengsizliklarni yeching.

1. $4^x > 256$

2. $\left(\frac{1}{3}\right)^x \le \frac{1}{729}$

3. $7^x < -49$

4. $13^x > -169$

5. $5^x < 0$

6. $8^{2x} > 0$

7. $10^x \le 0$

8. $\left(\frac{1}{3}\right)^{-\frac{x}{2}} > \sqrt{3}$

9. $\left(\frac{1}{6}\right)^{\frac{2x}{15}} < \sqrt[5]{6}$

10. $2^x > \left(\frac{1}{2}\right)^{x+1}$

11. n ning nechta natural qiymati $9 \le 3^n \le 79$ qo‘sh tengsizlikni qanoatlantiradi?

12. x ning qanday qiymatlarida $y = 5^x - 5$ funksiya musbat qiymatlarni qabul qiladi?

13. $\left(\frac{4}{9}\right)^x \cdot \left(\frac{3}{2}\right)^x > \left(\frac{2}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^{-2x}$ tengsizlikning eng katta butun yechimini toping.

14. $3 \cdot 9^{2x-2} > \left(\frac{1}{27}\right)^{3x-1}$

15. $\left(\frac{1}{4}\right)^{2x-3} > 4^{1-2x}$

16. $2 \cdot 8^{4-5x} < \left(\frac{1}{16}\right)^{x+2}$

17. $\left(\frac{3}{4}\right)^{\frac{x+1}{x+2}} > \frac{\sqrt{3}}{2}$

18. $6 \cdot 2^{x+3} - 5 \cdot 2^{x+2} + 4 \cdot 2^x > 128$

19. $7 \cdot 3^{x+4} + 2 \cdot 3^{x+3} - 5 \cdot 3^{x+2} \le 192$

20. $10 \cdot 3^{x+2} - 4 \cdot 10^{x+2} < 3^{x+4} - 3 \cdot 10^{x+2}$

21. $5^{x+2} - 5^{x+1} > 2^{x+2} + 2^{x+4}$

22. $\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64}$

23. $\left(\frac{1}{25}\right)^{2x} < \left(\sqrt{5}\right)^{x^2+2.75}$

24. $\left(\frac{1}{16}\right)^{x^2} < 8 \cdot \sqrt{2}^{16-2x}$

25. $2^{x^2} > \left(\frac{1}{2}\right)^{2x-3}$

26. $0{,}04^x - 26 \cdot (0{,}2)^x + 25 \le 0$

27. $25^x - 4 \cdot 5^x - 5 \ge 0$

28. $4^x - 10 \cdot 2^x + 16 < 0$

29. $3^{2x+1} + 1 < 4 \cdot 3^x$

30. $3^{8x} - 4 \cdot 3^{4x} \le -3$ tengsizlikning butun yechimlari yig‘indisini toping.

31. $x^2 \cdot 3^x - 3^{x+1} \le 0$ tengsizlikning butun sonlardan iborat yechimlari nechta?

LOGARIFM TUSHUNCHASI. LOGARIFMIK FUNKSIYA

Logarifm kundalik hayotda keng qo‘llanadi. Masalan, bankka qo‘yilgan mablag‘ biror miqdorga qancha vaqtda ko‘payishini topishda logarifmdan foydalaniladi. Yoki tovush balandligini baholashda logarifmik bog‘lanish ishlatiladi.

Logarifm tushunchasini va logarifmik funksiyani o‘rganish uchun:
1) ko‘rsatkichli funksiyani;
2) ko‘rsatkichli funksiyalarning xossalarini **bilish talab etiladi.**

Logarifm haqida tushuncha

1-misol. Tenglamani yeching: $3^x = 27$

Yechish

$3^x = 3^3 \Rightarrow x = 3$

Javob: $x = 3$.

2-misol. Tenglamani yeching: $2^x = 5$

Yechish

Bu tenglama ildizga ega va bu ildiz ratsional son emas. Shu kabi tenglamalarning ildizini ifodalash uchun ***logarifm*** tushunchasi kiritilgan. Berilgan tenglamaning ildizi 5 ning 2 asosga ko‘ra logarifmi deb ataluvchi kattalikka teng bo‘ladi va u $\log_2 5$ kabi yoziladi. Demak, $x = \log_2 5$

Javob: $x = \log_2 5$.

Umuman olganda, $a^x = b$ tenglamaning ildizi $x = \log_a b$ ga teng. Bu yerda $a > 0$, $a \neq 1$, $b > 0$.

Ta’rif

b sonning a asosga ko‘ra logarifmi deb b ni hosil qilish uchun a ni ko‘tarish kerak bo‘lgan daraja ko‘rsatkichiga aytiladi. b ning a asosga ko‘ra logarifmi $\log_a b$ orqali belgilanadi. Bu yerda a – logarifm asosi, b – logarifmosti ifodasi.

$\log_a b$ ifoda ***logarifm a asosga ko‘ra b*** deb o‘qiladi.

Masalan, $\log_2 5$ ifoda logarifm 2 asosga ko‘ra 5 deb o‘qiladi.

$\log_{10} b$ ifoda qisqacha $\lg b$ kabi belgilanadi va o‘nli logarifm deyiladi, ya’ni $\log_{10} b = \lg b$.

$\log_e b$ ifoda qisqacha $\ln b$ kabi belgilanadi va natural logarifm deyiladi, ya’ni $\log_e b = \ln b$. Bunda $e \approx 2{,}71$.

$a^x = b$ tenglamaning ildizi $\log_a b$ ni tenglamadagi x ning o‘rniga qo‘ysak,

$$a^{\log_a b} = b \qquad (a > 0, a \neq 1, b > 0)$$

tenglik hosil bo‘ladi. Bu tenglik ***asosiy logarifmik ayniyat*** deyiladi.

3-misol. Ta'rif bo'yicha hisoblang: a) $\log_2 32$; b) $\log_3 \frac{1}{9}$; c) lg100; d) $\ln e^3$.

Yechish

a) $\log_2 32 = 5$, chunki $2^5 = 32$ b) $\log_3 \frac{1}{9} = -2$, chunki $3^{-2} = \frac{1}{9}$

c) $\lg 100 = 2$, chunki $10^2 = 100$ d) $\ln e^3 = 3$, chunki $e^3 = e^3$

Javob: a) 5; b) –2; c) 2; d) 3.

4-misol. Hisoblang: $\log_{64} 32$.

Yechish

Ifodaning qiymatini ko'rsatkichli tenglama yordamida yechib topish mumkin. $\log_{64} 32 = x$ bo'lsin.

$$64^x = 32 \Rightarrow 2^{6x} = 2^5 \Rightarrow 6x = 5 \Rightarrow x = \frac{5}{6}$$

Javob: $\frac{5}{6}$.

5-misol. Asosiy logarifmik ayniyat yordamida hisoblang: $64^{\log_8 3}$.

Yechish

$$64^{\log_8 3} = \left(8^2\right)^{\log_8 3} = \left(8^{\log_8 3}\right)^2 = 3^2 = 9$$

Javob: 9.

Logarifmik funksiya va uning xossalari, grafigi

$a > 0$ va $a \neq 1$ shartlarni qanoatlantiradigan a haqiqiy sonni qaraylik. Ushbu

$$y = \log_a x$$

ko'rinishdagi funksiya ***logarifmik funksiya*** deyiladi.

Masalan, $y = \log_2 x$, $y = \log_{\frac{1}{3}} x$, $y = \lg x$, $y = \ln x$ kabi funksiyalar logarifmik funksiyalardir.

Logarifmik funksiyalar quyidagi xossalarga ega:

- $y = \log_a x$ logarifmik funksiyaning aniqlanish sohasi barcha musbat haqiqiy sonlar to'plamidan iborat:

$$D(y) = (0; +\infty)$$

- $y = \log_a x$ logarifmik funksiyaning qiymatlar to'plami esa barcha haqiqiy sonlar to'plamidan iborat:

$$E(y) = (-\infty; +\infty)$$

- $y = \log_a x$ davriy funksiya emas;
- $y = \log_a x$ funksiya juft ham emas, toq ham emas;
- $0<a<1$ bo'lganda $y = \log_a x$ funksiya kamayuvchi;
- $a > 1$ bo'lganda $y = \log_a x$ funksiya o'suvchi.

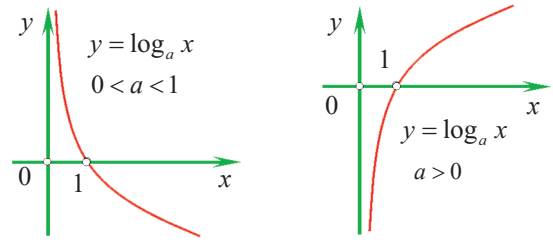
6-misol. Taqqoslang: a) $\log_{0,3}7$ va $\log_{0,3}8$
b) $\log_7 0{,}28$ va $\log_7 0{,}31$.

Yechish

$y=\log_{0,3}x$ funksiya kamayuvchi hamda $7<8$ ekanidan, $\log_{0,3}7>\log_{0,3}8$ bo‘ladi.

$y=\log_7 x$ funksiya o‘suvchi hamda $0{,}28<0{,}31$ ekanidan $\log_7 0{,}28<\log_7 0{,}31$ bo‘ladi.

1-rasm



Logarifmik funksiya grafigi

7-misol. Funksiyaning aniqlanish sohasini toping: $y=\log_7\left(x^2-5x+6\right)$.

Yechish

Logarifmosti ifoda musbat bo‘lishi kerak, bundan

$$x^2-5x+6>0\Rightarrow(x-2)(x-3)>0\Rightarrow x\in(-\infty;2)\cup(3;\infty)$$

Javob: $D(y)=(-\infty;2)\cup(3;\infty)$.

8-misol. $y=\log_{4-x}\left(x^2-9\right)$ funksiyaning aniqlanish sohasini toping.

Yechish

$$\begin{cases}4-x>0\\4-x\neq 1\end{cases}\Rightarrow\begin{cases}x<4\\x\neq 3\end{cases}\qquad x^2-9>0\Rightarrow x\in(-\infty;-3)\cup(3;\infty)$$

Javob: $D(y)=(-\infty;-3)\cup(3;4)$.

9-misol. Funksiyaning grafigini yasang:

$$y=-1+\log_2(x-1).$$

Yechish

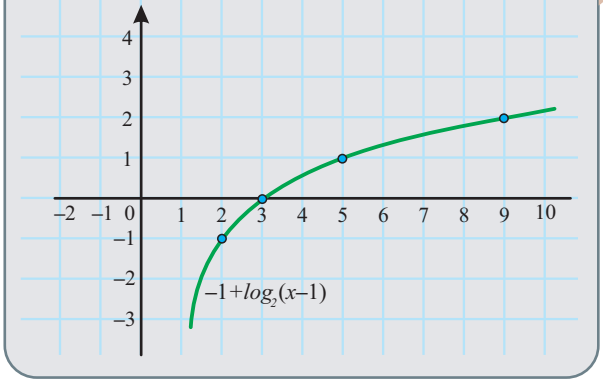
Aniqlanish sohasini topamiz:

$$x-1>0\Rightarrow x>1$$

Qiymatlar jadvalini tuzamiz:

x	2	3	5	9
y	-1	0	1	2

2-rasm



Topilgan nuqtalarni koordinata tekisligida belgilab, ularni egri chiziq bilan tutashtiramiz (2-*rasm*).

MISOLLAR

1. Berilgan funksiyalarning o‘suvchi yoki kamayuvchi ekanini aniqlang.

a) $y=\log_{0,075} x$ b) $y=\log_{\frac{\sqrt{3}}{2}} x$ c) $y=\lg x$

d) $y=\log_{11} x$ e) $y=-\log_{\frac{1}{e}} x$ f) $y=-\log_{\pi} x$

2. Taqqoslang.

a) $\log_e 0,5$ va $\log_e 0,35$ b) $\log_{0,1} 100$ va $\log_{0,1} 101$ c) $\log_{\frac{\sqrt{15}}{4}} \sqrt{37}$ va $\log_{\frac{\sqrt{15}}{4}} 6$

3. Sonlarni o‘sish tartibida joylashtiring.

a) $a=\log_{\frac{1}{5}} 10,\ b=\log_{\frac{1}{5}} 15,\ c=\log_{\frac{1}{5}} 20$

b) $a=\log_2 5,\ b=\log_{\frac{1}{4}} 3,\ c=\log_{\frac{1}{2}} 3$

c) $a=\log_{\frac{1}{6}} 4,\ b=\log_{\frac{1}{5}} 6,\ c=\log_{\frac{1}{5}} 4$

4. Ushbu mulohazalar to‘g‘ri ekanini misollar tuzib, tekshiring.

a) $a>1$ va $b>1$ bo‘lsa, $\log_a b>0$

b) $0<a<1$ va $0<b<1$ bo‘lsa, $\log_a b>0$

c) $a>1$ va $0<b<1$ bo‘lsa, $\log_a b<0$

d) $0<a<1$ va $b>1$ bo‘lsa, $\log_a b<0$

5. Berilgan sonlardan qaysilari musbat?

a) $a=\log_{0,2} 8$ b) $b=\log_3 0,8$ c) $c=\log_{0,9} 9$

d) $d=\log_4 2$ e) $p=\log_{0,9} 0,6$ f) $l=\log_{1,2} \frac{3}{8}$

g) $z=\log_{0,02} 0,001$ h) $p=\log_{|-13,08|} 2022$ i) $q=\log_{|-3|} 3$

6. $y=\log_2 x$ va $y=-\log_2 x$ funksiyalarning grafiklari abssissa o‘qiga nisbatan simmetrik ekanini ko‘rsating.

7. Funksiyaning aniqlanish sohasini toping.

a) $y=\log_4 x$ b) $y=\log_2(x-1)$

c) $y=\log_3(x^2-2x-3)$ d) $y=\log_4(x^2-4)$

e) $y=\lg(3-x)$ f) $y=-\log_2(x^2+5x-6)$

8. Funksiyaning grafigini yasang.

a) $y=\log_3 x$ b) $y=\log_{\frac{1}{2}} x$ c) $y=\lg x$ d) $y=\ln x$

9. Funksiyaning aniqlanish sohasini toping.

a) $y=\log_{x^2}(4-x)$ b) $f(x)=\log_{x^2}(x-1)+\sqrt{2-x}$

c) $f(x)=\sqrt{9-x^2}+\lg(x-1)-\sqrt{x}$ d) $f(x)=\sqrt{x+4}+\log_2(x^2-4)$

e) $f(x)=\dfrac{\log_{x^2+1}(6-x)}{\sqrt{x+2}}$ f) $y=\sqrt{2+\log_{\frac{1}{2}}(3-x)}$

10. Funksiyaning grafigini yasang.

a) $y=\log_2(x-1)$ b) $y=\log_3(5x+1)$ c) $y=\log_4(1-x)$

d) $y=\lg(x-3)$ e) $y=\log_6(3x-2)$ f) $y=1-\ln x$

g) $y=\log_8 x-4$ h) $y=\lg x+3$ i) $y=\log_6(x-2)-1$

11. Funksiya grafiklarining kesishish nuqtalari nechta?

a) $y=\log_2 x;\quad y=-x+1$ b) $y=\log_{\frac{1}{2}} x;\quad y=2x-5$ c) $y=\log_{\frac{1}{2}} x;\quad y=4x^2$

d) $y=\log_3 x;\quad y=2-\dfrac{1}{3}x^2$ e) $y=2^x;\quad y=\log_{0,5} x$ f) $y=\left(\dfrac{1}{3}\right)^x;\quad y=\log_3 x$

LOGARIFMIK IFODALARNI AYNIY ALMASHTIRISH

Logarifmik ifodalar ustida amallar bajarishda va ularni soddalashtirishda quyidagi ayniy almashtirishlardan foydalaniladi. Bu xossalarda qatnashadigan ifodalar logarifm aniqlangan bo‘lishi uchun talab etiladigan shartlarni qanoatlantiradi deb olamiz.

Logarifmning ta’rifidan uning quyidagi ***xossalari*** kelib chiqadi:

1°. $\log_a 1 = 0$.

2°. $\log_a a = 1$.

3°. $a^{\log_a b} = b$ $(a > 0, a \neq 1, b > 0)$.

4°. Ko‘paytmaning logarifmi ko‘paytuvchilar logarifmlarining yig‘indisiga teng:

$$\log_a (bc) = \log_a b + \log_a c.$$

5°. Bo‘linmaning logarifmi bo‘linuvchi va bo‘luvchi logarifmlarining ayirmasiga teng:

$$\log_a \left(\frac{b}{c}\right) = \log_a b - \log_a c.$$

6°. Darajaning logarifmi daraja ko‘rsatkichi bilan asos logarifmining ko‘paytmasiga teng:

$$\log_a b^p = p \log_a b.$$

7°. Bir asosdan boshqa asosga o‘tish formulasi: $\log_a b = \dfrac{\log_c b}{\log_c a}$.

8°. $\log_a b = \dfrac{1}{\log_b a}$.

9°. $\log_{a^k} b = \dfrac{1}{k} \log_a b$.

10°. $\log_{a^k} b^p = \dfrac{p}{k} \log_a b$.

Ko‘rsatkichli va logarifmik ifodalarni soddalashtirish

Logarifmning va logarifmik funksiyaning, shuningdek, darajaning va ko‘rsatkichli funksiyaning xossalari bilan tanishgan edik. Bu xossalardan logarifmik va ko‘rsatkichli ifodalarni soddalashtirishda foydalaniladi.

1-misol. Hisoblang: $\log_3 18 + \log_3 \dfrac{1}{54}$

Yechish

$$\log_3 18 + \log_3 \frac{1}{54} = \log_3 \left(18 \cdot \frac{1}{54}\right) = \log_3 \frac{1}{3} = -1$$

Javob: -1.

2-misol. Hisoblang: $3\log_2 8 - 2\log_3 9$

Yechish

$$3\log_2 8 - 2\log_3 9 = 3\log_2 2^3 - 2\log_3 3^2 = 3\cdot 3\cdot \log_2 2 - 2\cdot 2\cdot \log_3 3 = 9\cdot 1 - 4\cdot 1 = 5$$

Javob: 5.

3-misol. Hisoblang: $10^{1+\lg 5}$.

Yechish

$$10^{1+\lg 5} = 10^1 \cdot 10^{\lg 5} = 10\cdot 5 = 50$$

Javob: 50.

4-misol. Hisoblang: $\log_2 \log_5 \sqrt[8]{5}$.

Yechish

$$\log_2 \log_5 \sqrt[8]{5} = \log_2 \log_5 5^{\frac{1}{8}} = \log_2 \left(\frac{1}{8}\cdot \log_5 5\right) = \log_2 \frac{1}{8} = \log_2 2^{-3} = -3\log_2 2 = -3$$

Javob: –3.

5-misol. Hisoblang: $2^{\log_4\left(2-\sqrt{3}\right)^2} + 3^{\log_9\left(2+\sqrt{3}\right)^2}$.

Yechish

$$2^{\log_4\left(2-\sqrt{3}\right)^2} + 3^{\log_9\left(2+\sqrt{3}\right)^2} = 2^{\log_{2^2}\left(2-\sqrt{3}\right)^2} + 3^{\log_{3^2}\left(2+\sqrt{3}\right)^2} = 2^{\frac{1}{2}\cdot 2\log_2\left(2-\sqrt{3}\right)} + 3^{\frac{1}{2}\cdot 2\log_3\left(2+\sqrt{3}\right)} =$$

$$= 2^{\log_2\left(2-\sqrt{3}\right)} + 3^{\log_3\left(2+\sqrt{3}\right)} == 2-\sqrt{3}+2+\sqrt{3} = 4.$$

Javob: 4.

6-misol. Hisoblang: $\sqrt{5^{\frac{2}{\log_3 5}} + 0{,}5^{-\log_2 7}}$

Yechish

$$\sqrt{5^{\frac{2}{\log_3 5}} + 0{,}5^{-\log_2 7}} = \sqrt{5^{2\log_5 3} + \left(\frac{1}{2}\right)^{-\log_2 7}} = \sqrt{5^{\log_5 3^2} + 2^{\log_2 7}} = \sqrt{9+7} = \sqrt{16} = 4$$

Javob: 4.

7-misol. Hisoblang: $\dfrac{2}{1+\log_2 5} + \lg 25$

Yechish

$$\frac{2}{1+\log_2 5} + \lg 25 = \frac{2}{\log_2 2 + \log_2 5} + \lg 25 = \frac{2}{\log_2 (2\cdot 5)} + \lg 25 = \frac{2}{\log_2 10} + \lg 25 =$$

$$= 2\lg 2 + \lg 25 = \lg 2^2 + \lg 25 = \lg(4\cdot 25) = \lg 100 = 2$$

Javob: 2.

8-misol. $3^{2+\log_3 2}$ ni hisoblang.

Yechish. $a^{m+n}=a^n\cdot a^m$ va $a^{\log_a b}=b$ tengliklardan foydalanamiz:

$$3^{2+\log_3 2}=3^2\cdot 3^{\log_3 2}=9\cdot 2=18.$$

Ko‘rsatkichli va logarifmik ifodalarni soddalashtirishda keng qo‘llanadigan

$$\boxed{a^{\log_b c}=c^{\log_b a}}$$

tenglikni isbotlaymiz. Bu yerda a, b, $c>0$ va $b\neq 1$. Mazkur shartlar bajarilganda $\log_b c$ va $\log_b a$ ifodalar ma’noga ega bo‘ladi. Ravshanki,

$$\log_b c\log_b a=\log_b a\log_b c$$

tenglik o‘rinli. Bu ayniyatdan logarifmning $n\log_p q=\log_p q^n$ xossasiga ko‘ra

$$\log_b a^{\log_b c}=\log_b c^{\log_b a}$$

tenglik kelib chiqadi. Bundan esa

$$a^{\log_b c}=c^{\log_b a}$$

tenglikning o‘rinli bo‘lishini xulosa qilish mumkin.

9-misol. Agar $a=sin\frac{\pi}{6}$ bo‘lsa, $\log_4 a$ ni hisoblang.

Yechish

$a=sin\frac{\pi}{6}=\frac{1}{2}$ bo‘lgani uchun $\log_4 a=\log_4\frac{1}{2}=\log_{2^2}2^1=-\frac{1}{2}\log_2 2=-\frac{1}{2}\cdot 1=-\frac{1}{2}$

Javob: $-\frac{1}{2}$.

10-misol. Hisoblang: $\dfrac{\log_2^2 14+\log_2 14\cdot\log_2 7-2\log_2^2 7}{\log_2 14+2\log_2 7}$

Yechish

Suratni ko‘paytuvchilarga ajratib, quyidagini hosil qilamiz:

$$\frac{\log_2^2 14+\log_2 14\cdot\log_2 7-2\log_2^2 7}{\log_2 14+2\log_2 7}=\frac{(\log_2 14+2\log_2 7)(\log_2 14-\log_2 7)}{\log_2 14+2\log_2 7}=$$

$$=\frac{\cancel{(\log_2 14+2\log_2 7)}(\log_2 14-\log_2 7)}{\cancel{\log_2 14+2\log_2 7}}=\log_2 14-\log_2 7=\log_2\frac{14}{7}=\log_2 2=1$$

Javob: 1.

11-misol. $f(x)=\log_4\dfrac{x^2}{4}-2\log_4(4x^4)$ ifodani soddalashtiring va uning $x=-2$ dagi qiymatini toping.

Yechish

Berilgan ifoda ma’noga ega bo‘lishi uchun $x \neq 0$ bo‘lishi talab etiladi. Quyidagi tengliklar logarifmning xossalaridan kelib chiqadi:

$$f(x) = \log_4 \frac{x^2}{4} - 2\log_4\left(4x^4\right) = \log_4 x^2 - \log_4 4 - 2\left(\log_4 4 + \log_4 x^4\right) =$$

$$= 2\log_4|x| - 1 - 2\left(1 + 4\log_4|x|\right) = -6\log_4|x| - 3$$

$$f(-2) = -6\log_4|-2| - 3 = -6\log_{2^2} 2 - 3 = -\frac{6}{2}\log_2 2 - 3 = -3 - 3 = -6$$

12-misol. Agar $a = \log_{98} 112$ bo‘lsa, $\log_7 2$ ni a orqali ifodalang.

Yechish

$$a = \log_{98} 112 = \frac{\log_7 112}{\log_7 98} = \frac{\log_7\left(7\cdot 2^4\right)}{\log_7\left(7^2\cdot 2\right)} = \frac{\log_7 7 + \log_7 2^4}{\log_7 7^2 + \log_7 2} = \frac{1 + 4\log_7 2}{2 + \log_7 2}$$

$$\frac{1 + 4\log_7 2}{2 + \log_7 2} = a$$

$$1 + 4\log_7 2 = 2a + a\log_7 2$$

$$4\log_7 2 - a\log_7 2 = 2a - 1$$

$$(4 - a)\log_7 2 = 2a - 1$$

$$\log_7 2 = \frac{2a-1}{4-a}\ .$$

MISOLLAR

1. Logarifmik ifodalarning qiymatini toping.

a) $\log_2 4$ b) $\log_2 1$ c) $\log_2 16$ d) $\log_4 16$

e) $\log_2 64$ f) $\log_8 64$ g) $\log_4 64$ h) $\log_{64} 64$

2. Logarifmik ifodalarning qiymatini toping.

a) $\log_5 25$ b) $\log_{324} 18$ c) $\log_{128} 4$ d) $\log_{10}(0{,}001)$

e) $\log_9 3$ f) $\lg 1000$ g) $\ln e$ h) $\lg\left(\frac{1}{100}\right)$

3. Hisoblang.

a) $\log_2 8 + \log_2 4$ b) $\log_3 6 + \log_3 \frac{3}{2}$ c) $\log_2 15 - \log_2 \frac{15}{16}$

d) $\log_{\frac{1}{3}} 54 - \log_{\frac{1}{3}} 2$ e) $\log_{0,2} 75 - \log_{0,2} 3$ f) $\log_{36} 9 + \log_{36} 4$

4. Ushbu sonlardan qaysi biri qolgan uchtasiga teng emas?

a) $m = 2\log_2 8 - \log_2 4$ b) $n = \log_2 400 - 2\log_2 5$

c) $p = \log_5 125 + \log_5 5$ d) $q = \ln 12e - \ln 2$

5. Quyidagi sonlardan qaysi biri 2 dan kichik?

a) $M = \log_5 100 - \log_5 4$

b) $N = 4\log_2 3 - \log_2 9$

c) $P = \log_6 72 - \log_6 2$

d) $Q = \log_4 16 + \log_4 \frac{1}{8}$

6. Hisoblang.

a) $3 - \lg 50 + \frac{1}{2}\lg 25$

b) $\log_2 32 + \log_{32} 2$

c) $\frac{\log_4 13 + \log_4 25}{\log_{64} 325}$

d) $\frac{\log_4 11 + \log_4 23}{\log_8 253}$

e) $\frac{1}{\log_8 12} + \frac{1}{\log_{18} 12}$.

f) $\frac{1}{\log_{45} 15} + \frac{1}{\log_5 15}$

7. Hisoblang.

a) $81^{\log_3 5}$

b) $4^{-2\log_{\frac{1}{4}} 3}$

c) $32^{\log_8 27}$

d) $121^{\log_{11} 12}$

e) $3\log_{\sqrt{8}} 2 + 2^{-2\log_{\frac{1}{2}} 2}$

f) $3\log_{\sqrt{64}} 4 + 4^{-2\log_{\frac{1}{4}} 3}$

8. a ning berilgan qiymati uchun ifoda qiymatini hisoblang.

a) $3\log_{\frac{1}{3}} a,\ a = 2\cos\frac{\pi}{6}$.

b) $3\log_{\frac{1}{3}} a,\ a = \operatorname{tg}\frac{\pi}{3}$.

c) $4\log_3 a,\ a = \operatorname{tg}\frac{\pi}{3}$

9. Hisoblang.

a) $\log_{\frac{1}{3}} \log_3 27$

b) $\log_4 \log_3 \sqrt{81}$

c) $\log_3^2 \log_{\frac{1}{5}} \frac{1}{125}$

10. Hisoblang.

a) $\frac{2\log_3 12 - 4\log_3^2 2 + \log_3^2 12 + 4\log_3 2}{3\log_3 12 + 6\log_3 2}$

b) $\frac{\log_2^2 28 + \log_2 28 \cdot \log_2 7 - 2\log_2^2 7}{\log_2 28 + 2\log_2 7}$

c) $\frac{\log_{35}^2 7 - 2\log_{35} 7 \cdot \log_{35} 5 - 3\log_{35}^2 7}{2(\log_{35} 7 - 3\log_{35} 5)}$

d) $\frac{\log_2^2 12 - 2\log_2 12 + 2\log_2^2 3 - 3\log_2 3 \cdot \log_2 12 + 4\log_2 3}{\log_2 12 - 2\log_2 3}$

11. Quyidagi funksiyalarning aniqlanish sohalarini toping.

a) $y=\log_2(x+3)$ b) $y=\log_{0,2}(x^2-4x)$

c) $y=\log_{0,7}\left(2x-\frac{1}{8}\right)$ d) $y=\log_2(5-3x)$

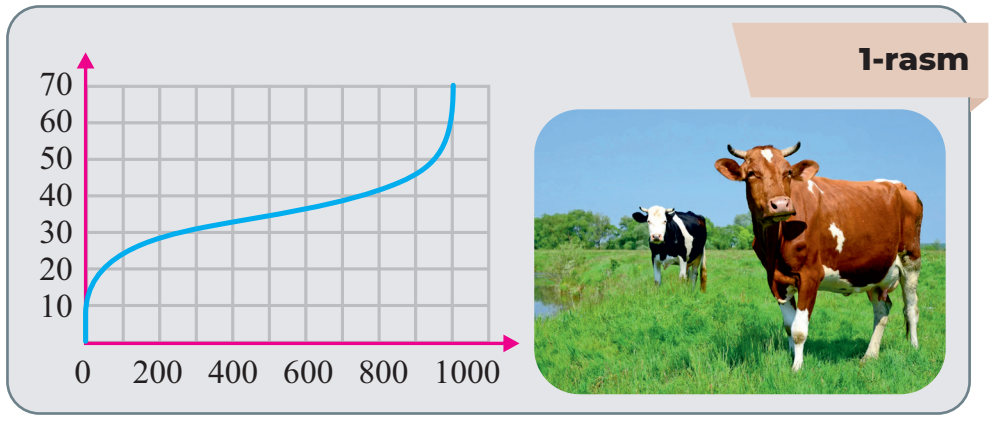
12. a orqali ifodalang .

a) $a=\log_2 3$ bo‘lsa, $\log_{36}108=?$ b) $a=\log_7 3$ bo‘lsa, $\log_{147}63=?$

c) $a=\log_{288}72$ bo‘lsa, $\log_3 2=?$ d) $a=\log_{441}189$ bo‘lsa, $\log_3 7=?$

13. Agar chorvadorning 1 000 bosh sigiridan bittasi yuqumli kasallikka chalingan bo‘lsa, u holda t kunda n ta sigirning kasallanish ko‘rsatkichi $t=-5\cdot\ln\left(\frac{1000-n}{999n}\right)$ formula bilan modellashtirilgan. 100 ta, 800 ta, 1 000 ta sigir necha kunda kasallanishini toping. Chizma asosida xulosa tayyorlang (1-*rasm*).

1-rasm



14. Jadvallar asosida funksiya grafigini yasang.

a)

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y=\log_2 x$	-2	-1	0	1	2	3

b)

x	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
$y=\log_{\frac{1}{2}} x$	2	1	0	-1	-2	-3

15. Quyidagi funksiyalarga teskari funksiyalarni aniqlang.

a) $f(x)=10^x$ b) $f(x)=\log_3(x+1)$

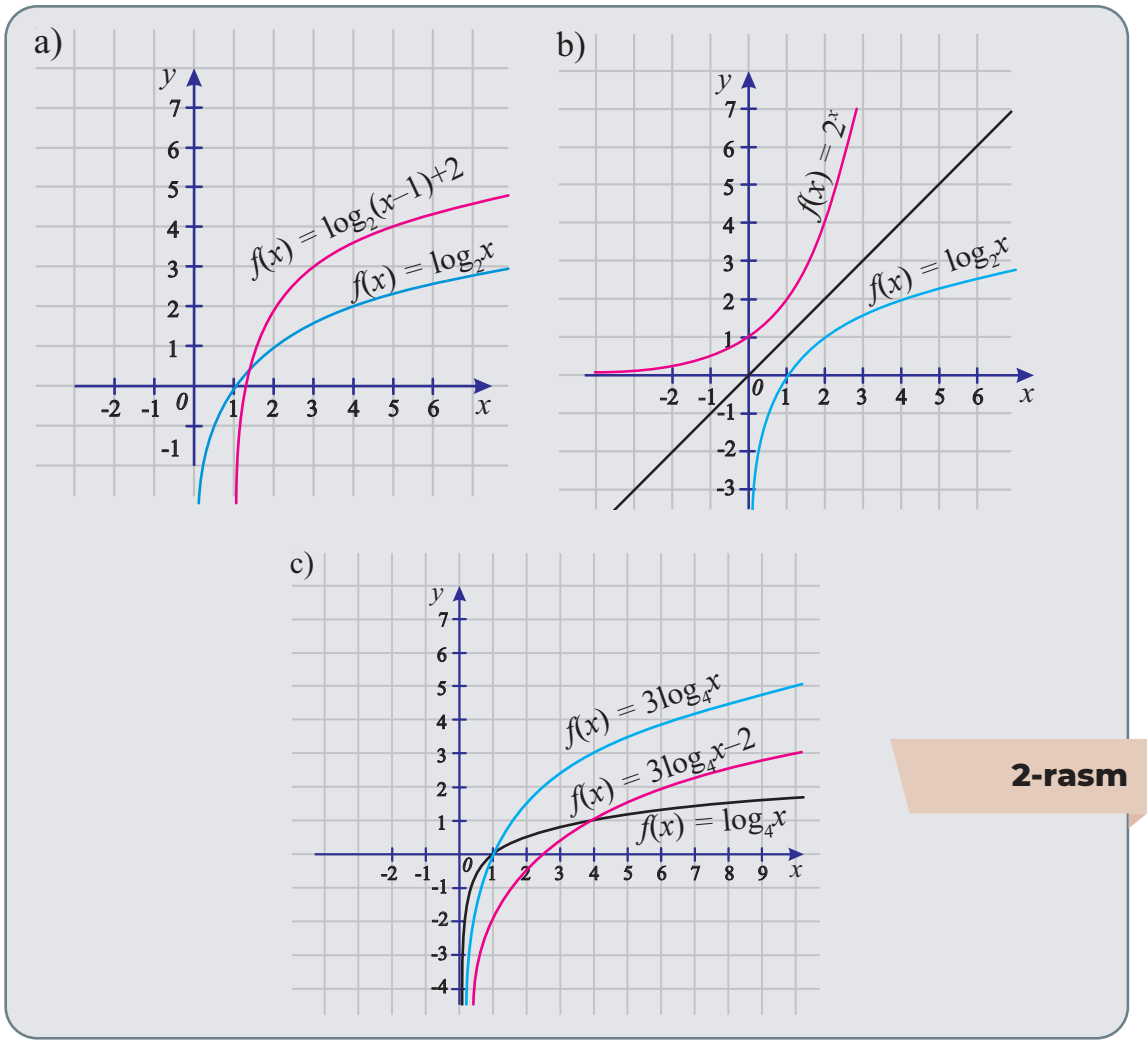
c) $f(x)=2+e^{x+4}$ d) $f(x)=5+\log_2(x-3)$

16. a va b larning qiymatini toping.

a) $\log_3 b=2$ b) $\log_a 8=3$ c) $\log b^2=\lg 4$ d) $\log_a 36=2$

17. $y=\ln e^x$ va $y=e^{\ln x}$ funksiyalarining grafigini yasang. O‘xshashlik va farqlarini tushuntiring.

