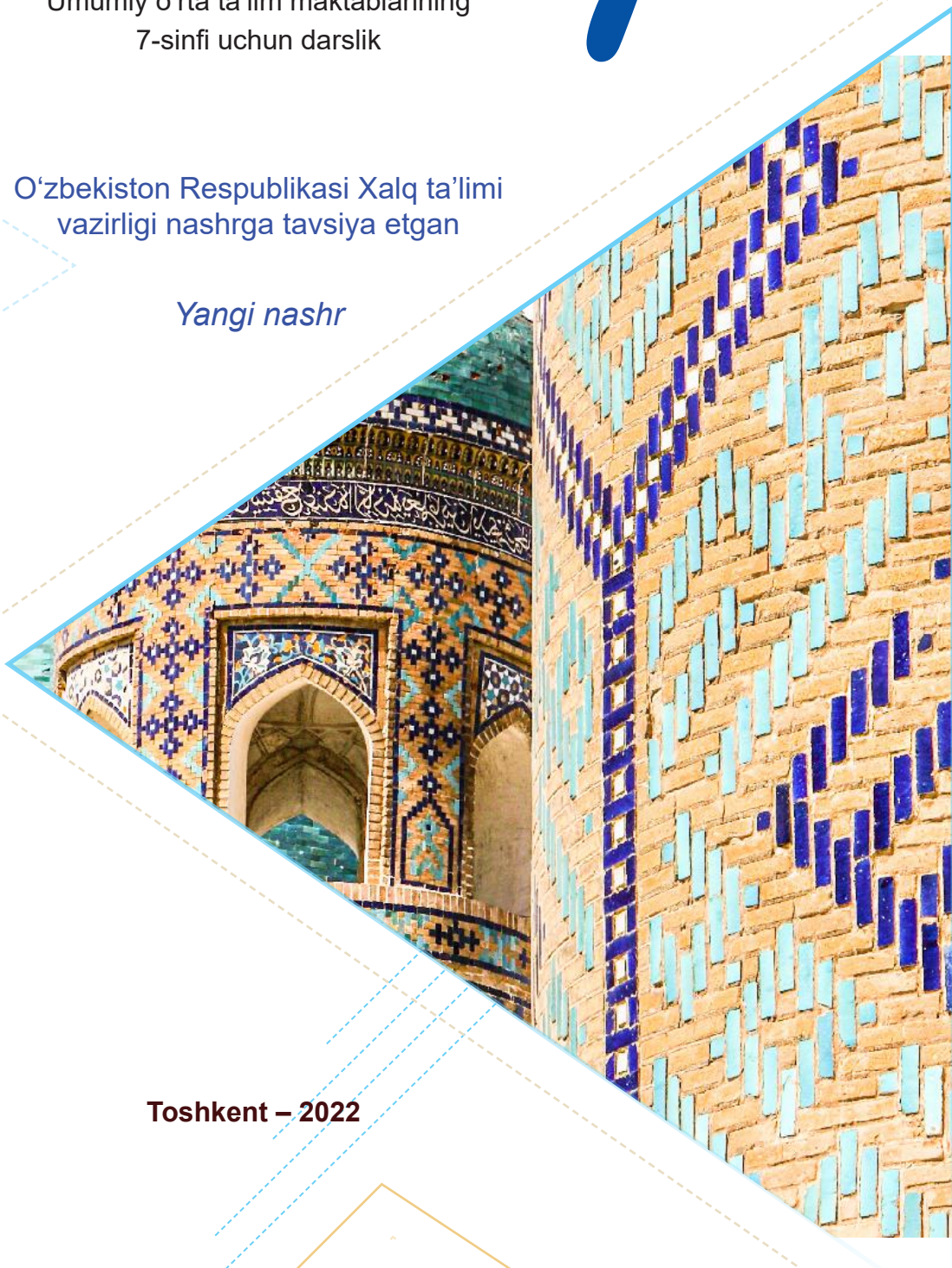
GEOMETRIYA

7

Umumiy o‘rta ta’lim maktablarining
7-sinfi uchun darslik

O‘zbekiston Respublikasi Xalq ta’limi
vazirligi nashrga tavsiya etgan

Yangi nashr



Toshkent – 2022

www.idum.uz saytidan yuklab olindi

UO‘K 514(075.3)
KBK 22.151ya72
G 35

Tuzuvchilar:

Boxodir Xaydarov
Nargiza Tashtemirova
Isak Asrorov

Taqrizchilar:

Z. R. Babayeva – Sirdaryo viloyati Guliston shahridagi 11-sonli umumiy o‘rta ta’lim maktabining matematika fani o‘qituvchisi.

M. X. Usmanov – Namangan viloyati xalq ta’limi boshqarmasi tasarrufidagi 3-IDUMI matematika fani o‘qituvchisi.

A. K. Alibekova – Toshkent shahri Shayxontohur tumanidagi 180-IDUM matematika fani o‘qituvchisi.

Geometriya 7-sinf [Matn]: darslik / B. Xaydarov, N. Tashtemirova, I. Asrorov − Toshkent: Respublika ta’lim markazi, 2022. − 192 b.

UNICEFning O‘zbekistondagi vakolatxonasi bilan hamkorlikda tayyorlandi.

O‘zbekiston Respublikasi Fanlar akademiyasi V. I. Romanovskiy nomidagi matematika instituti xulosasi asosida takomillashtirildi.

Original maket va dizayn konsepsiyasi
Respublika ta’lim markazi tomonidan ishlandi.

Respublika maqsadli kitob jamg‘armasi mablag‘lari hisobidan chop etildi.

Shartli belgilar:

 – geometrik tushuncha ta’rifi

 – teorema tavsifi

 – aksioma tavsifi

 – mavzu bo‘yicha savollar

 – faollashtiruvchi mashg‘ulot

 – masala yechish namunasi

 – amaliy mashq va tatbiq

 – geometrik tadqiqot

 – tarixiy lavhalar

 – geometrik boshqotirmalar

 – multimedia ilovalari

 – elektron resurslar

ISBN 978-9943-8353-0-6

© Respublika ta’lim markazi, 2022

www.idum.uz saytidan yuklab olindi



SO‘ZBOSHI

Aziz o‘quvchilar! *Siz matematikaning eng jozibali bo‘limi – geometriyani o‘rganishni boshlayapsiz. Quyi sinflarda ayrim geometrik shakllar va ularning xossalari bilan tanishgan edingiz. 7-sinfda geometrik shakllarning g‘aroyib olamiga sayohat qilib, uni atroflicha va tizimli ravishda o‘rganishga kirishasiz.*

Bizni geometrik shakllar olami o‘rab turadi. Geometrik shakllarga kundalik turmushda har qadamda duch kelamiz. Haqiqatan, to‘rtburchak shaklidagi deraza romi, qor parchasining hayratomuz shakli, turli shakldagi binolar, gugurt qutisi, velosiped g‘ildiragi, sovun pufaklari, gullar, daraxt barglari va shu kabi narsalar biror-bir shaklga ega.

Demak, ular geometriyaga aloqador. Shu bois geometriyani bilish, uning sirlaridan voqif bo‘lish har bir kishi uchun ahamiyatlidir.

Geometriya bilan tanishuv eng sodda geometrik shakllar va ularning xossalarini o‘rganishdan boshlanadi. Ular asosida o‘zingiz uchun yangidan yangi geometrik shakllar va ularning xossalarini kashf qilasiz.

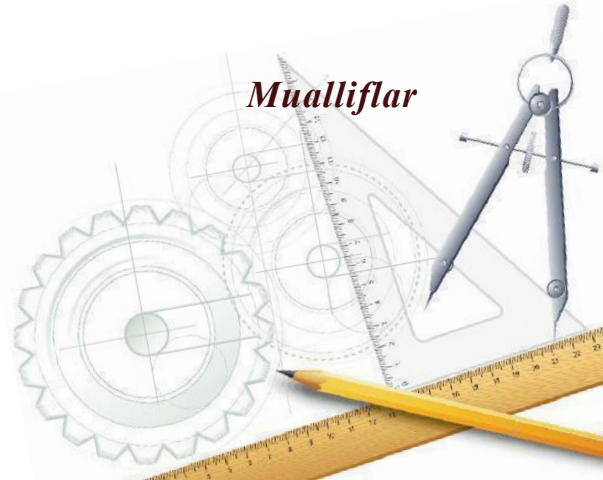
Shu bilan birga, ***geometriya sizni to‘g‘ri fikrlashga o‘rgatadi, mantiqiy mulohazalar yuritish va ular asosida oqilona qarorlar qabul qilish, to‘g‘ri xulosa chiqarish ko‘nikmalarini egallashingizga yordam beradi****.*

Bu esa barkamollikka intiluvchi har bir yigit-qiz uchun juda muhim fazilatdir.

Geometriyani shunchaki o‘qish emas, balki uqishga intilishingiz kerak bo‘ladi. Har bir jumlaning “mag‘zini chaqish”ga urining. Tushunmasangiz, qayta o‘qing, chizmalarga murojaat qiling. Imkoni boricha ko‘proq masala yeching. Chunki har bir yechilgan masala geometriyani o‘rganishning keyingi bosqichiga zamin tayyorlaydi. Shundagina geometriya o‘z sirlarini sizga ocha boradi.

Ishonchimiz komilki, siz geometriya bilan tez orada oshno bo‘lasiz va u siz uchun sevimli hamda qiziqarli bo‘lgan fanlardan biri bo‘lib qoladi.

Mualliflar



www.idum.uz saytidan yuklab olindi

MUNDARIJA



I bob. Geometriyaning boshlang‘ich tushunchalari

1. Eng sodda geometrik shakllar.....8
2. Kesma. Kesmalarni taqqoslash va o‘lchash 17
3. Burchak. Burchaklarni taqqoslash va o‘lchash 29
4. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring.....38
5. Burchakning turlari 45
6. Perpendikulyar to‘g‘ri chiziqlar 53
7. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring.....61



II bob. Uchburchaklar

8. Uchburchaklar, ularning turlari va elementlari.....72
9. Uchburchaklar tengligining birinchi alomati.....79
10. Teng yonli uchburchakning xossalari 82

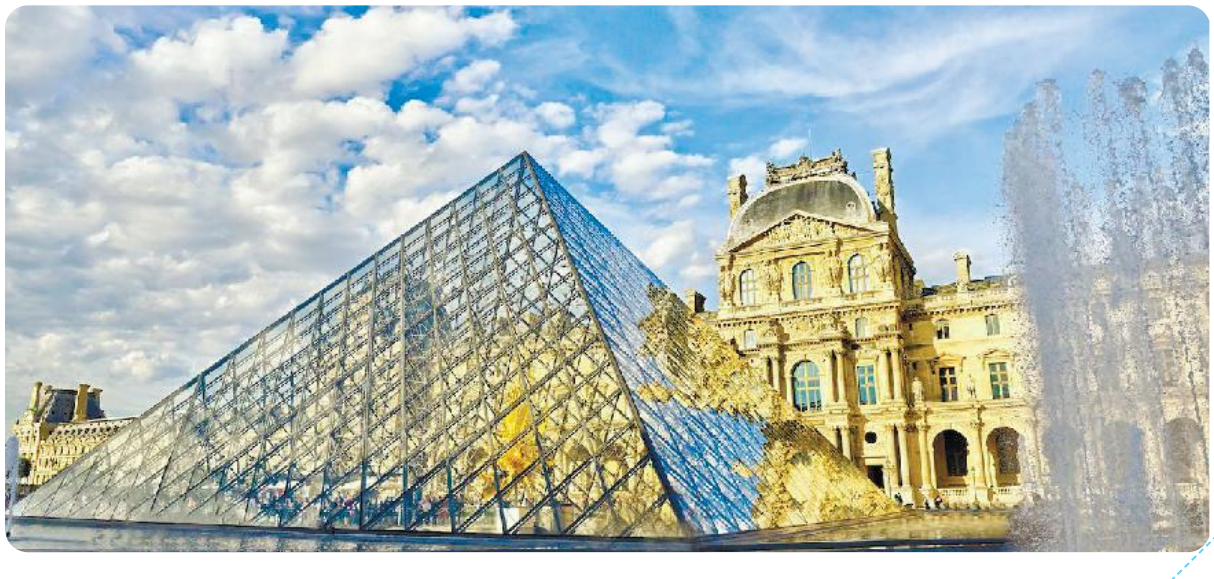
www.idum.uz saytidan yuklab olindi

11. Uchburchaklar tengligining ikkinchi alomati 85
12. Uchburchaklar tengligining uchinchi alomati.....87
13. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring 91