18. 2-rasmda funksiyalar ustida qanday almashtirishlar bajarilganini bayon qiling.



2-rasm

Logarifmik funksiyaning hayotda qo‘llanishi

Tovush intensivligi darajasi

Yuza birligi orqali vaqt birligida tovush to‘lqini olib o‘tayotgan energiya *tovushning intensivligi* deb ataladi. Elastik muhit bo‘ylab tovush tarqalganda u tarqalmagan paytdagiga nisbatan ortiqcha bosim hosil bo‘ladi, u *tovushning bosimi* deyiladi. Tovushning intensivligi tovush bosimining amplitudasiga hamda muhit xossasiga va to‘lqin shakliga bog‘liq. Ovoz balandligining intensivligi detsibelda (dB) o‘lchanadi.

I – tovushning intensivligi

I_0 – tovushning nisbiy intensivligi

L – ovoz intensivligining balandligi

$$L = 10\log_{10}\left(\frac{I}{I_0}\right) dB$$



Smartfon quloqliklariga uzatadigan tovush intensivligi 100 detsibeldan oshadi. Odam qulog‘i uchun 80 detsibeldan yuqori bo‘lgan tovush balandligi eshitish qobiliyatining buzilishiga yoki yo‘qolib borishiga olib keladi.

LOGARIFMIK TENGLAMALAR

Logarifmik tenglamalar

Noma’lum logarifmosti ifodada yoki logarifm asosida qatnashgan tenglama ***logarifmik tenglama*** deyiladi. Masalan, $\log_2 x = 3$, $\log_x 625 = 2$, $\log_x(x+2) = 2$, $\lg(2x-2) = \lg(x+2)$ tenglamalar logarifmik tenglamaga misol bo‘la oladi.

Noma’lumning berilgan logarifmik tenglamani to‘g‘ri tenglikka aylantiradigan qiymati logarifmik ***tenglamaning ildizi*** deyiladi.

Sodda logarifmik tenglamalarni yechish

$a > 0$, $a \neq 1$ bo‘lganda ushbu $\log_a x = b$ tenglama eng sodda logarifmik tenglama bo‘ladi. Bu tenglamaning yechimi $x = a^b$ bo‘ladi.

Logarifmik tenglamalarni yechishda ushbu qoida ishlatiladi:

$a > 0, a \neq 1$ bo‘lganda $\log_a f(x) = \log_a g(x)$ tenglamaning ildizlari $f(x) = g(x)$ tenglamaning $f(x) > 0$ (yoki $g(x) > 0$) shartni qanoatlantiruvchi ildizlaridan iborat bo‘ladi.

Quyida logarifmik tenglamalarni yechishning namunalarini keltiramiz.

1-misol. $\log_5 x = -2$ tenglamani yeching.

Yechish

Tenglamani yechishda $x > 0$ shart ostida logarifm ta’rifidan foydalanamiz:

$$\log_5 x = -2 \Rightarrow x = 5^{-2} \Rightarrow x = \frac{1}{25}$$

$x = \frac{1}{25} > 0$ ekanidan topilgan bu qiymat berilgan tenglamaning ildizi bo‘ladi.

Javob: $x = \frac{1}{25}$.

2-misol. $\log_3(x^2 - 4) = \log_3(5x - 8)$ logarifmik tenglamani yeching.

Yechish

Aniqlanish sohasini topamiz:

$$\begin{cases} x^2 - 4 > 0 \\ 5x - 8 > 0 \end{cases} \Rightarrow \begin{cases} (x-2)(x+2) > 0 \\ 5x > 8 \end{cases} \Rightarrow \begin{cases} x \in (-\infty; -2) \cup (2; \infty) \\ x > 1,6 \end{cases} \Rightarrow x \in (2; \infty)$$

Endi $x^2 - 4 = 5x - 8$ tenglamani yechamiz:

$$x^2 - 5x + 4 = 0 \Rightarrow (x-1)(x-4) = 0 \Rightarrow x_1 = 1,\ x_2 = 4$$

Noma’lumning $x_1 = 1$ qiymati $(2; \infty)$ to‘plamga tegishli emas, $x_2 = 4$ qiymati esa bu to‘plamga tegishli. Demak, $x_1 = 1$ qiymat berilgan tenglamaning chet ildizi, $x_2 = 4$ esa berilgan tenglamaning ildizi bo‘ladi.

Javob: $x = 4$.

3-misol. $\log_5^2 x - 3\log_5 x - 4 = 0$ logarifmik tenglamani yeching.

Yechish

Dastlab $x > 0$ aniqlanish sohasi bo‘lishini aniqlaymiz va $\log_5 x = t$ belgilash kiritamiz. U holda

$$t^2 - 3t - 4 = 0 \Rightarrow (t+1)(t-4) = 0 \Rightarrow t_1 = -1,\ t_2 = 4.$$

Demak, $\log_5 x = -1$ va $\log_5 x = 4$. Bundan $x_1 = \frac{1}{5} = 0,2$; $x_2 = 5^4 = 625$.

Javob: $x_1 = 0,2;\ x_2 = 625.$

4-misol. $\log_{x-1} 16 = 2$ tenglamani yeching.

Yechish

Dastlab

$$\begin{cases} x-1 > 0 \\ x-1 \neq 1 \end{cases} \Rightarrow \begin{cases} x > 1 \\ x \neq 2 \end{cases} \Rightarrow x \in (1;\ 2) \cup (2;\ \infty)$$

to‘plamga tegishli bo‘lishi kerak. Logarifm ta’rifidan foydalanamiz:

$\log_{x-1} 16 = 2 \quad \Rightarrow (x-1)^2 = 16$

$x - 1 = 4 \quad \Rightarrow x_1 = 5$

$x - 1 = -4 \quad \Rightarrow x_2 = -3$

Javob: $x = 5$.

5-misol. $\log_5 \log_2 \log_7 x = 0$ tenglamani yeching.

Yechish

Tenglamani yechishda logarifm ta’rifidan foydalanamiz:

$$\log_2 \log_7 x = 5^0 \Rightarrow \log_2 \log_7 x = 1 \Rightarrow \log_7 x = 2^1 \Rightarrow x = 7^2 = 49$$

Javob: $x = 49.$

6-misol. $\lg\left(x^2 - 3\right) \cdot \lg x = 0$ tenglamani yeching.

Yechish

Har bir ko‘paytuvchini 0 ga tenglashtiramiz:

$$\lg\left(x^2 - 3\right) = 0 \Rightarrow x^2 - 3 = 1 \Rightarrow x^2 = 4 \Rightarrow x_{1,2} = \pm 2$$

$$\lg x = 0 \Rightarrow x_3 = 1$$

Aniqlanish sohasiga ko‘ra, $x^2 - 3 > 0$ va $x > 0$ bo‘lishi kerak. Shu bois $x = 2$ ildiz bo‘la oladi.

Javob: $x = 2.$

MISOLLAR

1. Logarifmik tenglamalarni yeching.

a) $\log_2 x = -3$

b) $\log_4 2x = \frac{1}{2}$

c) $\lg \frac{5x}{2} = 1$

d) $\log_{\frac{1}{4}} x = -2$

e) $\log_3 (3x-1) = 2$

f) $\log_7 (x+3) = 2$

g) $\log_9 x^3 + \log_{\sqrt{3}} x = 3$

h) $\log_4 (2x-3) = 4$

i) $\log_2 x - 2\log_{\frac{1}{2}} x = 9$

2. Logarifmik tenglamalarni yeching.

a) $\log_5 x = 2\log_5 3 + 4\log_{25} 2$

b) $\log_{\frac{1}{2}} (7-8x) = 2$

c) $\log_2 x + \log_8 x = 0$

d) $\log_3 x = 9\log_{27} 8 - 3\log_3 4$

e) $\log_{0,5} (3x+1) = -2$

f) $\log_{0,2} (x+3) = -1$

g) $\log_{0,25} (x+30) = -2$

h) $\log_{\sqrt{3}} (1-2x) = 4$

i) $\log_2 \sqrt{x-1} = 1$

j) $\log_3 (x^2-4x+3) = \log_3 (3x+21)$

k) $\log_3 (2x-5) = \log_3 (20-3x)$

l) $\log_7 (9x-1) = \log_7 x$

m) $\log_3 (2x^2-3x) = 2\log_3 x$

n) $\lg(2x) = 2\lg(4x-15)$

3. $\lg(3x-11) + \lg(x-27) = 3$

4. $\log_{81} x - 2\log_3 x + 5\log_9 x = 1,5$

5. $\log_3 ((x-1)(2x-1)) = 0$

6. $3\lg x^2 - \lg^2 x = 9$

7. $\log_{\frac{1}{3}} \frac{x^2+4x}{2x-3} = 1$

8. $\log_{\frac{3}{4}} \frac{2x-1}{x+2} = 1$

9. $\log_\pi (\log_2 (\log_3 3x)) = 0$

10. $\log_2^2 x + 3 = \log_2 x^2$

11. $(x^2-6x-7)\log_2 (3x-1) = 0$

12. $(x^2-2x-15)\lg(4x-3) = 0$

13. $\log_5 (x+4) - \log_5 (1-2x) = -\log_5 (2x+3)$

14. $\log_2^2 x - 5\log_2 x = 4$

15. $\log_3 x + \log_x 9 = 3$

16. $\log_{x+2} 7 + 3\log_7 (x+2) = 4$

17. $\log_3 x \cdot \log_9 x \cdot \log_{27} x \cdot \log_{81} x = \frac{2}{3}$

18. $\log_5 \sqrt{x-9} + \log_5 \sqrt{2x-1} = \log_5 10$

KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI

Ko‘rsatkichli tenglamalar sistemasi va uni yechish

Ko‘rsatkichli ifoda qatnashgan tenglamalarni o‘z ichiga olgan tenglamalar sistemasi ***ko‘rsatkichli tenglamalar sistemasi*** deyiladi. Ko‘rsatkichli tenglamalar sistemasi turli xil ko‘rinishda bo‘ladi. Bunday sistemaning har birini yechishda o‘ziga xos yondashuv talab etiladi. Bunda ko‘rsatkichli va logarifmik ifodalarning xossalari keng qo‘llanadi.

1-misol. $\begin{cases} 3^x = 9^{y+1}, \\ 4y = 5 - x \end{cases}$ tenglamalar sistemasini yeching.

Yechish

$9 = 3^2$ ekanidan foydalanamiz. U holda $3^x = 3^{2(y+1)}$ bo‘lib, bu yerdan $x = 2y + 2$ kelib chiqadi. Sistemadagi ikkinchi tenglikda x o‘rniga $2y + 2$ ifodani qo‘yamiz:

$4y = 5 - (2y + 2) \Rightarrow y = \frac{1}{2}$. Endi $x = 2y + 2$ tenglikdagi y o‘rniga uning qiymatini qo‘yib, x ning qiymatini topamiz: $x = 2 \cdot \frac{1}{2} + 2 \Rightarrow x = 3$.

Javob: $\left(3;\ \frac{1}{2}\right)$.

2-misol. $\begin{cases} 9^{x+y} = 729, \\ 3^{x-y-1} = 1 \end{cases}$ tenglamalar sistemasini yeching.

Yechish

$729 = 9^3$ va $1 = 3^0$ ekanidan foydalanamiz. U holda

$$\begin{cases} 9^{x+y} = 9^3, \\ 3^{x-y-1} = 3^0 \end{cases} \Rightarrow \begin{cases} x + y = 3, \\ x - y - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 2, \\ y = 1 \end{cases} \Rightarrow (2;\ 1).$$

Javob: (2; 1).

3-misol. $\begin{cases} x^{y+1} = 27, \\ x^{2y-5} = \frac{1}{3} \end{cases}$ tenglamalar sistemasini yeching.

Yechish

Tenglamalar sistemasining berilishidan $x > 0$, $x \neq 1$ shartlar bajarilishi kelib chiqadi. Shuning uchun birinchi va ikkinchi tenglamalarning chap va o‘ng tarafidagi ifodalarni logarifmlash mumkin. Bu ifodalarni 3 asosga ko‘ra logarifmlaymiz va quyidagilarga ega bo‘lamiz:

$$\begin{cases} (y+1)\log_3 x = 3, \\ (2y-5)\log_3 x = -1 \end{cases} \Rightarrow \begin{cases} \log_3 x = \frac{3}{y+1}, \\ (2y-5)\frac{3}{y+1} = -1 \end{cases} \Rightarrow \begin{cases} x = 3, \\ y = 2 \end{cases}$$

Javob. (3; 2).

4-misol. $\begin{cases} 2^x + 2^y = 5, \\ 2^{x+y} = 4 \end{cases}$ tenglamalar sistemani yeching.

Yechish.

Birinchi tenglikdan $2^y = 5 - 2^x$ bog‘lanishni topamiz. $2^{x+y} = 2^x \cdot 2^y$ tenglikni e’tiborga olib, ikkinchi tenglikni $2^x \cdot 2^y = 4$ holatiga keltiramiz, bu yerdan $2^x\left(5 - 2^x\right) = 4$, undan esa $2^{2x} - 5 \cdot 2^x + 4 = 0$ tenglamaga ega bo‘lamiz. $t = 2^x$ belgilash kiritib, $t^2 - 5t + 4 = 0$ kvadrat tenglamaga ega bo‘lamiz. Bu yerda $t > 0$.

Bu kvadrat tenglamaning yechimi $t_1 = 1$, $t_2 = 4$ bo‘lib, x va y noma’lumlarning ularga mos qiymatlari

$t_1 = 1$: $1 = 2^{x_1} \Rightarrow x_1 = 0$; $2^{y_1} = 5 - 2^0 = 4, \Rightarrow y_1 = 2$

$t_2 = 4$: $4 = 2^{x_2} \Rightarrow x_2 = 2$; $2^{y_2} = 5 - 2^2 = 1, \Rightarrow y_2 = 0$ bo‘ladi.

Javob: (0; 2) va (2; 0).

Izoh. Yuqoridagi misollar har bir ko‘rsatkichli tenglamalar sistemasini yechish uchun ijodiy yondashish kerakligini ko‘rsatadi.

Logarifmik tenglamalar sistemasi va uni yechish

Logarifmik ifoda qatnashgan tenglamalarni o‘z ichiga olgan sistema ***logarifmik tenglamalar sistemasi*** deyiladi. Logarifmik tenglamalar sistemasi ham ko‘rsatkichli tenglamalar sistemasi kabi turli xil ko‘rinishda bo‘ladi. Ularning har birini yechishda ko‘rsatkichli va logarifmik ifodalarning xossalari keng qo‘llanadi hamda o‘ziga xos yondashuv talab etiladi.

5-misol. $\begin{cases} \log_9 \dfrac{x^2}{\sqrt{y}} = \dfrac{1}{2}, \\ \log_3 xy = 3 \end{cases}$ tenglamalar sistemasini yeching.

Yechish

Sistemadagi logarifmik ifodalar ma’noga ega bo‘lishi uchun

$$\begin{cases} \dfrac{x^2}{\sqrt{y}} > 0, \\ xy > 0 \end{cases}$$

tengsizliklar bajarilishi talab etiladi. $\dfrac{x^2}{\sqrt{y}} > 0$ tengsizlik $y > 0$ va $x \neq 0$ bo‘lgandagina o‘rinli. U holda sistemadagi ikkinchi $xy > 0$ tengsizlikdan $x > 0$ va $y > 0$ bo‘lishi zarurligi kelib chiqadi.

Endi logarifm xossalaridan foydalanib $x > 0$ va $y > 0$ bo‘lganda berilgan sistemani

$$\begin{cases} \log_9 x^2 - \log_9 \sqrt{y} = \dfrac{1}{2}, \\ \log_3 x + \log_3 y = 3 \end{cases}$$

kabi qayta yozish mumkin. $\log_9 x^2 = \log_{3^2} x^2 = \log_3 x$ hamda $\log_9 \sqrt{y} = \log_{3^2} y^{\frac{1}{2}} = \dfrac{1}{4}\log_3 y$

tengliklardan foydalansak, sistema ushbu ko‘rinishga keladi $\begin{cases} \log_3 x - \frac{1}{4}\log_3 y = \frac{1}{2}, \\ \log_3 x + \log_3 y = 3. \end{cases}$

Ikkinchi tenglamadan birinchi tenglamani ayirib, $\frac{5}{4}\log_3 y = \frac{5}{2}$ tenglikka ega bo‘lamiz. Bundan $\log_3 y = 2 \Rightarrow y = 9$ ekani kelib chiqadi. Endi sistemaning ikkinchi tenglamasiga y ning bu qiymatini qo‘yib, x noma’lumni topamiz:

$$\log_3 x + \log_3 9 = 3 \Rightarrow \log_3 x + 2 = 3 \Rightarrow \log_3 x = 1 \Rightarrow x = 3.$$

Javob: (3; 9).

6-misol. $\begin{cases} x^{\lg y} = 1000, \\ \log_y x = 3 \end{cases}$ tenglamalar sistemasini yeching.

Yechish

Sistemadagi ifodalar ma’noga ega bo‘lishi uchun $x > 0$, $x \neq 1$, $y > 0$, $y \neq 1$ shartlar bajarilishi zarur. $x^{\lg y} = 1000$ tenglikni 10 asosga ko‘ra logarifmlaymiz:

$$\lg x^{\lg y} = \lg 1000 \Rightarrow \lg y \lg x = 3$$

$\log_y x = 3$ tenglikning chap tomonidagi logarifmning asosini 10 asosga almashtiramiz:

$$\log_y x = \frac{\lg x}{\lg y} \Rightarrow \frac{\lg x}{\lg y} = 3 \Rightarrow \lg x = 3\lg y$$

Natijada $\lg^2 y = 1$ tenglamaga ega bo‘lamiz.

Bundan

$$\lg y = -1 \Rightarrow y = \frac{1}{10}, \quad \lg y = 1 \Rightarrow y = 10$$

ekani kelib chiqadi.

$\lg x = 3\lg y$ tenglikdan $x = y^3$ bog‘lanishni hosil qilib, x ning mos qiymatlarini topamiz:

$$y = \frac{1}{10} \Rightarrow x = \frac{1}{1000}$$
$$y = 10 \Rightarrow x = 1000$$

Javob: $\left(\frac{1}{1000}; \frac{1}{10}\right)$ va (1000; 10).

7-misol. $\begin{cases} \log_2 (x - y) = 1, \\ 2^x \cdot 3^{y+1} = 72 \end{cases}$ tenglamalar sistemasini yeching.

Yechish

Sistema aniqlangan bo‘lishi uchun $x - y > 0$, ya’ni $x > y$ bo‘lishi kerak. U holda sistemaning birinchi tenglamasidan

$$x - y = 2 \Rightarrow y = x - 2$$

bog‘lanish kelib chiqadi. Sistemaning ikkinchi tenglamasida y o‘rniga $x - 2$ ifodani qo‘yamiz:

$$2^x \cdot 3^{x-2+1} = 72 \Rightarrow 2^x \cdot 3^x = 3 \cdot 72 \Rightarrow 6^x = 216 \Rightarrow 6^x = 6^3 \Rightarrow x = 3$$

U holda $y = 1$.

Javob: (3; 1).

MISOLLAR

1. Tenglamalar sistemasini yeching.

a) $\begin{cases} 3^x \cdot 7^y = 63 \\ 3^x + 7^y = 16 \end{cases}$ b) $\begin{cases} 9^x - 3 \cdot 5^y = 3 \\ 9^x \cdot 5^y = 18 \end{cases}$ c) $\begin{cases} 3^y \cdot 2^x = 972 \\ y - x = 3 \end{cases}$

d) $\begin{cases} 4^{x+y} = 128 \\ 5^{3y-2x-3} = 1 \end{cases}$ e) $\begin{cases} 3^{-x} \cdot 2^y = 1152 \\ x + y = 5 \end{cases}$ f) $\begin{cases} 2^x \cdot 9^y = 648 \\ 3^x \cdot 4^y = 432 \end{cases}$

g) $\begin{cases} 3^x \cdot 2^y = \dfrac{1}{9} \\ \dfrac{1}{9} \cdot 3^y = 3^x \end{cases}$ h) $\begin{cases} 2^y \cdot 8^{-x} = 8\sqrt{2} \\ y + 3x = \dfrac{1}{2} \end{cases}$ i) $\begin{cases} 4^{y-1} \cdot 5^x = 6400 \\ y - x = 3 \end{cases}$

2. Tenglamalar sistemasini yeching.

a) $\begin{cases} \log_7 7x + \log_7 y = 2 \\ y - 5x = 2 \end{cases}$ b) $\begin{cases} \log_2 x - \log_2 y = 4 \\ y - x = 6 \end{cases}$ c) $\begin{cases} \log_2 (x^2 + y^2) = 5 \\ \log_2 x + \log_2 y = 4 \end{cases}$

d) $\begin{cases} \log_3 2x - \log_3 \left(\dfrac{2}{y}\right) = 1 \\ 4x - y = 1 \end{cases}$ e) $\begin{cases} \log_2 x - \log_4 y = 0 \\ \log_4 x + \log_2 y = 5 \end{cases}$ f) $\begin{cases} \log_2 2x + \log_2 \left(\dfrac{y}{2}\right) = -1 \\ x - y = -\dfrac{7}{4} \end{cases}$

3. Tenglamalar sistemasini yeching.

a) $\begin{cases} 3^x - 2^{y^2} = 77 \\ 3^{\frac{x}{2}} - 2^{\frac{y^2}{2}} = 7 \end{cases}$ b) $\begin{cases} 3 \cdot 2^x - 2^{x+y} = -2 \\ 5 \cdot 2^{x+1} - 2^{x+y+1} = 4 \end{cases}$ c) $\begin{cases} 9^x - 3 \cdot 2^y = 3 \\ 9^x \cdot 2^y = 18 \end{cases}$

d) $\begin{cases} \lg x (\lg x + \lg y) = 2 \\ \lg x - \lg y = 3 \end{cases}$ e) $\begin{cases} 3^x \cdot 25^y = 5625 \\ 5^x \cdot 9^y = 2025 \end{cases}$

4. $\begin{cases} x^{\sqrt{y}} = y \\ y^{\sqrt{y}} = x^4 \end{cases}$ sistemaning ildizlarini ifodalovchi nuqtalar orasidagi masofani toping.

5. $\begin{cases} 3^x \cdot 2^y = 972, \\ \log_{\sqrt{3}} (x - y) = 2 \end{cases}$ x va y tenglamalar sistemasi ildizlari bo‘lsa, xy ni toping.

6. $\begin{cases} x^{y+1} = 27, \\ x^{2y-5} = \dfrac{1}{3} \end{cases}$ x va y tenglamalar sistemasi ildizlari bo‘lsa, $x + y$ ni toping.

LOGARIFMIK TENGSIZLIKLAR

Logarifmik tengsizliklar

$a>0$ va $a\neq 1$ bo‘lsin. U holda

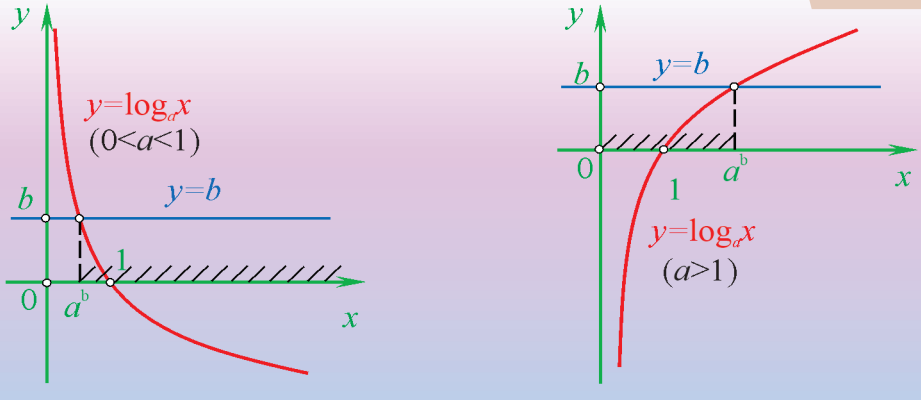
$$\log_a x<b,\ \log_a x>b,\ \log_a x\le b,\ \log_a x\ge b$$

tengsizliklar logarifmik tengsizliklar bo‘ladi. Ularni yechishda $y=\log_a x$ funksiyaning monotonligidan foydalaniladi.

$\log_a x<b$ tengsizlikni qaraylik. Bu tengsizlikning yechimi x o‘zgaruvchining shunday qiymatlari to‘plamiki, bu qiymatlarda $y=\log_a x$ funksiyaning Oxy koordinatalar sistemasidagi grafigi $y=b$ to‘g‘ri chiziqdan pastda joylashgan bo‘ladi.

$\log_a x<b$ tengsizlik yechimining geometrik talqini

1-rasm



a) $\log_a x<b$ *tengsizlikning* $0<a<1$ *bo‘lgandagi yechimi* $(a^b;+\infty)$ *oraliqdan iborat.*

b) $\log_a x<b$ *tengsizlikning* $a>1$ *bo‘lgandagi yechimi* $(0;a^b)$ *oraliqdan iborat.*

$\log_a x>b$, $\log_a x\le b$, $\log_a x\ge b$ tengsizliklar yechimlarining geometrik talqinlarini mustaqil ravishda keltiring.

Logarifmik tengsizliklarni yechish

Ushbu $\log_a f(x)<\log_a g(x)$ logarifmik tengsizlikning yechimi:

$0<a<1$ bo‘lganda $\begin{cases} f(x)>g(x), \\ g(x)>0 \end{cases}$

tengsizliklar sistemasining yechimidan;

$a>1$ bo‘lganda esa $\begin{cases} f(x)<g(x), \\ f(x)>0 \end{cases}$ tengsizliklar sistemasining yechimidan iborat bo‘ladi.

$\log_a f(x)\le\log_a g(x)$, $\log_a f(x)>\log_a g(x)$ va $\log_a f(x)\ge\log_a g(x)$ tengsizliklarning yechilishi quyidagi jadvalda keltirilgan.

Logarifmik tengsizliklar turi	$\log_a f(x) \le \log_a g(x)$	$\log_a f(x) < \log_a g(x)$	$\log_a f(x) > \log_a g(x)$	$\log_a f(x) \ge \log_a g(x)$
$0 < a < 1$ bo'lganda	$\begin{cases} f(x) \ge g(x) \\ g(x) > 0 \end{cases}$	$\begin{cases} f(x) > g(x) \\ g(x) > 0 \end{cases}$	$\begin{cases} f(x) < g(x) \\ f(x) > 0 \end{cases}$	$\begin{cases} f(x) \le g(x) \\ f(x) > 0 \end{cases}$
$a > 1$ bo'lganda	$\begin{cases} f(x) \le g(x) \\ f(x) > 0 \end{cases}$	$\begin{cases} f(x) < g(x) \\ f(x) > 0 \end{cases}$	$\begin{cases} f(x) > g(x) \\ g(x) > 0 \end{cases}$	$\begin{cases} f(x) \ge g(x) \\ g(x) > 0 \end{cases}$

1-misol. $\log_{27} x > \frac{1}{3}$ tengsizlikni yeching.

Yechish

Aniqlanish sohasi $x > 0$. Logarifm asosi 1 dan kattaligi va logarifm ta'rifidan foydalanamiz:

$$\log_{27} x > \frac{1}{3} \Rightarrow x > 27^{\frac{1}{3}} \Rightarrow x > \sqrt[3]{27} \Rightarrow x > 3 \begin{cases} x > 3 \\ x > 0 \end{cases} \Rightarrow x > 3 \Rightarrow x \in (3; \infty)$$

Javob: $x \in (3; \infty)$

2-misol. $\log_{0,5}(2x-3) > \log_{0,5}(x+1)$ tengsizlikni yeching.

Yechish

Tengsizlikni unga teng kuchli bo'lgan quyidagi sistemaga keltirib yechamiz:

$$\begin{cases} 2x-3 < x+1 \\ 2x-3 > 0 \end{cases} \Rightarrow \begin{cases} x < 4 \\ x > 1,5 \end{cases} \Rightarrow \begin{cases} x < 4 \\ x > 1,5 \end{cases} \Rightarrow x \in (1,5; 4)$$

Javob: $(1,5; 4)$

3-misol. $\log_7^2 x - 13\log_7 x + 42 \ge 0$ tengsizlikni yeching.

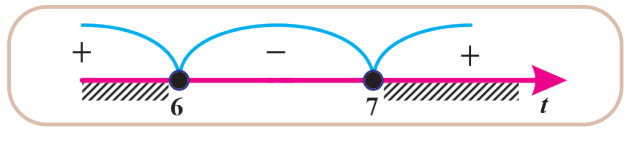
Yechish

$t = \log_7 x$ belgilash kiritamiz. Natijada

$$t^2 - 13t + 42 \ge 0$$

tengsizlik hosil bo'ladi.

$(t-6)(t-7) \ge 0$ tengsizlikni yechamiz.



Demak, $t \le 6$ yoki $t \ge 7$ ekan. $\log_7 x \le 6$ yoki $\log_7 x \ge 7$ bo'ladi. Bundan $x \le 7^6$ yoki $x \ge 7^7$ tengsizliklar hosil bo'ladi. $x > 0$ shartni e'tiborga olsak,

$$x \in \left(0; 7^6\right] \cup \left[7^7; \infty\right)$$

bo'ladi.

Javob: $x \in \left(0; 7^6\right] \cup \left[7^7; \infty\right)$.

$A\cdot\log_a^2 x+B\cdot\log_a x+C<0$ kabi tengsizliklar $\log_a x$ belgilash bilan kvadrat tengsizlikka keltirib yechiladi.

4-misol. Tengsizlikni yeching: $\log_3^2 x-3\log_3 x+2\le 0$.

Yechish

Aniqlanish sohasi $x>0$.

$\log_3 x=t$ belgilash kiritamiz,

$t^2-3t+2\le 0$ tengsizlikni yechamiz,

$(t-1)(t-2)\le 0$, bundan $1\le t\le 2$

$1\le\log_3 x\le 2 \Rightarrow \log_3 3\le\log_3 x\le\log_3 9 \Rightarrow 3\le x\le 9$

Javob: $[3;9]$

5-misol. $\log_{x+1}\left(x^2+2x+1\right)^{x^2+2x+5}>4x+28$ tengsizlikni yeching.

Yechish

Tengsizlikni quyidagicha yozib olamiz:

$$\log_{x+1}(x+1)^{2(x^2+2x+5)}>4x+28$$

Bu yerda ikkita hol bo‘lishi mumkin:

1-hol. $0<x+1<1\Rightarrow -1<x<0$

Bunda: $2(x^2+2x+5)<4x+28 \Rightarrow x^2+2x+5<2x+14 \Rightarrow x^2-9<0 \Rightarrow x\in(-3;\ 3)$

$-1<x<0$ ekanidan $x\in(-1;0)$.

2-hol. $x+1>1\Rightarrow x>0$

Bunda:

$2(x^2+2x+5)>4x+28 \Rightarrow x^2+2x+5>2x+14 \Rightarrow x^2-9>0 \Rightarrow x\in(-\infty;-3)\cup(3;\infty)$

$x>0$ ekanidan $x\in(3;\infty)$.

1- va 2-hollarni birlashtirsak, tengsizlikning yechimi quyidagicha bo‘ladi:

$$x\in(-1;0)\cup(3;\infty)$$

Javob: $x\in(-1;0)\cup(3;\infty)$.

MISOLLAR

1. Tengsizliklarni yeching.

a) $\log_2 x > 3$ b) $\log_{0,5} x > 2$ c) $\log_2 8 > x$

d) $\log_5 x > 3$ e) $\log_3 x > 4$ f) $\log_3 x \geq \log_6 36$

g) $\log_2 x < \log_{49} 7$ h) $\log_{\frac{1}{5}}(x-5) > -2$ i) $\log_3(x+20) < 3$

j) $\log_3(4x+2) - \log_3 10 < 0$ k) $\log_8 64 > \log_{\frac{1}{5}} x$ l) $\log_4(5-x^2) > 1$

m) $\log_5(3x-2x^2) > 0$ n) $\log^2_{\frac{1}{2}} x - 9 \leq 0$ o) $5^{\log_5(x-7)} < 4$

2. $\log_2(4-x) - \log_2 7 < 0$ tengsizlikni qanoatlantiradigan butun sonlar nechta?

3. Tengsizliklarni yeching.

a) $\log_{\frac{4}{3}}(x+6) - \log_{\frac{4}{3}} 9 < \log_{\frac{4}{3}} 2 - \log_{\frac{4}{3}} 6$ b) $\log_2(x-1) < \log_2(3x-1)$

c) $\log_{\frac{1}{3}}(2x-4) \geq \log_{\frac{1}{3}}(x+1)$ d) $\log_{\frac{1}{2}}(x^2-5x-6) \geq -3$

e) $\lg^2 x + 11 \cdot \lg x + 10 < 0$ f) $\log_2^2 x - 6\log_2 x + 8 \leq 0$

g) $\log_2 \log_{\sqrt{2}}(x+1) < 1$ h) $2\log_{\frac{1}{5}}(x-2) + 3\log_5(x-2) < 1$

i) $\log_x x^2 + x > 1$ j) $\lg(x+2) + \lg(x-3) \leq \lg x^2$

4. Tengsizliklarni yeching.

a) $\lg 10^{\lg(x^2+21)} > 1 + \lg x$ b) $\left(\frac{2}{5}\right)^{\log_{0,25}(x^2-5x+8)} \leq 2,5$

c) $\log_{\frac{1}{\sqrt{5}}}\left(6^{x+1} - 36^x\right) \geq -2$ d) $\log_{\frac{x-1}{5x-6}}\left(\sqrt{6}-2x\right) > 0$

e) $x^{1+\lg\sqrt{x}} < 0,1^{-2}$ f) $\sqrt{x^{4\lg x}} < 10x$

5. $\log_{0,2}(x^4+2x^2+1) > \log_{0,2}(6x^2+1)$ tengsizlikning barcha manfiy yechimlari to‘plamini toping.

KO‘RSATKICHLI VA LOGARIFMIK FUNKSIYALARNING TATBIQI

Murakkab foiz formulasi va uning tatbiqlari

Aytaylik, biror Q_0 miqdordagi pul qarz olinmoqchi. Qarz beruvchi belgilangan muddatda dastlabki miqdorni biror P foyda bilan qaytarishni talab etishi mumkin. Demak, ko‘rsatilgan muddatda qarz oluvchi qaytaradigan miqdor

$$Q_1 = Q_0 + P$$

bo‘ladi. Muddat sifatida bir kun, ikki kun, ..., bir hafta, ikki hafta, ..., bir oy, ikki oy va hokazo olinishi mumkin. Bunda ushbu

$$p = \frac{P}{Q_0} \cdot 100\%$$

kattalik *olingan qarzni o‘z muddatida qaytarish foizi* deyiladi.

1. Oddiy foiz formulasi. Agar foiz faqatgina olingan Q_0 miqdorga qo‘llansa, birinchi muddat oxirida qarz miqdori

$$Q_1 = Q_0 + \frac{p}{100} \cdot Q_0 = \left(1 + \frac{p}{100}\right) Q_0$$

bo‘ladi. Bunda

$$P_1 = \frac{p}{100} \cdot Q_0 = P$$

formula – qarz beruvchining birinchi muddat oxiridagi foydasi. Bu jarayonni n marta takrorlab, n-muddat oxirida qarz miqdori

$$Q_n = \left(1 + \frac{np}{100}\right) Q_0$$

bo‘lishi, qarz beruvchining n-muddat oxiridagi foydasi

$$P_n = \frac{np}{100} \cdot Q_0 \quad (\text{ravshanki, } P_n = nP)$$

bo‘lishi topiladi. Bunday hisoblanadigan foiz ***oddiy foiz***,

$$Q_n = \left(1 + \frac{np}{100}\right) Q_0$$

formula esa ***oddiy foiz formulasi*** deyiladi.

2. Murakkab foiz formulasi. Foizni olingan qarzga hosil bo‘lgan foydani qo‘shib qo‘llash mumkin. Bunda birinchi muddat oxirida qarz miqdori

$$Q_1 = Q_0 + \frac{p}{100} \cdot Q_0 = \left(1 + \frac{p}{100}\right) Q_0$$

bo‘ladi. Bunda

$$P_1 = \frac{p}{100} \cdot Q_0 = P$$

formula – qarz beruvchining birinchi muddat oxiridagi foydasi. Bu jarayonni n marta takrorlab, n-muddat oxirida qarz miqdori

$$Q_n = \left(1 + \frac{p}{100}\right)^n Q_0$$

bo‘lishi, qarz beruvchining n-muddat oxiridagi foydasi

$$P_n = Q_n - Q_0 = \left(\left(1+\frac{p}{100}\right)^n - 1\right)Q_0$$

bo‘lishi topiladi. Bunday hisoblanadigan foiz ***murakkab foiz***,

$$Q_n = \left(1+\frac{p}{100}\right)^n Q_0$$

formula esa ***murakkab foiz formulasi*** deyiladi.

Oddiy va murakkab foiz formulalari ishlatilishiga oid ko‘plab amaliy misol va masalalar uchraydi. Hozir *kredit, ipoteka qarzi* kabi iboralarga ko‘p duch kelamiz. Ipoteka qarzini hisoblash bo‘yicha masala yechish namunasini keltiramiz.

Odatda qarz beruvchi bank, qarz oluvchi esa mijoz shaklida namoyon bo‘ladi. Banklar uy-joy olishda (ipoteka), transport vositasi yoki xo‘jalik mollari (televizor, sovitkich, uyali aloqa telefoni va boshqalar) olishda (kredit) qarzni bir necha yilgacha bo‘lgan uzoq muddatga beradi va mijozdan har oyda qarzning ma’lum miqdorini to‘lab borishni talab etadi.

1-misol. Dastlabki narxi 360 000 000 so‘m bo‘lgan xonadonni yosh oila yillik 20% bilan 15 yilga ipoteka krediti orqali olmoqchi. 15 yil davomida bankka qancha mablag‘ qaytariladi? Bunda bank qancha foyda ko‘radi?

Yechish

Dastlabki 360 000 000 so‘m mablag‘ bank tilida ***asosiy qarz*** deyiladi. 1 yil 12 oydan iborat. Shuning uchun mijoz bankka har oyda asosiy qarzning

$$\frac{360\ 000\ 000}{15\cdot 12} = 2\ 000\ 000$$

so‘m miqdorini qaytarishi kerak. Qaytarishning birinchi oyida paydo bo‘ladigan foiz quyidagicha topiladi:

$$a_1 = 360\ 000\ 000\cdot\frac{20\%}{100\%}\cdot\frac{1}{12} = 6\ 000\ 000 \text{ so‘m.}$$

Demak, mijoz birinchi oy oxirida jami

$$2\ 000\ 000 + 6\ 000\ 000 = 8\ 000\ 000$$

so‘m qaytarishi kerak. Shundan so‘ng qolgan qarz miqdori

$$360\ 000\ 000 - 2\ 000\ 000 = 358\ 000\ 000$$

so‘m bo‘ladi. Ikkinchi oy oxirida mijoz asosiy qarzning 2 000 000 so‘m miqdorini va paydo bo‘lgan ushbu

$$360\ 000\ 000 - 2\ 000\ 000 = 358\ 000\ 000$$

so‘m foiz miqdorini, jami esa

$$2\ 000\ 000 + 5\ 966\ 667 = 7\ 966\ 667$$

so‘m mablag‘ni qaytarishi lozim.

$(n-1)$-oyning mablag‘i to‘langach,

$$360\ 000\ 000 - 2\ 000\ 000\cdot(n-1)$$

so‘m miqdorda asosiy qarz qoladi. n-oy oxirida mijoz bankka $2\ 000\ 000+a_n$ miqdorda pul to‘laydi. Bu yerda a_n ifodasi n-oyning foizi bo‘lib,

$$a_n=\left(360\ 000\ 000-2\ 000\ 000\cdot\left(n-1\right)\right)\cdot\frac{20\%}{100\%}\cdot\frac{1}{12}=\left(181-n\right)\cdot\frac{100\ 000}{3}$$

tenglik orqali topiladi. Ko‘rinib turibdiki, a_1, a_2, ... , a_n kamayuvchi arifmetik progressiya bo‘lib, uning ayirmasi

$$d=a_n-a_{n-1}=\left(181-n\right)\cdot\frac{100\ 000}{3}-\left(181-\left(n-1\right)\right)\cdot\frac{100\ 000}{3}=-\frac{100\ 000}{3},\ \text{ya’ni}\ d=-\frac{100\ 000}{3}$$

ga teng.

Ravshanki, 15 yil 180 oydan iborat, shuning uchun $1\le n\le 180$ bo‘ladi. Demak,

$$a_1=6\ 000\ 000,\ a_{180}=\frac{100\ 000}{3}$$

bo‘lib, arifmetik progressiyaning 180 ta hadining yig‘indisi

$$S_{180}=\frac{a_1+a_{180}}{2}\cdot 180=\frac{6\ 000\ 000+\frac{100\ 000}{3}}{2}\cdot 180=$$

$$=\frac{18\ 000\ 000+100\ 000}{3}\cdot 90=18\ 100\ 000\cdot 30=543\ 000\ 000$$

bo‘ladi. Demak, mijoz bankka jami

$$360\ 000\ 000+543\ 000\ 000=903\ 000\ 000$$

so‘m qaytarishi kerak ekan. Bunda bankning foydasi 543 000 000 so‘m bo‘ladi.

Radioaktiv yemirilish

Yarim yemirilish davri. Ayrim kimyoviy elementlar o‘z yadrolaridan zarralar chiqarib turadi. Bunday elementlar *radioaktiv elementlar* deb yuritiladi, ularning o‘z yadrolaridan zarra chiqarish jarayoni ***radioaktiv yemirilish*** deyiladi. Radioaktiv yemirilish natijasida dastlabki kimyoviy element boshqa kimyoviy elementga aylanib qoladi.

Dastlabki kimyoviy element massasi m_0 bo‘lib, uning yarmi yemirilishiga ketadigan vaqt T_1 bo‘lsin. U holda $t_1=T_1$ vaqtdan keyin yemirilmay qolgan element massasi $m_1=\frac{m_0}{2}$ bo‘lib, m_1 massaning yarmi yemirilishi uchun T_2 vaqt sarflansin. $t_2=T_1+T_2$ vaqtdan keyin yemirilmay qolgan element massasi $m_2=\frac{m_1}{2}=\frac{m_0}{2^2}$ bo‘lib, massaning yarmi yemirilishi uchun T_3 vaqt sarflansin. Xuddi shuningdek, $t_3=T_1+T_2+T_3$ vaqtdan keyin yemirilmay qolgan element massasi $m_3=\frac{m_2}{2}=\frac{m_0}{2^3}$ bo‘lib, m_3 massaning yarmi yemirilishi uchun T_4 vaqt sarflansin. Bu jarayon cheksiz davom ettirilgan bo‘lsin.

Uzoq yillik tajriba natijasida $T_1=T_2=T_3=\ldots=T_n=T_{n+1}=\ldots$ bo‘lishi isbotlangan. Demak, aynan bitta element massasining yarmi yemirilishi uchun ketadigan vaqt o‘zgarmas miqdor ekan. Bu miqdor ***elementning yarim yemirilish davri*** deyiladi va T orqali belgilanadi:

$$T=T_1=T_2=T_3=\ldots=T_n=T_{n+1}=\ldots .$$

Natijada

$$t_1 = T_1 = T,$$
$$t_2 = T_1 + T_2 = 2T,$$
$$t_3 = T_1 + T_2 + T_3 = 3T,\ldots,$$
$$t_n = T_1 + T_2 + T_3 + \ldots + T_n = nT$$

tengliklar hosil bo‘ladi. Dastlabki massasi m_0 bo‘lgan elementning $t_n = nT$ vaqtdan keyin yemirilmay qolgan qismining massasi $m_n = \frac{m_0}{2^n} = 2^{-n} m_0$ bo‘lar ekan. Bu yerda $n = \frac{t_n}{T}$ ekanini e’tiborga olsak, $m_n = 2^{-\frac{t_n}{T}} m_0$ tenglikka ega bo‘lamiz. Bu formula ixtiyoriy t moment uchun ham o‘rinli:

$$m(t) = 2^{-\frac{t}{T}} m_0.$$

Shunday qilib, radioaktiv elementning yemirilmay qolgan qismining massasi vaqtning ko‘rsatkichli funksiyasi ekan.

2-misol. Sutkaning dastlabki 8 soatida radioaktiv moddaning aktivligi 4 marta kamaydi. Sutka davomida moddaning aktivligi necha marta kamayadi?

Yechish. Qaralayotgan moddaning dastlabki massasi m_0 bo‘lib, yarim yemirilish davri T bo‘lsin. 8 soatdan keyin uning massasi $m(8) = \frac{m_0}{4}$ bo‘lgan. Bu berilganlar uchun $m(t) = 2^{-\frac{t}{T}} m_0$ formulani qo‘llab, moddaning yarim yemirilish davri topiladi:

$$m(8) = 2^{-\frac{8}{T}} m_0 \Rightarrow \frac{m_0}{4} = 2^{-\frac{8}{T}} m_0 \Rightarrow 2^{\frac{8}{T}} = 2^2 \Rightarrow \frac{8}{T} = 2 \Rightarrow T = 4 \text{ soat.}$$

Endi $m(t) = 2^{-\frac{t}{T}} m_0$ formula yana bir bor $t = 24$ (bir sutka = 24 soat) uchun ishlatilib, radioaktiv moddaning aktivligi sutka davomida necha marta kamaygani topiladi:

$$m(24) = 2^{-\frac{24}{T}} m_0 = 2^{-6} m_0 = \frac{m_0}{64}.$$

Shunday qilib, sutka davomida radioaktiv moddaning aktivligi 64 marta kamayadi.

Qo‘shilgan qiymat solig‘i

Qo‘shilgan qiymat solig‘i qisqacha QQS deb nomlanadi.

Siz *soliq* tushunchasi bilan tanishsiz. Tovarlarni ishlab chiqaruvchi yoki import qiluvchi (ulgurji sotuvchi yoki chakana sotuvchi) davlatga savdo solig‘ini to‘lashi kerak. *Qo‘shilgan qiymat solig‘i* – ishlab chiqaruvchidan tortib to chakana sotuvchiga qadar ta’minot zanjirining ko‘p nuqtalarida hukumat tomonidan amalga oshiriladigan soliq. Har bir bosqichda faqat tovarga qo‘shilgan qiymat savdo solig‘iga tortiladi. Savdo solig‘ining yakuniy holati iste’molchida qoladi.

Bu asl ishlab chiqaruvchidan sotuvchiga tovarlarning har bir o‘tkazilishida qo‘shilgan qiymatga soliq.

Aytaylik, QQS stavkasi 10% va tadbirkor 8 000 000 so‘mga mahsulot sotib oldi, u to‘laydigan soliq = 8 000 000 so‘mning 10 foizi = 800 000 so‘m.

Xuddi shu mahsulotni 11 500 000 so‘mga sotsa, undan undiradigan soliq = 11 500 000 ning 10 foizi = 1 150 00 so‘m.

Tadbirkor uchun QQS = 1 150 000 – 800 000 = 350 000 so‘m bo‘ladi.

MISOLLAR *(kalkulyatordan foydalanish mumkin)*

1. Dastlabki narxi 360 million so‘m bo‘lgan xonadonni yillik 18% bilan 20 yilga ipoteka qarzi orqali olgan oila muddat oxirida bankka qancha mablag‘ qaytargan bo‘ladi? Bank foydasi qancha bo‘ladi?

2. Yillik 8% bilan 3 yil muddatga 5000 AQSh dollari bo‘yicha murakkab foizlarni toping.

3. Vohid 50 million so‘m qarz oldi va birinchi, ikkinchi va uchinchi yil uchun mos ravishda 10%, 12% va 14% stavkada foiz to‘lashga rozi bo‘ldi. 3 yildan keyin to‘lashi kerak bo‘lgan umumiy miqdorni toping.

4. Bir kishi bankka 100 million so‘m qo‘ygan. Buning evaziga u 133,1 million so‘m oldi. Bank yiliga 10% foiz berdi. U pulni qancha vaqt bankda saqlagan?

5. Omonatchi 26 million so‘mni bank hisobiga o‘tkazdi. 18 oydan keyin uning hisobida 32 million so‘m bo‘ldi. Yillik foiz stavkasi qancha?

6. Menda 400 dollar bor. Do‘stim menga bankka sarmoya kiritishni taklif qildi. Men yillik 13% xorijiy valyutadagi hisob raqamiga va har oyda 1% to‘ldiriladigan summa hisobiga sarmoya kiritdim.

a) Agar valyuta hisob raqamiga pul kiritsam, bir yilda qancha olaman?

b) Agar men bu pulni so‘mga to‘liq aylantirib, summa hisobiga qo‘ygan bo‘lsam, bir yilda qanday miqdorda dollar olaman? Dollar va so‘m kursi o‘zgarishini hisobga oling.

7. Murod 10 million so‘mga tovar sotib olsa, 7% soliq to‘laydi. U xuddi shu tovarni 13 million so‘mga sotsa, 9% soliq oladi. Murod to‘laydigan QQSni toping.

8. Tadbirkor buyumni 7,5 millionga sotsa, xaridordan 12% stavkada savdo solig‘i oladi. Agar u 180 000 so‘m miqdorida QQS to‘lasa, tadbirkor to‘lagan soliqni hisobga olgan holda dastlabki narxni aniqlang.

9. Ishlab chiqaruvchi o‘z mahsulotining narxini har biri uchun 12 million deb e’lon qildi. U ulgurji sotuvchiga 30% chegirmaga ruxsat berdi, ulgurji sotuvchi esa, o‘z navbatida, chakana sotuvchiga e’lon qilingan narxdan 20% chegirmaga ruxsat berdi. Agar tovar uchun belgilangan savdo solig‘i stavkasi 10% bo‘lsa va chakana sotuvchi uni iste’molchiga e’lon qilingan narxda sotsa, ulgurji va chakana sotuvchi to‘lagan qo‘shilgan qiymat solig‘ini toping.

10. Chakana sotuvchi ulgurji sotuvchidan buyumni 80 000 so‘mga sotib oldi va ulgurji sotuvchi belgilangan 8% miqdorida savdo solig‘ini oldi. Chakana sotuvchi narxni 100 000 so‘m qilib belgilab qo‘ydi va xuddi shu stavkada savdo solig‘ini iste’molchidan undiradi. Chakana sotuvchi davlatga qancha QQS to‘laydi?



4-BOB. TRIGONOMETRIK FUNKSIYALAR

- TRIGONOMETRIK FUNKSIYALAR. DAVRIY JARAYONLAR
- TESKARI TRIGONOMETRIK FUNKSIYALAR

TRIGONOMETRIK FUNKSIYALAR VA ULARNING XOSSALARI, GRAFIGI. DAVRIY JARAYONLAR

Trigonometrik funksiyalar. Davriy jarayonlar

Tabiatda, texnikada, ishlab chiqarishda va boshqa sohalarda vaqt o‘tishi bilan takrorlanadigan hodisa va jarayonlar ko‘plab uchraydi. Masalan, quyosh chiqishi, fasllar almashinuvi, ichki yonuv dvigatelida porshen harakati va boshqalar vaqt o‘tishi bilan takrorlanadi. Bunday jarayonlar ***davriy jarayonlar*** deb ataladi. Davriy jarayonlar trigonometrik funksiyalar orqali tavsiflanadi.

Trigonometrik funksiyalarni o‘rganishda:

1) burchak kattaligining gradus o‘lchovini;

2) 1° burchakning 60 dan bir qismi 1 *minut* (belgilanishi 1′), 1′ ning 60 dan bir qismi 1 *sekund* (belgilanishi 1″) ekanini, ya’ni

$$1' = \frac{1^\circ}{60},\ 1'' = \frac{1'}{60} = \frac{1^\circ}{3600}$$

tengliklarni;

3) burchak kattaligining radian o‘lchovini;

4) burchakning radian o‘lchovidan gradus o‘lchoviga o‘tish

$$\alpha\ rad = \left(\frac{180}{\pi}\cdot\alpha\right)^\circ$$

formulasini;

5) burchakning gradus o‘lchovidan radian o‘lchoviga o‘tish

$$\alpha^\circ = \left(\frac{\pi}{180}\cdot\alpha\right) rad$$

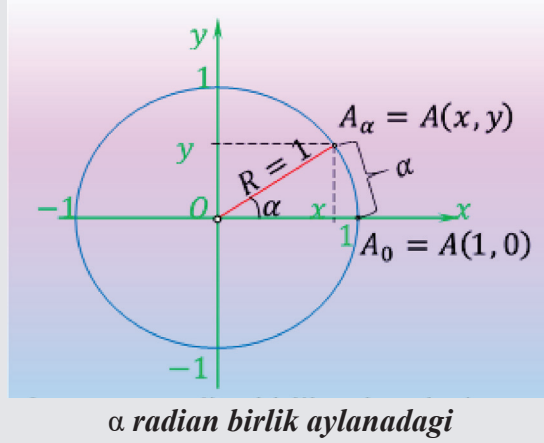
formulasini;

6) keltirish formulalarini bilish talab etiladi.

Burchakning sinusi, kosinusi, tangensi va kotangensi

Oxy Dekart koordinatalar sistemasi kiritilgan tekislikda markazi koordinatalar boshida bo‘lgan **birlik aylana** (ya’ni radiusi 1 ga teng aylana)ni ko‘rib chiqamiz. $A_\alpha = A\ (1;\ 0)$ nuqtani tayinlab olamiz. Aylanada A_0 nuqtadan soat mili harakatiga qarshi (ya’ni **musbat**) yo‘nalishda uzunligi α ga teng yoy ajratib olamiz va uning oxirini A_α orqali belgilaymiz (1-*rasm*). Burchak kattaligining radian o‘lchovi aniqlanishiga ko‘ra A_0OA_α burchakning kattaligi α radianga teng bo‘ladi:

1-rasm



α radian birlik aylanadagi uzunligi α bo‘lgan $\overset{\frown}{A_0A_\alpha}$ yoy markaziy burchagining burchak kattaligidir

$$\alpha = \angle A_0 O A_\alpha.$$

Diqqat qiling! A_α nuqta Oxy tekisligida biror koordinataga ega bo‘ladi.

Aytaylik, A_α nuqtaning Oxy tekisligidagi koordinatalari $(x; y)$ bo‘lsin.

Ta’rif

1) x kattalik α burchakning *kosinusi* deyiladi va $cos\alpha$ orqali belgilanadi.

2) y kattalik α burchakning *sinusi* deyiladi va $sin\alpha$ orqali belgilanadi.

3) $\frac{y}{x}$ nisbat α burchakning *tangensi* deyiladi va $tg\alpha$ orqali belgilanadi.

4) $\frac{x}{y}$ nisbat α burchakning *kotangensi* deyiladi va $ctg\alpha$ orqali belgilanadi.

Demak, ta’rifga ko‘ra:

$$cos\alpha = x, \qquad sin\alpha = y, \qquad tg\alpha = \frac{y}{x}, \qquad ctg\alpha = \frac{x}{y} \tag{1}$$

bo‘ladi.

Eslatma! Agar birlik aylana o‘rniga ixtiyoriy R radiusli aylana olinsa, u holda

$$cos\,\alpha = \frac{x}{R}, \quad sin\,\alpha = \frac{y}{R}, \quad tg\,\alpha = \frac{y}{x}, \quad ctg\,\alpha = \frac{x}{y} \tag{1'}$$

tengliklar hosil bo‘ladi.

Ravshanki, aylanadagi A_0 nuqtani berilgan burchakka quyidagicha ikkita yo‘nalishda markaziy burish mumkin:



Musbat burish: burish soat mili harakatiga qarshi yo‘nalishda bajariladi.



Manfiy burish: burish soat mili harakati yo‘nalishi bo‘ylab bajariladi.

$y = sinx, y = cosx, y = tgx, y = ctgx$ funksiyalar va ularning xossalari, grafigi

Har bir x songa birlik aylanadagi A_0 nuqtadan boshlab x burchakka burishda hosil bo‘ladigan A_x nuqtani mos qo‘yaylik. U holda aylanadagi A_x nuqta uchun $sinx$, $cosx$, tgx, $ctgx$ qiymatlarni hisoblash mumkin. Natijada x songa $sinx$, $cosx$, tgx, $ctgx$ qiymatlarni mos qo‘yuvchi va ***trigonometrik funksiyalar*** deb ataluvchi ushbu

$$y = sinx, \; y = cosx, \; y = tgx, \; y = ctgx$$

funksiyalarga ega bo‘lamiz.

Bu funksiyalar davriy, ya’ni har bir $k \in Z$ uchun quyidagi tengliklar o‘rinli bo‘ladi:

$$sin(x + 2\pi k) = sin\,x$$

$$cos(x + 2\pi k) = cos\,x$$

$$tg(x + \pi k) = tgx$$

$$ctg(x + \pi k) = ctgx$$

Demak, $y = sinx$ va $y = cosx$ funksiyalarning asosiy davri $T_0 = 2\pi$ hamda $y = tgx$ va $y = ctgx$ funksiyalarning asosiy davri $T_0 = \pi$ ekan.

$y = cosx$ funksiya juft:

$$f(-x) = cos(-x) = cos\, x = f(x).$$

$y = sinx$, $y = tgx$, $y = ctgx$ funksiyalar esa toq:

$$f(-x) = sin(-x) = -sin\, x = -f(x)$$

$$f(-x) = tg(-x) = -tg\, x = -f(x)$$

$$f(-x) = ctg(-x) = -ctg\, x = -f(x)$$

Ravshanki,

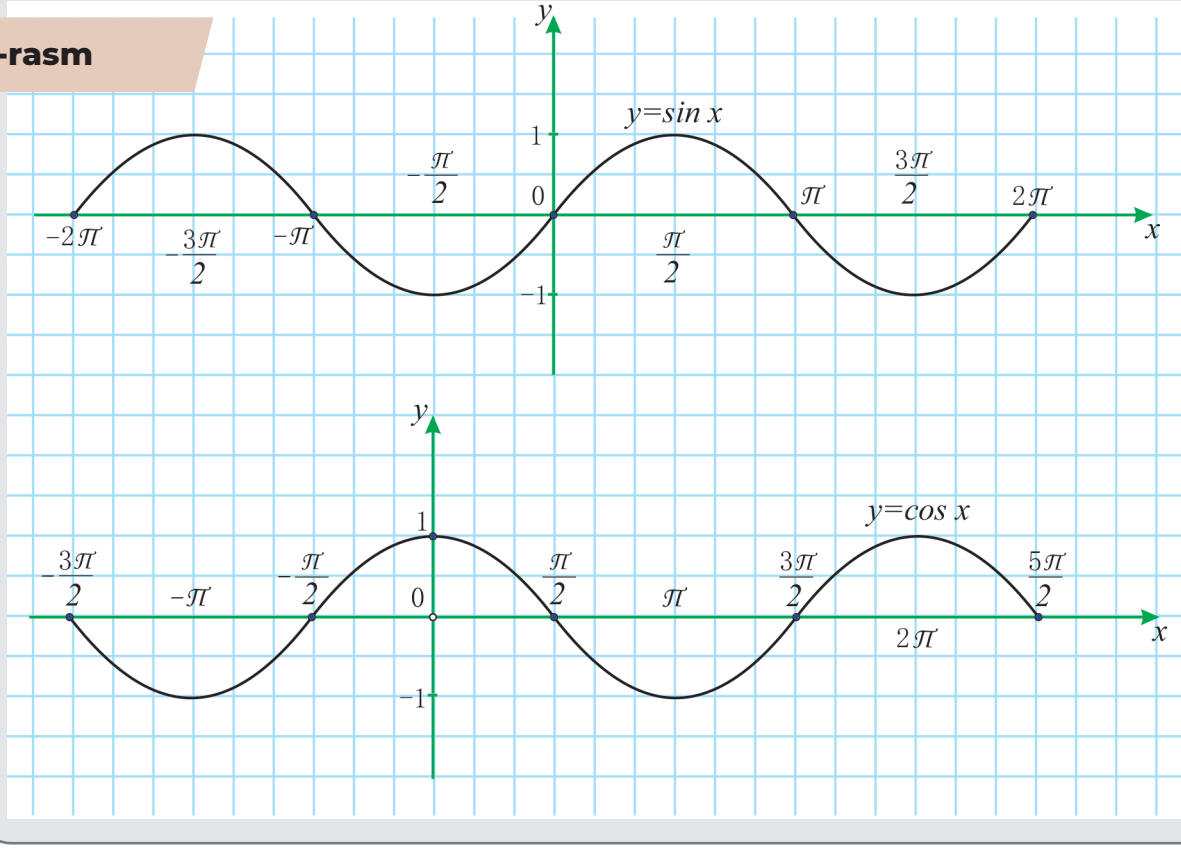
$y = sinx$ va $y = cosx$ funksiyalar uchun $D(y) = (-\infty; +\infty)$, $E(y) = [-1; 1]$,

$y = tgx$ funksiya uchun $D(y) = \left(-\frac{\pi}{2} + \pi k; \frac{\pi}{2} + \pi k\right)$, $k \in Z$, $E(y) = (-\infty, +\infty)$,

$y = ctgx$ funksiya uchun $D(y) = (\pi k;\ \pi + \pi k)$, $k \in Z$, $E(y) = (-\infty, +\infty)$ bo‘ladi.

Quyidagi rasmlarda trigonometrik funksiyalar grafiklari keltirilgan.

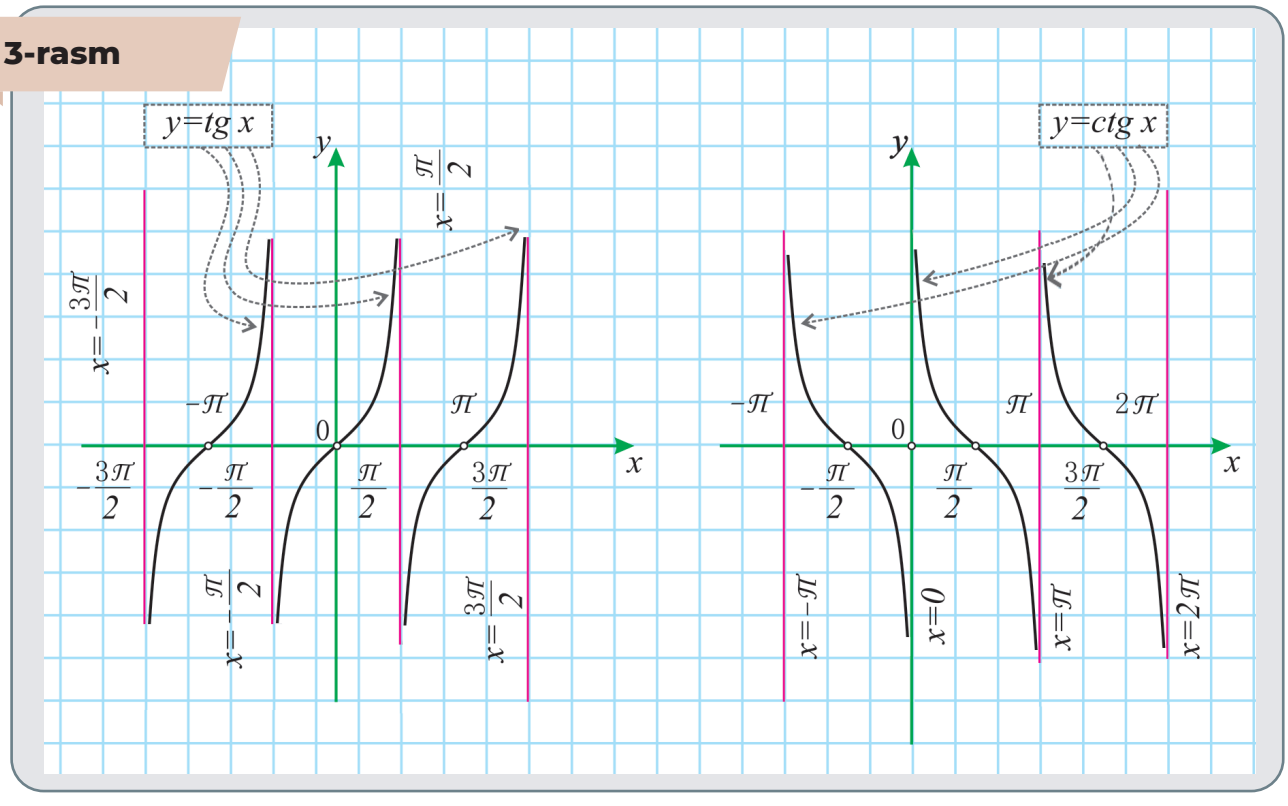
2-rasm



Bu grafiklardan quyidagi muhim xulosalar kelib chiqadi:

1) $y = sinx$ funksiya $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ oraliqda o‘sadi va bu oraliqdan olingan har bir x ga y ning $[-1; 1]$ kesmadagi yagona qiymati mos keladi;

3-rasm



2) $y = cosx$ funksiya $[0; \pi]$ oraliqda kamayadi va bu oraliqdan olingan har bir x ga y ning $[-1; 1]$ kesmadagi yagona qiymati mos keladi;

3) $y = tgx$ funksiya $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda o‘sadi va bu oraliqdan olingan har bir x ga y ning $(-\infty; +\infty)$ oraliqdagi yagona qiymati mos keladi;

4) $y = ctgx$ funksiya $(0; \pi)$ oraliqda kamayadi va bu oraliqdan olingan har bir x ga y ning $(-\infty; +\infty)$ oraliqdagi yagona qiymati mos keladi.

Aniqlanish sohasini topishga oid misollarni yechishda ayrim hollarda funksiya aniqlanmagan nuqtalarni ko‘rsatish yetarli bo‘ladi.

1-misol. $y = 2tg(3x-1)$ funksiyaning aniqlanish sohasini toping.

Yechish. Ma’lumki, $y = tgx$ funksiya $x = \frac{\pi}{2} + \pi n,\ n \in Z$ nuqtalarda aniqlanmagan, shuning uchun $y = 2tg(3x-1)$ funksiya argumentning $3x-1 = \frac{\pi}{2} + \pi n$ qiymatlarida aniqlanmagan. Bu yerdan $x \neq \frac{\pi}{6} + \frac{1}{3} + \frac{\pi n}{3}, n \in Z$.

Javob: $y = 2tg(3x-1)$ funksiya $x = \frac{\pi}{6} + \frac{1}{3} + \frac{\pi n}{3}, n \in Z$ nuqtalardan boshqa barcha haqiqiy sonlarda aniqlangan.

2-misol. $y = 2 - \frac{1}{3}cos(5x-4)$ funksiyaning qiymatlar to‘plamini toping.

Yechish. Bu misolni yechishda

$$-1 \le cos\, x \le 1$$

qo‘shtengsizlik x ning barcha qiymatlarida o‘rinli bo‘lishidan foydalanamiz. Demak,

$$-1 \le cos(5x-4) \le 1$$

Yuqoridagi qo‘shtengsizlikni $-\frac{1}{3}$ ga ko‘paytiramiz va quyidagi qo‘shtengsizlikni hosil qilamiz:

$$-\frac{1}{3} \le -\frac{1}{3}cos(5x-4) \le \frac{1}{3}$$

Bu qo‘shtengsizlikning har bir tarafiga 2 ni qo‘shsak,

$$2-\frac{1}{3} \le 2-\frac{1}{3}cos(5x-4) \le 2+\frac{1}{3}$$

$$1\frac{2}{3} \le 2-\frac{1}{3}cos(5x-4) \le 2\frac{1}{3}$$

yoki

$1\frac{2}{3} \le y \le 2\frac{1}{3}$ hosil bo‘ladi.

Javob: Berilgan funksiyaning qiymatlar to‘plami $\left[1\frac{2}{3};2\frac{1}{3}\right]$ kesmadan iborat, yoki $E(y)=\left[1\frac{2}{3};2\frac{1}{3}\right]$.

Ma’lumki, $y=f(x)$ funksiyaning asosiy davri T bo‘lsa, $y=af(kx+b)$ funksiya uchun $\frac{T}{|k|}$ miqdor eng kichik musbat davri bo‘ladi. k noldan farqli son.

3-misol. $y=2\sin\left(\frac{4}{3}x+7\right)$ funksiyaning eng kichik musbat davrini toping.

Yechish. $y=\sin x$ funksiyaning asosiy davri 2π ga teng. Shuning uchun $y=2\sin\left(\frac{4}{3}x+7\right)$ funksiyaning eng kichik musbat davri

$$T=\frac{2\pi}{\frac{4}{3}}=\frac{3\pi}{2}$$ bo‘ladi.

Javob: Berilgan funksiyaning eng kichik musbat davri $\frac{3\pi}{2}$.