III bob. Parallel to‘g‘ri chiziqlar

14. Parallel to‘g‘ri chiziqlar 100
15. Ikki to‘g‘ri chiziqning parallellik alomatlari 105
16. Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan burchaklar 109
17. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring.....114



IV bob.Uchburchakning tomonlari va burchaklari orasidagi munosabatlar

18. Uchburchakning ichki burchaklari yig‘indisi 124
19. To‘g‘ri burchakli uchburchaklar 131

20. Burchak bissektrisasining xossasi 135
21. Uchburchakning tomonlari va burchaklari orasidagi munosabatlar 138
22. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring..... 142



V bob. Yasashga doir masalalar

23. Sirkul va chizg‘ich yordamida geometrik yasashga doir masalalar.....152
24. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring.....162



VI bob. Takrorlash

25. Takrorlashga doir masalalar 168
26. Amaliy mashq va tatbiq. Bilimingizni sinab ko‘ring.....174
7-sinf uchun geometriyaga oid asosiy ma’lumotlar.....180
Javoblar va ko‘rsatmalar.....186

7-SINF “GEOMETRIYA” DARSLIGI UCHUN TA’LIMIY O‘YINLAR

7-SINF “GEOMETRIYA” DARSLIGI UCHUN VIDEODARSLAR



I BOB

BOSHLANG‘ICH GEOMETRIK MA’LUMOTLAR

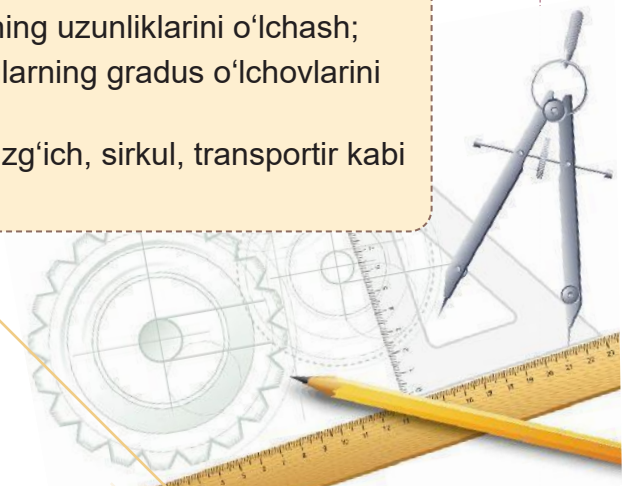
Bu bobni o‘rganib chiqqach, quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilim:

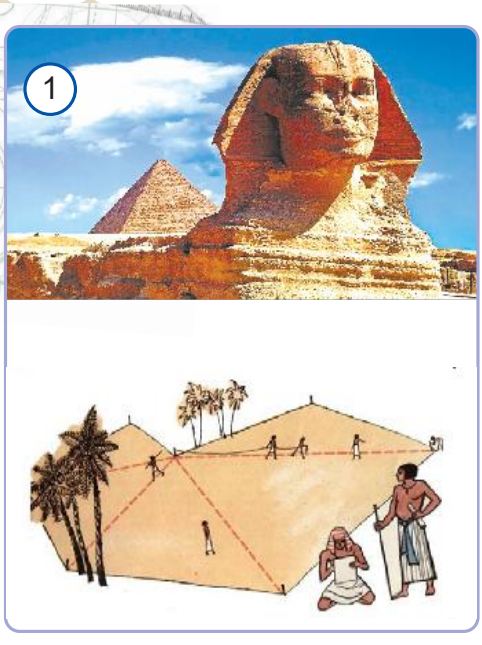
– geometriya tarixiga oid asosiy ma’lumotlar;
– nuqta, to‘g‘ri chiziq, tekislik, kesma, nur, burchak kabi boshlang‘ich geometrik tushunchalar;
– eng sodda geometrik shakllarning xossalari;
– geometriya va planimetriyaning ta’rifi;
– aylana, doira va ularning elementlari ta’riflari;
– to‘g‘ri, o‘tkir va o‘tmas burchaklar;
– qo‘shni va vertikal burchaklar hamda ularning xossalari;
– ta’rif, aksioma, teorema va isbot tushunchalarining mohiyati;
– teskarisini faraz qilib isbotlash usuli.

Amaliy ko‘nikmalar:

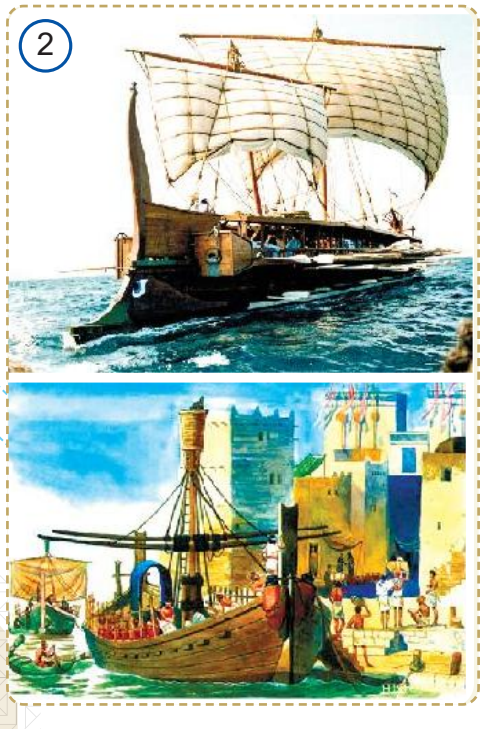
– eng sodda geometrik shakllarni tekislikda tasvirlash, belgilash, taniy olish va belgilariga ko‘ra o‘qish;
– kesmalarni nurga qo‘yish, o‘zaro taqqoslash va ularning uzunliklarini o‘lchash;
– burchaklarni yarimtekislikka qo‘yish, taqqoslash va ularning gradus o‘lchovlarini topish;
– geometrik shakllarni yasash va o‘lchash ishlarida chizg‘ich, sirkul, transportir kabi o‘quv qurollaridan foydalana olish.



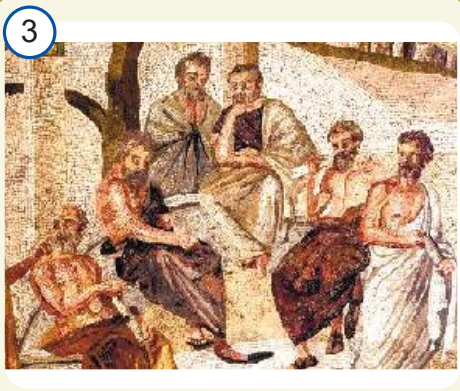
1 ENG SODDA GEOMETRIK SHAKLLAR



1



2



3

1.1. Geometriyaning paydo bo‘lishi

Geometriyaga oid dastlabki tushunchalar bundan 4–5 ming yil muqaddam qadimgi Misrda paydo bo‘lgan. O‘sha kezlarda Nil daryosining suvi har yili toshib, ekin maydonlarini yuvib turgan. Shu sababli ekinzorlarni qayta taqsimlash va soliq miqdorini aniqlash uchun bu maydonlarda belgilash va o‘lchash ishlarini bajarishga to‘g‘ri kelgan *(1-rasm)*. Qadimgi yunon olimlari yer o‘lchash usullarini misrliklardan o‘rganib, uni *geometriya* deb ataganlar. “Geometriya” yunoncha so‘z bo‘lib, *geo* –“yer”, *metrio* – “o‘lchash” degan ma’noni anglatadi.

Geometriyaga oid dastlabki tushunchalar Qadimgi Bobilda ham paydo bo‘lgan. Xususan, tarixchilar Pifagor teoremasi Bobilda topilgan deb hisoblashadi. Miloddan avvalgi VII–VI asrlarda Qadimgi Xorazmda ham Misrdagi kabi Amudaryoning quyi qismida yer o‘lchash ishlari bajarilgan.

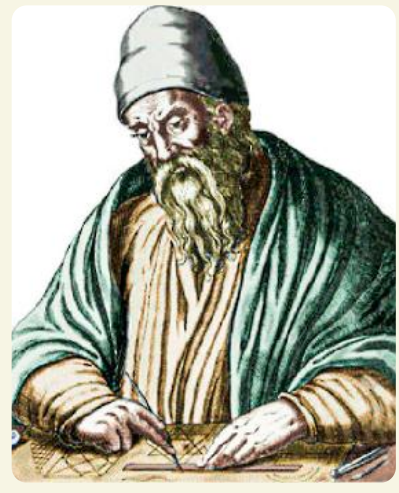
Qadimgi Misrda geometriya bahaybat piramidalar, qasrlar va turarjoylarni qurishda ham zarur bo‘lgan. Yunonlar geometriyadan qurilishdan tashqari, dengizda suzishda ham foydalangan *(2-rasm)*. Mana shunday amaliy ehtiyojlar insonni turli shakllarni va ularning xossalarini o‘rganishga undagan. Qadimgi Yunonistonning yetti donishmandlaridan biri Miletlik Fales geometriyaning dastlabki teoremalarini isbotlagan.

Miloddan avvalgi IV asrga kelib, geometriyaga oid o‘rganilgan ko‘plab tushunchalar va ularning xossalari yig‘ilib qolgan. Yunon olimi Platon geometriyada ajoyib bir qonuniyatni payqagan: oldin o‘rganilgan, to‘g‘riligi tasdiqlangan xossalardan mantiqiy fikrlash, mushohada yuritish orqali yangi xossalarni keltirib chiqarsa bo‘lar ekan. Bunday mashg‘ulot o‘quvchilarning fikrlash qobiliyatini o‘stirgani uchun geometriya maktablarda asosiy fanga aylangan. Hatto Platon akademiyasining peshtoqiga “Geometriyani bilmaydiganlar uchun bu maktabga yo‘l yo‘q!” degan shior osib qo‘yilgan ekan *(3-rasm)*.

Qadimgi yunon olimi Yevklid o‘sha paytgacha ma’lum bo‘lgan barcha geometrik tushuncha va xossalarni tartibga keltirib, “Negizlar” deb nomlangan

kitobida bayon etdi. Bu kitob ikki ming yil mobaynida maktablar uchun eng muhim darslik vazifasini o‘tadi va fan taraqqiyotida ulkan ahamiyatga ega bo‘ldi.

O‘tmishda yashab o‘tgan deyarli barcha olimlar geometriya bilan shug‘ullangan. Buyuk vatandoshlarimiz – Muhammad ibn Muso al-Xorazmiy, Ahmad al-Farg‘oniy, Abu Rayhon Beruniy, Abu Ali ibn Sino ham Yevklid “Negizlar”ini puxta o‘rganib, bu fan rivojiga o‘z hissalarini qo‘shgan. Sharq mamlakatlarida geometriya *handasa* deb atalgan va unga katta ahamiyat berilgan. Bu fikrni muhandis (injener) so‘zining o‘zagi “handasa” ekanligi ham tasdiqlab turibdi.