MISOLLAR

1. Funksiyaning aniqlanish sohasini toping.

a) $y=\cos 3x$ b) $y=\sin\frac{2x-1}{5}$ c) $y=\sin\frac{1}{x+5}$ d) $y=\sin\sqrt{\frac{1-x}{x+3}}$

e) $y=tg3x$ f) $y=ctg\frac{2x}{5}$ g) $y=tg\frac{1}{x}$

2. Funksiyaning qiymatlar to‘plamini toping.

a) $y=-1+\cos x$ b) $y=-6\sin 3x\cos 3x$ c) $y=2+\cos x$ d) $y=-3\sin 2x+2$

e) $y=5tg4x$ f) $y=3-4\cos 5x$ g) $y=-5+\frac{1}{2}\cos x\sin x$

3. Funksiyaning juft yoki toqligini aniqlang.

a) $y=2xtgx$ b) $y=x^3-tg^3x$ c) $y=tgx\sin^2 x$ d) $y=tg2x+2\sin x$

e) $y = x^2 + tg^2x$ f) $y = tg10|x|$ g) $y = \frac{x^2 + \cos x}{2}$ h) $y = \frac{\sin x + \cos x}{x+5}$

4. $y = sinx$ funksiya grafigidan foydalanib quyidagi funksiyalarning grafiklarini yasang.

a) $y = -sinx$ b) $y = 2sinx$ c) $y = -0{,}5sinx$ d) $y = |sinx|$

e) $y = sin\left(x - \frac{\pi}{3}\right)$ f) $y = |sin|x||$ g) $y = 1 + sinx$ h) $y = sin2x$

5. $y = cosx$ funksiya grafigidan foydalanib quyidagi funksiyalarning grafiklarini yasang.

a) $y = -cosx$ b) $y = 0{,}5cosx$ c) $y = cos2x$ d) $y = |cosx|$

e) $y = cos\left(x + \frac{\pi}{6}\right)$ f) $y = |cos|x||$ g) $y = 2 - cosx$ h) $y = cos4x$

6. Funksiya grafigini yasang.

a) $y = tg2x$ b) $y = ctg\frac{x}{2}$ c) $y = 2tgx$ d) $y = \frac{1}{3}ctgx$

7. Funksiyaning juft yoki toqligini aniqlang.

a) $y = \frac{cos\,2x - sin^2 x}{x^2}$ b) $y = ctg3x + 5sinx$ c) $y = sin5x$

d) $y = 2sin^2x$ e) $y = sin^2x + sinx$ f) $y = 5sin^3x + 2sinx$

8. $f(x)$ funksiya $(-\infty;\infty)$ oraliqda aniqlangan bo‘lsin:

a) $f(x) + f(-x)$ juft funksiya ekanini ko‘rsating.

b) $f(x) - f(-x)$ toq funksiya ekanini ko‘rsating.

9. Funksiyaning eng kichik musbat davrini toping.

a) $f(x) = cos(3x+1)$ b) $f(x) = sin\left(\frac{x}{4} - 3\right)$ c) $f(x) = tg(2x+1)$

d) $f(x) = sin2\pi x$ e) $f(x) = cos\sqrt{3x}$ f) $f(x) = tg(4\pi x - 3)$

10. Berilgan $f(x)$ funksiyaning eng kichik musbat davrini toping:

a) $f(x) = sin\frac{3x}{2} + tg7x$ b) $f(x) = cos\,x + 2\,sin\left(\frac{3x}{5} + \frac{\pi}{6}\right)$

c) $f(x) = ctg(x-1) - 3sin3x$ d) $f(x) = \sin 3x + \cos\frac{3x}{4} + \frac{1}{2}tg\frac{9x}{5}$

11. $T = -5\pi$ soni $f(x) = sin\,6x$ funksiyaning davri bo‘lishini ko‘rsating.

12. $T = \pi$ soni $f(x) = \sqrt{sin\,2x + 1}$ funksiyaning davri bo‘lishini ko‘rsating.

13. Quyidagi funksiyalardan qaysilarini eng kichik musbat davri π ga teng.

a) $y = sinx$ b) $y = cosx$ c) $y = tgx$ d) $y = ctgx$

14. Funksiya grafigini yasang.

a) $y = |sin\,x|$ b) $y = |cos\,x|$ c) $y = |tgx|$ d) $y = |ctgx|$

TESKARI TRIGONOMETRIK FUNKSIYALAR VA ULARNING XOSSALARI, GRAFIGI

Teskari trigonometrik funksiyalar

Kundalik hayotimizda inshootlar, ko‘priklar, transport vositalari, elektr stansiyalari, samolyot va boshqa qurilmalarni vayronaga aylantiruvchi rezonans hodisasi uchrab turadi. Rezonans hodisasi davriy jarayonlarning o‘zaro uyg‘unlashuvi natijasida kuzatiladi. Bunday hollarning oldini olish uchun trigonometrik funksiyalar berilgan qiymatni argumentning qanday qiymatida qabul qilishini, ya’ni teskari trigonometrik funksiyalarni bilish lozim.

Teskari trigonometrik funksiyalarni o‘rganishda quyidagilarni bilish talab etiladi:

1) trigonometrik funksiyalarning davriyligini va ularning asosiy davrlarini;

2) $y = sinx$ funksiya $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda o‘sadi va bu oraliqdan olingan har bir x ga y ning $[-1; 1]$ kesmadagi yagona qiymati mos keladi;

3) $y = cosx$ funksiya $[0; \pi]$ oraliqda kamayadi va bu oraliqdan olingan har bir x ga y ning $[-1; 1]$ kesmadagi yagona qiymati mos keladi;

4) $y = tgx$ funksiya $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda o‘sadi va bu oraliqdan olingan har bir x ga y ning $(-\infty; +\infty)$ oraliqdagi yagona qiymati mos keladi;

5) $y = ctgx$ funksiya $(0; \pi)$ oraliqda kamayadi va bu oraliqdan olingan har bir x ga y ning $(-\infty; +\infty)$ oraliqdagi yagona qiymati mos keladi.

$y = arcsinx$ funksiya va uning xossalari, grafigi

$$y = sinx$$

tenglama $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu ildiz

$$x = arcsiny$$

ko‘rinishda yoziladi. Bu tenglik bilan $[-1; 1]$ to‘plamning har bir y elementiga $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ to‘plamning yagona x elementini mos qo‘yuvchi arksinus funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

$$y = arcsinx$$

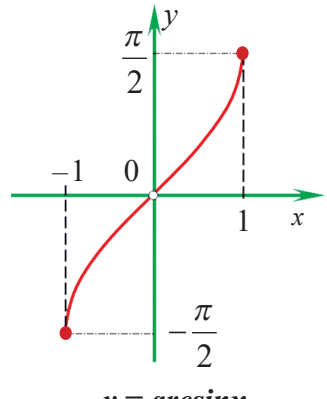
ko‘rinishda yozamiz (1-*rasm*).

$y = arcsinx$ funksiya $y = sinx$ funksiyaga teskari funksiya bo‘ladi:

$$sin(arcsinx) = x, \quad x \in [-1; 1]$$

$$arcsin(sinx) = x, \quad x \in \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$$

1-rasm



***y = arcsinx* funksiyaning grafigi**

$y = arcsinx$ funksiya xossalari:

- $D(y) = [-1;1]$;
- $E(y) = \left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;
- $y = arcsinx$ – o‘suvchi funksiya;
- $y = arcsinx$ funksiyaning eng katta qiymati $\frac{\pi}{2}$ ga, eng kichik qiymati $-\frac{\pi}{2}$ ga teng;
- $y = arcsinx$ funksiya grafigi koordinata boshidan o‘tadi;
- $y = arcsinx$ – toq funksiya, ya’ni $arcsin(-x) = -arcsinx$;
- $y = arcsinx$ funksiya davriy emas.

1-misol. $arcsin\frac{\sqrt{3}}{2}$ ifodaning qiymatini toping.

Yechish. Aytaylik, $arcsin\frac{\sqrt{3}}{2} = x$ bo‘lsin. U holda berilgan topshiriqni boshqacha qo‘yish mumkin: $sinx = \frac{\sqrt{3}}{2}$ tenglikni qanoatlantiruvchi x ning $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqdagi qiymatini toping. Ma’lumki, $sinx = \frac{\sqrt{3}}{2}$ tenglik $x = \frac{\pi}{3}$ bo‘lganda bajariladi. Demak, $arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$.

Quyidagi jadvalda $arcsinx$ ifodaning ayrim qiymatlari keltirilgan.

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$arcsinx$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

$y = arccosx$ funksiya va uning xossalari, grafigi

$$y = cosx$$

tenglik $[0; \pi]$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu yechim

$$\boldsymbol{x = arccosy}$$

ko‘rinishda yoziladi. Bu tenglik bilan $[-1; 1]$ to‘plamning har bir y elementiga $[0; \pi]$ to‘plamning yagona x elementini mos qo‘yuvchi arkkosinus funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

$$\boldsymbol{y = arccosx}$$

ko‘rinishda yozamiz (2-*rasm*).

$\boldsymbol{y = arccosx}$ funksiya $\boldsymbol{y = cosx}$ funksiyaga teskari funksiya bo‘ladi:

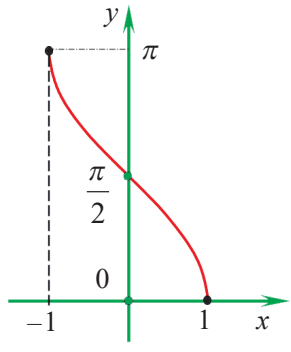
$$cos(arccosx) = x, \quad x \in [-1; 1]$$

$$arccos(cosx) = x, \quad x \in [0; \pi]$$

$y = arccosx$ funksiya quyidagi xossalarga ega:

- $D(y) = [-1;1]$;
- $E(y) = [0;\pi]$;
- $y = arccosx$ – kamayuvchi funksiya;
- $y = arccosx$ funksiyaning eng katta qiymati π ga, eng kichik qiymati 0 ga teng;
- $y = arccosx$ funksiya grafigi Ox o‘qini abssissasi $x = 1$ bo‘lgan (1; 0) nuqtada, Oy o‘qini esa ordinatasi $y = \frac{\pi}{2}$ bo‘lgan $\left(0;\frac{\pi}{2}\right)$ nuqtada kesib o‘tadi;
- $y = arccosx$ – toq ham emas, juft ham emas. Bu yerda $arccos(-x) = \pi - arccosx$ tenglik o‘rinli bo‘ladi;
- $y = arccosx$ funksiya davriy emas.

2-rasm



y = arccosx funksiyaning grafigi

2-misol. $arccos\frac{\sqrt{2}}{2}$ ifodaning qiymatini toping.

Yechish. Aytaylik, $arccos\frac{\sqrt{2}}{2} = x$ bo‘lsin. U holda berilgan vazifani quyidagicha ifodalash mumkin: $cosx = \frac{\sqrt{2}}{2}$ tenglikni qanoatlantiruvchi x ning $[0; \pi]$ oraliqdagi qiymatini toping. Ma’lumki, $cosx = \frac{\sqrt{2}}{2}$ tenglik $x = \frac{\pi}{4}$ bo‘lganda bajariladi. Demak,

$$arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}.$$

Quyidagi jadvalda $arccosx$ ifodaning ayrim qiymatlari keltirilgan.

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$arccosx$	π	$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	0

$y = arctgx$ funksiya va uning xossalari, grafigi

$$y = tgx$$

tenglik $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu yechim

$$x = arctgy$$

ko‘rinishda yoziladi. Bu tenglik bilan $R = (-\infty; +\infty)$ to‘plamning har bir y elementiga $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$

to‘plamning yagona x elementini mos qo‘yuvchi arktangens funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

$$y = arctgx$$

ko‘rinishda yozamiz (3-*rasm*).

$y = arctgx$ funksiya $y = tgx$ funksiyaga teskari funksiya bo‘ladi:

$$tg(arctgx) = x, \quad x \in (-\infty; +\infty)$$

$$arctg(tgx) = x, \quad x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right).$$

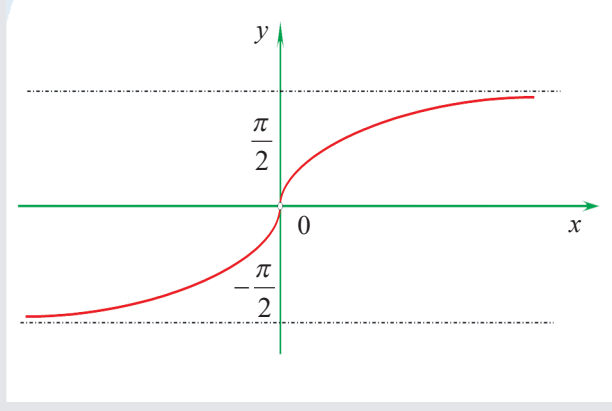
$y = arctgx$ funksiya quyidagi xossalarga ega:

- $D(y) = (-\infty; +\infty)$;
- $E(y) = \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$;
- $y = arctgx$ – o‘suvchi funksiya;
- $y = arctgx$ funksiya eng katta va eng kichik qiymatlarga erishmaydi;
- $y = arctgx$ funksiya grafigi koordinata boshidan o‘tadi;
- $y = arctgx$ – toq funksiya, ya’ni

$$arctg(-x) = -arctgx;$$

- $y = arctgx$ funksiya davriy emas.

3-rasm



y = arctgx funksiyaning grafigi

3-misol. $arctg\left(-\frac{\sqrt{3}}{3}\right)$ ifodaning qiymatini toping.

Yechish. Aytaylik, $arctg\left(-\frac{\sqrt{3}}{3}\right) = x$ bo‘lsin. U holda $tgx = -\frac{\sqrt{3}}{3}$ tenglikni qanoatlantiruvchi x ning qiymatini topish talab etiladi.

Ma’lumki, $tg\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$ bo‘ladi. Demak, $arctg\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

Quyidagi jadvalda $arctgx$ ifodaning ayrim qiymatlari keltirilgan.

x	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$arctgx$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$

$y = arcctgx$ funksiya va uning xossalari, grafigi

$$y = ctgx$$

tenglik $(0; \pi)$ oraliqda x o‘zgaruvchiga nisbatan bir qiymatli yechiladi va bu yechim

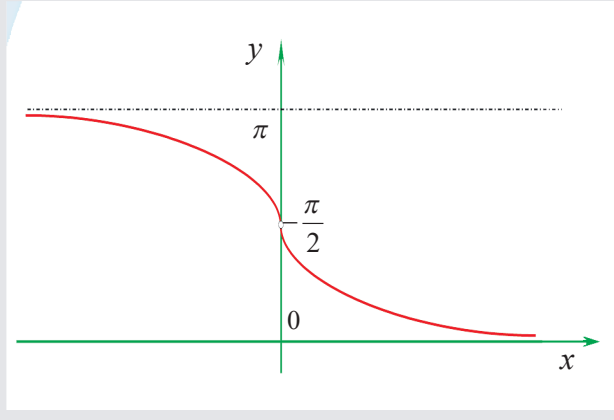
$$x = arcctgy$$

ko‘rinishda yoziladi. Bu tenglik bilan $R = (-\infty; +\infty)$ to‘plamning har bir y elementiga $(0; \pi)$ to‘plamning yagona x elementini mos qo‘yuvchi arkkotangens funksiyasi aniqlanadi. Aniqlangan bu moslikda argumentni x orqali, funksiyani esa y orqali belgilab, uni

$$y = arcctgx$$

ko‘rinishda yozamiz (4-*rasm*).

4-rasm



y = arcctgx funksiyaning grafigi

$y = arcctgx$ funksiya $y = ctgx$ funksiyaga teskari funksiya bo‘ladi:

$$ctg(arcctgx) = x, \quad x \in (-\infty; +\infty), \quad arcctg(ctgx) = x, \quad x \in (0; \pi).$$

$y = arcctgx$ funksiya quyidagi xossalarga ega:

- $D(y) = (-\infty; +\infty)$;
- $E(y) = (0; \pi)$;
- $y = arcctgx$ – kamayuvchi funksiya;
- $y = arcctgx$ funksiya eng katta va eng kichik qiymatlarga erishmaydi;
- $y = arcctgx$ funksiya grafigi Ox o‘qi bilan kesishmaydi, Oy o‘qi bilan esa ordinatasi $y = \frac{\pi}{2}$ bo‘lgan $\left(0; \frac{\pi}{2}\right)$ nuqtada kesishadi;
- $y = arcctgx$ – toq ham emas, juft ham emas. Bu funksiya uchun $arcctg(-x) = \pi - arcctg\, x$ tenglik bajariladi;
- $y = arcctgx$ funksiya davriy emas.

4-misol. $arcctg\sqrt{3}$ ifodaning qiymatini toping.

Yechish. Aytaylik, $arcctg\sqrt{3} = x$ bo‘lsin. U holda $ctgx = \sqrt{3}$ tenglikni qanoatlantiruvchi x ning qiymatini topish talab etiladi. Ma’lumki, $ctg\frac{\pi}{6} = \sqrt{3}$ bo‘ladi. Demak, $arcctg\sqrt{3} = \frac{\pi}{6}$.

Quyidagi jadvalda $arcctgx$ ifodaning ayrim qiymatlari keltirilgan.

x	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$arcctgx$	$\frac{5\pi}{6}$	$\frac{3\pi}{4}$	$\frac{2\pi}{3}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$

MISOLLAR

1. Quyidagi ifodalar ma'noga egami?

a) $arcsin(\sqrt{3}-1)$ b) $arcsin(4-\sqrt{5})$ c) $arccos\left(-\frac{\pi}{3}\right)$

d) $arccos\left(\sqrt{2}\right)$ e) $arctg(\sqrt{2})$ f) $arcctg(-100)$

2. Hisoblang:

a) $arcsin\frac{1}{2}$ b) $arcsin\,(-1)$ c) $arcsin\frac{1}{\sqrt{2}}$ d) $arcsin 0$

e) $arcsin\frac{\sqrt{3}}{2}$ f) $arcsin\left(-\frac{\sqrt{2}}{2}\right)$ g) $arccos\left(-\frac{\sqrt{2}}{2}\right)$ h) $arccos\frac{1}{2}$

i) $arccos 0$ j) $arccos\frac{1}{\sqrt{2}}$ k) $arccos\left(-\frac{\sqrt{3}}{2}\right)$ l) $arctg0$

m) $arctg(-1)$ n) $arctg\frac{1}{\sqrt{3}}$ o) $arctg\sqrt{3}$ p) $arctg1$

q) $arctg(-\sqrt{3})$ r) $arcctg\sqrt{3}$ s) $arcctg(-1)$ t) $arcctg\frac{1}{\sqrt{3}}$

3. Hisoblang.

a) $arcsin\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}-1}-\frac{\sqrt{2}-\sqrt{3}}{1+\sqrt{2}-\sqrt{3}}\right)$

b) $arccos\left(\frac{10+5\sqrt{5}}{\frac{5}{2}\left(3+\sqrt{5}\right)}-\frac{\frac{5}{2}\left(3+\sqrt{5}\right)}{10+5\sqrt{5}}\right)$

c) $arctg\left(\frac{1-2(2+\sqrt{3})}{3+\sqrt{3}}-\frac{1}{1+\sqrt{3}}\right)$

d) $arcctg\left(\frac{1+\sqrt{\frac{7}{3}-\frac{2}{\sqrt{3}}}}{1-\sqrt{\frac{7}{3}+\frac{2}{\sqrt{3}}}}-\frac{1-\sqrt{\frac{7}{3}+\frac{2}{\sqrt{3}}}}{1+\sqrt{\frac{7}{3}-\frac{2}{\sqrt{3}}}}\right)$

4. Hisoblang.

a) $cos\left(arctg2\right)$ b) $sin\left(arctg7\right)$ c) $cos\left(arcsin\frac{1}{4}\right)$

d) $ctg\left(arctg5\right)$ e) $sin\left(arctg11\right)$ f) $sin\left(arccos\frac{1}{5}\right)$

g) $cos\left(arcctg\left(-4\right)\right)$ h) $ctg\left(arcsin\left(-0,9\right)\right)$ i) $sin\left(arccos\left(-\frac{4}{7}\right)\right)$

j) $ctg\left(arctg\left(-15\right)\right)$ k) $tg\left(arccos\left(-0,3\right)\right)$ l) $ctg\left(arccos\left(-\frac{\pi}{7}\right)\right)$

5. Hisoblang.

a) $cos\left(2arcsin0,2\right)$ b) $sin\left(2\,arccos\left(-\frac{2}{3}\right)\right)$ c) $sin\left(2arcctg\sqrt{26}\right)$

d) $tg\left(2arccos0,6\right)$ e) $tg\left(2arcsin\frac{7}{9}\right)$ f) $cos\left(2arccos\left(-0,8\right)\right)$

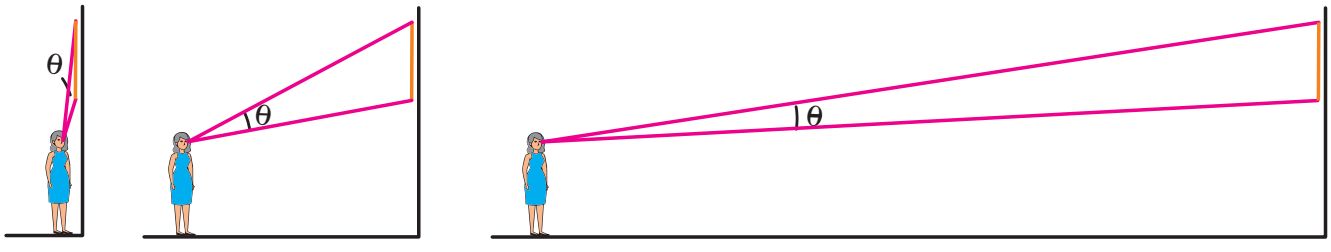
g) $tg\left(2arcctg\left(-3\right)\right)$ h) $sin\left(2arcsin\left(-0,1\right)\right)$ i) $tg\left(2arcctg20\right)$

LOYIHA ISHI

KINOTEATRDA QAYERDA O‘TIRISH KERAK?

Obyektning ko‘rinadigan o‘lchami uning tomoshabindan uzoqligiga bog‘liqligini hamma biladi. Obyekt qanchalik uzoqda bo‘lsa, uning ko‘rinadigan o‘lchami shunchalik kichik bo‘ladi. Ko‘rinadigan o‘lcham obyektning tomoshabin ko‘ziga qaragan burchagi bilan belgilanadi.

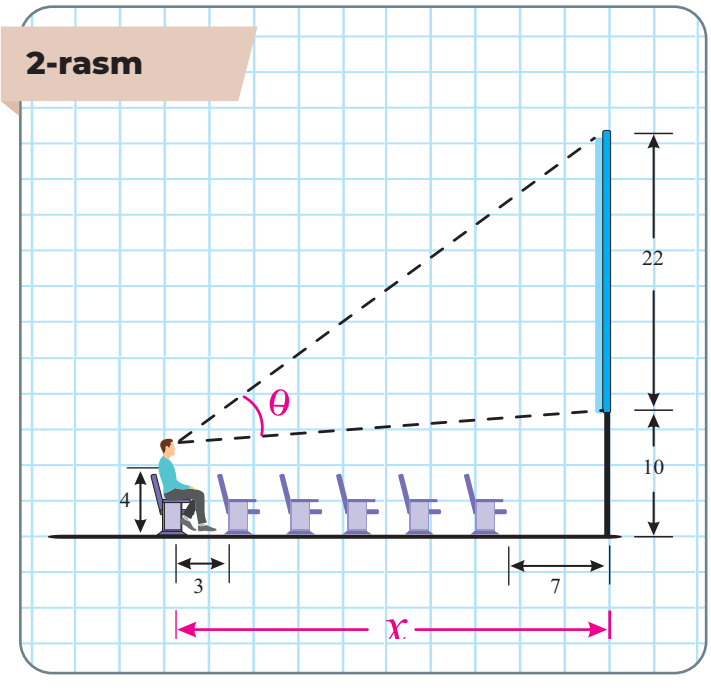
Devorga osilgan rasm ko‘zingizga maksimal ko‘rinishga ega bo‘lishi uchun undan qancha masofa uzoqlikda turishingiz kerak? Rasm ko‘z darajasidan yuqorida osilgan bo‘lsa, unda 1-rasmda ko‘rsatilgandek siz juda yaqin yoki juda uzoq bo‘lsangiz, ko‘z nazari ostidagi burchak kichik ekani aniq.



Xuddi shu holat kinoteatrda o‘rindiq tanlashda ham sodir bo‘ladi.

1. Kinoteatrdagi ekran 22 fut balandlikda va tekis poldan 10 fut balandlikda joylashgan. O‘rindiqlarning birinchi qatori ekrandan 7 fut, qatorlar esa 3 fut masofada joylashgan. Siz maksimal ko‘rinishga ega bo‘lgan qatorga o‘tirishga qaror qildingiz, ya’ni ekranning ko‘zingiz nazaridagi burchagi θ maksimal bo‘lgan joyda. Aytaylik, ko‘zlaringiz rasmdagi kabi poldan 4 fut balandlikda va siz ekrandan x masofada o‘tiribsiz (1 fut = 0,3048 m) (2-*rasm*).

2-rasm



Quyidagini isbotlang: $\theta = arctg6x - arctg28x$

Quyidagini keltirib chiqarish uchun ayirmaning tangensi formulasidan foydalaning:

$$\theta = arctg\left(\frac{22x}{x^2+168}\right)$$

Geogebra ilovasidan foydalanib θ ning x ga nisbatan funksiya sifatida grafigini tuzing. x ning qaysi qiymati θ ni maksimal darajada oshiradi? Qaysi qatorda o‘tirish kerak? Ushbu qatordagi ko‘rish burchagi qanday?

Endi faraz qilaylik, birinchi qatordagi o‘rindiqlardan boshlab o‘rindiqlar gorizontal tekislikdan yuqorida va kinoteatr poli α burchak ostida qiyalikda. Siz o‘tirgan joydan polgacha masofa, rasmda ko‘rsatilganidek, x ga teng (3-rasm).

1. Quyidagini keltirib chiqarish uchun kosinuslar formulasidan foydalaning:

$$\theta = arccos\left(\frac{a^2+b^2-484}{2ab}\right).$$

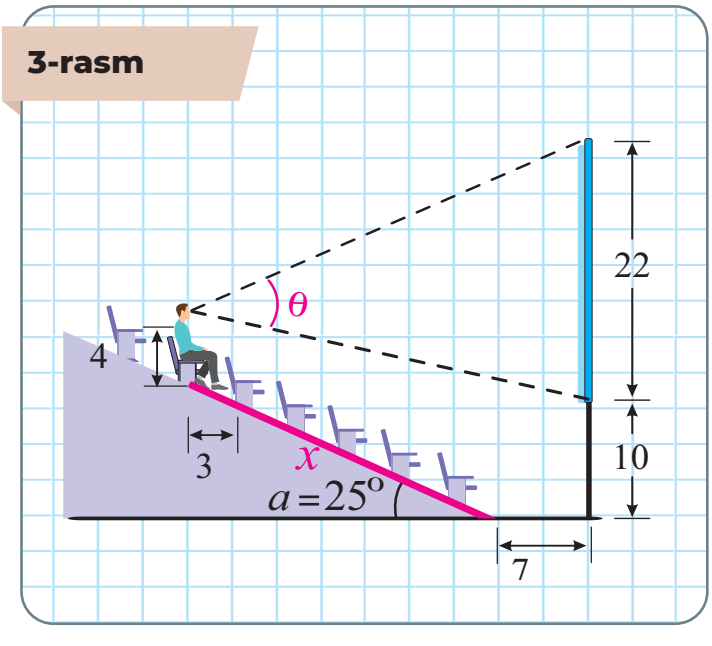
Bu yerda

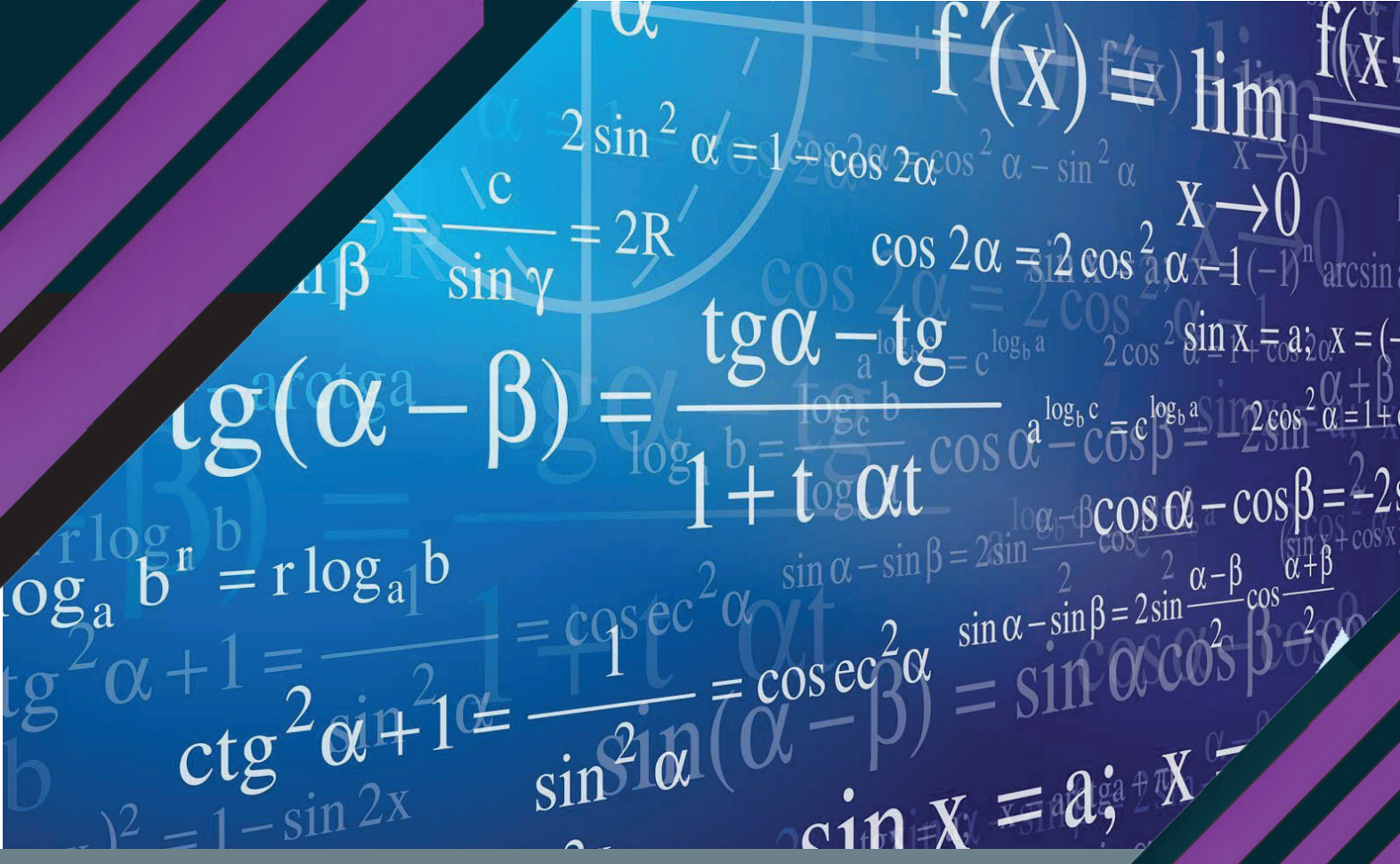
$$a^2 = (7+x\cos\alpha)^2 + (28-x\sin\alpha)^2$$

va $b^2 = (7+x\cos\alpha)^2 + (x\sin\alpha-6)^2$

2. ***Geogebra*** grafik ilovasidan foydalanib θ ning x ning funksiyasi sifatida grafigini tuzing va θ ni maksimallashtiruvchi x ning qiymatini toping. Qaysi qatorda o‘tirish kerak? Bu qatordagi ko‘rish burchagi qanday?

3-rasm





5-BOB. TRIGONOMETRIK TENGLAMALAR VA TENGSIZLIKLAR

- TRIGONOMETRIK TENGLAMALAR
- BA’ZI TRIGONOMETRIK TENGLAMALARNI YECHISH USULLARI
- TRIGONOMETRIK TENGSIZLIKLAR

TRIGONOMETRIK TENGLAMALAR

Eng sodda trigonometrik tenglamalar

Davriy funksiyalar bilan tavsiflanadigan jarayonlar qachon qanday qiymat qabul qilishini bilish muhim ahamiyatga ega. Buning uchun davriy funksiyalar qatnashgan

$$\sin x = a,\ \cos x = a,\ tgx = a,\ ctgx = a$$

ko‘rinishdagi eng sodda trigonometrik tenglamalarni yechishni bilish zarur.

Eng sodda trigonometrik tenglamalarni yechishni o‘rganish uchun:

1) tenglama tushunchasini;

2) tenglamaning ildizi tushunchasini; ildizlar to‘plami “yechim” deb atalishini;

3) trigonometrik funksiyalar davriyligi, trigonometrik tenglamaning ildizlari cheksiz ko‘p bo‘lishini;

4) topilgan cheksiz ko‘p ildizlarni umumlashtirib, qisqa formulalar orqali yoza olishni (bunda har bir k butun son uchun $n = 2k$ ifoda juft sonni, $n = 2k+1$ ifoda esa toq sonni anglatishini) bilish talab etiladi.

1-misol. $\sin x = \frac{1}{2}$ tenglamani yeching.

Yechish. Ma’lumki, $\sin\frac{\pi}{6} = \frac{1}{2}$ bo‘ladi. $\sin x = \frac{1}{2}$ tenglik x ning $\frac{5\pi}{6} = \pi - \frac{\pi}{6}$ qiymatida ham bajariladi (1-*rasm*). Sinus davriy funksiya ekani sababli har qanday n butun son uchun

$$x = \frac{\pi}{6} + 2\pi n$$

yoki

$$x = \left(\pi - \frac{\pi}{6}\right) + 2\pi n = \frac{5\pi}{6} + 2\pi n$$

bo‘lganda ham $\sin x = \frac{1}{2}$ bo‘ladi (2-*rasm*). Bu ikkita tenglikni quyidagicha umumlashtirish mumkin:

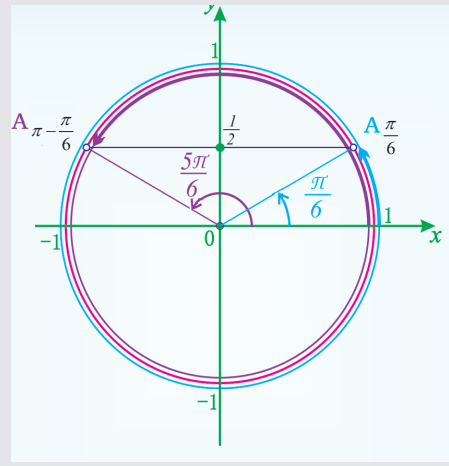
$$x = (-1)^n \frac{\pi}{6} + \pi n,\ n \in \mathbb{Z}.$$

Haqiqatan ham, n juft bo‘lsa, $x = \frac{\pi}{6} + 2\pi n$ tenglikka; n toq bo‘lsa, $x = \frac{5\pi}{6} + 2\pi n$ tenglikka ega bo‘lamiz. Shunday qilib, $\sin x = \frac{1}{2}$ tenglik x ning

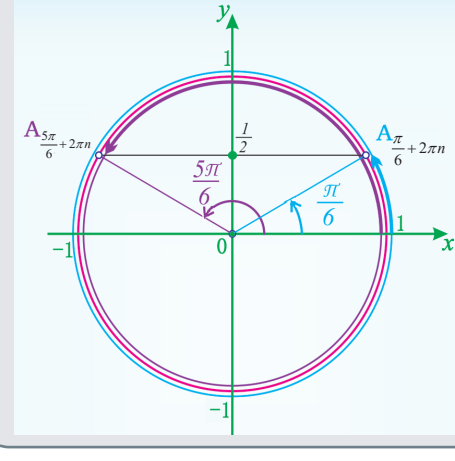
$$x = (-1)^n \frac{\pi}{6} + \pi n,\ n \in \mathbb{Z}.$$

qiymatlarida bajarilar ekan.