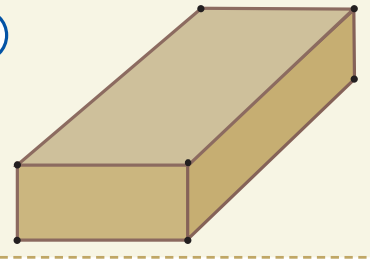


Yevklid (miloddan oldingi III asr) – qadimgi yunon olimi, geometriya fani shakllanishida katta o‘rin tutgan “Negizlar” asari bilan mashhur.

1.2. Geometriyaga kirish

Bizni o‘rab turgan har bir predmet qandaydir shaklga ega. Masalan, g‘ishtni olaylik. U 5-sinfdan sizga tanish to‘g‘ri burchakli parallelepiped shaklidadir *(4-rasm)*. Uning 8 ta uchi bor – bular nuqtalar, 12 ta qirrasi bor – bular kesmalar, 6 ta yog‘i bor – bular to‘g‘ri to‘rtburchaklar.

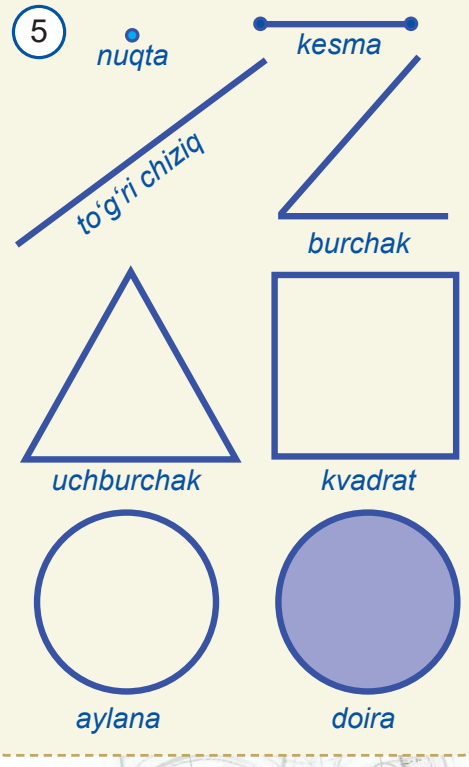
4



Nuqta, to‘g‘ri chiziq, kesma, burchak, uchburchak, kvadrat, aylana, kub, shar kabi qator geometrik shakllar bilan siz quyi sinflarda tanishgansiz *(5–6-rasmlar)*.

6-rasmda tasvirlangan shakllar tabiatdagi turli jismlarning geometrik timsolidan iborat. Jismlarni geometrik nuqtayi nazardan o‘rganganimizda ularning faqat shaklini inobatga olamiz.

5



Biz nuqta, kesma, burchak, uchburchak kabi yassi shakllarni daftar varag‘iga chiza olamiz. Kub, piramida, shar kabi fazoviy geometrik shakllarni esa aynan chiza olmaymiz. Ammo ularning ko‘rinishini daftarda tasvirlashimiz mumkin.

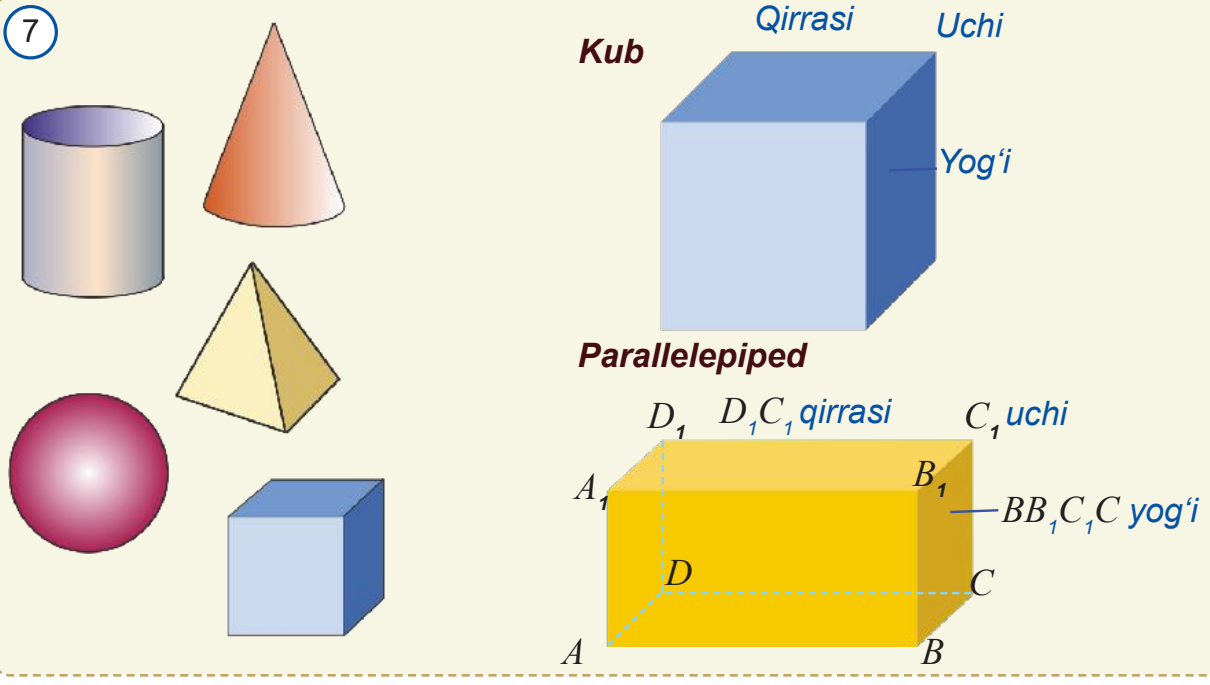
Geometriya – geometrik shakllar va ularning xossalari haqidagi fan.

Planimetriya geometriyaning bo‘limi bo‘lib, u tekislikdagi geometrik shakllarning xossalarini o‘rganadi. Bu shakllarni *yassi* (ya’ni ikki o‘lchamli) *shakllar* deb ham ataymiz.

6



7



Fazoviy shakllarning xossalarini esa geometriyaning *stereometriya* deb ataladigan bo‘limi o‘rganadi. Tevarak-atrofimizdagi barcha predmetlar uch o‘lchamli bo‘lib, ularning shakli qaysidir fazoviy geometrik jismga o‘xshab ketadi *(6-rasm)*.

Quyi sinflarda siz bunday fazoviy (uch o‘lchamli) jismlar bilan tanishgansiz. Shunday bo‘lsa-da, bu geometrik shakllarning ba’zi planimetriyaga oid xususiyatlari borki, kezi kelganda ularni ham o‘rganib boramiz.

Shu bois ba’zi bir fazoviy jismlar elementlari haqidagi ma’lumotlarni qisqacha eslatib o‘tishni lozim topdik *(7-rasm)*.

1.3. Geometriyaning boshlang‘ich tushunchalari

Nuqta, *to‘g‘ri chiziq* va *tekislik* – geometriyaning boshlang‘ich tushunchalari. Ularni ta’rifsiz qabul qilamiz.

Qalam uchini qog‘ozga, bo‘rni doskaga tekkizganda qolgan iz yoki osmondagi yulduzlarni

(*8-rasm*) olib qaraydigan bo‘lsak, ular ko‘zimizga shu qadar kichik ko‘rinadiki, ularning o‘lchamlarini hisobga olmasa ham bo‘ladi. *Nuqta* – ana shunday o‘lchamlarini hisobga olmasa ham bo‘ladigan narsalarning geometrik timsoli. Yevklid “Negizlar” deb nomlangan asarida nuqtani hech bir qismga ega bo‘lmagan shakl sifatida keltirgan. Har qanday nuqtalar to‘plami geometrik shakl sifatida qaraladi.

Tekis yotqizilgan temiryo‘l relslari *(9-rasm)*, simyog‘ochga tarang tortilgan elektr simlari, tarang tortilgan dor simi kabi jismlarning geometrik timsoli *to‘g‘ri chiziq* bo‘ladi. Yorug‘lik nuri to‘g‘ri chiziq bo‘ylab tarqaladi. Aslida to‘g‘ri chiziq cheksiz shakldir. Biz uni qog‘oz, sinf doskasida tasvirlaganda uning kichik bo‘laginigina chizamiz xolos. Lekin to‘g‘ri chiziqni doim har ikki tomonga cheksiz davom etgan deb tasavvur qilish kerak.

Pol, stolning ustki qismi, devor, daftar varag‘i sirti, sokin ko‘ldagi suv sathi *(10-rasm)* kabilarning geometrik timsoli *tekislik* bo‘ladi.

Nuqtalar katta lotin harflari $A, B, C, D, \ldots$, to‘g‘ri chiziqlar kichik lotin harflari $a, b, c, d, \ldots$ bilan belgilanadi va *“A nuqta”, “a to‘g‘ri chiziq”* tarzida o‘qiladi *(11-rasm)*.

Tekislikda qanday to‘g‘ri chiziq olinmasin, unda bu to‘g‘ri chiziqqa tegishli bo‘lgan nuqtalar ham, tegishli bo‘lmagan nuqtalar ham mavjud.

Masalan, 11-rasmda A nuqta a to‘g‘ri chiziqqa tegishli, B va C nuqtalar a to‘g‘ri chiziqqa tegishli emas. Buni qisqacha $A \in A$ va $B \notin a$, $C \notin a$ tarzda belgilaymiz va “A *tegishli* a” va “B *tegishli emas* a” deb o‘qiymiz.

Agar O nuqta b to‘g‘ri chiziqqa ham, undan farqli bo‘lgan c to‘g‘ri chiziqqa ham tegishli bo‘lsa, b va c to‘g‘ri chiziqlar O nuqtada *kesishadi* deymiz *(12-rasm)*. O nuqta b va c *to‘g‘ri chiziqlarning kesishish nuqtasi* deb ataladi.

13-rasmda tasvirlangan to‘g‘ri chiziq A va B nuqtalardan o‘tadi.

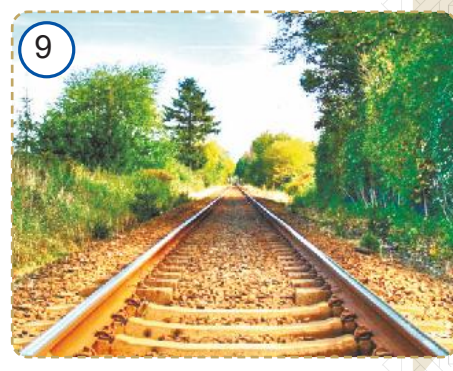
Har qanday ikki nuqtadan faqat bitta to‘g‘ri chiziq o‘tadi.

Bu xossaga ko‘ra, to‘g‘ri chiziqning ikkita nuqtasi ko‘rsatilsa, bu to‘g‘ri chiziq aniqlangan bo‘ladi. Shuning uchun to‘g‘ri chiziqni unda yotgan ikki nuqta yordamida ham belgilash mumkin. 13-rasmda AB *to‘g‘ri chiziq* tasvirlangan. Xuddi shu to‘g‘ri chiziq ba’zida BA to‘g‘ri chiziq tarzida ham belgilanadi.

8



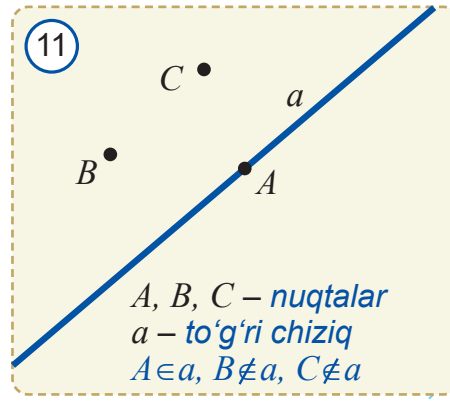
9



10

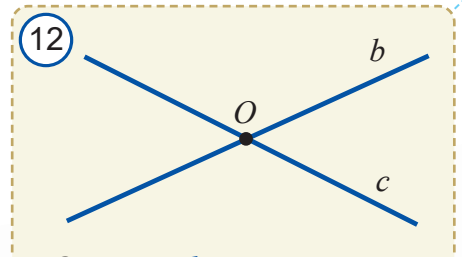


11



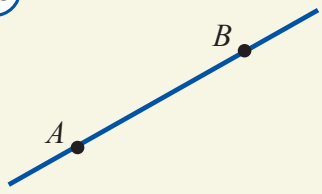
A, B, C – *nuqtalar*
a – *to‘g‘ri chiziq*
$A \in a$, $B \notin a$, $C \notin a$

12



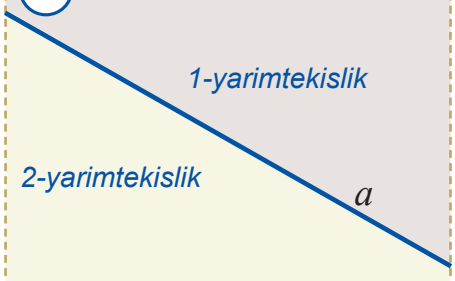
O *nuqta* – b *va* c *to‘g‘ri chiziqlarning kesishish nuqtasi*

13



AB – to‘g‘ri chiziq

14



Har bir to‘g‘ri chiziq tekislikni ikki bo‘lakka – ikkita yarimtekislikka ajratadi.

Qaralayotgan to‘g‘ri chiziq yarimtekisliklarning har ikkalasiga ham tegishli deb qaraladi va ularning umumiy chegarasi bo‘ladi *(14-rasm)*.

Eslatma. *Kelgusida ikki to‘g‘ri chiziq (ikki nuqta, ikki yarimtekislik, ...) deyilganda har xil ikkita to‘g‘ri chiziq (ikkita nuqta, ikkita yarimtekislik, ...) tushuniladi.*

Masala. Ikkita a va b to‘g‘ri chiziq A nuqtada kesishadi. a to‘g‘ri chiziq B nuqtadan o‘tadi. b to‘g‘ri chiziq ham B nuqtadan o‘tadimi?

Yechilishi. Yo‘q, b to‘g‘ri chiziq B nuqtadan o‘ta olmaydi. Chunki agar b to‘g‘ri chiziq ham B nuqtadan o‘tadigan bo‘lsa, bu holda a va b to‘g‘ri chiziqlarning ikkalasi ham A va B nuqtalardan o‘tib qoladi. Bu esa ikki nuqtadan faqat bitta to‘g‘ri chiziq o‘tkazish mumkin degan xossaga zid. Shu bois b to‘g‘ri chiziq B nuqtadan o‘tishi mumkin emas.

Shunday qilib, to‘g‘ri chiziqlarning quyidagi yana bir muhim xossasini bilib oldik.

Xossa. ***Agar ikki to‘g‘ri chiziq kesishsa, ular faqat bitta nuqtada kesishadi.***

Mavzu bo‘yicha savollar

1. *Geometriyaga oid dastlabki ma’lumotlar qayerda va qanday paydo bo‘lgan?*

2. *“Geometriya” so‘zining ma’nosi nima?*

3. *Geometriya faniga asos solgan va uning rivojiga hissa qo‘shgan qaysi olimlarni bilasiz?*

15



4. *Geometriya fani nimani o‘rganadi?*

5. *Planimetriya geometriyaning qanday bo‘limi? Stereometriya-chi?*

6. *Tekislikning qanday xossalarini bilasiz?*

7. *15–16-rasmlarda keltirilgan zamonaviy binolar rasmida qanday geometrik shakllarni ko‘ryapsiz?*

8. *17-rasmda futbol maydoni tasvirlangan. Unda qanday geometrik shakllarni ko‘ryapsiz?*

9. *18-rasmda davlatimiz gerbidagi timsol tasvirlangan. U qanday geometrik shakllardan tuzilgan?*

Amaliy mashq va tatbiq

1. Quyidagi ifodalarni o‘qing va izohlang: a) $A \in b$; b) $C \notin b$; c) $C \in AB$. Bu ifodalarga mos chizmalarni chizing.

2. Ikkita kesishuvchi to‘g‘ri chiziq chizing: a) ularning har ikkisiga tegishli; b) faqat bittasiga tegishli; c) birortasiga ham tegishli bo‘lmagan nuqtalarni belgilang. Kiritilgan belgilar yordamida bu munosabatlarni yozing.

3. 19-rasmdan imkoni boricha ko‘proq nuqta, to‘g‘ri chiziq, tekislik va yarimtekisliklar orasidagi munosabatlarni aniqlang va ularni kiritilgan belgilar yordamida yozing.

4. 20-rasmdan:

a) a to‘g‘ri chiziqqa tegishli;

b) b to‘g‘ri chiziqqa tegishli;

c) har ikkala to‘g‘ri chiziqqa tegishli;

d) a to‘g‘ri chiziqqa tegishli, lekin b to‘g‘ri chiziqqa tegishli bo‘lmagan;

e) b to‘g‘ri chiziqqa tegishli, lekin a to‘g‘ri chiziqqa tegishli bo‘lmagan;

f) a to‘g‘ri chiziqqa ham, b to‘g‘ri chiziqqa ham tegishli bo‘lmagan nuqtalarni aniqlang.

5. A va B nuqtalar c to‘g‘ri chiziqqa tegishli, C nuqta esa c to‘g‘ri chiziqqa tegishli emas. AB va AC to‘g‘ri chiziqlar haqida nima deyish mumkin?

6. AB va AK to‘g‘ri chiziqlar nechta umumiy nuqtaga ega bo‘lishi mumkin?

7. Tekislikda b to‘g‘ri chiziq chizing va unda A nuqtani belgilang. b to‘g‘ri chiziqdan farqli AB to‘g‘ri chiziqni o‘tkazing. B nuqta b to‘g‘ri chiziqda yotadimi?

8. Tekislikda yotgan ixtiyoriy: a) bitta; b) ikkita; c) uchta nuqtadan o‘tuvchi nechta to‘g‘ri chiziq o‘tkazish mumkin? Javobingizni asoslang.

9. Tekislikni: a) bitta to‘g‘ri chiziq nechta yarimtekislikka; b) ikkita to‘g‘ri chiziq nechta bo‘lakka ajratadi?

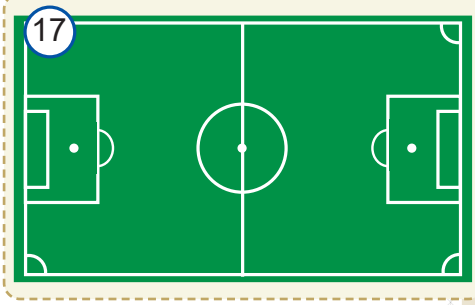
10. AB, BC, AC to‘g‘ri chiziqlarni chizing. Ular tekislikni necha bo‘lakka ajratadi?

11. Daftaringizga A, B, C va D nuqtalarni belgilang

16



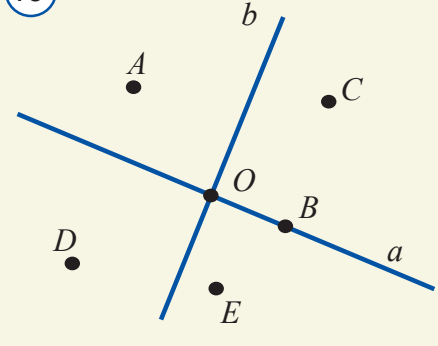
17



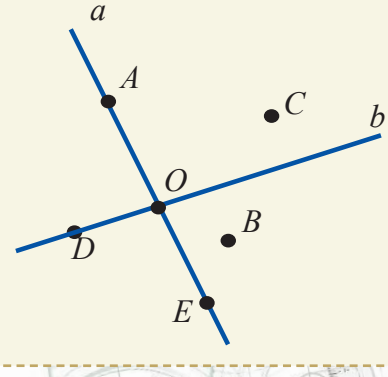
18



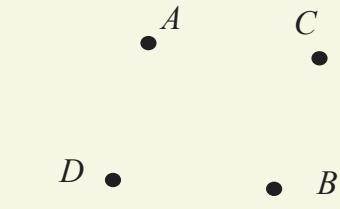
19



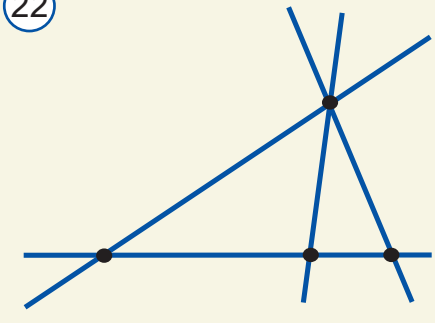
20



21



22



(21-rasm). Ularning har ikkisidan o‘tuvchi barcha to‘g‘ri chiziqlarni chizing. Jami nechta to‘g‘ri chiziq hosil bo‘ldi? Bu to‘g‘ri chiziqlar tekislikni necha bo‘lakka ajratadi?

12*. Ixtiyoriy uchtasi bir to‘g‘ri chiziqda yotmaydigan: a) uchta; b) to‘rtta nuqta orqali shu nuqtalarni juft-jufti bilan tutashtiruvchi nechta to‘g‘ri chiziq o‘tkazish mumkin?

13*. To‘rtta to‘g‘ri chiziqning har ikkitasi kesishgan nuqtalar belgilandi. Nuqtalar soni ko‘pi bilan nechta bo‘ladi? To‘g‘ri chiziqlar beshta bo‘lsa-chi?

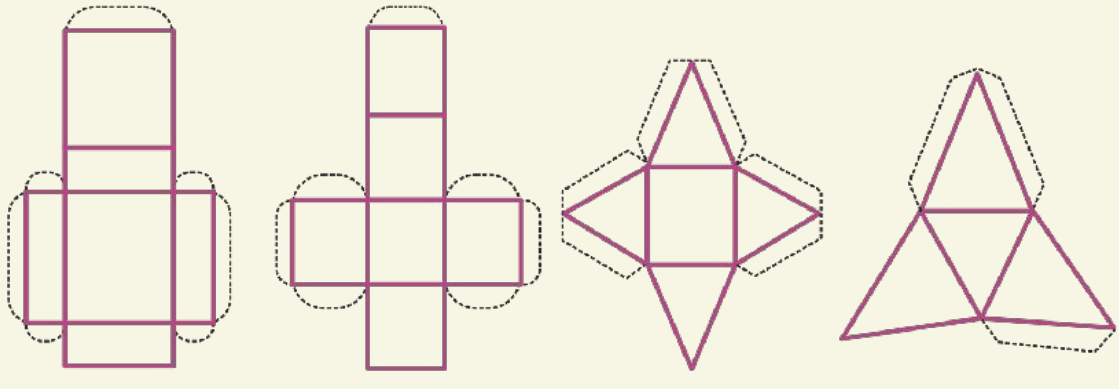
14*. 22-rasmda to‘rtta to‘g‘ri chiziq to‘rtta nuqtada kesishyapti. Shunday chizma chizingki, ular: a) 5 ta; b) 6 ta nuqtada kesishsin. To‘rtta to‘g‘ri chiziq uchta nuqtadan o‘tishi mumkinmi?