1-rasm



2-rasm



$sinx = a$ ko‘rinishdagi tenglama

$a > 1$ yoki $a < -1$ bo‘lsa, u holda $sin\,x = a$ tenglama ildizga ega bo‘lmaydi. Shuning uchun bunday hollarda $sin\,x = a$ tenglamaning yechimi bo‘sh to‘plam $\varnothing$ dan iborat degan javob yoziladi;

$-1 \le a \le 1$ bo‘lsa, $sin\,x = a$ tenglamaning yechimi

$$x = (-1)^n \, arcsin\, a + \pi n,\ n \in Z$$

ko‘rinishda bo‘ladi.

Xususiy hollar.

1) $sinx = -1$ tenglamaning yechimi x ning

$$x = -\frac{\pi}{2} + 2\pi n,\ \ n \in \mathbb{Z}$$

qiymatlaridan iborat.

2) $sinx = 0$ tenglamaning yechimi x ning

$$x = \pi n,\ n \in \mathbb{Z}$$

qiymatlaridan iborat.

3) $sinx = 1$ tenglamaning yechimi x ning

$$x = \frac{\pi}{2} + 2\pi n,\ \ n \in \mathbb{Z}$$

qiymatlaridan iborat.

$cosx = a$ ko‘rinishdagi tenglama

$\boldsymbol{cos\,x = a}$ ko‘rinishdagi tenglamada

agar $a > 1$ yoki $a < -1$ bo‘lsa, u holda $cos\,x = a$ tenglama ildizga ega bo‘lmaydi. Bunday hollarda $cos\,x = a$ tenglamaning yechimi $\varnothing$ degan javob yoziladi;

$-1 \le a \le 1$ bo‘lsa, $cos\,x = a$ tenglamaning yechimi

$$x = \pm\, arccos\, a + 2\pi n,\ n \in Z$$

bo‘ladi.

Xususiy hollar

1) $cosx = -1$ tenglamaning yechimi x ning

$$x = \pi + 2\pi n,\ \ n \in \mathbb{Z}$$

qiymatlaridan iborat.

2) $cosx = 0$ tenglamaning yechimi x ning

$$x = \frac{\pi}{2} + \pi n,\ n \in \mathbb{Z}$$

qiymatlaridan iborat.

3) $cosx = 1$ tenglamaning yechimi x ning

$$x = 2\pi n,\ \ n \in \mathbb{Z}$$

qiymatlaridan iborat.

2-misol. $\cos x = \frac{\sqrt{2}}{2}$ tenglamani yeching.

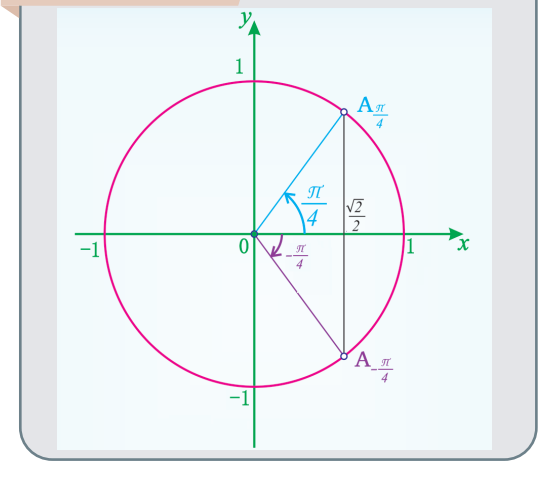
Yechish

$\cos\frac{\pi}{4} = \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ bo‘lishi ma’lum (3-*rasm*). Kosinus davriy funksiya ekani sababli har qanday n butun son uchun

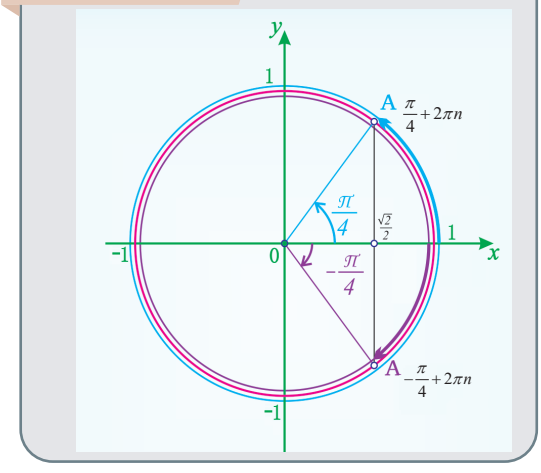
$$x = \frac{\pi}{4} + 2\pi n \text{ yoki } x = -\frac{\pi}{4} + 2\pi n$$

bo‘lganda ham $\cos x = \frac{\sqrt{2}}{2}$ bo‘ladi (4-*rasm*). Bu ikkita tenglikni quyidagicha umumlashtirish mumkin: $x = \pm\frac{\pi}{4} + 2\pi n,\ n \in \mathbb{Z}$.

3-rasm



4-rasm



tgx = a ko‘rinishdagi tenglama

Har bir n butun son uchun x ning

$$x = arctga + \pi n$$

qiymatlari $tgx = a$ tenglamaning ildizi bo‘ladi. Bu holda yechim

$$x = arctga + \pi n,\ n \in Z$$

ko‘rinishda bo‘ladi.

ctgx = a ko‘rinishdagi tenglamalar

Har bir n butun son uchun x ning

$$x = arcctga + \pi n$$

qiymatlari $ctgx = a$ tenglamaning ildizi bo‘ladi. Bu holda yechim

$$x = arcctga + \pi n,\ n \in Z$$

ko‘rinishida bo‘ladi.

3-misol. Tenglamani yeching: $tg\left(x+\frac{\pi}{7}\right)=-1$

Yechish

$tg\left(x+\frac{\pi}{7}\right)=-1,\ x+\frac{\pi}{7}=arctg(-1)+\pi k, k\in Z,\ x+\frac{\pi}{7}=-\frac{\pi}{4}+\pi k, k\in Z,\ x=-\frac{11\pi}{28}+\pi k, k\in Z.$

Javob: $x=-\frac{11\pi}{28}+\pi k, k\in Z.$

4-misol. Tenglamani yeching: $ctg\frac{3x}{2}=\sqrt{3}.$

Yechish

$ctg\frac{3x}{2}=\sqrt{3}\ ,\ \frac{3x}{2}=arcctg\sqrt{3}+\pi k,\ k\in Z,\ \frac{3x}{2}=\frac{\pi}{6}+\pi k, k\in Z\ ,\ 3x=\frac{\pi}{3}+2\pi k, k\in Z\ ,$

$x=\frac{\pi}{9}+\frac{2}{3}\pi k, k\in Z.$

Javob: $x=\frac{\pi}{9}+\frac{2}{3}\pi k, k\in Z.$

MISOLLAR

1. Tenglamalarni yeching.

a) $sin2x=1$ b) $sin\frac{x}{3}=-1$ c) $sin\left(2x-\frac{\pi}{5}\right)=0$

d) $2sin4x=\sqrt{5}$ e) $sin(4x-1)=-\frac{\pi}{3}$ f) $sinx=\frac{1}{2}$

g) $sinx=-\frac{\sqrt{3}}{2}$ h) $sin4x=1$ i) $sin2x=\frac{\sqrt{2}}{2}$

j) $sin\left(x+\frac{\pi}{7}\right)=\frac{\sqrt{3}}{2}$ k) $sin\frac{2x}{3}=-1$ l) $sin\left(2x+\frac{\pi}{5}\right)=0$

m) $sin(3x+1)=-\frac{\sqrt{2}}{2}$ n) $sin(-x)=-\frac{1}{2}$ o) $sin\left(\frac{3\pi}{4}-x\right)=1$

2. Tenglamalarni yeching.

a) $cos\frac{2x}{5}=1$ b) $cos\left(2x-\frac{\pi}{7}\right)=-1$ c) $cos8x=0$

d) $cos3x=1,2$ e) $2cos(x-1)=\frac{11}{2}$ f) $cosx=\frac{\sqrt{3}}{2}$

g) $cosx=-\frac{1}{2}$ h) $cosx=-1$ i) $cos\frac{x}{2}=-\frac{\sqrt{2}}{2}$

j) $cos\left(x-\frac{\pi}{5}\right)=\frac{1}{2}$ k) $cos\frac{3x}{4}=0$ l) $cos\,4x=-\frac{\sqrt{3}}{2}$

m) $\sqrt{3}+2cos\frac{\pi x}{9}=0$ n) $1-2cos\frac{3\pi x}{4}=0$ o) $cos\left(\pi\left(x-3\right)\right)=1$

p) $sin^2\frac{2}{3}x=\frac{3}{4}$ q) $cos^2\frac{3}{2}x=\frac{1}{4}$ r) $2cos\left(2x-\frac{\pi}{6}\right)+\sqrt{3}=0$

3. Tenglamalarni yeching.

a) $tgx=\frac{1}{\sqrt{3}}$ b) $tgx=-\frac{1}{\sqrt{3}}$ c) $tgx=-1$

d) $tgx=\sqrt{3}$ e) $tg\frac{2x}{5}=-\sqrt{3}$ f) $tg\left(x+\frac{7\pi}{3}\right)=1$

g) $tg(\frac{\pi}{4}(x-1))=0$ h) $1-\sqrt{3}tg\frac{2\pi x}{7}=0$ i) $tg9x=tg45°$

j) $tg6x=tg\frac{2\pi}{3}$ k) $3tg\left(x+\frac{5\pi}{36}\right)+\sqrt{3}=0$

4. Tenglamalarni yeching.

a) $ctgx=\sqrt{3}$ b) $ctgx=-\frac{1}{\sqrt{3}}$ c) $ctg4x=sin0°$

d) $ctg(\pi\left(2x+3\right))=cos0°$ e) $\sqrt{3}+ctg\frac{\pi x}{5}=0$ f) $ctg7x=-\sqrt{3}$

g) $ctg\frac{3x}{2}=1$ h) $ctg3x=\sqrt{3}$ i) $ctg\left(2x-\frac{\pi}{5}\right)=0$

5. Tenglamalarning berilgan kesmadagi ildizlarini toping.

a) $sin3x=\frac{\sqrt{2}}{2}$, $[0;2\pi]$ b) $cos\,3x=\frac{\sqrt{3}}{2}$, $[-\pi;\pi]$

c) $tg\frac{x}{2}=\frac{\sqrt{3}}{3}$, $[0;\pi]$ d) $ctg4x=-1$, $[-3\pi;3\pi]$

6. a ning qanday qiymatlarida $tgx=\frac{a+1}{a-1}$ tenglik o‘rinli bo‘lishi mumkin?

7. a ning qanday qiymatlarida $sin\,x=a+\frac{1}{a}$ tenglik o‘rinli bo‘lishi mumkin?

8. a ning qanday qiymatlarida $5cos\left(2x-3\right)=a-\frac{6}{a}$ tenglama yechimga ega?

9. a ning qanday qiymatlarida $5sin\left(x-7\right)=a-\frac{4}{a}$ tenglama yechimga ega emas?

BA'ZI TRIGONOMETRIK TENGLAMALARNI YECHISH USULLARI

Kvadrat tenglamaga keltiriladigan tenglamalar

1-misol. Tenglamani yeching: $2\sin^2 x - 3\sin x + 1 = 0$.

Yechish

Bu tenglama $\sin x$ ga nisbatan kvadrat tenglamadir. $\sin x = t$ deb belgilasak, $2t^2 - 3t + 1 = 0$ bundan $t_1 = 1,\ \ t_2 = \frac{1}{2}$ kelib chiqadi.

1) $\sin x = 1,\ x = \frac{\pi}{2} + 2\pi n,\ n \in Z$ 2) $\sin x = \frac{1}{2},\ x = (-1)^n \cdot \frac{\pi}{6} + \pi n,\ n \in Z$

Javob: $x = \frac{\pi}{2} + 2\pi n,\ x = (-1)^n \cdot \frac{\pi}{6} + \pi n,\ \ n \in Z$.

2-misol. Tenglamani yeching: $2\cos^2 x - 5\sin x + 1 = 0$.

Yechish

$\cos^2 x$ ni $1 - \sin^2 x$ bilan almashtirib, $2(1 - \sin^2 x) - 5\sin x + 1 = 0$ yoki $2\sin^2 x + 5\sin x - 3 = 0$ ni,

$\sin x = y$ belgilash kiritib, $2y^2 + 5y - 3 = 0$ ni hosil qilamiz. Bundan $y_1 = -3; y_2 = \frac{1}{2}$.

$\sin x = -3$ tenglama yechimga ega emas, chunki $|-3| > 1$.

$\sin x = \frac{1}{2}$ tenglamani yechamiz. Bundan $x = (-1)^n \arcsin\frac{1}{2} + \pi n = (-1)^n \frac{\pi}{6} + \pi n,\ n \in Z$ ni hosil qilamiz.

Javob: $x = (-1)^n \frac{\pi}{6} + \pi n,\ \ n \in Z.$

3-misol. Tenglamani yeching: $tgx - 2ctgx + 1 = 0$.

Yechish

$tgx - \frac{2}{tgx} + 1 = 0$, bundan $tg^2 x + tgx - 2 = 0$. $tgx = t$ deb belgilasak,

$t^2 + t - 2 = 0$

$t_1 = 1,\ \ t_2 = -2$

$1)\ tgx = 1,\ x = arctg1 + \pi n,\ n \in Z,\ x = \frac{\pi}{4} + \pi n,\ \text{n} \in Z$

$2)\ tgx = -2,\ x = -arctg2 + \pi n,\ \ n \in Z$

Javob: $x = \frac{\pi}{4} + \pi n,\ \ x = -arctg2 + \pi n,\ \ n \in Z$.

4-misol. Tenglamani yeching: $3\sin^2 x+5\sin x\cos x+2\cos^2 x=0$.

Yechish

Tenglamani hadma-had $\cos^2 x$ ga bo‘lamiz. $3tg^2x+5tgx+2=0$

$tgx=t$ deb belgilasak, $3t^2+5t+2=0$. Bundan $t_1=\frac{-5-1}{6}=-1,\quad t_2=\frac{-5+1}{6}=-\frac{2}{3}$

1) $tgx=-1,\ x=-\frac{\pi}{4}+\pi n,\ n\in Z$

2) $tgx=-\frac{2}{3},\ x=-arctg\frac{2}{3}+\pi n,\ n\in Z$

Javob: $x=-\frac{\pi}{4}+\pi n,\ x=-arctg\frac{2}{3}+\pi n,\ n\in Z$.

$a\sin x+b\cos x=c$ ko‘rinishdagi tenglamalar

5-misol. Tenglamani yeching: $3\sin x-2\cos x=0$.

Yechish

1) Tenglamaning ikkala tarafini $\cos x$ ga bo‘lib, $3tgx-2=0$ tenglamani hosil qilamiz.

2) $3tgx-2=0$, $tgx=\frac{2}{3},\ x=arctg\frac{2}{3}+\pi n,\ n\in Z$.

$a\sin x+b\cos x=0$ tenglamani $\cos x$ (yoki $\sin x$) ga bo‘lganda berilgan tenglamaga teng kuchli tenglama hosil bo‘ladi ($\cos x=0$ va $\sin x=0$ tengliklar bir vaqtda bajarilmaydi).

Javob: $x=arctg\frac{2}{3}+\pi n,\ n\in Z$.

6-misol. Tenglamani yeching: $2\sin x+\cos x-2=0$.

Yechish

$\sin x=2\sin\frac{x}{2}\cos\frac{x}{2},\quad \cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2},\quad 2=2\cdot1=2\left(\sin^2 x+\cos^2 x\right)$ formulalarga ko‘ra,

$4\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}=2\sin^2\frac{x}{2}+2\cos^2\frac{x}{2}\quad\Rightarrow\quad 3\sin^2\frac{x}{2}-4\sin\frac{x}{2}\cos\frac{x}{2}+\cos^2\frac{x}{2}=0$

Tenglamani $\cos^2\frac{x}{2}$ ga bo‘lamiz. $3tg^2\frac{x}{2}-4tg\frac{x}{2}+1=0$ $tg\frac{x}{2}=t$ deb belgilash kiritamiz.

$3t^2-4t+1=0,\quad D=16-12=4$

$t_1=\frac{4+2}{6}=1,\quad t_2=\frac{4-2}{6}=\frac{1}{3}$

1) $tg\frac{x}{2}=1,\ x=\frac{\pi}{2}+2\pi n,\ n\in Z$

2) $tg\frac{x}{2}=\frac{1}{3},\ x=2arctg\frac{1}{3}+2\pi n,\ n\in Z$

Javob: $x=\frac{\pi}{2}+2\pi n,\ x=2arctg\frac{1}{3}+2\pi n,\ n\in Z$.

7-misol. Tenglamani yeching: $sinx + cosx = 1$.

Yechish

Tenglamaning ikkala tarafini $\sqrt{1^2+1^2} = \sqrt{2}$ ga bo'lamiz.

$$\frac{1}{\sqrt{2}} sin\, x + \frac{1}{\sqrt{2}} cos\, x = \frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{2}} = sin\frac{\pi}{4} = cos\frac{\pi}{4}$ bo'lgani uchun tenglamani quyidagicha yozib olamiz:

$$sin\frac{\pi}{4} sin\, x + cos\frac{\pi}{4} cos\, x = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad cos\left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad x - \frac{\pi}{4} = \pm arccos\frac{1}{\sqrt{2}} + 2\pi n,\ n \in Z$$

$$x = \frac{\pi}{4} \pm \frac{\pi}{4} + 2\pi n,\ n \in Z \ \Rightarrow\ x = 2\pi n,\ x = \frac{\pi}{2} + 2\pi n,\ n \in Z.$$

Javob: $x = 2\pi n,\ x = \frac{\pi}{2} + 2\pi n,\ n \in Z.$

Chap qismini ko'paytuvchilarga ajratib yechiladigan tenglamalar

8-misol. Tenglamani yeching: $sin\, 9x - sin\, x = cos\, 5x$.

Yechish

$$2\, sin\, 4x\, cos\, 5x = cos\, 5x \quad \Rightarrow \quad cos\, 5x\left(2\, sin\, 4x - 1\right) = 0$$

1) $cos\, 5x = 0,\ x = \frac{\pi}{10} + \frac{\pi n}{5},\ n \in Z$

2) $sin\, 4x = \frac{1}{2},\ 4x = (-1)^n \cdot \frac{\pi}{6} + \pi n,\ n \in Z,\ x = (-1)^n \cdot \frac{\pi}{24} + \frac{\pi n}{4},\ n \in Z$

Javob: $x = \frac{\pi}{10} + \frac{\pi n}{5},\ x = (-1)^n \cdot \frac{\pi}{24} + \frac{\pi n}{4},\ \mathrm{n} \in Z$.

9-misol. Tenglamani yeching: $2\, sin\, x\, cos\, x + 5\, sin\, x + 5\, cos\, x + 1 = 0$.

Yechish

$2\, sin\, x\, cos\, x + 5\left(sin\, x + cos\, x\right) + 1 = 0$, $sin\, x + cos\, x = t$ deb belgilasak, $2\, sin\, x\, cos\, x = t^2 - 1$ bo'ladi.

$$t^2 - 1 + 5t + 1 = 0 \ \Rightarrow\ t^2 + 5t = 0 \ \Rightarrow\ t(t+5) = 0 \ \Rightarrow\ t_1 = -5;\ t_2 = 0$$

1) $sin\, x + cos\, x = -5$ tenglama ildizlarga ega emas

2) $sin\, x + cos\, x = 0,\ tgx + 1 = 0,\ tgx = -1,\ x = -\frac{\pi}{4} + \pi n,\ n \in Z$

Javob: $x = -\frac{\pi}{4} + \pi n,\ \mathrm{n} \in Z$.

MISOLLAR

Tenglamalarni yeching.

1. $2\cos^2 x - 5\cos x + 2 = 0$
2. $\cos^2 x - \cos x - 2 = 0$
3. $2ctg^2 3x - 3ctg3x + 1 = 0$
4. $tg^2 x - 2tgx = 3$
5. $2\cos^2 x + \sin x - 1 = 0$
6. $3\sin^2 2x + 7\cos 2x - 3 = 0$
7. $2\cos x = 1 - \sqrt{\cos x}$
8. $\sin 2x = \cos^4 x - \sin^4 x$
9. $\sin 5x = \frac{2}{3}\cos^2 5x$
10. $\cos^4 \frac{x}{5} + \sin^2 \frac{x}{5} = 1$
11. $3tg2x - 2ctg2x - 1 = 0$
12. $2tgx - 2ctgx = 3$
13. $\sqrt{3}\sin x - \cos x = 0$
14. $\sin 2x + \cos 2x = 0$
15. $\sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right) = 0$
16. $\sqrt{3}\sin\left(x + \frac{\pi}{12}\right) + \cos\left(x + \frac{\pi}{12}\right) = 0$
17. $\cos x - \sin x = 1$
18. $\sin x + \cos x = \sqrt{2}$
19. $\sin \frac{x}{2} + \cos \frac{x}{2} = -1$
20. $\sqrt{3}\sin x + \cos x = \sqrt{2}$
21. $3\sin x + 4\cos x = 3$
22. $\sin 4x + \cos 4x = 4$
23. $\sin 2x = \cos^4 \frac{x}{2} - \sin^4 \frac{x}{2}$
24. $\cos 2x = \sqrt{2}(\cos x - \sin x)$
25. $\cos 3x \cos 2x = \sin 3x \sin 2x$
26. $\sin 5x \cos 4x - \cos 5x \sin 4x = \frac{\sqrt{3}}{2}$
27. $\cos 9x - \cos 7x + \cos 3x - \cos x = 0$
28. $\cos 7x + \sin 8x = \cos 3x - \sin 2x$
29. $\sin 3x + \sin 5x = \sin 4x$
30. $\sin 2x \sin 6x = \cos x \cos 3x$
31. $(2\cos x - 3) \cdot ctgx = 0$
32. $(tgx - 3)(\cos x - \frac{1}{2}) = 0$
33. $tg3x\cos x = 0$
34. $\sin 2x tgx = 0$
35. $\frac{\cos 2x}{1 + tgx} = 0$
36. $\frac{1 - 2\cos 2x}{\cos 2x - 2} = 0$
37. $\frac{tgx}{\sin 5x} = 0$
38. $\frac{\cos x}{1 - \cos 4x} = 0$
39. $|\cos 2x - 1| - 2|\cos 2x + 2| = 0$
40. $\sin^3 x + \cos^4 x = 1$
41. $\cos x \sqrt{\sin x} = 0$
42. $\cos 3x + 2\cos x = 0$
43. $\sin^{13} x + \cos^{13} x = 1$
44. $\sin 9x = 2\sin 3x$

TRIGONOMETRIK TENGSIZLIKLAR

Eng sodda trigonometrik tengsizliklarni yechishda:

1) *Oy* o'qi **sinuslar o'qi** deb atalishini;
2) *Ox* o'qi **kosinuslar o'qi** deb atalishini;
3) x o'zgaruvchining har bir qiymatida $-1 \le \sin x \le 1$ bo'lishini;
4) x o'zgaruvchining har bir qiymatida $-1 \le \cos x \le 1$ bo'lishini bilish talab etiladi.

Aytaylik, $f(x)$ yozuv $\sin x$, $\cos x$, tgx yoki $ctgx$ trigonometrik funksiyalardan birini anglatsin, ya'ni $f(x) = \sin x$, $f(x) = \cos x$, $f(x) = tgx$ yoki $f(x) = ctgx$ bo'lsin.

U holda biror a soni uchun $f(x) < a$, $f(x) \le a$, $f(x) > a$, $f(x) \ge a$ ko'rinishdagi tengsizliklar ***trigonometrik tengsizliklar*** deb yuritiladi.

$\sin x < a$ va $\sin x \le a$ tengsizliklarni yechish

1) $a \le -1$ bo'lsa, $\sin x < a$ tengsizlikning yechimi $\varnothing$ bo'ladi (1a-*rasm*).

2) $a > 1$ bo'lsa, $\sin x < a$ tengsizlikning yechimi $(-\infty; +\infty)$ bo'ladi (1b-*rasm*).

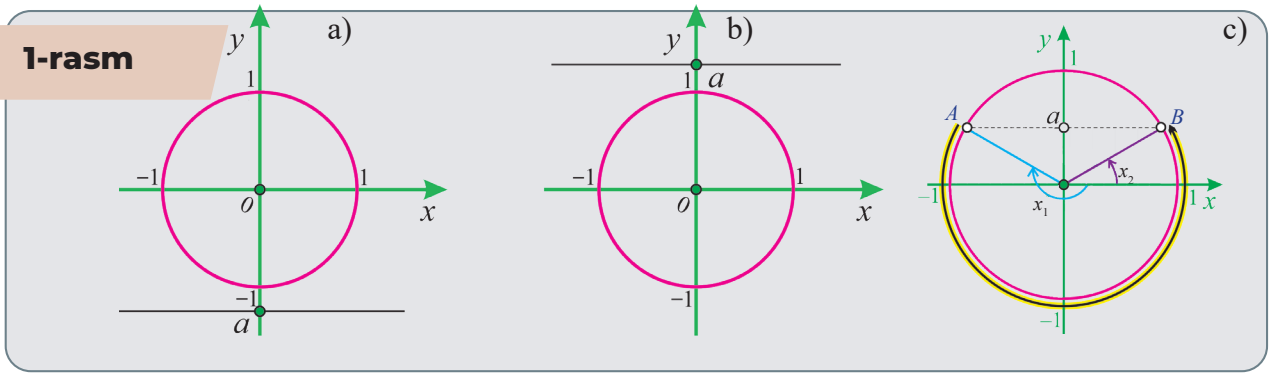
3) $a < -1$ bo'lsa, $\sin x \le a$ tengsizlikning yechimi $\varnothing$ bo'ladi.

4) $a \ge 1$ bo'lsa, $\sin x \le a$ tengsizlikning yechimi $(-\infty; +\infty)$ bo'ladi.

5) $a = -1$ bo'lsa, $\sin x \le -1$ tengsizlikning yechimi $x = -\frac{\pi}{2} + 2\pi n,\ n \in Z$ bo'ladi.

6) $a = 1$ bo'lsa, $\sin x < 1$ tengsizlikning yechimi $x \ne \frac{\pi}{2} + 2\pi n,\ n \in Z$ bo'ladi.

1-rasm



7) $-1 < a < 1$ bo'lganda $\sin x < a$ tengsizlikning yechimi (1c-*rasm*).

$$x_1 < x < x_2$$

$$x_1 = -\pi - \arcsin a$$

$$x_2 = \arcsin a$$

$$-\pi - \arcsin a + 2\pi n < x < \arcsin a + 2\pi n,\ n \in Z$$

8) $-1 < a < 1$ bo'lsa, u holda $\sin x \le a$ tengsizlikning yechimi

$$-\pi - \arcsin a + 2\pi n \le x \le \arcsin a + 2\pi n,\ n \in Z$$

1-misol. Tengsizlikni yeching: $\sin x \le -\frac{\sqrt{3}}{2}$.

Yechish

$$-\pi - arcsin\left(-\frac{\sqrt{3}}{2}\right) + 2\pi n \le x \le arcsin\left(-\frac{\sqrt{3}}{2}\right) + 2\pi n,\ n \in Z$$

$$-\frac{2\pi}{3} + 2\pi n \le x \le -\frac{\pi}{3} + 2\pi n,\ n \in Z$$

Javob: $\left[-\frac{2\pi}{3} + 2\pi n;\ -\frac{\pi}{3} + 2\pi n\right]$, $n \in Z$.

cosx < *a* va *cosx* ≤ *a* tengsizliklarni yechish

1) $a \le -1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $\varnothing$ bo‘ladi (2a-*rasm*).

2) $a > 1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi $(-\infty; +\infty)$ bo‘ladi (2b-*rasm*).

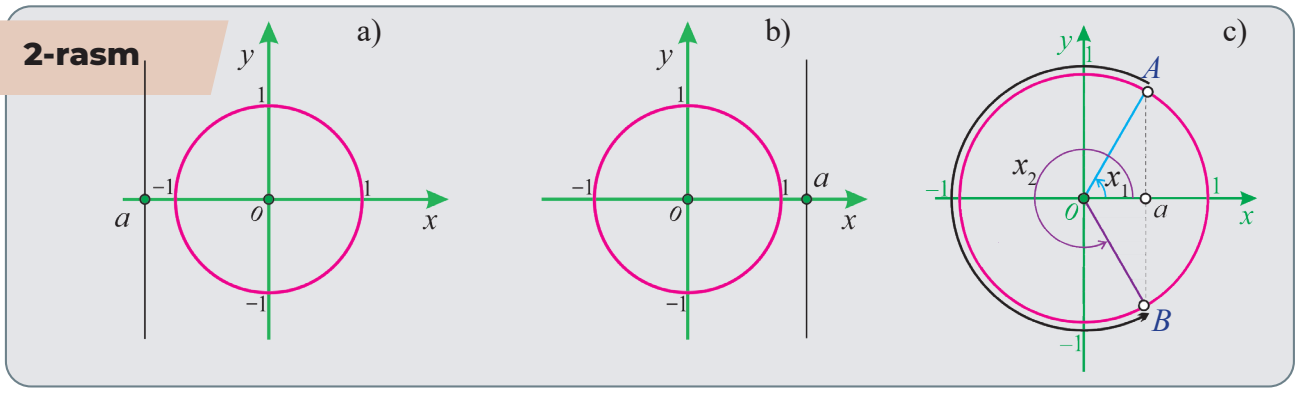
3) $a < -1$ bo‘lsa, $\cos x \le a$ tengsizlikning yechimi $\varnothing$ bo‘ladi.

4) $a = -1$ bo‘lsa, $\cos x \le -1$ tengsizlikning yechimi $x = \pi + 2\pi n,\ n \in Z$ nuqtalardan iborat.

5) $a = 1$ bo‘lsa, $\cos x < 1$ tengsizlikning yechimi $x \ne 2\pi n,\ n \in Z$ bo‘ladi.

6) $a \ge 1$ bo‘lsa, $\cos x \le a$ tengsizlikning yechimi $(-\infty; +\infty)$ bo‘ladi.

2-rasm



7) $-1 < a < 1$ bo‘lsa, $\cos x < a$ tengsizlikning yechimi (2c-*rasm*).

$$x_1 < x < x_2$$

$$x_1 = arccos a$$

$$x_2 = 2\pi - arccos a$$

$$\boldsymbol{arccos a + 2\pi n < x < 2\pi - arccos a + 2\pi n,\ n \in Z}$$

8) $-1 < a < 1$ bo‘lsa, $\cos x \le a$ tengsizlikning yechimi

$$\boldsymbol{arccos a + 2\pi n \le x \le 2\pi - arccos a + 2\pi,\ n \in Z.}$$

2-misol. Tengsizlikni yeching: $\cos x \le \frac{1}{2}$.

Yechish

$$-\arccos\left(-\frac{1}{2}\right)+2\pi n \le x \le \arccos\left(-\frac{1}{2}\right)+2\pi n,\ n \in Z$$

$$-\frac{2\pi}{3}+2\pi n \le x \le \frac{2\pi}{3}+2\pi n,\ n \in Z$$

Javob: $\left[-\frac{2\pi}{3}+2\pi n;\frac{2\pi}{3}+2\pi n\right],\ n \in Z$.

$tgx < a$ va $tgx \le a$ tengsizlikni yechish

$tgx < a$ tengsizlikni $y = tgx$ funksiya grafidan foydalanib yechaylik.

3-rasmdan ravshanki, $tgx < a$ tengsizlik x ning

$$-\frac{\pi}{2} < x < arctga$$

qo‘shtengsizlikni qanoatlantiruvchi qiymatlarida bajariladi. $y = tgx$ funksiya davriy ekanligidan $tgx < a$ tengsizlikning yechimi

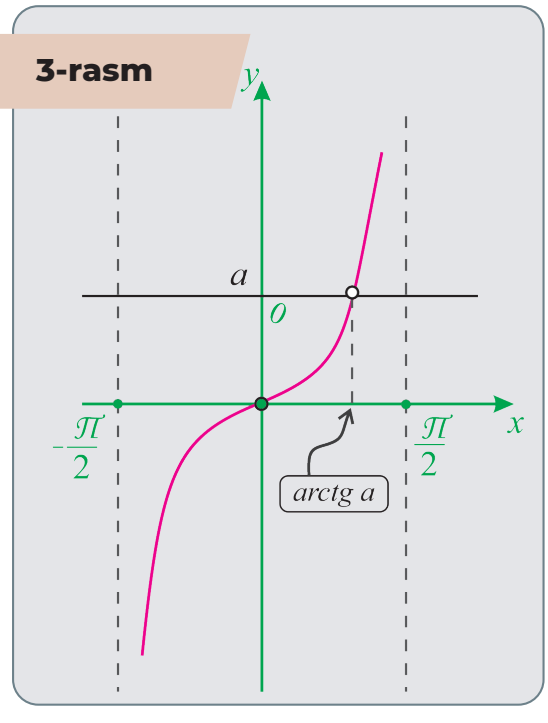
$$-\frac{\pi}{2}+\pi n < x < arctga+\pi n,\ n \in Z$$

bo‘lishi kelib chiqadi. Xuddi shuningdek, $tgx \le a$ tengsizlikning yechimi

$$-\frac{\pi}{2}+\pi n < x \le arctga+\pi n,\ n \in Z$$

bo‘ladi.

3-rasm



3-misol. Tengsizlikni yeching: $tg\frac{x}{4} \le -1$.

Yechish

$$-\frac{\pi}{2}+\pi n < \frac{x}{4} \le arctg(-1)+\pi n,\ n \in Z$$

$$-\frac{\pi}{2}+\pi n < \frac{x}{4} \le -\frac{\pi}{4}+\pi n,\ n \in Z$$

$$-2\pi+4\pi n < x \le -\pi+4\pi n,\ n \in Z$$

Javob: $\left(-2\pi+4\pi n;-\pi+4\pi n\right],\ n \in Z$.

ctgx < a va *ctgx ≤ a* tengsizliklarni yechish

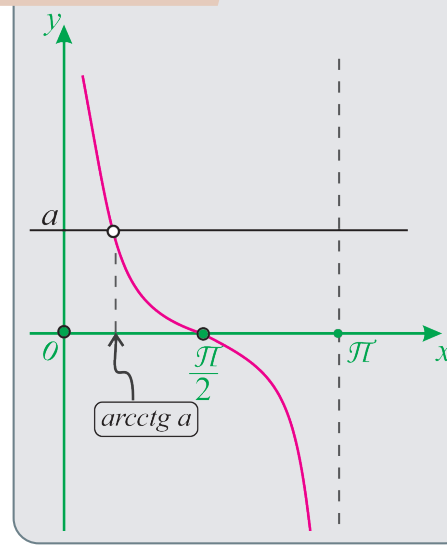
ctgx < a tengsizlikni *y = ctgx* funksiya grafigidan foydalanib yechamiz.

Ravshanki, *ctgx < a* tengsizlik *x* o‘zgaruvchining

$$arcctga < x < \pi$$

qo‘shtengsizlikni qanoatlantiruvchi qiymatlarida bajariladi (4-*rasm*). *y=ctgx* funksiya davriy ekani sababli *ctgx < a* tengsizlikning yechimi kelib chiqadi.

4-rasm



$$\boldsymbol{arcctga + \pi n < x < \pi + \pi n,\ n \in Z}$$

Xuddi shuningdek, *ctgx ≤ a* tengsizlikning yechimi

$$\boldsymbol{arcctga + \pi n \le x < \pi + \pi n,\ n \in Z}$$

bo‘ladi.

4-misol. Tengsizlikni yeching: $ctg\frac{x}{5} < -\sqrt{3}$.

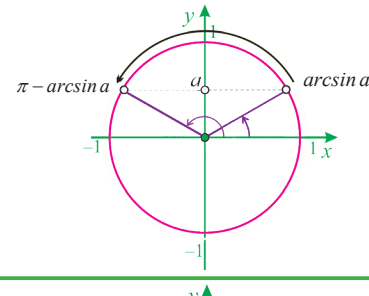
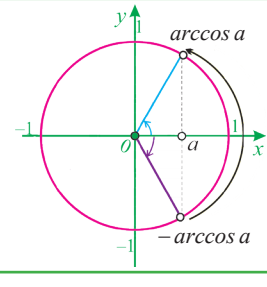
Yechish

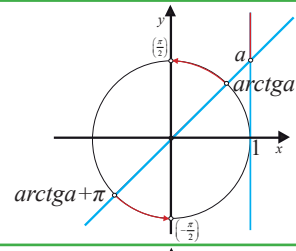
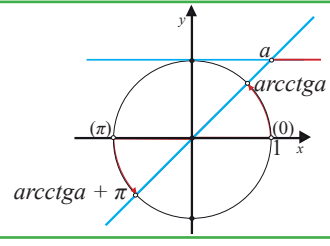
$$arcctg\left(-\sqrt{3}\right) + \pi n < \frac{x}{5} < \pi + \pi n,\ n \in Z \Rightarrow \frac{5\pi}{6} + \pi n < \frac{x}{5} < \pi + \pi n,\ n \in Z \Rightarrow$$

$$\frac{25\pi}{6} + 5\pi n < x < 5\pi + 5\pi n,\ n \in Z$$

Javob: $\left(\frac{25\pi}{6} + 5\pi n; 5\pi + 5\pi n\right),\ n \in Z$.

Ba’zi tengsizliklarning yechimi

Tengsizlik	Yechimi	Trigonometrik aylanadagi tasviri
sinx > a	$arcsin\,a + 2\pi n < x < \pi - arcsin\,a + 2\pi n,\ n \in Z$	
cosx > a	$-arccos\,a + 2\pi n < x < arccos\,a + 2\pi n,\ n \in Z$	

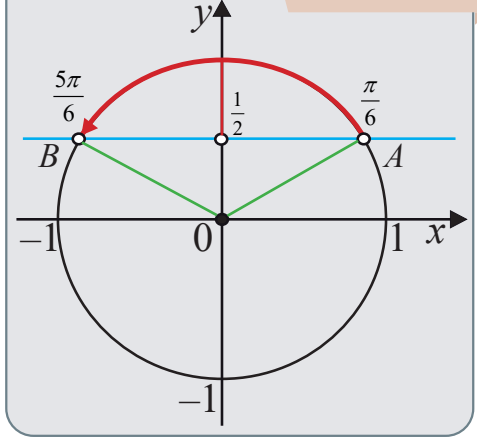
$tgx > a$	$arctga + \pi n < x < \frac{\pi}{2} + \pi n,\ n \in Z$	
$ctgx > a$	$\pi n < x < arcctga + \pi n,\ n \in Z$	

5-misol. Tengsizlikni yeching: $sinx > \frac{1}{2}$.

Yechish

Birlik aylanani (5-*rasm*) A va B nuqtalarda kesib o‘tuvchi $y = \frac{1}{2}$ to‘g‘ri chiziqni o‘tkazamiz. $sinx$ ning so‘ralayotgan qiymatlari shu to‘g‘ri chiziqning yuqorisida joylashgan bo‘ladi. $y=sinx$ va $y=\frac{1}{2}$ lar $x = \frac{\pi}{6}$ va $x = \frac{5\pi}{6}$ da kesishadi. Rasmdan ko‘rinib turibdiki, x ning $\frac{\pi}{6}$ dan katta va $\frac{5\pi}{6}$ dan kichik qiymatlarida $sinx$ ifoda $\frac{1}{2}$ dan katta bo‘ladi.

5-rasm



Shunday qilib, $\frac{\pi}{6} < x < \frac{5\pi}{6}$ bo‘ladi. $\sin x > \frac{1}{2}$ tengsizlikning barcha yechimlari ushbu

$\frac{\pi}{6} + 2\pi n < x < \frac{5\pi}{6} + 2\pi n,\ n \in Z$ formula bilan topiladi.

Javob: $\frac{\pi}{6} + 2\pi n < x < \frac{5\pi}{6} + 2\pi n,\ n \in Z.$

6-misol. Tengsizlikni yeching: $cos\,x\,cos\frac{\pi}{4} - sin\,x\,sin\frac{\pi}{4} \geq -\frac{\sqrt{2}}{2}$.

Yechish

Tengsizlikning chap tarafini yig‘indining kosinusi formulasidan foydalanib soddalashtirib olamiz:

$$cos\left(x + \frac{\pi}{4}\right) \geq -\frac{\sqrt{2}}{2}$$

Birlik aylanada (6-*rasm*) $x = -\frac{\sqrt{2}}{2}$ to‘g‘ri chiziqni o‘kazamiz. Bu to‘g‘ri chiziq aylanani $x + \frac{\pi}{4}$ ning $-\frac{3\pi}{4}$ va $\frac{3\pi}{4}$ qiymatlariga mos nuqtalarda kesib o‘tadi.

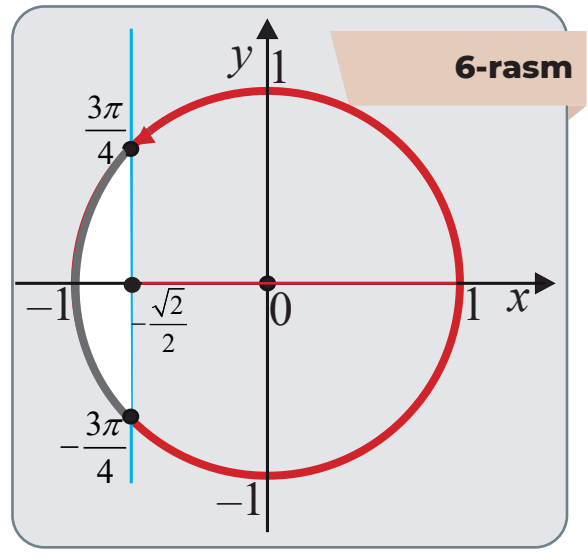
Bizga $x+\frac{\pi}{4}$ ning ushbu qiymatlari kerak:

$$-\frac{3\pi}{4}+2\pi n \le x+\frac{\pi}{4} \le \frac{3\pi}{4}+2\pi n,\ n\in Z.$$

Qo‘sh tengsizlikning har bir hadidan $\frac{\pi}{4}$ ni ayiramiz va quyidagini hosil qilamiz:

$$-\pi+2\pi n \le x \le \frac{\pi}{2}+2\pi n,\ n\in Z$$

Javob: $x\in\left[-\pi+2\pi n;\ \frac{\pi}{2}+2\pi n\right], n\in Z.$



6-rasm

7-misol. Tengsizlikni yeching: $tg\left(2x-\frac{\pi}{4}\right)\ge 1$.

Yechish

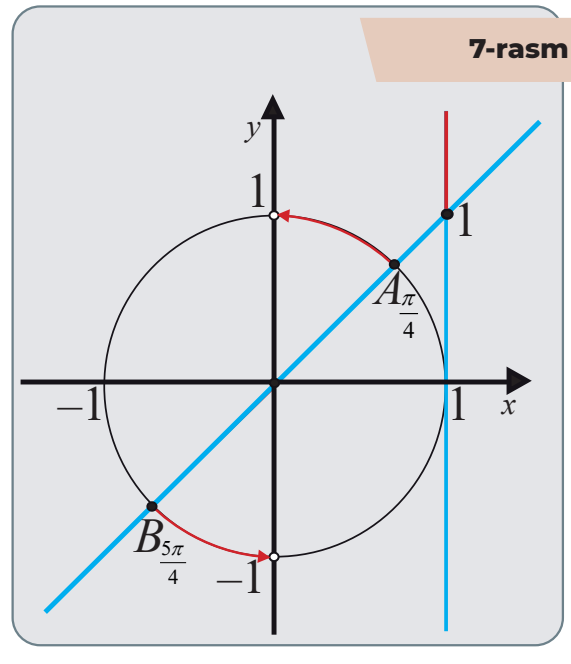
7-rasmdan $2x-\frac{\pi}{4}$ miqdor ushbu shartlarni bajarishi kelib chiqadi:

$$\frac{\pi}{4}+\pi n \le 2x-\frac{\pi}{4} < \frac{\pi}{2}+\pi n,\quad n\in Z$$

$$\frac{\pi}{2}+\pi n \le 2x < \frac{3\pi}{4}+\pi n,\quad n\in Z$$

$$\frac{\pi}{4}+\frac{\pi n}{2} < x < \frac{3\pi}{8}+\frac{\pi n}{2},\quad n\in Z$$

Javob: $\left[\frac{\pi}{4}+\frac{\pi n}{2};\frac{3\pi}{8}+\frac{\pi n}{2}\right),\quad n\in Z$



7-rasm

8-misol. Tengsizlikni yeching: $\sin x > \cos x$.

Yechish

$$\sin x-\sin\left(\frac{\pi}{2}-x\right)>0 \Rightarrow 2\sin\left(x-\frac{\pi}{4}\right)\cos\frac{\pi}{4}>0 \Rightarrow \sin\left(x-\frac{\pi}{4}\right)>0$$

$$\arcsin 0+2\pi n < x-\frac{\pi}{4} < \pi-\arcsin 0+2\pi n,\ n\in Z$$

$$2\pi n < x-\frac{\pi}{4} < \pi+2\pi n,\ n\in Z \Rightarrow \frac{\pi}{4}+2\pi n < x < \frac{5\pi}{4}+2\pi n,\ n\in Z$$

Javob: $\left(\frac{\pi}{4}+2\pi n;\frac{5\pi}{4}+2\pi n\right),\ n\in Z.$

MISOLLAR

1. Tengsizliklarni yeching.

a) $\sin x > 1$ b) $\sin x \geq 1$ c) $\sin x < 1$ d) $\sin x \leq 1$

e) $\sin x > -1$ f) $\sin x \geq -1$ g) $\sin x < -1$ h) $\sin x \leq -1$

i) $\sin x > 1{,}5$ j) $\sin x \geq -1{,}2$ k) $\sin x < 1{,}1$ l) $\sin x \leq -2$

2. Tengsizliklarni yeching.

a) $\cos x > 1$ b) $\cos x \geq 1$ c) $\cos x < 1$ d) $\cos x \leq 1$

e) $\cos x > -1$ f) $\cos x \geq -1$ g) $\cos x < -1$ h) $\cos x \leq -1$

i) $\cos x > 2$ j) $\cos x \geq -1{,}6$ k) $\cos x < 1{,}4$ l) $\cos x \leq -1{,}7$

3. Tengsizliklarni yeching.

a) $\sin 2x \geq 0$ b) $\cos 3x \leq 0$ c) $\cos x \leq \frac{1}{2}$ d) $\sin x > -\frac{\sqrt{3}}{2}$

e) $\cos 2x > \frac{\sqrt{2}}{2}$ f) $\sin 3x < \frac{\sqrt{3}}{2}$ g) $\sqrt{2} - 2\sin x > 0$ h) $2\cos x + \sqrt{3} \leq 0$

4. Tengsizliklarni yeching.

a) $\sin\left(3x - \frac{\pi}{4}\right) < 0$ b) $\cos\left(2x - \frac{\pi}{6}\right) \leq 0$

c) $\cos\left(2x + \frac{\pi}{6}\right) < -\frac{\sqrt{3}}{2}$ d) $\sin\left(2x - \frac{\pi}{3}\right) > -\frac{\sqrt{2}}{2}$

5. Tengsizliklarni yeching.

a) $tg\left(2x + \frac{\pi}{3}\right) \geq \sqrt{3}$ b) $tg\left(2x - \frac{5\pi}{6}\right) > 0$ c) $ctg\left(2x - \frac{\pi}{4}\right) > -\frac{\sqrt{3}}{3}$

d) $ctg\left(2x + \frac{\pi}{3}\right) \geq 0$ e) $ctg\left(3x - \frac{\pi}{3}\right) < 0$ f) $tg\left(4x - \frac{\pi}{6}\right) \leq -1$

6. $\sin\left(2x + \frac{\pi}{3}\right) < -\frac{1}{2}$ tengsizlikning $[0;\ \pi]$ oraliqdagi yechimlarini toping.

7. $tg\left(2x - \frac{\pi}{6}\right) < -\sqrt{3}$ tengsizlikning $\left[-\frac{3}{8}; \frac{21}{8}\right]$ oraliqdagi yechimlarini toping.

8. Tengsizliklarni yeching.

a) $\cos^2 x - 3\cos x < 0$ b) $2\sin^2 x - 5\sin x + 3 \geq 0$

c) $3\cos^2 x + 7\cos x + 4 \leq 0$ d) $tg^2 x - 4tgx + 3 < 0$

9. Tengsizliklarni yeching.

a) $\cos\left(3\sin\left(x - \frac{\pi}{6}\right)\right) < -\frac{\sqrt{3}}{2}$ b) $\sin\left(\cos\left(x + \frac{\pi}{4}\right)\right) > -\frac{1}{2}$



6-BOB. EHTIMOLLIKLAR NAZARIYASI

- TASODIFIY HODISALAR
- EHTIMOLLIK TA’RIFLARI
- TAKRORLASH

TASODIFIY HODISALAR

Ehtimolliklar nazariyasi matematikaning bo‘limi bo‘lib, hozirgi zamon matematikasining asosiy yo‘nalishlaridan biridir. Ehtimolliklar nazariyasi predmeti tasodifiy hodisalar bilan bosh-qariladigan qonuniyatlarni o‘rganishdan iborat. Uning asosiy tushunchalari tajriba va hodisa hisoblanadi.

Tajriba deyilganda aniq shartlar majmuini amalga oshirish tushuniladi. Tajriba natijasi ***hodisa*** deyiladi.

Hodisalar uch xil, ya’ni mumkin bo‘lmagan (hech qachon bajarilmaydi), muqarrar (har doim bajariladi) va tasodifiy (bajarilishi ham mumkin, bajarilmasligi ham mumkin) bo‘lib, bulardan eng asosiysi tasodifiy hodisa ehtimolliklarini hisoblashni o‘rganishdir.

Tabiat va jamiyat qonunlarida uchraydigan har qanday hodisalar tasodifiylikka bog‘liqdir. Masalan, ulardan ayrimlarini oldindan aytish mumkin, ayrimlari esa taqribiy bashorat qilinadi: ob-havo, narx-navo, hosilning mo‘l bo‘lish-bo‘lmasligi va hokazolarni oldindan aniq aytish qiyin.

XVII asr o‘rtalarida tavakkalchilikka asoslangan o‘yinlarida kuzatilayotgan hodisalarning ba’zi qonuniyatlarini o‘rganishga Paskal, Ferma, Bernulli kabi olimlar jiddiy e’tibor berib, jarayonlarni o‘rganganlar va natijada ehtimolliklar nazariyasi deb ataluvchi fanning vujudga kelishiga ulkan hissa qo‘shganlar. Ehtimolliklar nazariyasi turli tarmoqlarda, jumladan, iqtisodiyot, biologiya, tibbiyot, qishloq xo‘jaligi, texnika va boshqa sohalarda keng ko‘lamda qo‘llanadi.

Har qanday hodisani kuzatish yoki tajriba tariqasida o‘rganish ma’lum sinovlarni o‘tkazish orqali amalga oshiriladi.

Hodisalar haqida tushuncha

Ta’rif. Tajriba sinovlarining har qanday natijasi (yoki oqibati)ga ***hodisa*** deyiladi. Hodisalar lotin alifbosining bosh harflari bilan – ***A, B, C, ...*** tarzida belgilanadi.

Odatiy turmushda, amaliy faoliyatda hamda ilmiy tekshirishlarda natijalarni to‘la ishonch bilan oldindan aytish mumkin bo‘lmagan tajribalar va sinovlar tez-tez uchrab turadi.

Masalan, tangani tashlaganda u yoki bu tomonining tushishini to‘la ishonch bilan aytish mumkin emas; nishonga o‘q uzganda tegish yoki tegmasligi aniq emas; shashqol (kubik) tashlandi, bunda 6 raqamining tushishi oldindan ma’lum emas; biror raqamli lotereya chiptasiga yutuq chiqishini ham oldindan aytib bo‘lmaydi.

Ta’rif. Tajriba natijasida albatta ro‘y beradigan hodisa ***muqarrar hodisa*** deyiladi va u odatda **Ω** harfi bilan belgilanadi.

Masalan, shashqol tashlanganda 1 dan 6 gacha bo‘lgan butun sonlarning tushishi, tavakkaliga tanlangan so‘zda 1000 dan ortiq bo‘lmagan harfning bo‘lishi, kundan so‘ng tun kelishi va hokazolar muqarrar hodisalardir.

Ta’rif. Tajriba natijasida hech qachon ro‘y bermaydigan hodisaga esa ***mumkin bo‘lmagan hodisa*** deyiladi va odatda Ø belgisi bilan belgilanadi.

Masalan, bitta lotereyaga ikkita yutuq chiqishi, kosmik kemaning quyoshga qo‘nib qaytib ke-

lishi va hokazolar mumkin bo‘lmagan hodisalardir.

Ta’rif. Tajriba natijasida ro‘y berishi ham, ro‘y bermasligi ham mumkin bo‘lgan hodisa ***tasodifiy hodisa*** deyiladi.

Masalan, tanga tashlaganda gerbli tomonning tushishi, o‘q uzilganda nishonga tegishi, lotereya chiptasiga yutuq chiqishi, shashqol tashlanganda 6 raqamining tushishi va hokazolar tasodifiy hodisalarga misol bo‘ladi.

Ta’rif. Biri ro‘y berganda boshqasi ro‘y bermaydigan hodisalar ***birgalikdamas (birgalikda bo‘lmagan)*** hodisalar deyiladi.

1-misol. Detallar solingan qutidan tavakkaliga bitta detal olindi. Bunda sifatli detal chiqishi sifatsiz detal chiqishini yo‘qqa chiqaradi yoki aksincha. “Sifatli detal chiqdi” va “sifatsiz detal chiqdi” hodisalari birgalikda emas.

2-misol. Tanga tashlashda gerbli tomoni tushishi raqamli tomon tushishini yo‘qqa chiqardi. “Gerbli tomon tushdi” va “Raqamli tomon tushdi” hodisalari birgalikda emas.

Ta’rif. Agar hodisalar bir paytda ro‘y berishi mumkin bo‘lsa, bunday hodisalar ***birgalikda bo‘lgan hodisalar*** deyiladi.

Masalan, “quyosh chiqdi” va “kun sovuq” – bu hodisalar birgalikda bo‘lishi mumkin bo‘lgan hodisalar bo‘ladi.

Ta’rif. Tajribaning har bir natijasini ifodalovchi hodisa ***elementar hodisa*** deyiladi.

Ta’rif. Elementar hodisalarga ajratish mumkin bo‘lgan hodisa ***murakkab hodisa*** deyiladi.

Ta’rif. Agar bir necha hodisalardan istalgan birining tajriba natijasida ro‘y berishi boshqalariga qaraganda kattaroq imkoniyatga ega deyishga asos bo‘lmasa, bunday hodisalar ***teng imkoniyatli hodisalar*** deyiladi.

Masalan, tanga tashlanganda gerbli yoki raqamli tomoni tushishi yoki shashqol tashlanganda bir raqamining tushishi, ikki raqamining tushishi,... olti raqamining tushishi – bularning barchasi teng imkoniyatli hodisalar bo‘ladi.

Ta’rif. A hodisaga ***qarama-qarshi hodisa*** deb A hodisaning ro‘y bermasligidan iborat hodisaga aytiladi va $\overline{A}$ kabi belgilanadi.

Bog‘liq va bog‘liq bo‘lmagan hodisalar haqida tushuncha

Ta’rif. Agar ikkita hodisadan birining ro‘y berishi ikkinchi hodisaning ro‘y berish yoki ro‘y bermasligiga bog‘liq bo‘lmasa, bu hodisalar ***erkli (bog‘liq bo‘lmagan) hodisalar*** deyiladi.

3-misol. Tanga ikki marta tashlangan. Birinchi tashlashda gerbli tomon tushish (A hodisa) ehtimolligi ikkinchi tashlashda gerbli tomon tushish yoki tushmasligiga (B hodisa) bog‘liq emas. O‘z navbatida, ikkinchi tajribada gerbli tomon tushish ehtimolligi birinchi tajriba natijasiga bog‘liq emas. Shunday qilib, A va B hodisalar erkli.

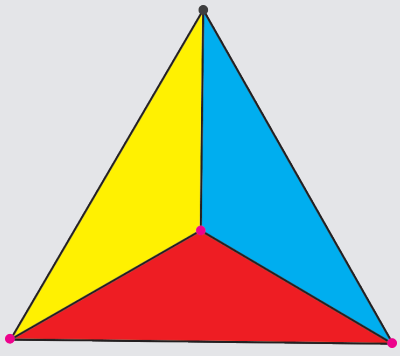
4-misol. Tanga va shashqol tashlashda A – tangada gerbli tomoni, B – shashqolda juft raqami tushish hodisalari bo‘lsin. Bu yerda A va B bog‘liqsiz hodisalardir.

5-misol. Ikkita shashqol tashlashda A – birinchi shashqolda, B – ikkinchi shashqolda juft raqami tushish hodisalari bo‘lsin. Bu yerda A va B bog‘liqsiz hodisalardir.

Ta’rif. Bir nechta hodisaning ixtiyoriy ikkitasi bog‘liq bo‘lmasa, ular ***juft-juft erkli*** deyiladi.

6-misol. Tanga 3 marta tashlangan. A, B, C mos ravishda birinchi, ikkinchi va uchinchi tajribalarda gerbli tomon tushish hodisasi bo‘lsin. Ravshanki, ko‘rilayotgan hodisalardan har ikkitasi (ya’ni A va B, A va C, B va C) bog‘liq emas. Shunday qilib, A, B va C juft-juft erkli.

1-rasm



7-misol. Muntazam tetraedrning bir yog‘i qizil, boshqa bir yog‘i sariq, uchinchi yog‘i ko‘k hamda to‘rtinchi yog‘i shu uchta rangda (1-*rasm*). Tetraedrni tashlashda qizil rangning tushishi A hodisa, sariq rangning tushishi B hodisa, ko‘k rangning tushishi C hodisalar juft-juft erklidir.

Ta’rif. Agar ikki hodisadan birining ro‘y berishi ikkinchi hodisaning ro‘y berish yoki ro‘y bermasligiga bog‘liq bo‘lsa, bu hodisalar ***bog‘liq*** deyiladi.

8-misol. Idishda 80 ta oq, 20 ta qora shar bor. Tavakkaliga bitta shar olinib, qaytarib qo‘yilmaydi. Agar birinchi olishda oq shar chiqishi A hodisa bo‘lsa, u holda ikkinchi olishdagi sharning oq chiqishi B hodisasining ro‘y berishi A hodisaga bog‘liq bo‘ladi, ya’ni A va B hodisalar bog‘liqdir.

EHTIMOLLIK TA'RIFLARI

"Ehtimollik" tushunchasi ehtimollar nazariyasining asosiy tushunchalaridan biridir.

Idishda yaxshilab aralashtirilgan sharlar bo'lib, ulardan 5 tasi qizil, 4 tasi qora va qolgan 3 tasi oq rangda. Idishdan olingan sharning qizil yoki qora bo'lishi imkoniyati oq rangli bo'lishi imkoniyatidan ko'proq. Bu imkoniyatni son bilan tavsiflash mumkinmi? Ha, mumkin. Mana shu son ***hodisaning ehtimolligi*** deb ataluvchi kattalikdir.

Shunday qilib, ehtimollik – hodisaning ro'y berish imkoniyatini tavsiflovchi sondir.

Biz o'z oldimizga tavakkaliga olingan sharning qizil yoki qora rangli bo'lish imkoniyatini miqdoriy baholash vazifasini qo'yaylik. Qizil yoki qora rangli shar chiqishini A hodisa sifatida qaraymiz. Tajribada (tajriba idishdan shar olishdan iborat) mumkin bo'lgan natijalarning har birini, ya'ni ro'y berishi mumkin bo'lgan har bir hodisani elementar hodisa deb ataymiz. Elementar hodisalarni $E_1, E_2, E_3, E_4, \ldots$ bilan belgilaymiz. Bizning misolda quyidagi 12 ta elementar hodisa bo'lishi mumkin: E_1, E_2, E_3, E_4, E_5 – qizil shar chiqadi; E_6, E_7, E_8, E_9 – qora shar chiqadi; E_{10}, E_{11}, E_{12} – oq shar chiqadi.

Osongina ko'rish mumkinki, bu natijalar yagona mumkin bo'lgan (bitta shar, albatta, chiqadi) va teng imkoniyatli (shar tavakkaliga olinadi, sharlar bir xil va yaxshilab aralashtirilgan) hodisadir.

Bizni qiziqtirayotgan hodisaning ro'y berishiga olib keladigan elementar hodisalarni bu hodisaning ro'y berishiga qulaylik tug'diruvchi deymiz. Bizning misolda A (qizil yoki qora rangli shar chiqishi) hodisaning ro'y berishiga quyidagi 9 ta elementar hodisa qulaylik tug'diradi: $E_1, E_2, E_3, E_4, E_5, E_6, E_7, E_8, E_9$. A hodisaning ro'y berishiga qulaylik tug'diruvchi elementar hodisalar sonining ularning umumiy soniga nisbati A hodisaning ehtimolligi deyiladi va $P(A)$ bilan belgilanadi. Ko'rilayotgan misolda elementar hodisalar jami 12 ta, ulardan 9 tasi A hodisaga qulaylik tug'diradi. Demak, olingan sharning qizil yoki qora bo'lish ehtimolligi: $P(A)=\frac{9}{12}=\frac{3}{4}$. Topilgan son (ehtimol) biz oldimizga qo'ygan masaladagi qizil yoki qora shar chiqishi mumkinligining miqdoriy bahosini beradi.

Ehtimollikning turli ta'riflari mavjud. Bular klassik, statistik va geometrik ta'riflardir.

Ehtimollikning klassik ta'rifi

Ta'rif. ***A hodisaning ehtimolligi*** deb, tajribaning bu hodisa ro'y berishiga qulaylik tug'diruvchi natijalari soni – m ning tajribaning mumkin bo'lgan barcha elementar hodisalari soni – n ga nisbatiga aytiladi va quyidagi ko'rinishda belgilanadi:

$$P(A)=\frac{m}{n}$$

Ehtimollik ta'rifidan quyidagi xossalar kelib chiqadi:

1. Muqarrar hodisaning ehtimolligi 1 ga teng, ya'ni $P(\Omega)=1$.

Haqiqatan ham, agar hodisa muqarrar bo'lsa, u holda tajribaning har qanday natijasi shu hodisaning ro'y berishiga qulaylik tug'diradi. Bu holda, $m=n$. Demak:

$$P(\Omega)=\frac{m}{n}=\frac{n}{n}=1$$

1-misol. Idishda 20 ta shar bo'lib, ular 1 dan 20 gacha raqamlangan. Idishdan tavakkaliga bitta shar olindi. Bu sharning tartib raqami 20 dan katta bo'lmaslik (A hodisa) ehtimoli qanday?

Yechish. Yashikdagi istalgan sharlarning tartib raqami 20 dan oshmaydi. Shuning uchun bu hodisaning ro'y berishiga qulaylik tug'diruvchi hodisalar soni va barcha mumkin bo'lgan hollar soni o'zaro teng: $m = n = 20$ va $P(A)=\frac{m}{n}=1$. Bu holda A hodisa muqarrar hodisadir.

2. Mumkin bo'lmagan hodisaning ehtimolligi nolga teng.

Haqiqatan ham, agar hodisa ro'y bermaydigan bo'lsa, u holda tajribaning hech bir elementar natijasi bu hodisaning ro'y berishiga qulaylik tug'dirmaydi. Bu holda, $m = 0$. Demak:

$$P(\varnothing)=\frac{m}{n}=\frac{0}{n}=0$$

2-misol. Qutida 10 ta shar bo'lib, ulardan 4 tasi oq, qolganlari qora rangda. Shu qutidan tavakkaliga bitta shar olindi. Uning qizil shar bo'lish (A hodisa) ehtimoli qanday?

Yechish. Qutida qizil shar yo'q, ya'ni $m = 0$, lekin $n=10$. Demak, $P(A)=\frac{m}{n}=\frac{0}{n}=0$. Bu holda A hodisa mutlaqo yuz bermaydigan, ya'ni mumkin bo'lmagan hodisadir.

3. Tasodifiy hodisaning ehtimolligi musbat son bo'lib, u 0 va 1 oralig'ida bo'ladi.

Haqiqatan ham, tasodifiy hodisaning ro'y berishiga tajribaning barcha elementar hodisalarining bir qismigina qulaylik tug'diradi. Bu holda $0<m<n$. Shuning uchun $0<\frac{m}{n}<1$. Demak, $0<P(A)<1$.

Shunday qilib, istalgan hodisaning ehtimolligi quyidagi qo'sh tengsizlikni qanoatlantiradi:

$$0\le P(A)\le 1$$

Quyidagi misollarni yechishdan oldin bitta formulani keltirib o'tamiz.

Idishda n ta shar bo'lib, ulardan n_1 tasi oq, n_2 tasi qora, n_3 tasi qizil va hokazo n_k tasi sariq. Shu idishdan tavakkaliga m ta shar olinganda, ulardan m_1 tasi oq, m_2 tasi qora, m_3 tasi qizil va hokazo m_k tasi sariq bo'lish A hodisasining ehtimolligini topish formulasi:

$$P(A)=\frac{C_{n_1}^{m_1}\cdot C_{n_2}^{m_2}\cdot C_{n_3}^{m_3}\cdot\ldots\cdot C_{n_k}^{m_k}}{C_n^m},$$

Eslatma: $P_n=n!$, $A_n^m=\frac{n!}{(n-m)!}$, $C_n^m=\frac{n!}{m!(n-m)!}$

3-misol. Qopda 12 ta shar mavjud: 3 ta oq, 4 ta qora va 5 ta qizil. Tavakkaliga bitta shar olindi. Uning qora shar bo'lishi (A hodisa) ehtimolligini toping.

Yechish. Bizga qulaylik tug‘diruvchi elementar hodisalar soni $m = 4$, hamda jami elementar hodisalar soni $n = 12$, demak, A hodisaning ehtimolligi:

$$P(A) = \frac{m}{n} = \frac{4}{12} = \frac{1}{3}$$

4-misol. Yashikda 10 ta shar bor: 6 ta oq va 4 ta qora. Tavakkaliga 2 ta shar olindi. Ikkala shar ham oq bo‘lishi (A hodisa) ehtimolligini toping.

Yechish. Bu masalada mumkin bo‘lgan barcha holatlar soni $n = C_{10}^2 = \frac{10 \cdot 9}{1 \cdot 2} = 45$ ga teng.

A hodisaga qulaylik tug‘diruvchi hollar soni esa $m = C_6^2 = \frac{5 \cdot 6}{1 \cdot 2} = 15$ ga teng. Bundan kelib chiqadiki, $P(A) = \frac{m}{n} = \frac{15}{45} = \frac{1}{3}$.

5-misol. 2 000 lotereya chiptasi sotilgan. Bunda 1 ta chiptaga 100 000 so‘m, 4 ta chiptaga 50 000 so‘m, 10 ta chiptaga 20 000 so‘m, 20 ta chiptaga 10 000 so‘m, 165 ta chiptaga 5 000 so‘m, 400 ta chiptaga 1 000 so‘mdan yutuq chiqishi belgilangan, qolgan chiptalar yutuqsiz. Bitta chiptaga 10 000 so‘mdan kam bo‘lmagan yutuq chiqish ehtimoli qanday?

Yechish. Bu yerda $m = 1 + 4 + 10 + 20 = 35$, $n = 2000$. Chunki, 35 ta chiptaga 10 000 so‘mdan yuqori yutuqlar belgilangan. Shuning uchun,

$$P(A) = \frac{m}{n} = \frac{35}{2000} = 0{,}0175.$$

6-misol. Do‘konda 6 erkak va 4 ayol kishi ishlaydi. Tabeldagi tartib raqami bo‘yicha tasodifiy ravishda 7 kishi tanlab olindi. Tanlab olinganlar orasida 3 kishi ayol bo‘lishi ehtimolligini toping.

Yechish. Umumiy ro‘y berishlar soni, ya’ni 10 kishidan 7 kishini necha xil usulda tanlash mumkinligi. Bu esa, $n = C_{10}^7 = \frac{8 \cdot 9 \cdot 10}{1 \cdot 2 \cdot 3} = 120$ ga teng va endi qulaylik tug‘diruvchi elementar hodisalar sonini topish kerak. Buning uchun 7 kishilik jamoani quyidagi ko‘rinishda tuzamiz: 4 ta ayoldan 3 tasini va 6 ta erkakdan 4 tasini olishimiz kerak, ya’ni $m = C_4^3 \cdot C_6^4 = \frac{4}{1} \cdot \frac{5 \cdot 6}{1 \cdot 2} = 60$. Demak, bu hodisaning ehtimolligi $P(A) = \frac{m}{n} = \frac{60}{120} = \frac{1}{2}$ ga teng.