Amaliy mashq va tatbiq

Tekislikda beshta nuqtani shunday joylashtiringki, ularning har ikkitasi orqali to‘g‘ri chiziq o‘tkazganda to‘g‘ri chiziqlar beshta bo‘lsin.

Fazoviy jismlarni yaxshiroq tasavvur qilish uchun ularning modelidan foydalangan ma’qul. Fazoviy jismlarning modelini ularning yoyilmasidan foydalanib yasash mumkin *(23-rasm)*. Ko‘rib turganingizdek, fazoviy jismlarning yoyilmasi yassi geometrik shakllardan iborat. Quyidagi yoyilmalardan foydalanib to‘g‘ri burchakli parallelepiped, kub va piramidalar modelini yasang.

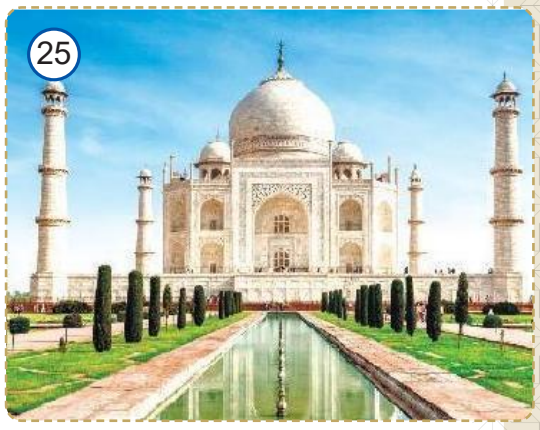
23

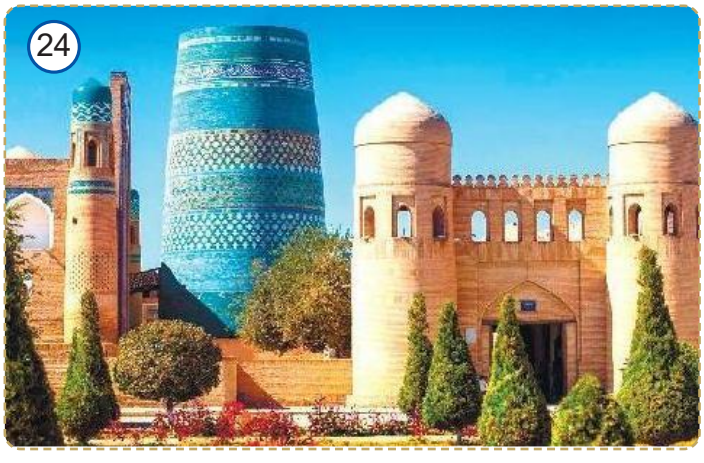


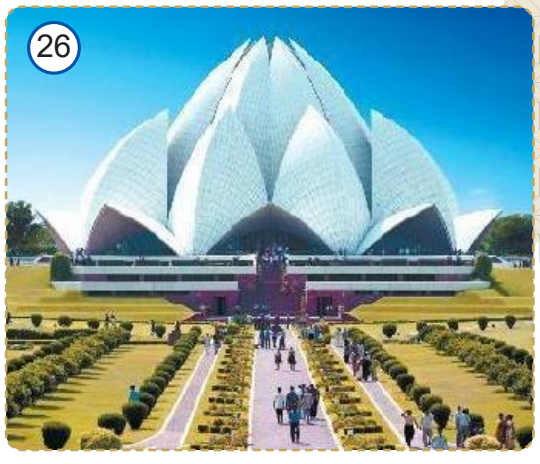
Geometrik joziba

O‘tmishda qadimiy arxitektura yodgorliklarini qurgan ota-bobolarimiz katta geometrik bilim va salohiyatga ega bo‘lgan. Buni birgina Samarqand shahridagi Registon maydonida qurilgan tarixiy yodgorliklardan ham bilib olish mumkin (23-rasm). Bu mo‘jizalarni qurishda ular qanday geometrik bilim va ko‘nikmalardan foydalangan deb o‘ylaysiz?











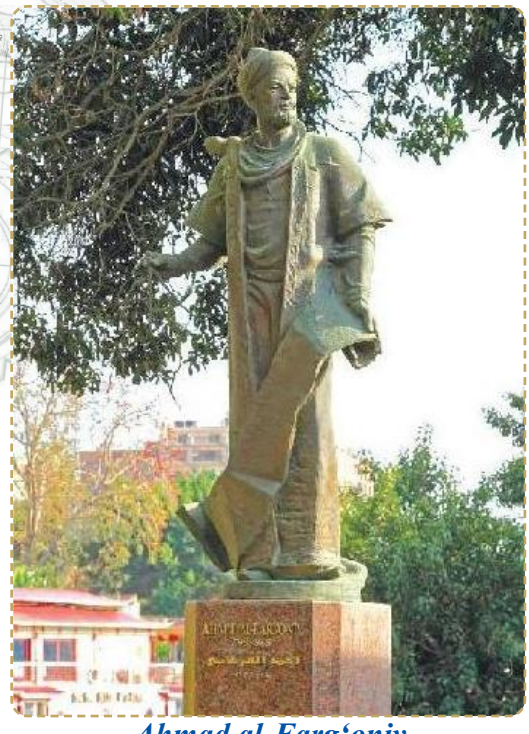


Xiva shahridagi Ichanqal'a rasmida (24-rasm) qanday geometrik shakllarni ko'ryapsiz?

Tojmahal – Hindistonning Agra shahrida boburiy hukmdor Shoh Jahon qurdirgan qadimiy yodgorlik (25-rasm). Uni qurgan ustalar geometriyadan mukammal bilimga ega bo'lganliklari kundek ravshan.

Sidney shahri opera teatri (26-rasm) – Avstraliyada qurilgan zamonaviy me'morchilik namunasi. O'zining g'aroyib geometrik ko'rinishi bilan diqqatga sazovordir.

Go'zal geometrik tasavvurlar egasi, iroqlik mashhur arxitektor ayol Zaha Hadidning loyihasi asosida Xitoy potaxti Pekin shahrida qad rostlagan "Galaxy Soho" dam olish kompleksining ajabtovur ko'rinishidan zavq olmaslikning iloji yo'q (27-rasm).



Ahmad al-Farg‘oniy

Taxminan 797–875-yillarda yashab ijod qilgan buyuk astronom, geograf va matematik olim.

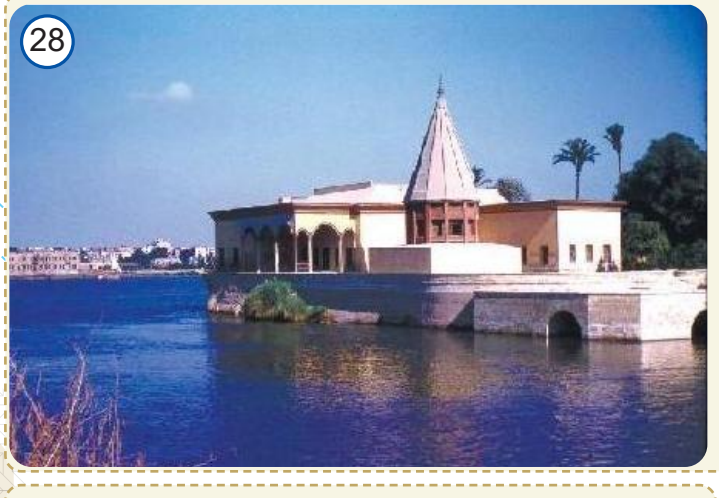
Tarixiy lavhalar

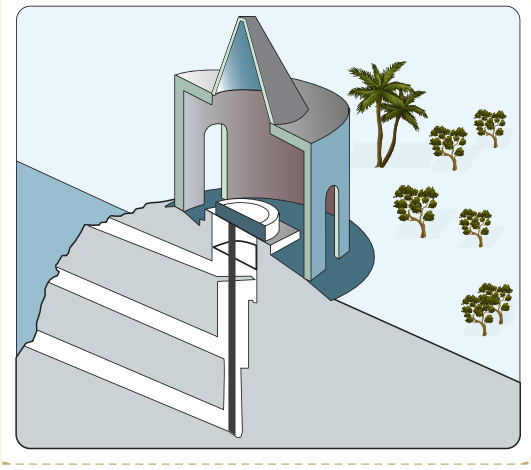
Nilni jilovlagan farg‘onalik alloma

Tarixiy ma’lumotlarga ko‘ra, yurtimizdan yetishib chiqqan Ahmad al-Farg‘oniy 861-yili Qohira shahri yaqinida Nil daryosidagi suv sathini o‘lchaydigan “Nilometr” (ya’ni “Nil o‘lchagich”) deb nomlangan inshootni barpo etgan (28-rasm). Ilmiy-texnik va me’moriy jihatdan g‘oyat mukammal hisoblangan hamda o‘zida noyob geometrik yechimlarni mujassam etgan bu qurilmada olib borilgan o‘lchash ishlari uzoq vaqtlar davomida dehqonchilik uchun juda zarur bo‘lgan va u hozirgacha saqlanib qolgan.

Ahmad al-Farg‘oniy o‘zining “Usturlab yasash haqida risola” asarida astronomiya uchun muhim xossa – Ptolemey teoremasining nafis isbotini bergan. O‘rta asr Yevropa ilmiy adabiyotida uni Alfraganus deb ataganlar. Ahmad al-Farg‘oniy sharafiga Oyda topilgan krater nomlangan va Qohira shahrida haykal o‘rnatilgan.

28





29



Eng qadimgi ilmiy risola

29-rasmda miloddan avvalgi III asrda yashab o‘tgan yunon xattoti Axmes tomonidan papirus bargiga yozilgan va bizgacha yetib kelgan ilk geometrik risola keltirilgan.

2 KESMA. KESMALARNI TAQQOSLASH VA O‘LCHASH

2.1. Kesma va nur

Agar a to‘g‘ri chiziqda uchta – A, B, C nuqtalar olinsa (*1-rasm*), ularning faqat bittasi – B nuqta qolgan ikkitasi, ya’ni A va C nuqtalarning orasida yotadi. A va B nuqtalar C nuqtadan bir tomonda, B va C nuqtalar esa A nuqtadan bir tomonda yotadi.

Bir to‘g‘ri chiziqda olingan istalgan uchta nuqtaning faqat bittasi qolgan ikkitasining orasida yotadi.

Kesma deb to‘g‘ri chiziqning ikki nuqtasi va ular orasida yotgan nuqtalardan iborat qismiga aytiladi. 2-rasmda kesma tasvirlangan. A va B nuqtalar *kesmaning uchlari* yoki *chetki nuqtalari* deyiladi. Ular orasidagi nuqtalar esa kesmaning *ichki nuqtalari* deb yuritiladi.

Kesma o‘zining chetki nuqtalari yordamida AB *kesma* tarzida belgilanadi. Xuddi shu kesmani BA *kesma* tarzida yozish ham mumkin.

Nur deb to‘g‘ri chiziqning biror O nuqtasi va undan bir tomonda yotgan barcha nuqtalaridan iborat qismiga aytiladi. O nuqta esa bu *nurning uchi* yoki *boshlang‘ich nuqtasi* deb ataladi. 3-rasmda nur tasvirlangan.

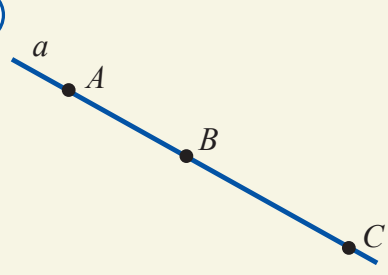
Nur O uchi va biror-bir A nuqtasi orqali “OA *nur*” tarzida belgilanadi *(3-rasm)*. Bunday yozuvda nurning uchi birinchi o‘rinda yoziladi.

Ayrim hollarda OA nur O *nuqtadan chiquvchi nur* deb ham aytiladi.

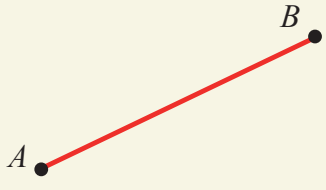
a to‘g‘ri chiziqda yotgan O nuqta bu to‘g‘ri chiziqni ikkita *bir-birini to‘ldiruvchi nurlarga* ajratadi. 4-rasmda bir-birini to‘ldiruvchi OA va OB nurlar tasvirlangan.

Nurga yorug‘lik nurining geometrik timsoli sifatida qarash mumkin. Kechasi osmonga qaratilgan lazer yoki kuchli projektor nuri bunga misol bo‘la oladi *(5-rasm)*. “Nur” atamasi shundan kelib chiqqan.

1

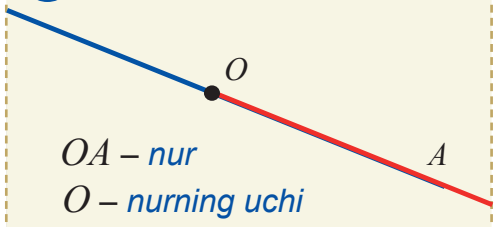


2



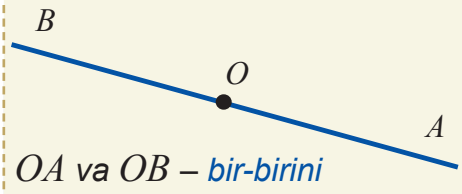
AB – *kesma*
A, B – *kesmaning uchlari*

3



OA – *nur*
O – *nurning uchi*

4

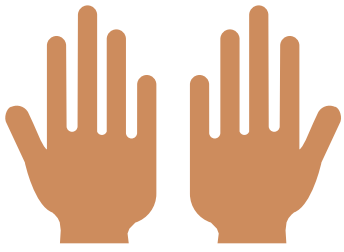


OA va OB – *bir-birini to‘ldiruvchi nurlar*

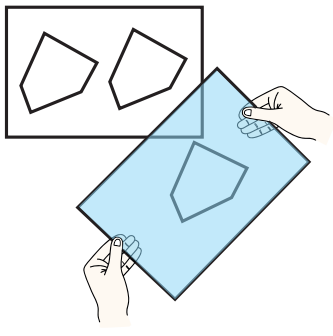
5



6

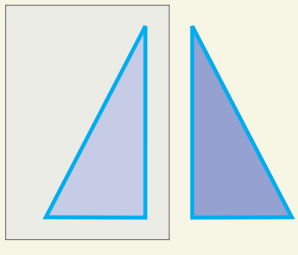


7

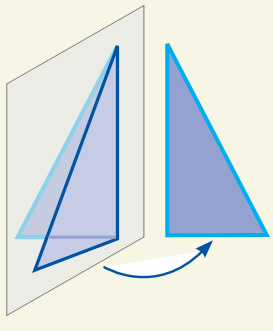


8

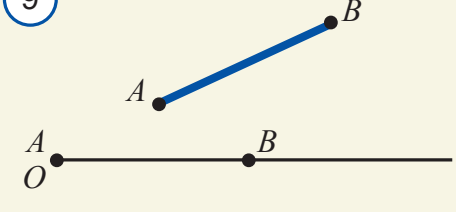
a)



b)



9



2.2. Kesmalarni taqqoslash

Faollashtiruvchi mashq

1. Tevarak-atrofingizdan shakli ham, o‘lchamlari ham bir xil bo‘lgan narsalarga misollar keltiring.

2. Ikkita daftar varag‘ining o‘lchamlari bir xil ekanini qanday amaliy usul bilan aniqlash mumkin?

3. 6-rasmda chap va o‘ng qo‘l tasviri berilgan. Bu shakllarning birini ikkinchisiga aynan ustma-ust tushadigan qilib qo‘yish mumkinmi? Qanday qilib? Buni o‘z qo‘llaringiz bilan bajarib ko‘ring.

Teng shakllar deb birini ikkinchisining ustiga aynan ustma-ust tushadigan qilib qo‘yish mumkin bo‘lgan shakllarga aytiladi.

Bir geometrik shaklni ikkinchisining ustiga qo‘yish tushunchasi bilan faollashtiruvchi mashqlarda tanishdik. Bu tushunchani amalda quyidagicha tasavvur etish mumkin. Bir shaklni ikkinchisining ustiga qo‘yish uchun oldin shaffof plyonkaga birinchi shaklning nusxasini ko‘chirib andoza olamiz. So‘ng shaffof plyonkani tekislik bo‘ylab siljitib, birinchi shakl andozasini ikkinchi shakl bilan aynan ustma-ust tushadigan qilib qo‘yishga harakat qilamiz *(7-rasm)*. Agar shakllar aynan ustma-ust tushsa, bu shakllar teng bo‘ladi.

Ba’zida bir shaklni ikkinchisiga aynan ustma-ust qo‘yish uchun oldin shakl nusxasi tasvirlangan shaffof plyonkani ag‘darib olishga to‘g‘ri keladi. 8-rasmda shunday holat tasvirlangan.

Uchi O nuqtada bo‘lgan nur va ixtiyoriy AB kesma berilgan bo‘lsin. Ravshanki, bir uchi shu nur uchi bilan ustma-ust tushadigan, ikkinchi uchi esa nurda yotadigan va AB kesmaga teng bo‘lgan kesmani nurining ustiga qo‘yish mumkin (*9-rasm*). Bunday kesma yagona bo‘ladi va bu amal *AB kesmani O nurga qo‘yish* deyiladi. Buni kelgusida qisqacha *kesmani nurga qo‘yish* deb ham yuritamiz.

Bir xil uzunlikdagi taxtalarni arralab, kesib olinayotganda ham shunday qilinadi *(10-rasm)*.

Ixtiyoriy nurga uning uchidan boshlab berilgan kesmaga teng yagona kesmani qo‘yish mumkin.

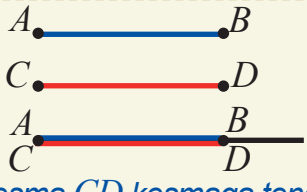
Ikkita kesmani o‘zaro taqqoslash uchun ular bitta nurga qo‘yiladi. Keyin esa quyidagi hollardan qaysi biri bo‘lishiga qarab, kesmalarning o‘zaro tengligi yoki uzun-qisqaligi (ya’ni katta-kichikligi) haqida xulosa chiqariladi *(11-rasm)*:

10



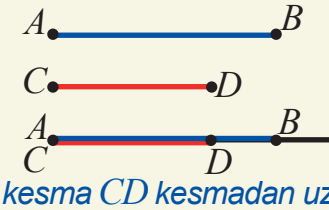
11

a)



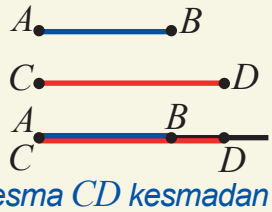
AB kesma CD kesmaga teng;

b)



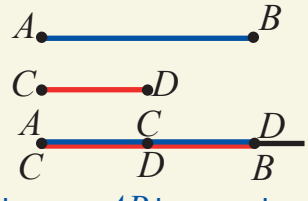
AB kesma CD kesmadan uzun;

c)



AB kesma CD kesmadan qisqa;

d)



CD kesma *AB* kesmaning yarmi;

Kesmaning o‘rtasi deb uni o‘zaro teng ikkita kesmaga ajratuvchi nuqtaga aytiladi.

12-rasmda AB kesmaning o‘rtasi bo‘lgan C nuqta tasvirlangan. Shaklda teng kesmalar bir xil sondagi chiziqchalar bilan belgilanadi.

12



2.3. Kesmaning uzunligi va uning xossalari. Kesmalarni o‘lchash

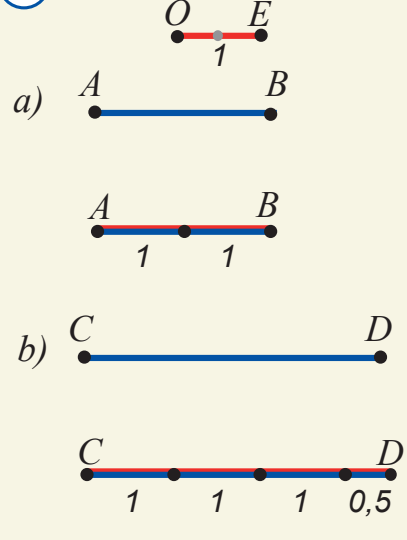
Kesmalarni nurga qo‘yish orqali taqqoslash u qadar qulay emas. Kesmalarning qaysi biri uzun yoki qisqaligini (ya’ni katta yoki kichikligini) ularning uzunliklarini taqqoslash asosida aniqlash ham mumkin.

Biror kesmani birlik kesma deb olib, uning uzunligini 1 ga teng deb qabul qilamiz. Qolgan kesmalar uzunliklarini shu birlik kesma uzunligiga nisbatan aniqlaymiz.

Kesmaning uzunligi musbat son bo‘lib, u kesmaga birlik kesma va uning bo‘laklarini necha marta joylash mumkinligini ko‘rsatadi.

Aniqki, 12-rasmdagi OE kesmani birlik kesma deb olsak, ya’ni uzunligini 1 ga teng desak, u holda AB

13



kesma uzunligi 2 ga teng bo‘ladi. Chunki AB kesmaga OE kesma ikki marta joylashyapti. CD kesma uzunligi 3,5 ga teng bo‘ladi. Chunki bu kesmaga OE kesma butunligicha uch marta va uning yarmi joylashyapti.

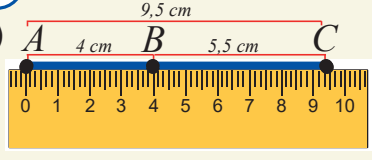
Har qanday kesma noldan farqli tayin uzunlikka ega bo‘lib, u musbat son bilan ifodalanadi.

Faollashtiruvchi mashq

14*a*-rasmda berilgan kesmalarning uzunligi chizg‘ich yordamida o‘lchangan. Bu kattaliklar o‘zaro pastdagi formula yordamida bog‘langanini tekshiring.

14

a)



$AC = AB + BC;$

b)



$AC = AB + BC;$

c)



$AC < AB + BC.$

Agar to‘g‘ri chiziqda B nuqta A va C nuqtalar orasida joylashgan bo‘lsa, AC kesma uzunligi AB va BC kesmalar uzunliklarining yig‘indisiga teng bo‘ladi (14b-rasm):

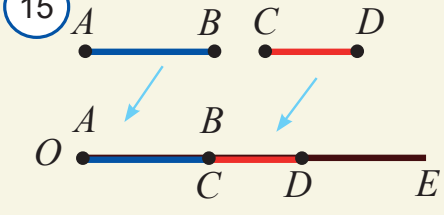
$$AC = AB + BC.$$

Agar B nuqta AC kesmada yotmasa, AC kesma uzunligi AB va BC kesmalar uzunliklari yig‘indisidan kichik bo‘ladi *(14c-rasm)*: $AC < AB + BC$.