7-misol. 2 ta matematika, 2 ta fizika va 2 ta kimyo kitoblari javonning bir tokchasiga qo‘yilmoqda. Kimyo kitoblarining yonma-yon kelish ehtimolligi nimaga teng?

Yechish. Barcha o‘rin almashishlar sonini topib olamiz, ya’ni 6 ta kitobning o‘rin almashishlari soni $n = P_6 = 6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$ ta. Endi kimyo kitoblari yonma-yon kelishi uchun kimyo kitoblarini 1 ta kitob deb qarab, barcha o‘rin almashishlar soni – $P_5 = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$ ni topamiz va kimyo kitoblarini ham o‘rin almashishlar sonida hisobga olishimiz shart, ya’ni $P_2 = 2! = 1 \cdot 2 = 2$. Bundan esa, $m = P_5 \cdot P_2 = 120 \cdot 2 = 240$ hosil bo‘ladi. Demak, ehtimollikning

klassik ta'rifi bo'yicha $P(A)=\frac{m}{n}=\frac{240}{720}=\frac{1}{3}$ ga teng ekan.

8-misol. Abonent telefon raqamini terayotganda oxirgi uchta raqamni eslay olmadi. Lekin raqamlar turli ekanini biladi. Barcha terishlardan to'g'ri raqamni terish ehtimolligi nimaga teng bo'ladi?

Yechish. To'g'ri raqamni terish hodisasini A bilan, uning ehtimolligini esa $P(A)$ bilan belgilaymiz.

Oxirgi uchta raqamni A_{10}^3 usul bilan terish mumkin. Shunda jami sinovlar soni $n=A_{10}^3=\frac{10!}{(10-3)!}=8\cdot 9\cdot 10=720$ ga teng bo'ladi. Izlanilayotgan telefon raqami shu 720 tadan bittasi bo'ladi, ya'ni $m=1$. Ehtimollikning klassik ta'rifi bo'yicha $P(A)=\frac{m}{n}=\frac{1}{720}$ bo'ladi.

Ehtimollikning statistik ta'rifi

Nisbiy chastota ehtimol bilan bir qatorda ehtimollar nazariyasining asosiy tushunchalaridan biri hisoblanadi.

Ta'rif. ***Hodisaning nisbiy chastotasi*** deb, hodisa ro'y bergan tajribalar sonining aslida o'tkazilgan jami tajribalar soniga nisbatiga aytiladi. Shunday qilib, A hodisaning nisbiy chastotasi quyidagi formula bilan aniqlanadi:

$$W(A)=\frac{M}{N}$$

bu yerda M soni – A hodisaning N tajribada ro'y berishlari soni.

Ta'rif. Statistik ehtimollik – tajribalar sonining katta qiymatlardagi nisbiy chastotasi.

Ehtimollik va nisbiy chastota ta'riflarini solishtirib quyidagi xulosaga kelamiz: ehtimollikning ta'rifida tajribalarning haqiqatan o'tkazilganligi talab qilinmaydi, nisbiy chastotaning ta'rifida esa tajribalarning aslida o'tkazilganligi talab qilinadi. Soddaroq aytganda, ehtimol tajribadan oldin (ilgari), nisbiy chastota esa tajribadan keyin (so'ng) hisoblanadi.

Agar $M=N$ bo'lsa, ya'ni o'tkazilgan tajribalar soni hodisaning ro'y berishlar soniga teng bo'lsa, bu hodisani shartli ravishda muqarrar hodisa deb atash mumkin.

Agar $M=0$ bo'lsa, ya'ni o'tkazilgan tajriba natijasida hodisa biror marta ham sodir bo'lmasa, u holda bu hodisa shartli ravishda mumkin bo'lmagan hodisa deb hisoblash mumkin.

1-misol. Mergan nishonga qarata 30 ta o'q uzdi. Bunda ulardan 23 tasi nishonga teggani ma'lum bo'lsa, mergan o'qlarining nishonga tegishining nisbiy chastotasini toping.

Yechish. Mergan o'qlarining 23 tasi nishonga tegdi, demak, hodisaning ro'y berishlar soni $M=23$ va jami uzilgan o'qlar soni $N=30$, demak, bu hodisaning nisbiy chastotasi $W(A)=\frac{23}{30}$ bo'ladi.

2-misol. Dastlabki 1000 ta natural sonlar ichidan olingan sonning 5 ga karrali bo‘lishining nisbiy chastotasini toping.

Yechish. Bu yerda sonning 5 ga karrali chiqish hodisasini A bilan, uning nisbiy chastotasini esa $W(A)$ bilan belgilaymiz. O‘tkazilgan jami sinovlar soni $N = 1000$ ga, dastlabki 1000 ta natural sonlar ichida 5 ga karrali 200 ta natural son bor, demak, $M = 200$, nisbiy chastota esa $W(A) = \frac{200}{1000} = \frac{1}{5}$

3-misol. Bir mamlakatga xorijdan kelgan sayyohlar va shu mamlakat hududida sayohat qilgan fuqarolar (ichki sayyohlar) haqida quyidagi ma’lumotlar berilgan bo‘lsin.

Yillar	Xorijiy sayyohlar soni	Ichki sayyohlar soni	Jami sayyohlar soni
2018	610 623	403 989	1 014 612
2019	746 224	348 953	1 095 177
2020	822 558	316 897	1 139 455
2021	774 262	346 103	1 120 365
2022	811 314	351 028	1 162 342
$\sum$	3 764 981	1 766 970	5 531 951

Qaralayotgan yillarda mamlakat ichida sayohat qilgan mamlakat fuqarolari sonining nisbiy chastotasini toping.

Mamlakat ichida sayohat qilgan mahalliy fuqarolar soni: $M = 1\,766\,970$.

Xorijiy sayyohlar soni: $K = 3\,764\,981$.

Umumiy sayyohlar soni: $N = 1\,766\,970 + 3\,764\,981 - 5531951$.

$$W = \frac{M}{N} = \frac{1766970}{5531951} \approx 0{,}3194.$$

Ehtimollikning geometrik ta’rifi

Barcha nuqtalari teng imkoniyatga ega bo‘lgan biror Ω soha (kesma, shakl yoki jism) berilgan bo‘lib, bu sohaga tashlangan nuqtaning unga tushishi muqarrar bo‘lsin. Shu berilgan sohadan kichkina ω soha (kesma, shakl yoki jism) ajrataylik. Ω sohaga tashlangan nuqtaning ajratilgan ω sohaga tushish ehtimolligi so‘ralgan bo‘lsin. Ajratilgan soha qancha katta bo‘lsa, tushish ehtimolligi ham kattalashib boradi, ω soha Ω sohaga tenglashganda esa tushish ehtimolligi muqarrar hodisaga aylanadi. Demak, tashlangan nuqtaning ω sohachaga tushish ehtimolligi ω sohacha kattaligiga to‘g‘ri proporsional bo‘lib, uni geometrik nuqtayi nazardan talqin qilish kerak bo‘ladi. Bunday hollarda ehtimollikning geometrik ta’rifidan foydalanish qulaydir.

Agar tashlangan nuqtaning Ω sohaga tushishi muqarrar bo‘lsa, u holda bu nuqtaning shu sohadan ajratilgan ω sohachaga tushish ehtimolligi ω sohacha o‘lchovining Ω soha o‘lchoviga

nisbatiga teng bo'ladi:

$$P(A)=\frac{m(\omega)}{m(\Omega)}$$

Bu yerda ω – sohaning o'lchovi, ya'ni bir o'lchovli holda uzunlik, ikki o'lchovlida yuza, uch o'lchovlida hajm va hokazo.

Agar Ω sohaning o'lchovi L kesma va ω sohani l kesma deb olsak, L kesmagacha tashlangan nuqtaning l kesmaga tushish ehtimolligi quyidagicha bo'ladi:

$$P(A)=\frac{l}{L}$$

Agar Ω sohaning S yuza va ω sohani s yuza deb olsak, S yuzaga tashlangan nuqtaning s yuzaga tushish ehtimolligi quyidagicha bo'ladi:

$$P(A)=\frac{s}{S}$$

Agar Ω sohani V hajm va ω sohani v hajm deb olsak, V hajmga tashlangan nuqtaning v hajmga tushish ehtimolligi quyidagicha bo'ladi:

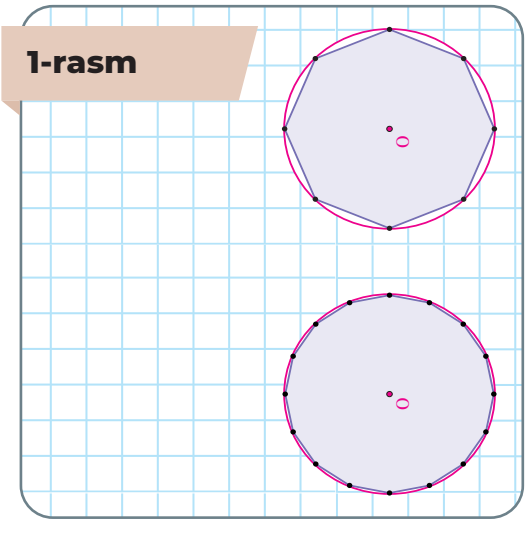
$$P(A)=\frac{v}{V}$$

Geometrik ta'rifdan vaqtga nisbatan ham foydalanish mumkin. Agar voqea T vaqt ichida sodir bo'lishi muqarrar bo'lsa, bu voqeaning t vaqt ichida sodir bo'lish ehtimolligi quyidagicha bo'ladi:

$$P(A)=\frac{t}{T}$$

1-misol. R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan muntazam n-burchak ichiga tushish ehtimolligini toping.

1-rasm



Yechish. $S(D_n)$ – n-burchakning yuzi, $S(D)$ – doiraning yuzi (1-*rasm*). U holda

$$P(B_n)=\frac{S(D_n)}{S(D)}=\frac{n\cdot\frac{R^2}{2}\cdot\sin\frac{2\pi}{n}}{\pi R^2}=\frac{n\cdot\sin\frac{2\pi}{n}}{2\pi}=\frac{\sin\frac{2\pi}{n}}{\frac{2\pi}{n}}$$

a) R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan muntazam uchburchak ichiga tushish ehtimolligini toping.

Yechish. $S(D_3)$ – uchburchakning yuzi, $S(D)$ – doiraning yuzi (2-*rasm*).

B_3 – nuqtaning uchburchakka tushish hodisasi. U holda

$$P(B_3)=\frac{S(D_3)}{S(D)}=\frac{3\sqrt{3}R^2}{4\pi R^2}=\frac{3\sqrt{3}}{4\pi}\approx 0,4137$$

b) R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan kvadrat ichiga tushish ehtimolligini toping.

Yechish. $S(D_4)$ – kvadratning yuzi, $S(D)$ – doiraning yuzi (3-*rasm*).

B_4 – nuqtaning kvadratga tushish hodisasi. U holda

$$P(B_4)=\frac{S(D_4)}{S(D)}=\frac{2R^2}{\pi R^2}=\frac{2}{\pi}\approx 0,637$$

c) R radiusli doiraga nuqta tavakkaliga tashlangan. Tashlangan nuqtaning doiraga ichki chizilgan muntazam oltiburchak ichiga tushish ehtimolligini toping.

Yechish. $S(D_6)$ – oltiburchakning yuzi, $S(D)$ – doiraning yuzi (4-*rasm*).

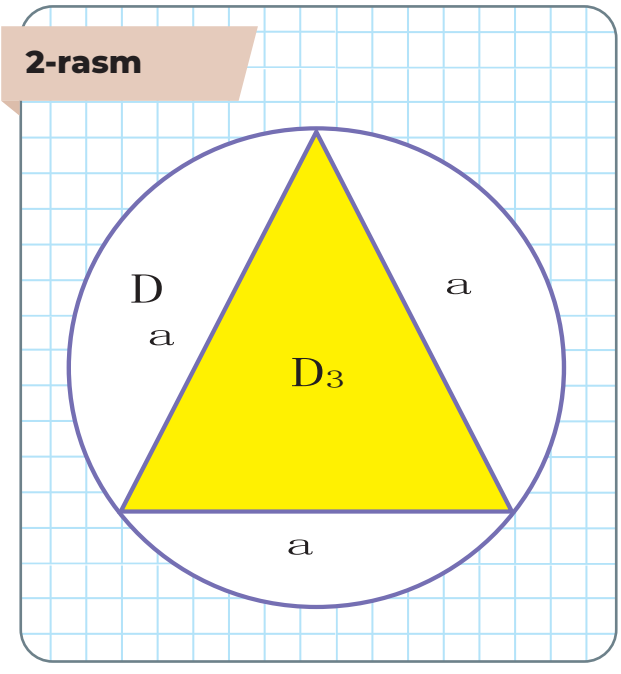
B_6 – nuqtaning oltiburchakka tushish hodisasi. U holda

$$P(B_6)=\frac{S(D_6)}{S(D)}=\frac{3\sqrt{3}R^2}{2\pi R^2}=\frac{3\sqrt{3}}{2\pi}\approx 0,8274$$

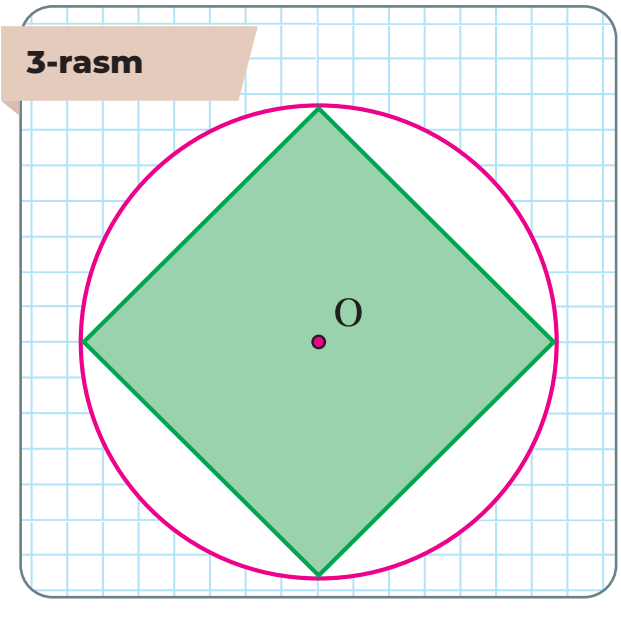
2-misol. Uzunligi 30 cm bo‘lgan L kesmaga uzunligi 12 cm bo‘lgan l kesma joylashtirilgan. Katta kesmaga tavakkaliga qo‘yilgan nuqtaning kichik kesmaga ham tushish ehtimolligini toping. Nuqtaning kesmaga tushish ehtimolligi kesmaning uzunligiga to‘g‘ri proporsional bo‘lib, uning joylashishiga bog‘liq emas, deb faraz qilinadi.

Yechish. Tashlangan nuqtaning L kesmaga tushishi muqarrar. $P(E)$ – bu L kesmada joylashgan l kesmaga tushish ehtimolligini topamiz (5-*rasm*). Rasmda faqatgina uch holat ko‘rsatil-

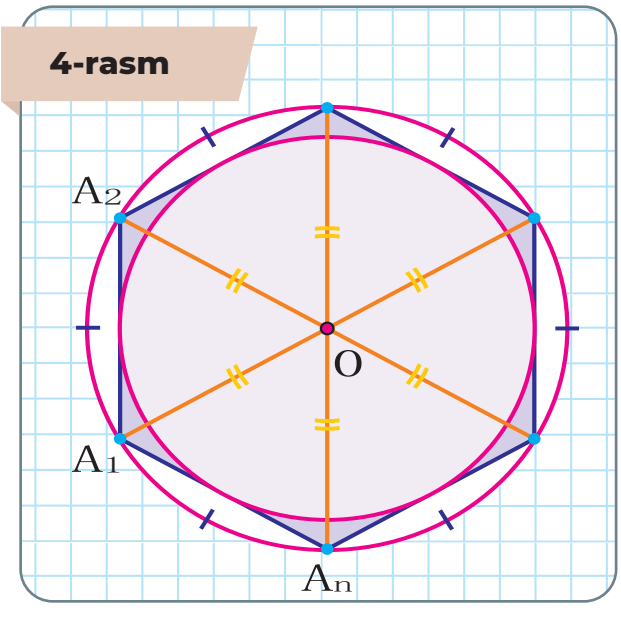
2-rasm



3-rasm



4-rasm

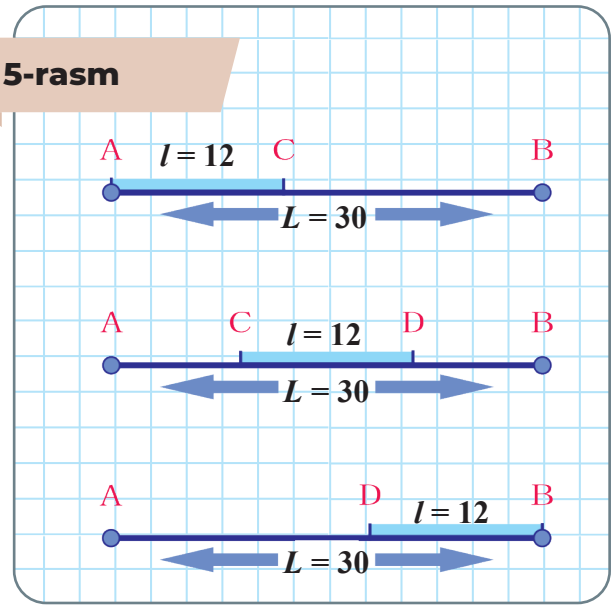


gan. Lekin l kesma L ning istalgan qismida joylashgan bo‘lishi mumkin.

$$P(E)=\frac{l}{L}=\frac{12}{30}=\frac{2}{5}$$

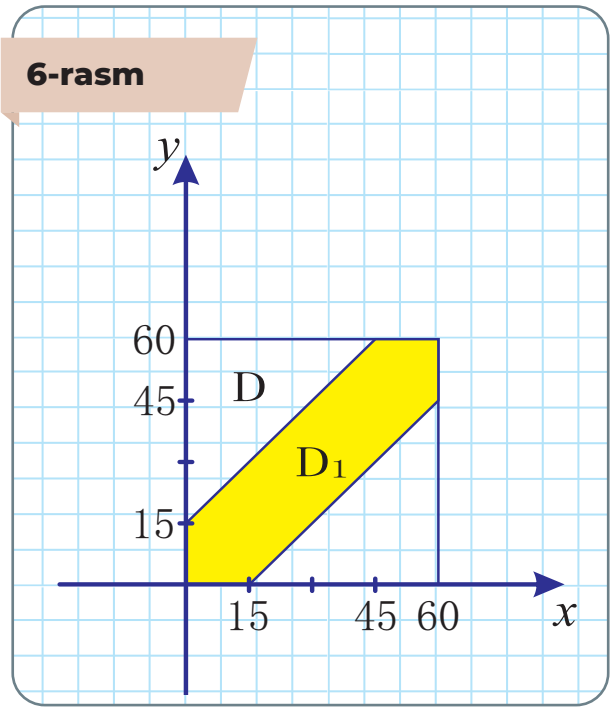
3-misol. Ikki do‘st soat 9 bilan 10 orasida uchrashmoqchi bo‘lishdi. Birinchi kelgan kishi do‘stini 15 minut davomida kutishi avvaldan shartlashib olindi. Agar bu vaqt mobaynida do‘sti kelmasa, u ketishi mumkin. Agar ular soat 9 bilan 10 orasidagi ixtiyoriy paytda kelishlari mumkin bo‘lib, kelish paytlari ko‘rsatilgan vaqt mobaynida tasodifiy bo‘lsa va o‘zaro kelishib olingan bo‘lmasa, bu ikki do‘stning uchrashish ehtimolligi nimaga teng?

5-rasm



Yechish. Birinchi kishining kelish vaqt momenti x, ikkinchisiniki esa y bo‘lsin. Ularning uchrashishlari uchun $|x-y|\le 15$ tengsizlikning bajarilishi zarur va yetarlidir. x va y larni tekislikdagi Dekart koordinatalari sifatida tasvirlaymiz va masshtab birligi deb minutlarni olamiz. Ro‘y berishi mumkin bo‘lgan barcha imkoniyatlar tomonlari 60 bo‘lgan kvadrat nuqtalaridan va uchrashishga qulaylik tug‘diruvchi imkoniyatlar bo‘yalgan soha nuqtalaridan iborat (6-*rasm*).

6-rasm



Demak, ehtimollikning geometrik ta’rifiga ko‘ra, izlanayotgan ehtimollik bo‘yalgan soha yuzasining kvadrat yuzasiga bo‘lgan nisbatiga teng:

$S(D_1)$ – bo‘yalgan sohaning yuzi, $S(D)$ – kvadratning yuzi bo‘lsin (6-*rasm*). A – do‘stlarning uchrashish hodisasi.

$$S(D_1)=60\cdot 60-2\cdot\frac{45\cdot 45}{2}=1575 \qquad S(D)=60\cdot 60=3600.$$

Izlanayotgan ehtimollik:

$$P(A)=\frac{S(D_1)}{S(D)}=\frac{1575}{3600}=\frac{7}{16}; \qquad P(A)=\frac{7}{16}$$

MASALALAR

1. Lotereyada 1000 ta chipta bor. Ulardan 500 tasi yutuqli, 500 tasi yutuqsiz. Ikkita chipta sotib olindi. Ikkala chiptaning ham yutuqli bo‘lish ehtimolligini toping.

2. Tanga ikki marta tashlandi. Ikki marta ham gerbli tomoni tushish ehtimolligini toping.

3. 20 ta kitob javonlarga tavakkaliga taxlandi. 20 ta kitobdan aniq 5 tasining yonma-yon turishi (A hodisa) ehtimolligi nimaga teng?

4. Ikkita shashqoltosh tashlangan. Ularning yoqlarida chiqqan raqamlar yig‘indisi – juft son. Shu bilan birga, tashlangan shashqollarning kamida bittasida har doim 6 raqami tushish ehtimolligini toping.

5. Ikkita shashqol tashlangan. Ularning yoqlarida chiqqan raqamlar yig‘indisi 5 ga, ko‘paytmasi esa 4 ga teng bo‘lish ehtimolligini toping.

6. Ikkita shashqol tashlangan bo‘lib, ularning yoqlarida chiqqan raqamlar yig‘indisi 7 ga teng bo‘lish ehtimolligini toping.

7. Raqamlari har xil ikki xonali son o‘ylangan. O‘ylangan son tasodifan aytilgan ikki xonali son bo‘lish ehtimolligini toping.

8. Qutida 20 ta shar bor: 10 ta qora va 10 ta oq. Qutidan tasodifiy ravishda bir shar olindi. Bu shar: a) oq; b) qora shar bo‘lish ehtimolligini toping.

9. Texnik nazorat bo‘limi tasodifan ajratib olingan 100 ta kitobdan iborat partiyada 5 ta yaroqsiz kitob topdi. Yaroqsiz kitoblar chiqishining nisbiy chastotasini toping.

10. Nishonga 20 ta o‘q uzilgan. Shundan 18 tasi nishonga teggani qayd qilindi. Nishonga tegishning nisbiy chastotasini toping.

11. Buyumlar partiyasini sinashda yaroqli buyumlar nisbiy chastotasi 0,9 ga teng bo‘ldi. Agar hammasi bo‘lib 200 ta buyum tekshirilgan bo‘lsa, yaroqli buyumlar sonini toping.

12. Bir shaharda 920 ta odamdan ishga qanday yetib borishlarini so‘rashganda, ulardan 350 tasi mashinada, 420 tasi jamoat transportida, 80 tasi velosipedda, 70 tasi piyoda borishlari ma’lum bo‘ldi.

1) mashinada;
2) jamoat transportida;
3) velosipedda;
4) piyoda boruvchilar sonining nisbiy chastotasini toping.

13. Radiusi 20 cm bo‘lgan doira ichida bir-biri bilan kesishmaydigan va birining radiusi 5 cm, ikkinchisiniki 10 cm bo‘lgan ikkita aylana o‘tkazilgan. Katta doira ichida tavakkaliga olingan nuqta kichik aylanalardan birining ichida bo‘lish ehtimolligini toping.

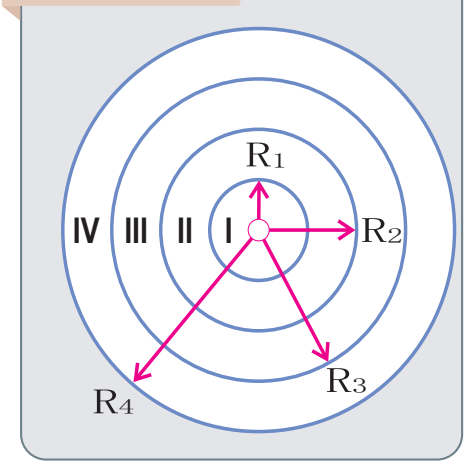
14. Ikki do‘st ma’lum joyda soat 10 bilan 11 orasida uchrashishga kelishishdi. Birinchi kelgan ikkinchisini 20 daqiqa davomida kutadi, shundan so‘ng ketadi. Agar ko‘rsatilgan vaqt oralig‘ida do‘stlarning kelish momentlari teng imkoniyatli bo‘lsa, ularning uchrashish ehtimolligini toping.

15. Qattiq bo'ron natijasida 40- va 70-kilometrlar oralig'ida telefon simi uzilgan. Uzilish 50- va 55-kilometrlar orasida sodir bo'lish ehtimolligini toping.

16. Doiraga kvadrat ichki chizilgan. Doira ichiga tavakkaliga qo'yilgan nuqta kvadrat ichida bo'lib qolish ehtimolligi qancha?

17. Nishon radiuslari $R_1 = r,\ R_2 = 2r,\ R_3 = 3r,\ R_4 = 4r$ bo'lgan konsentrik doiradan iborat. Agar nishonga otilgan nayzaning doiraga tegishi muqarrar bo'lsa, u holda nayzaning har bir sohaga tushish ehtimolligini toping (7-*rasm*).

7-rasm



18. Ikkita shashqol baravar tashlandi. Tushgan sonlar yig'indisining beshga teng bo'lish ehtimolligini toping.

19. Guruhda 30 ta talaba bo'lib, ulardan 10 tasi matematika to'garagiga qatnashadi. Guruh ichidan tavakkaliga 6 ta talaba tanlab olindi. Ularning ichidan hech bo'lmaganda bittasi matematika to'garagiga qatnashadigan talaba bo'lish ehtimolligini toping.

20. 3 ta ko'k va 4 ta yashil sharlardan ixtiyoriy tanlangan 3 ta sharning 2 tasi ko'k, 1 tasi yashil rangda bo'lishi ehtimolligini toping.

21. Shashqol bir marta tashlanganda juft raqam tushish ehtimolligini toping.

22. Tashish vaqtida 10 000 ta tarvuzdan 26 tasi yorilgan. Yorilgan tarvuzlar sonining nisbiy chastotasini toping.

23. Qutida 7 ta oq, 3 ta qora shar bor. Undan tavakkaliga olingan sharning oq bo'lish ehtimolligini toping.

24. Telefonda raqam terayotgan abonent oxirida ikkita raqamini esdan chiqarib qo'ydi va faqat bu raqamlar har xil ekanini bilgan holda ularni tavakkal terdi. Kerakli raqamlar terilganligi ehtimolligini toping.

25. Qurilma 5 ta elementdan iborat bo'lib, ularning 2 tasi eskirgan. Qurilma ishga tushirilganda tasodifiy ravishda 2 ta element ulandi. Ishga tushirishda eskirmagan elementlar ulangan bo'lish ehtimolligini toping.

26. Qutida m ta oq va n ta qora sharlar bor. Qutidan tavakkal bitta shar olingan. Olingan sharning oq bo'lishi ehtimolligini toping.

27. Tavakkaliga 20 dan katta bo'lmagan natural son tanlanganda, uning 5 ga karrali bo'lish ehtimolligini toping.

TAKRORLASH

FUNKSIYA VA UNING XOSSALARI

1. Funksiyalarning aniqlanish sohasini toping.

a) $f(x)=\dfrac{x-3}{x^2-4}$ b) $y=\sqrt{3x-x^3}$ c) $y=\dfrac{1}{\sqrt{x-5}-\sqrt{9-x}}$

d) $y=\sqrt{\dfrac{(x-1)(3-x)}{x(4-x)}}$ e) $y=\sqrt{\dfrac{x(x+1)}{(x-2)(4-x)}}$ f) $y=\sqrt{25-x^2}+\dfrac{2x-3}{x+1}$

2. Agar $f(x)=x^2$ va $g(x)=2x-1$ bo‘lsa, x ning nechta qiymatida $f(g(x))=g(f(x))$ bo‘ladi?

3. Agar $f(x+1)=x^2-3x+2$ bo‘lsa, $f(x)=?$

4. Agar $f(x)=\sqrt{x^3-1}$ bo‘lsa, $f\left(\sqrt[3]{x^2+1}\right)$ nimaga teng?

5. Agar $f(x)=\dfrac{1-x}{1+x}$ bo‘lsa, $f\left(\dfrac{1}{x}\right)+\dfrac{1}{f(x)}$ nimaga teng?

6. Funksiyalar qanday qiymatlarni qabul qiladi?

a) $f(x)=\dfrac{3}{x-4}$ b) $f(x)=\dfrac{2x}{1+x^2}$ c) $f(x)=\dfrac{|x-2|}{x-2}+2$

d) $y=-x^4+2x^2+5$ e) $y=\dfrac{x^2-4x+9}{x^2-4x+5}$ f) $y=\sqrt{x^2-6x+11}$

7. Berilgan funksiyalardan qaysi biri juft funksiya?

a) $y=\dfrac{5x^2}{(x-3)^2}$ b) $y=\dfrac{x(x-2)(x-4)}{x^2-6x+8}$ c) $y=x^2+|x+1|$

d) $f(x)=x^3-\dfrac{2}{x^3}$ e) $y=\begin{cases}-x^2, x<0\\ x^2, x\geq 0\end{cases}$ f) $y=\sqrt{x^2-6x+11}$

8. Berilgan funksiyalardan qaysi biri toq funksiya?

a) $y=3x^5+x^3$ b) $y=(0,25)^x+(0,25)^{-x}$ c) $y=\begin{cases}x, x<0\\ -x, x\geq 0\end{cases}$

d) $y=|x|-1$ e) $y=\dfrac{x^4-2x^2}{3x}$ f) $y=\sqrt{3-x^2-2x}$

RATSIONAL TENGLAMALAR

Tenglamalarni yeching (2-21)

1. t ning qanday qiymatlarida $18x+7=5$ va $18x+7+t=5+t$ tenglamalar teng kuchli bo‘ladi?

2. $\dfrac{4x^2-7x-2}{x^2-5x+6}=0$

3. $2+\dfrac{4}{x^2}=\dfrac{9}{x}$

4. $1-\dfrac{15}{x}=\dfrac{16}{x^2}$

5. $\dfrac{9}{x}+\dfrac{13}{2x}=2$

6. $\dfrac{2}{x-3}=\dfrac{x}{x+3}$

7. $\dfrac{x^3-3x^2}{x+2}\cdot\dfrac{x^2-4}{x^2}=0$

8. $\dfrac{1-x}{(2-x)(x-3)}+1=\dfrac{1}{2-x}$

9. $\dfrac{1}{x^2-9}+\dfrac{1}{3x-x^2}=\dfrac{3}{2x+6}$

10. $\left(\dfrac{x-1}{x+1}\right)^2-\left(\dfrac{x+2}{x-2}\right)^2=0$

11. $\dfrac{1}{x}+\dfrac{36}{9x-x^2}-\dfrac{x-5}{9-x}=0$

12. $\dfrac{x-49}{50}+\dfrac{x-50}{49}=\dfrac{49}{x-50}+\dfrac{50}{x-49}$

13. $5-\dfrac{x^2-14x-51}{x^2-x-12}=\dfrac{3}{x-4}$

14. $\dfrac{2}{x^2-4}+\dfrac{x-4}{x^2+2x}=\dfrac{1}{x^2-2x}$

15. $\dfrac{30}{x^2-1}-\dfrac{13}{x^2+x+1}=\dfrac{18x+7}{x^3-1}$

16. $\dfrac{x^2-3x}{x-2}+\dfrac{x-2}{x^2-3x}=2,5$

17. $\dfrac{4}{x^2-3x+2}-\dfrac{3}{2x^2-6x+1}+1=0$

18. $\dfrac{x-2}{x+1}+\dfrac{4(x+1)}{x-2}=5$

19. $\dfrac{x^2-x}{x^2-x+1}-\dfrac{x^2-x+2}{x^2-x-2}=1$

20. $\dfrac{1}{x^2+2x-3}+\dfrac{18}{x^2+2x+2}=\dfrac{18}{x^2+2x+1}$

21. $x^2+\dfrac{x^2}{(x+1)^2}=\dfrac{40}{9}$

22. Poyezd yo‘lda 30 minut to‘xtab qoldi. Poyezd jadval bo‘yicha yetib kelishi uchun haydovchi 80 km masofada tezlikni 8 km/h ga oshirdi. Poyezd jadval bo‘yicha qanday tezlik bilan yurishi kerak edi?

23. Daryo oqimi bo‘yicha motorli qayiqda 28 km va oqimga qarshi 25 km o‘tildi. Bunda butun yo‘lga sarflangan vaqt turg‘un suvda 54 km ni bosib o‘tish uchun ketgan vaqtga teng. Agar daryo oqimining tezligi 2 km/h bo‘lsa, motorli qayiqning turg‘un suvdagi tezligini toping.