Bu tengsizlikka keyingi boblarda yana to‘xtalamiz.

Yuqorida keltirilgan xossa kesmalar ustida qo‘shish va ayirish amallarini aniqlash imkonini beradi. OE nur, AB va CD kesmalar berilgan bo‘lsin. Oldin OE nurga AB kesmani qo‘yamiz. So‘ng BE nurga CD kesmani qo‘yamiz *(15-rasm)*.

15



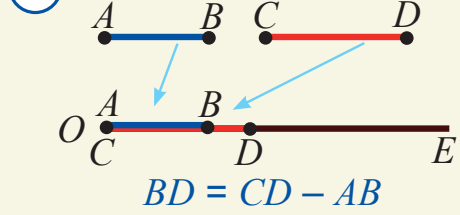
$AD = AB + CD$

Natijada hosil bo‘lgan AD kesma AB va CD *kesmalarning yig‘indisi* deb ataladi va $AD = AB + CD$ tenglik o‘rinli bo‘ladi.

Xuddi shunga o‘xshab, kesmalarni bir-biridan ayirish amalini ham kiritish mumkin.

Aytaylik, OE nur, AB va CD kesmalar berilgan hamda $CD > AB$ bo‘lsin *(16-rasm)*. OE nurga oldin uzun CD kesmani, OE nurga AB kesmani qo‘yamiz. Hosil bo‘lgan DB kesma CD va AB *kesmalar ayirmasi* deb ataladi va $BD = CD - AB$ tenglik o‘rinli bo‘ladi.

16



$BD = CD - AB$

AB kesmaning uzunligi A va B nuqtalar orasidagi *masofa* deb ham yuritiladi.

Bir xil uzunlikka ega kesmalar o‘zaro teng bo‘lishi aniq ko‘rinib turibdi.

2.4. Aylana va doira

Bir yoki bir nechta berilgan xossalarni qanoatlantiruvchi barcha nuqtalardan iborat shakl *nuqtalarning geometrik o‘rni* deb ataladi.

Nuqtalarning geometrik o‘rniga aylana va doira misol bo‘la oladi.

Berilgan nuqtadan teng uzoqlikda yotgan nuqtalarning geometrik o‘rni *aylana* deb ataladi. Bu nuqta aylananing *markazi* deyiladi.

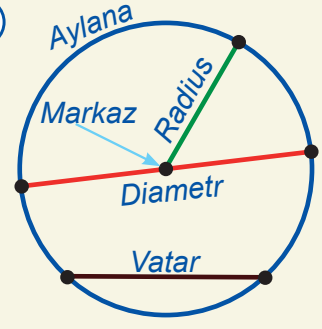
17-rasmda aylana shaklidagi “Charxpalak” ko‘ngilochar qurilmasi keltirilgan.

17



Aylananing ixtiyoriy nuqtasidan uning markazigacha bo‘lgan masofa esa aylananing *radiusi* deb ataladi *(18-rasm)*. Shuningdek, aylananing markazini uning ixtiyoriy nuqtasi bilan tutashtiruvchi kesmani ham radius deb yuritamiz.

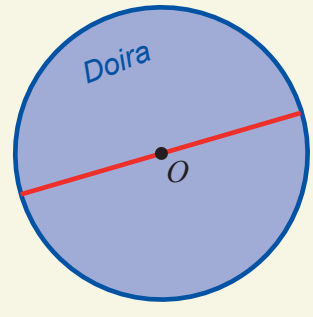
18



Aylananing ixtiyoriy ikki nuqtasini tutashtiruvchi kesma aylana *vatari* deb ataladi. Markazdan o‘tuvchi vatar esa *diametr* deb ataladi.

Doira deb tekislikda berilgan nuqtadan berilgan sondan katta bo‘lmagan masofada yotgan nuqtalarning geometrik o‘rniga aytiladi *(19-rasm)*. Berilgan nuqta doiraning *markazi*, berilgan son esa uning *radiusi* deb ataladi.

19



Aylana (doira) diametri markazdan o‘tgani uchun u ikkita radiusdan iborat bo‘ladi *(19-rasm)*. Demak, diametr uzunligi ikkita radius uzunligiga teng.

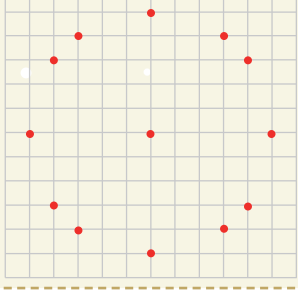
Aylanani katakli daftarda sirkulsiz chizish yo‘l-yo‘rig‘i

- *Katak daftarga 20-rasmda ko‘rsatilgandek qilib nuqtalarni belgilang.*
- *Hosil bo‘lgan 12 ta nuqtani ketma-ket yoysimon chiziq bilan tutashtirib chiqing.*

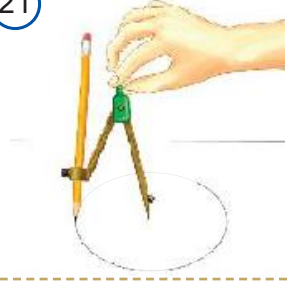
Natijada markazi O nuqtada bo‘lgan aylananing taxminiy tasviri hosil bo‘ladi (tekshirib ko‘ring).

Aylana sirkul yordamida chiziladi. Markazi O nuqtada berilgan, radiusi AB kesmadan iborat aylanani sirkul va arqon yordamida chizish 21- va 22-rasmlarda ko‘rsatilgan.

20



21



22





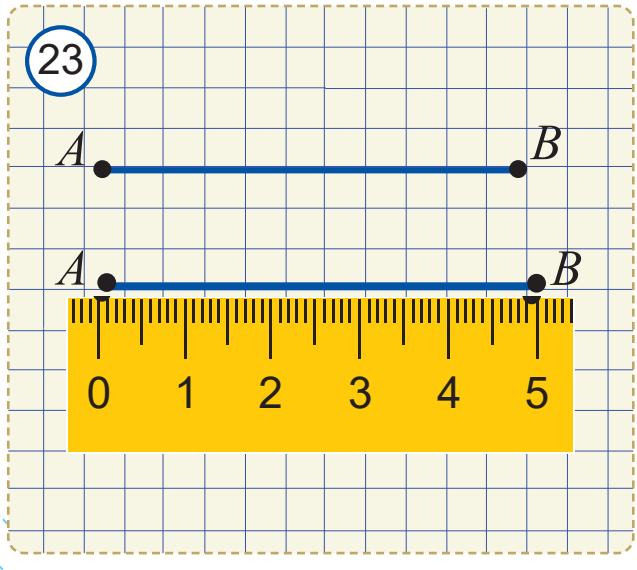
2.5. Uzunlik o‘lchov birliklari

Qadimdan odamlar uzunlikni o‘lchashda turli xil uzunlik birliklaridan foydalanib keladi. Masalan, O‘rta Osiyoda bo‘g‘in, qarich, quloch, chaqirim kabi uzunlik birliklari qo‘llangan. Turli o‘lchov birliklaridan foydalanish noqulayliklar tug‘dirgan. Shu bois XVIII asrdan boshlab dunyo bo‘yicha xalqaro uzunlik o‘lchov birligi sifatida *metr* qabul qilingan.

Siz uzunlik namunasi bo‘lgan metr etaloni bilan 7-sinf “Fizika” darsligida tanishgansiz. U yerda metrga nisbatan ancha katta yoki kichik uzunliklarni o‘lchash uchun foydalaniladigan birliklar ham keltirilgan edi. Jumladan: 1 km = 1 000 m; 1 cm = 0,01 m; 1 mm = 0,001 m.

Kesmalarning uzunligi turli xil asboblar yordamida o‘lchanadi. Ularning eng soddasi shkalali, ya’ni bo‘linish nuqtalariga ega bo‘lgan chizg‘ichdir. Kesma uzunligining qiymati tanlangan uzunlik o‘lchov birligiga bog‘liq bo‘ladi. Agar uzunlik birligi sifatida uzunligi 1 cm ga teng kesmani oladigan bo‘lsak, 23-rasmda tasvirlangan kesmaning uzunligi 5 cm ga teng bo‘ladi va $AB = 5\ cm$ deb yoziladi. Agar uzunlik o‘lchov birligi sifatida uzunligi 1 millimetrga teng kesmani oladigan bo‘lsak, AB = 50 mm bo‘ladi.

23



Ayrim hollarda kesma uzunligi o‘lchov birligi ko‘rsatilmasdan yoziladi. Masalan, AB = 5. Bunda AB kesma uzunligi 5 o‘lchov birligiga teng deb tushuniladi.

Daftarda turli kesma uzunliklarini o‘lchash uchun millimetrli bo‘lmalarga ega bo‘lgan o‘quv chizg‘ichidan *(24-rasm)* foydalanib keldingiz. Doskada kesmalarni chizish uchun santimetrli bo‘lmalarga ega maktab chizg‘ichidan foydalaniladi. Yer ustida turli o‘lchash ishlarini amalga oshirish uchun tasmali o‘lchov asbobi – ruletkadan, dalada esa hakka – dala sirkulidan foydalaniladi.

Hozirda muhandis-ustalar uzunlikni juda

24



qulay va aniq o‘lchaydigan elektron ruletkalardan foydalanmoqda (*25a-rasm*).

25*b*-rasmda devorgacha, 25*c*-rasmda esa simyog‘ochning asosi va uchigacha bo‘lgan masofani o‘lchash jarayoni tasvirlangan. Bu rasmlardan foydalanib lazerli elektron ruletkaning qanday ishlashini sharhlashga urinib ko‘ring.

Masala. Bir to‘g‘ri chiziqda yotuvchi A, B va C nuqtalar uchun $AB = 8$ va $BC = 11$ bo‘lsa, AC kesmaning uzunligini toping.

Yechish. Quyidagi hollarni ko‘rib chiqamiz:

1. A, B, C nuqtalar a to‘g‘ri chiziqda 26*a*-rasmda tasvirlangan tartibda joylashgan bo‘lsin.

Kesmalar uzunliklarining xossasiga ko‘ra, $AC = AB + BC = 8 + 11 = 19$ bo‘ladi.

2. A, B va C nuqtalar a to‘g‘ri chiziqda 26*b*-rasmda tasvirlangan tartibda joylashgan bo‘lsin.

U holda kesmalar uzunliklarining xossasiga ko‘ra, $BA + AC = BC$ yoki

$AC = BC - BA = 11 - 8 = 3$ bo‘ladi.

3. C nuqta 26*c*-rasmdagidek B va A nuqtalar orasida joylasha olmaydi. Chunki $AB < BC$. Demak, AC kesmaning uzunligi nuqtalarning o‘zaro joylashuviga qarab 19 yoki 3 ga teng bo‘ladi.

Javob: 19 yoki 3.