Tenglamalarni yeching (24-38)

24. $\dfrac{x}{2}+\dfrac{2}{x}=\dfrac{x}{3}+\dfrac{3}{x}$

25. $\dfrac{1+x}{6}-\dfrac{6}{1+x}=\dfrac{4}{x+1}-\dfrac{x+1}{4}$

26. $\dfrac{x-2}{x+2}+\dfrac{x+2}{x-2}=3\dfrac{1}{3}$

27. $\dfrac{2x+1}{2x-1}+\dfrac{2x-1}{2x+1}=5,2$

28. $\dfrac{x^2-2x}{x-1}-\dfrac{2x-1}{1-x}=3$

29. $\dfrac{2}{x-4}+\dfrac{4}{x^2-4x}=0,625$

30. $\dfrac{(x^2+1)x}{(x^2-x+1)^2}=\dfrac{10}{9}$

31. $\dfrac{1}{x-1}+\dfrac{2}{x-2}+\dfrac{3}{x-3}=\dfrac{6}{x+6}$

32. $x^2+\dfrac{1}{x^2}-3\left(x+\dfrac{1}{x}\right)+4=0$

33. $x^4-6x^3+10x^2-6x+1=0$

34. $31\left(\dfrac{24-5x}{x+1}+\dfrac{5-6x}{x+4}\right)+370=29\left(\dfrac{17-7x}{x+2}+\dfrac{8x+55}{x+3}\right)$

35. $\dfrac{x+3}{4x^2-9}-\dfrac{3-x}{4x^2+12x+9}=\dfrac{2}{2x-3}$

36. $\dfrac{30}{x^2-1}+\dfrac{7-18x}{x^3+1}=\dfrac{13}{x^2-x+1}$

37. $\dfrac{2x+7}{x^2+5x-6}+\dfrac{3}{x^2+9x+18}=\dfrac{1}{x+3}$

38. $2x^4+x^3-x^2+x+2=0$

RATSIONAL TENGLAMALAR SISTEMASI

Tenglamalar sistemasini yeching (1-8)

1. $\begin{cases}\dfrac{x}{y}+\dfrac{y}{x}=2,5\\ x^2-y^2=3\end{cases}$

2. $\begin{cases}x+y+\dfrac{x}{y}=9\\ \dfrac{(x+y)x}{y}=20\end{cases}$

3. $\begin{cases}2xy-\dfrac{3x}{y}=15\\ xy+\dfrac{x}{y}=15\end{cases}$

4. $\begin{cases}\dfrac{1}{x+y}+\dfrac{2}{x-y}=3\\ \dfrac{3}{x+y}+\dfrac{4}{x-y}=7\end{cases}$

5. $\begin{cases}\dfrac{x}{y}-\dfrac{3y}{x}=\dfrac{1}{2}\\ x^3-\dfrac{y^3}{8}=-28\end{cases}$

6. $\begin{cases}\dfrac{x}{y}+\dfrac{y}{x}=\dfrac{25}{12}\\ x^2-y^2=7\end{cases}$

7. $\begin{cases}\dfrac{4y}{x}+\dfrac{x}{y}=5\\ xy=4\end{cases}$

8. $\begin{cases}x+y=5\\ \dfrac{x}{y}+\dfrac{y}{x}=-\dfrac{13}{6}\end{cases}$

Tenglamalar sistemasini yeching (9-13)

9. $\begin{cases}\dfrac{2}{2x-y}+\dfrac{3}{x-2y}=\dfrac{1}{2}\\ \dfrac{2}{2x-y}-\dfrac{1}{x-2y}=\dfrac{1}{18}\end{cases}$

10. $\begin{cases}\dfrac{x+y}{x-y}+\dfrac{x-y}{x+y}=\dfrac{10}{3}\\ 2x^2+y^2=27\end{cases}$

11. $\begin{cases}\dfrac{6}{x+y}+\dfrac{5}{x-y}=7\\ \dfrac{3}{x+y}-\dfrac{2}{x-y}=-1\end{cases}$

12. $\begin{cases} \dfrac{11}{2x-3y}+\dfrac{18}{3x-2y}=13 \\ \dfrac{27}{3x-2y}-\dfrac{2}{2x-3y}=1 \end{cases}$

13. $\begin{cases} \dfrac{2}{x}+\dfrac{y}{3}=3 \\ \dfrac{x}{2}+\dfrac{3}{y}=\dfrac{3}{2} \end{cases}$

RATSIONAL TENGSIZLIKLAR

Tengsizliklarni yeching.

1. $(-4x+3)(-5x+4)>0$

2. $(x^2-16)^3(x+7)<0$

3. $(x-2)^2(x-1)(x+7)(x-5)\geq 0$

4. $\dfrac{(x+6)^3(x-4)}{(7-x)^5}<0$

5. $(x^2-1)(x^2+5x+6)(x^2-5x+6)\leq 0$

6. $\left(2x+\dfrac{1}{x}\right)^2+2x+\dfrac{1}{x}-12<0$

7. $\dfrac{5x+4}{x-2}<1$

8. $\dfrac{3x+2}{x-3}>1$

9. $\dfrac{x-4}{x^2-9x+14}>0$

10. $\dfrac{x^4-10x^2+9}{6-2x}<0$

11. $\dfrac{x^2+1}{x-3}>0$

12. $\dfrac{x+3}{x^2+7}<0$

13. $(3-\sqrt{10})(2x-7)<0$

14. $\dfrac{(x^2-x-2)^2}{x^2+7x-8}\geq 0$

15. $\dfrac{3x-1}{x^2+x+1}\leq 0$

16. $\dfrac{x^2+2x-15}{3x^2+5x-8}\leq 0$

17. $\dfrac{2}{x+2}<\dfrac{1}{x-3}$

18. $\dfrac{3}{2-x}>\dfrac{1}{x+3}$

19. $\dfrac{2}{x+3}<\dfrac{1}{2x-1}$

20. $\dfrac{x+1}{x-2}>\dfrac{3}{x-2}-\dfrac{1}{2}$

21. $\dfrac{x^3-5x^2+8x-4}{x-3}\leq 0$

22. $\dfrac{6}{x-1}\leq\dfrac{3}{x+1}+\dfrac{7}{x+2}$

23. $\dfrac{14x(2x+3)}{x+1}<\dfrac{(9x-30)(2x+3)}{x-4}$

24. $\dfrac{(5x+4)(3x-2)}{x+3}\leq\dfrac{(3x-2)(x+2)}{1-x}$

25. $(x-3)^2+\dfrac{1}{x^2-6x+9}>2$

26. $\dfrac{2x-3}{4\sqrt{6}-10}>5+2\sqrt{6}$

RATSIONAL TENGSIZLIKLAR SISTEMASI

Tengsizliklar sistemasini yeching.

1. $\begin{cases} 2x-14<0 \\ -3x+9<0 \end{cases}$

2. $\begin{cases} 6x-1>9-4x \\ 3-2x<x+16 \end{cases}$

3. $\begin{cases} 3(2-3x)+2(3-2x)>x \\ 6<x^2-x(x-8) \end{cases}$

4. $\begin{cases} 5\left(1-\dfrac{x-4}{4}\right)-7(2x-3)>0 \\ \dfrac{3x-14}{5}-\dfrac{3x-10}{20}-0,7(x+8)<0 \end{cases}$

5. $\begin{cases} x^2 \le 9 \\ x+1>0 \end{cases}$

6. $\begin{cases} 4x+\dfrac{2x-3}{2}>\dfrac{7x-5}{2} \\ \dfrac{7x-2}{3}-2x>\dfrac{5(x-2)}{4} \end{cases}$

7. $\begin{cases} \dfrac{5x-4}{4}-\dfrac{4x+1}{3}\ge\dfrac{x+2}{4}-7 \\ \dfrac{4x}{3}-1-\dfrac{6x+2}{2}>x+\dfrac{6}{5} \end{cases}$

8. $\begin{cases} x^2+5x-6<0 \\ x+3\ge 0 \end{cases}$

9. $\begin{cases} 13-\dfrac{3-7x}{10}+\dfrac{x+1}{2}<14-\dfrac{7-8x}{2} \\ 7(3x-5)+4(17-x)>18-\dfrac{5(2x-6)}{2} \end{cases}$

10. $\begin{cases} \dfrac{x}{3}-\dfrac{3x-1}{6}<\dfrac{2-x}{12}-\dfrac{x+1}{2}+3 \\ x>\dfrac{5x-4}{10}-\dfrac{3x-1}{5}-2,5 \end{cases}$

11. $\begin{cases} \dfrac{3}{4}(x-1)+\dfrac{7}{8}<\dfrac{1}{4}(x-1)+\dfrac{5}{2} \\ \dfrac{x}{4}-\dfrac{2x-3}{3}<2 \end{cases}$

12. $\begin{cases} x^2+x+8<0 \\ x^2+6x+5\ge 0 \end{cases}$

13. $\begin{cases} 5x\ge 2 \\ -0,3x^2+4,8<0 \\ -2x^2+17x+19\ge 0 \end{cases}$

14. $\begin{cases} 2-5x\le 0 \\ x-x^2\ge 0 \\ -4x^2-5x+21\ge 0 \end{cases}$

15. $\begin{cases} \dfrac{x-4}{4}-x+1<\dfrac{x-2}{2}-\dfrac{x-3}{3} \\ 3-x>2x-10 \end{cases}$

16. $\begin{cases} \dfrac{2x}{3}-1<3-2(1-2x) \\ 3x-5>1-2(1-x) \\ 1-2x<3(2x-1) \end{cases}$

17. $\begin{cases} 7<2x+1<11 \\ \dfrac{x+2}{x-5}<\dfrac{x-6}{x-3} \end{cases}$

18. $\begin{cases} -2<2-x<1 \\ \dfrac{x+3}{1-x}\le\dfrac{8-x}{x-4} \end{cases}$

19. $\begin{cases} \dfrac{2}{7}<2^n<3 \\ 3^n>2 \end{cases}$

20. $\begin{cases} 1<\left(\dfrac{2}{3}\right)^n<3 \\ \left(\dfrac{3}{4}\right)^n<1,5 \end{cases}$

21. $\begin{cases} \dfrac{1}{5}<3^n<4 \\ 2<\left(\dfrac{1}{3}\right)^n<10 \end{cases}$

IRRATSIONAL TENGLAMALAR

Tenglamalarni yeching.

1. $\sqrt{x-1}=-4$
2. $\sqrt{x}=8$
3. $\sqrt{x}=-16$
4. $\sqrt[3]{x+1}=2$
5. $\sqrt[4]{x-7}=-3$
6. $\sqrt{x^2+2x-6}\cdot\sqrt{x-9}=0$
7. $1+\sqrt{x+3}=0$
8. $\sqrt[3]{2x-1}+\sqrt[3]{x-1}=1$
9. $\sqrt{x-4}+\sqrt{x^2-3}=0$
10. $\sqrt{1+4x-x^2}=x-1$
11. $\sqrt{x-3}+\sqrt{2x+4}=-11$
12. $\sqrt{2x^2+8x+7}-2=x$
13. $\sqrt{x^2-7x+12}=2x-6$
14. $\sqrt{4-x}=\sqrt{x-7}$
15. $\sqrt{3+\sqrt{5-x}}=\sqrt{x}$
16. $\sqrt{4-x}+\sqrt{5+x}=3$
17. $2\sqrt{x+18}+\sqrt{4x-3}=15$
18. $\sqrt{x+20}-\sqrt{x-1}=3$
19. $\sqrt{x-5}+\sqrt{1-x}=7$
20. $(x^2-5x+6)\cdot\sqrt{2-x}=0$
21. $(2-x)\cdot\sqrt{x^2-x-20}=12-6x$
22. $(x-1)\cdot\sqrt{\dfrac{x-2}{x^2-1}}=0$
23. $(4x-x^2-3)\cdot\sqrt{x^2-2x}=0$
24. $\sqrt[3]{9x+1}=1+3x$
25. $\sqrt{x+5}+\sqrt[4]{x+5}=12$
26. $x^2+11+\sqrt{x^2+4}=42$
27. $x^2+5x+\sqrt{x^2+5x-5}=17$
28. $\sqrt{x^2-x}+\sqrt{2-x-x^2}=\sqrt{x}-1$
29. $\sqrt{4-x}+\sqrt{x-4}=0$
30. $\sqrt{7-5x}+\sqrt{5x-7}=29$
31. $\sqrt{\dfrac{x-1}{2x+1}}+\sqrt{\dfrac{2x+1}{x-1}}=\dfrac{10}{3}$
32. $\sqrt{5+2x}=10-3\sqrt[4]{5+2x}$
33. $\sqrt{3-x}+\sqrt{x-2}=(x-7)^2\cdot(x-5)$
34. $2\sqrt{x-1}-5=\dfrac{3}{\sqrt{x-1}}$
35. $\dfrac{\sqrt{x^2-3x-4}}{x+2}=\dfrac{\sqrt{x^2-3x-4}}{4-x}$
36. $x^2+\sqrt{x^2+20}=22$
37. $\sqrt{x^3+4x-1-8\sqrt{x^4-x}}=\sqrt{x^3-1}+2\sqrt{x}$
38. $6x^2+7x\sqrt{1+x}=24(1+x)$
39. $\sqrt{(x^2+8x)^2}=x^2+8x$
40. $\sqrt{(4x^2-5x)^2}=5x-4x^2$
41. $\sqrt{x-4\sqrt{x-4}}=2-\sqrt{x-4}$
42. $\sqrt{x^2+\dfrac{1}{x^2}-2}=x-\dfrac{1}{x}$

43. $\sqrt{5-x}+\sqrt{x-6}=x^2+2x$

44. $\sqrt{x+6\sqrt{x-9}}+\sqrt{x-6\sqrt{x-9}}=6$

45. $\sqrt{x+8\sqrt{x-16}}+\sqrt{x-8\sqrt{x-16}}=2\sqrt{x-16}$

46. $\dfrac{\sqrt[4]{x^4-16}+\sqrt[6]{x^3-8}}{3x-x^2-2}=0$

IRRATSIONAL TENGLAMALAR SISTEMASI

Tenglamalar sistemasini yeching.

1. $\begin{cases}\sqrt{x}+\sqrt{y}=8\\ \sqrt{x}\cdot\sqrt{y}=15\end{cases}$

2. $\begin{cases}\sqrt{xy}=12\\ \sqrt{x}+\sqrt{y}=7\end{cases}$

3. $\begin{cases}\sqrt{x}+\sqrt{y}=6\\ x-y=12\end{cases}$

4. $\begin{cases}\sqrt{x+3y+6}=2\\ \sqrt{2x-y+2}=1\end{cases}$

5. $\begin{cases}x\sqrt{y}+y\sqrt{x}=30\\ \sqrt{x}+\sqrt{y}=5\end{cases}$

6. $\begin{cases}3\sqrt{x}-\sqrt{y}=8\\ \sqrt{x}+2\sqrt{y}=19\end{cases}$

7. $\begin{cases}25y+x=100-10\sqrt{xy},\\ \sqrt{x}-\sqrt{y}=4,\end{cases}$

8. $\begin{cases}xy=64\\ x-y+\sqrt{xy}=20\end{cases}$

9. $\begin{cases}\sqrt{x+y-1}=1\\ \sqrt{x-y+2}=2y-2\end{cases}$

10. $\begin{cases}x+y-\sqrt{xy}=7\\ xy=9\end{cases}$

11. $\begin{cases}\sqrt{x}+\sqrt{y}=26\\ \sqrt[4]{x}+\sqrt[4]{y}=6\end{cases}$

12. $\begin{cases}\sqrt[3]{x}+\sqrt[3]{y}=-3\\ xy=8\end{cases}$

13. $\begin{cases}\sqrt[3]{x}-\sqrt[3]{y}=3\dfrac{3}{4}\\ xy=1\end{cases}$

14. $\begin{cases}\sqrt{x}-\sqrt{y}=5\\ \sqrt[4]{x}-\sqrt[4]{y}=1\end{cases}$

15. a) $\begin{cases}5x+3\sqrt{xy}+4y=12\\ 3x+2\sqrt{xy}+3y=8\end{cases}$ b) $\begin{cases}2x-6\sqrt{xy}+7y=9\\ x-4\sqrt{xy}+5y=6\end{cases}$

16. a) $\begin{cases}\sqrt{x}+\sqrt{y}=5\\ x+y+4\sqrt{xy}=37\end{cases}$ b) $\begin{cases}\sqrt{x}+\sqrt{y}=4\\ x+y-3\sqrt{xy}=1\end{cases}$

17. a) $\begin{cases}x\sqrt{x}+12y\sqrt{x}=28\\ 8y\sqrt{y}+6x\sqrt{y}=36\end{cases}$ b) $\begin{cases}x\sqrt{x}+27y\sqrt{x}=36\\ 27y\sqrt{y}+9x\sqrt{y}=28\end{cases}$

KO‘RSATKICHLI TENGLAMALAR

1. Tenglamani yeching.

a) $\left(\frac{1}{2}\right)^x = \frac{1}{1024}$ b) $\left(\frac{5}{4}\right)^{2x-1} = (0{,}8)^{x-2}$ c) $0{,}5^{\sqrt{x+1}} \cdot 0{,}5^{-1} = 0{,}5^{\sqrt{x}}$

d) $4^{x-1} - 4^{x+1} + 4^{x+2} = 49$ e) $2^{x+1} + 3 \cdot 2^{x-1} - 5 \cdot 2^x + 6 = 0$ f) $5^{x+1} + 3 \cdot 5^{x-1} - 6 \cdot 5^x + 10 = 0$

2. $49^x + 7^x + 1 = 57$ tenglama nechta ildizga ega?

3. $9^x - 3^{x+1} + 2 = 0$ tenglamaning eng katta ildizini toping.

KO‘RSATKICHLI TENGSIZLIKLAR

1. Tengsizlikni yeching.

a) $\left(\frac{2}{3}\right)^{2x^2-3x} \le \frac{3}{2}$ b) $\left(\frac{1}{7}\right)^{3x+4} \cdot 7\sqrt{7} < \frac{1}{7}$ c) $(0{,}04)^{2x} > (0{,}2)^{x(3-x)}$

d) $25^x + 5^x > 0$ e) $3^{\frac{x-1}{x+1}} > 27$ f) $3{,}2^{2\left(x-\frac{1}{2}\right)} \ge 3{,}2\sqrt{3{,}2}$

g) $7^{2x-9} > 7^{3x-6}$ h) $0{,}5^{4x+3} \le 0{,}5^{6x-1}$ i) $2\sqrt{2} \cdot 2^{x-3} \ge \frac{1}{2}$

2. Tengsizlikning yechimini qanoatlantiruvchi natural sonlar nechta?

a) $8^{-2x+8} > 512$ b) $2^{5x-7} \le 16$ c) $2^{5x-7} \ge 16$ d) $0{,}1^{4x-5} > 0{,}001$

3. Tengsizlikning eng katta butun yechimini toping.

a) $2{,}5^{2x+3} \le 6{,}25$ b) $1{,}1^{5x-3} < 1{,}21$ c) $0{,}7^{9x+4} > 0{,}343$ d) $\left(\frac{2}{5}\right)^{7x-9} \ge \frac{8}{125}$

LOGARIFM TUSHUNCHASI. LOGARIFMIK FUNKSIYA

1. $A(-2;-1)$ nuqta qaysi funksiya grafigiga tegishli emas?

1) $y = \log_2\left(-\frac{1}{x}\right)$ 2) $y = \log_2|x|$ 3) $y = \log_{\frac{1}{2}}|x|$ 4) $y = -\log_2(-x)$

2. Funksiyaning aniqlanish sohasini toping.

a) $y = \lg(x+2) + \lg(3-x)$ b) $y = \ln(x + |x|)$

3. $y = \log_x(x+1)$ funksiya $x \in \{2;3;4;5;6\}$ bo‘lganda argumentning qaysi qiymatida eng katta qiymatga erishadi?

4. $y = \log_{\frac{1}{3}} x$ funksiya grafigini $y = \log_3 x$ funksiya grafigidan qanday usul bilan hosil qilish mumkin?

LOGARIFMIK IFODALARNI AYNIY ALMASHTIRISH

1. Ifodaning qiymatini hisoblang.

a) $\log_{12}\sqrt[5]{144}$ b) $\log_3 5-\log_3\frac{5}{27}$ c) $\frac{\log_{27}2}{\log_3 8}$

d) $\frac{\log_{11}12}{\log_{11}6}+\frac{\log_5 3}{\log_5 6}$ e) $\frac{3\log_7 2-\frac{1}{2}\log_7 64}{4\log_5 2+\frac{1}{3}\log_5 27}$ f) $81^{\frac{1}{\log_5 3}}+27^{\log_3 4}+3^{\frac{4}{\log_7 9}}$

g) $\frac{1}{2}\log_3\log_5 125$ h) $-\log_2\log_2\sqrt{\sqrt[4]{2}}$

LOGARIFMIK TENGLAMALAR

1. Tenglamani yeching.

a) $\log_5 x=2$ b) $\log_{0,2} x=4$ c) $\log_{\frac{1}{3}} x=-1$ d) $\log_7 x=\frac{1}{3}$

2. Tenglamani yeching.

a) $\log_3 x+\log_{\frac{1}{3}} x+2=0$ b) $3\log_{\frac{1}{7}} x=\log_{\frac{1}{7}}9+\log_{\frac{1}{7}}3$

c) $\log_2(3x-6)=\log_2(2x-3)$ d) $\log_6(14-4x)=\log_6(2x+2)$

3. Tenglamani yeching.

a) $\log_2^2\left(x+\frac{1}{x}\right)-1=0$ b) $\log_{\frac{1}{2}}^2\left(x^2+x\right)+\log_{\frac{1}{2}}\left(x^2+x\right)=0$

c) $\lg^2 x-\lg x+1=\frac{9}{\lg 10x}$ d) $\log_2^2 x+7\log_2 x+49=\frac{-218}{\log_2\frac{x}{128}}$

4. Tenglamani yeching.

a) $x^{5+\log_2 x}=\frac{1}{16}$ b) $5^{2(\log_5 2+x)}-2=5^{x+\log_5 2}$

c) $\lg\left(625\sqrt[5]{5^{x^2-15}}\right)=0$ d) $x^{\lg 2}+2^{\lg x}=4$

5. $\log_2(x+4)=-2\log_2\frac{1}{2-x}$ tenglamaning nechta butun ildizi bor?

KO‘RSATKICHLI VA LOGARIFMIK TENGLAMALAR SISTEMASI

1. Tenglamalar sistemasini yeching.

a) $\begin{cases}3\cdot 7^x+3^y=12\\ 7^x\cdot 3^y=15\end{cases}$ b) $\begin{cases}3^x\cdot 2^y=\frac{4}{9}\\ x+y=4\end{cases}$ c) $\begin{cases}2^x+2y=1\\ 3y-6y^2=2^{x-1}\end{cases}$

d) $\begin{cases}2^x+2y=1\\ 3y-6y^2=2^{x-1}\end{cases}$ e) $\begin{cases}3\cdot 2^x+y=13\\ 2^{2x+1}+3y=35\end{cases}$ f) $\begin{cases}3\cdot 7^x-3^y=12\\ 7^x\cdot 3^y=15\end{cases}$

2. Tenglamalar sistemasini yeching.

a) $\begin{cases}\log_5(x+y)=1\\ 2^x+2^y=12\end{cases}$ b) $\begin{cases}\log_4 x+\log_4 y=1+\log_4 9\\ 2^{\frac{x+y}{2}}=1024\end{cases}$

LOGARIFMIK TENGSIZLIKLAR

1. Tengsizlikni yeching.

a) $\log_4(x+5)<0$ b) $\log_3(2-5x)<1$ c) $\log_{\frac{1}{7}}(x+5)>-1$

d) $\log_{0,2}(x-3)+2\geq 0$ e) $\log_{\frac{1}{2}}(2x+3)>\log_{\frac{1}{2}}(x+1)$ f) $\log_{\frac{1}{3}}(x^2+x+1)\leq 0$

g) $\log_3(13-4^x)>2$ h) $2\log_3 x-\log_x 81<2$ i) $\log_2(x-1)+\log_2(x+1)\geq 3$

2. $\frac{1}{\log_3 x-2}>\frac{1}{\log_3 x}$ tengsizlikning 5 dan kichik natural yechimlari nechta?

3. $2\log_5 x-\log_x 125<1$ tengsizlikning natural yechimlari yig‘indisini toping.

4. $\log_x(3-x)>1$ tengsizlikning nechta butun yechimi bor?

TRIGONOMETRIK FUNKSIYALAR

1. Funksiyaning aniqlanish sohasini toping.

a) $y=\frac{1}{\sin x}$ b) $y=\frac{1}{\cos x}$ c) $y=\frac{\cos x}{\sin x-2\sin^2 x}$

d) $y=\frac{3x}{2\cos x-1}$ e) $y=\cos x+\sin x$ f) $y=\cos x+ctgx$

2. Funksiyaning qiymatlar to‘plamini toping.

a) $y=3\cos x-1$ b) $y=2-\sin x$ c) $y=1-2\sin^2 x$ d) $y=2\cos^2 x-1$

3. Berilgan funksiyaning juft yoki toqligini aniqlang.

a) $y=\frac{\sin x}{x}$ b) $y=x\cos x$ c) $y=\sin x+x^2$ d) $y=\cos x-x^2$

4. Funksiyaning eng kichik musbat davrini toping.

a) $y=\sin\frac{x}{2}$ b) $y=\cos(3x-1)$ c) $y=tg2x$ d) $y=\cos\frac{x}{3}$

5. Funksiyaning eng katta va eng kichik qiymatini toping.

a) $y=\cos^4 x-\sin^4 x$ b) $y=\cos\left(x+\frac{\pi}{4}\right)\cos\left(x-\frac{\pi}{4}\right)$ c) $y=1-2|\sin 3x|$

6. Funksiya nollarini toping.

a) $y=\sin x-2$ b) $y=2\cos x+1$ c) $y=x\cos x$ d) $y=\cos\left(x+\frac{\pi}{6}\right)$

TESKARI TRIGONOMETRIK FUNKSIYALAR

1. Funksiyaning aniqlanish sohasini toping.

a) $y = arccos\dfrac{2x+3}{4}$ b) $y = arcsin(2+3x)$

c) $y = arcsin\left(3\sqrt{x}+2\right)$ d) $y = arccos\dfrac{4-x}{3}$

2. Taqqoslang.

a) $arccos\dfrac{\sqrt{3}}{2}$ va $arcsin\left(-\dfrac{1}{2}\right)$ b) $arctg(-1)$ va $arccos\left(-\dfrac{1}{2}\right)$

c) $arccos\sqrt{3}$ va $arcsin1$ d) $arccos\left(-\dfrac{\sqrt{3}}{2}\right)$ va $arcsin\dfrac{1}{2}$

3. Ifodalarning qiymatini toping.

a) $2\,arcsin\left(-\dfrac{\sqrt{3}}{2}\right)+arctg(-1)+arccos\dfrac{\sqrt{2}}{2}$ b) $arcsin\dfrac{1}{2}+4\,arccos\left(-\dfrac{1}{\sqrt{2}}\right)+arctg\left(-\sqrt{3}\right)$

c) $arctg\left(-\sqrt{3}\right)+arccos\left(-\dfrac{\sqrt{3}}{2}\right)+arccos1$ d) $arcsin1-\dfrac{1}{2}+arccos\left(-\dfrac{1}{2}\right)+6arctg\sqrt{3}$

4. Hisoblang.

a) $2\,arcsin\dfrac{\sqrt{3}}{2}+3\,arcsin\left(-\dfrac{1}{2}\right)$ b) $2\,arc\,cos\left(-\dfrac{1}{2}\right)+arcsin\dfrac{\sqrt{3}}{2}$

c) $2\,arc\,tg1+3arctg\left(-\dfrac{\sqrt{3}}{3}\right)$ d) $2\,arc\,tg(-1)+3arctg\left(\sqrt{3}\right)$

5. Hisoblang.

a) $sin\left(arccos\dfrac{\sqrt{3}}{2}\right)$ b) $tg\left(arccos\dfrac{1}{2}\right)$ c) $tg\left(arccos\dfrac{\sqrt{2}}{2}\right)$

d) $sin(4arcsin1)$ e) $cos(arcsin1)$ f) $sin\left(arcsin\dfrac{\sqrt{3}}{2}\right)$

TRIGONOMETRIK TENGLAMA VA TENGSIZLIKLAR

1. $0 \le x < 360^\circ$ oraliqda tenglamalarni yeching.

a) $sinx = -0{,}3$ b) $sinx = 0{,}15$ c) $cosx = 0{,}6$ d) $cosx = -0{,}43$

2. Oraliqlarni hisobga olgan holda x ning qiymatini toping.

a) $4sinx + 2 = 0,\ 0 \le x < 2\pi$ b) $ctgx - \sqrt{3} = 0,\ 0 \le x < 2\pi$

c) $2sin^2x + 5sinx = 3\ \ 0 \le x < 2\pi$ d) $cos2x = -\dfrac{1}{\sqrt{2}},\ 0 \le x < 2\pi$

3. $0 \le x < 360^\circ$ oraliqda tenglamalarni yeching.

a) $7 - 6\cos^2 x = 5\sin x$ b) $7 + 2cosx = 8sin^2x$ c) $2sinx - 3cosx = 0$

4. Tenglamalarni yeching.

a) $sin10x = -\frac{\sqrt{3}}{2}$ b) $cos10x = \frac{\sqrt{3}}{2}$ c) $tg10x = \sqrt{3}$ d) $ctg10x = \frac{\sqrt{3}}{3}$

5. Tenglamalarni yeching.

a) $sin4xcos3xtg8x = 0$ b) $cos4x = -cos5x$ c) $tg5x = -tg\frac{x}{3}$

6. Tenglamalarni yeching.

a) $2sin^2x + cos^2x - 2 = 0$ b) $2sin^2x + cosx = 0$ c) $sinxcosx = 0$

7. Tenglamalarni yeching.

a) $sin^2x - 2sinxcosx + cos^2x = 0$ b) $7cos^2x - 3sin^2x = 0$

c) $cos^22x - 10sin2xcos2x + 21sin^22x = 0$ d) $8sin^2x - cos^2x = 0$

8. O‘rniga qo‘yish usulidan foydalanib yeching.

a) $cos^22x + 1 = 2cos^2x$ b) $3cos^2xsinx + 1 = 3cos^2x + sinx$

c) $6cos^2x + 6sin^2x - 3cosx - 3 = 0$ d) $5sin^2xcosx + 6cos^2x - 10cosx + 6 = 0$

9. Tenglamalarni yeching.

a) $cos2x + cosx = 0$ b) $cos3x = 2cos2x - 1$

c) $2cos^2x = 4sinxcosx - 1$ d) $cos^2x - 3sinxcosx = -1$

10. Tenglamalarni $sinx + cosx = t$ almashtirish yordamida yeching.

a) $2(sinx + cosx) + sin2x + 1 = 0$ b) $\sin x + \cos x = 1 + \frac{\sin 2x}{2}$

11. Tenglamalarni baholash usuli bilan yeching.

a) $2sin^8x - 3cos^8x = 5$ b) $(cos2x - cos4x)^2 = 4 - 4cos^23x$

12. Tenglamalarni yordamchi burchak kiritish usuli bilan yeching.

a) $12cosx - 5sinx = -13$ b) $sinx + cosx = \sqrt{2}$

13. Tengsizliklarni yeching.

a) $\sqrt{2}\cos 2x \le 1$ b) $2sin3x > -1$ c) $\sin\left(x + \frac{\pi}{4}\right) \le \frac{\sqrt{2}}{2}$

d) $\cos\left(x - \frac{\pi}{6}\right) \ge \frac{\sqrt{3}}{2}$ e) $\sin\left(\frac{x}{4} - 3\right) < \frac{\sqrt{2}}{2}$ f) $\cos\left(\frac{x}{3} + 2\right) < \frac{1}{2}$

14. Tengsizlikni yeching.

a) $sin^2x + 2sinx > 0$ b) $cos^2x - cosx < 0$

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EHTIMOLLIKLAR NAZARIYASI

1. Lotereyada 2000 ta chipta bo‘lib, ulardan 400 tasi yutuqli. Tavakkaliga olingan 2 ta chiptadan faqat bittasi yutuqli bo‘lish ehtimolligini toping.
2. Ikkita shashqoltosh tashlangan bo‘lib, ularning yoqlarida chiqqan ochkolar yig‘indisi 7 ga, ko‘paytmasi 6 ga teng bo‘lish ehtimolligini toping.
3. 7 ta qora va 8 ta oq shar bo‘lgan idishdan tasodifiy ravishda bitta shar olinganda uning: a) oq; b) qora shar bo‘lish ehtimolligini toping.
4. Tavakkaliga 25 dan katta bo‘lmagan natural son tanlanganda uning 3 ga karrali bo‘lish ehtimolligini toping.
5. Nashriyotda tasodifan ajratib olingan 1000 ta kitobdan iborat partiyada 7 tasi yaroqsiz deb topildi. Yaroqsiz kitoblar chiqishining nisbiy chastotasini toping.
6. Kvadratga doira ichki chizilgan. Kvadratga tavakkaliga qo‘yilgan nuqtaning doira ichida bo‘lib qolish ehtimolligini toping.
7. Ikkita shashqoltosh baravar tashlanganda tushgan sonlar yig‘indisi oltidan kichik bo‘lish ehtimolligini toping.
8. Yashikda 11 ta oq va 9 ta qora shar bor. Tavakkaliga olingan 4 ta shardan 2 tasi oq bo‘lish ehtimolligini toping.
9. Idishda 7 ta qizil va 13 ta ko‘k koptok bor. Tavakkaliga olingan 2 ta koptokning har xil rangli bo‘lish ehtimolligini toping.
10. Qutida 7 ta oq, 3 ta qora shar bor. Undan tavakkaliga olingan 2 ta sharning har xil rangda bo‘lish ehtimolligini toping.
11. Telefon raqamini terayotgan abonent oxirida uchta raqamini esdan chiqarib qo‘ydi. Tavakkaliga raqamni terganda kerakli raqamlar terilgani ehtimolligini toping.
12. Qutida 100 ta lampochka bo‘lib, ularning 10 tasi yaroqsiz. Tavakkaliga 4 ta lampochka olinganda, ulardan 2 tasi yaroqsiz bo‘lish ehtimolligini toping.
13. 3 ta ko‘k, 4 ta qizil va 5 ta yashil shardan ixtiyoriy tanlangan 3 ta sharning turli rangda bo‘lish ehtimolligini toping.
14. Yaroqliligining nisbiy chastotasi 0,8 ga teng bo‘lgan buyumlar partiyasida 250 ta buyum tekshirilgan bo‘lsa, yaroqli buyumlar sonini toping.
15. Xaltada 5 ta ko‘k va 7 ta sariq shar bo‘lib, tasodifiy ravishda olingan ikkita shar har xil rangda bo‘lish ehtimolligi toping.
16. Savatda 5 ta yashil, 7 ta sariq va 8 ta qizil olma bor. Ixtiyoriy ravishda olingan 3 ta olma har xil rangda bo‘lish ehtimolligini toping.
17. Qutichada 6 ta bir xil raqamlangan donacha bor. Tavakkaliga bitta-bittadan barcha donachalar olinganda donachalarning raqamlari kamayib borish tartibida chiqishi ehtimolligini toping.
18. Qutida 12 ta oq, 18 ta qizil shar bor. Tavakkaliga olingan 4 ta sharning 3 tasi qizil bo‘lish ehtimolligini toping.
19. Sinfda 36 ta o‘quvchi bo‘lib, ulardan 13 tasi shaxmat to‘garagiga qatnashadi. Shu sinfdan tavakkaliga olingan 7 ta o‘quvchidan hech bo‘lmaganda bittasi shaxmat to‘garagiga qatnashadigan bo‘lishi ehtimolligini toping.

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O‘quv nashri

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№	O‘quvchining ismi va familiyasi	O‘quv yili	Darslikning olingandagi holati	Sinf rahbari-ning imzosi	Darslikning topshirilgan-dagi holati	Sinf rahbari-ning imzosi
1						
2						
3						
4						
5						
6						

Darslik ijaraga berilib, o‘quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to‘ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko‘chmagan, betlarida yozuv va chiziqlar yo‘q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta’mirlangan. Ko‘chgan varaqlari qayta ta’mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqova chizilgan, yirtilgan, asosiy qismidan ajralgan yoki butunlay yo‘q, qoniqarsiz ta’mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo‘yab tashlangan. Darslikni tiklab bo‘lmaydi.