25

a)



b)

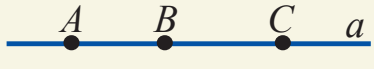


c)



26

a)



b)



c)

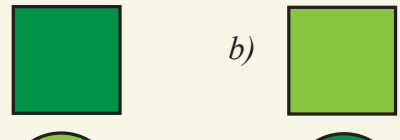


27

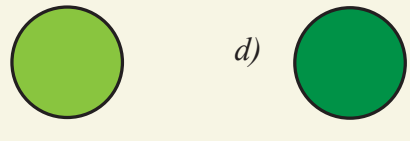


28

a) *b)*



c) *d)*



Mavzu bo‘yicha savollar

1. 27-rasmda B nuqta qaysi nuqtalar orasida yotibdi? Qaysi nuqtalar C nuqtadan bir tomonda yotibdi?
2. Kesma va nurga ta’rif bering. Ular qanday belgilanadi?
3. To‘g‘ri chiziqda C va D nuqtalar berilgan. CD va DC kesmalar ustma-ust tushadimi? CD va DC nurlar-chi?
4. Kesma, nur va to‘g‘ri chiziq bir-biridan nimasi bilan farq qiladi?
5. Qanday shakllarni o‘zaro teng deymiz?
6. 28-rasmda qaysi shakllar o‘zaro teng?
7. Kesmani nurga qo‘yish deganda nimani tushunasiz?
8. Ixtiyoriy nurga uning uchidan boshlab berilgan kesmaga teng nechta kesmani qo‘yish mumkin?
9. Kesmalar qanday taqqoslanadi va o‘lchanadi?
10. Kesmaning o‘rtasi nima?
11. Kesma uzunligining asosiy xossalarini ayting.
12. Kesmalarning yig‘indisi va ayirmasini tavsiflang.
13. Nuqtalarning geometrik o‘rni nima?
14. Aylana va doiraga ta’rif bering va ularning elementlarini ayting.

Amaliy mashq va tatbiq

1. 27-rasmda nechta kesma bor?

2. Uchlari to‘g‘ri chiziqdagi A, B, C, D nuqtalarda bo‘lgan jami nechta kesma bor? Ularni yozing.

3. 29-rasmdagi barcha nurlarni yozing. Ularning qaysilari bir-birini to‘ldiruvchi?

4. 30-rasmda AB to‘g‘ri chiziq MN va PQ to‘g‘ri chiziqlarni mos ravishda C va D nuqtalarda kesib o‘tadi. a) C nuqtadan chiquvchi; b) D nuqtadan chiquvchi va bir-birini to‘ldiruvchi nurlarni yozing.

5. Bir to‘g‘ri chiziqda yotgan 2 ta nuqta shu to‘g‘ri chiziqda yotgan nechta nurni aniqlaydi? 3 ta nuqta-chi?

6. 31a-rasmdagi shaklni qog‘ozga o‘lchamlarini o‘zgartirmagan holda chizing va qirqib oling. Uni 31b-rasmdagi shaklning ustiga qo‘yish orqali ularni taqqoslang.

7. Quyidagi a) harf; b) raqam belgilarining qaysilari geometrik shakl sifatida o‘zaro teng?

a) **a, b, g, d, i, y, n, o, p, u, q;**

b) 1234567890

8. 32-rasmdagi teng shakllarni toping.

9*. To‘g‘ri chiziqda yotgan: a) 1; b) 2; c) 3; d) 10; e) n ta nuqta uni nechta bo‘lakka bo‘ladi?

10*. Tekislikda yotgan; a) bitta; b) ikkita; c) uchta;

d) to‘rtta to‘g‘ri chiziq uni ko‘pi bilan necha qismga ajratadi?

11*. To‘g‘ri chiziq va unda yotmaydigan A, B, C nuqtalar berilgan. AB kesma berilgan to‘g‘ri chiziqni kesib o‘tadi, AC kesma esa kesib o‘tmaydi. BC kesma bu to‘g‘ri chiziqni kesib o‘tadimi?

12. 33-rasmdagi AB kesmani birlik kesma deb olib, qolgan kesmalar uzunliklarini toping.

13. 34-rasmdan AB, AC, AD, BC, BD, CD kesmalarning uzunligini aniqlang.

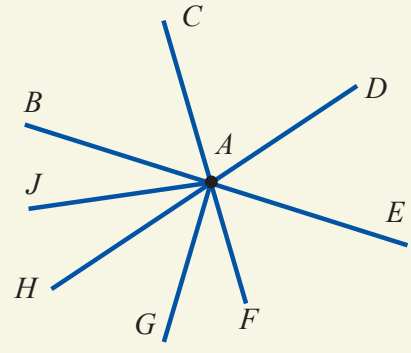
14. 35-rasmdagi ma’lumotlardan foydalanib noma’lumlarni toping.

15. Agar $B \in AC$, AB=7,2 cm, AC=2 dm bo‘lsa, BC ni toping.

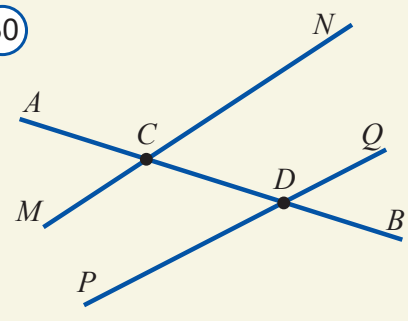
16. Agar $C \in AB$, $D \in AB$, AB= 5, AC= 2,2 va BD = 3,6 bo‘lsa, CD ni toping.

17. To‘g‘ri chiziqda ko‘z bilan chamalab: a) 3 cm; b) 7 cm; c) 10 cm bo‘lgan kesma ajrating.

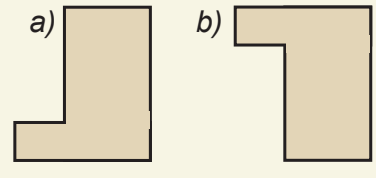
29



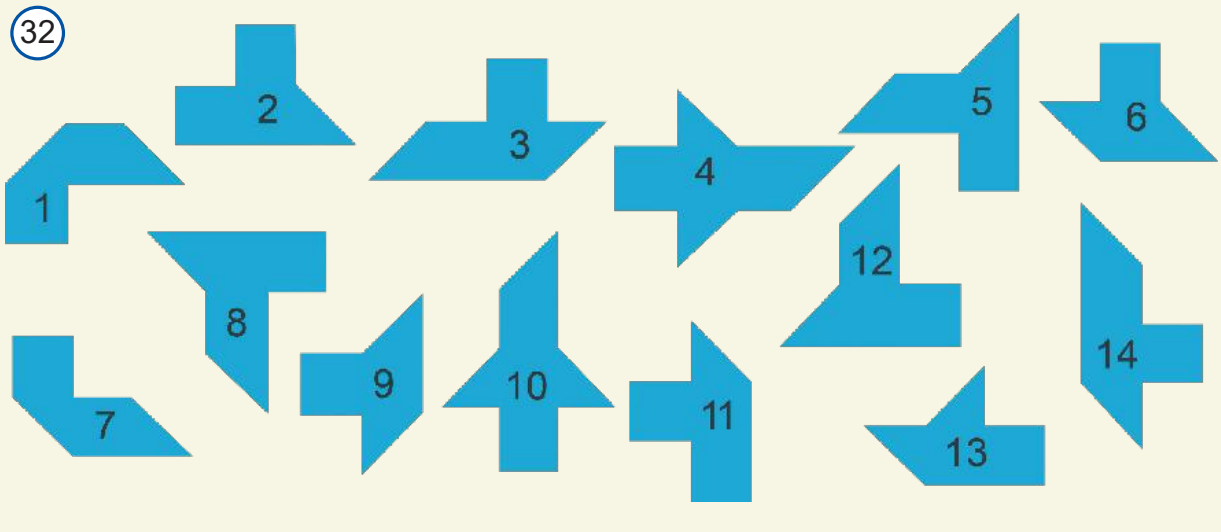
30



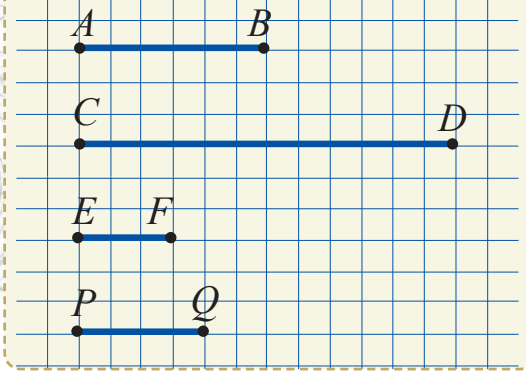
31



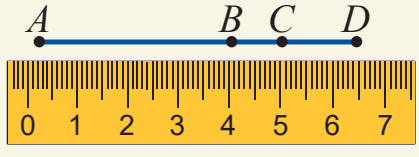
32



33

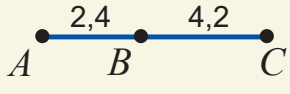


34



35

a) $AC=?$



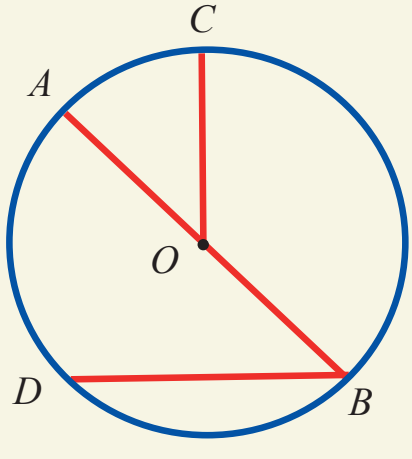
b) $AB=3$, $AC=2BC$, $BC=?$



c) $AB=24$, $BC=AC+6$, $AC=?$



36



So‘ng ishni qanchalik aniq bajarganingizni chizg‘ich bilan tekshiring.

18. To‘g‘ri chiziqdagi A, B, C nuqtalar uchun $AB=600\ m$, $BC=200\ m$ bo‘lsa, AC ni toping.

19. 36-rasmda tasvirlangan aylananing markazi, barcha radiuslari, diametri va vatarlarini aniqlang va yozing.

20. Aylananing radiusi 23 cm bo‘lsa, uning eng uzun vatarini toping.

21. Aylananing: a) radiusi 12 cm bo‘lsa, diametrini; b) diamerti 28 dm bo‘lsa, radiusini toping.

22. To‘g‘ri chiziqdagi A, B, C va D nuqtalar uchun $AB=2$, $AC=CB$, $2AD=3BD$ bo‘lsa, CD ni toping.

23. Markazi O nuqtada bo‘lgan aylana a) OA to‘g‘ri chiziq; b) OA nur bilan nechta nuqtada kesishishi mumkin?

24*. Nur va uzunliklari $AB=1{,}2\ cm$, $CD=2{,}8\ cm$ bo‘lgan kesmalar berilgan. Bu kesmalardan foydalanib shu nurga uzunligi: a) 4 cm; b) 1,6 cm; c) 0,4 cm; d) 2,6 cm bo‘lgan kesmalarni qo‘ying.

25. C nuqta AB kesmaning o‘rtasi va $CB=13$ bo‘lsa, AB kesma uzunligini toping.

26. O‘zining yarmidan 46 cm uzun bo‘lgan kesmaning uzunligini toping.

27. Darsligingiz bo‘yi va enini chizg‘ich yordamida o‘lchang va ularni: a) mm; b) dm; c) m; d) km larda ifodalang.

28. C nuqta AB kesmani 1 : 3 nisbatda bo‘ladi. Agar AC kesma uzunligi: a) 2,5 cm; b) 3,9 m; c) 1,4 km bo‘lsa, CB kesma uzunligini toping;

29. D nuqta EF kesmani 1 : 2 nisbatda bo‘ladi. Agar DE kesma uzunligi: a) 35 cm; b) 93 dm; c) 41 m bo‘lsa, EF kesma uzunligini toping.

30*. Uzunligi 9 cm bo‘lgan AB kesma chizing. AB nurda shunday C nuqtani belgilangki,

a) $AC - BC = 1\ cm$; b) $AC + BC = 11\ cm$; c) $AC + BC = 10\ cm$ bo‘lsin.

31*. AB kesma berilgan. Uzunligi: a) $2AB$; b) $AB:2$; c) $AB:4$; d) $0{,}75AB$ bo‘lgan kesmalarni yasang.

32*. To‘g‘ri chiziqdagi A, B, C nuqtalar uchun $AB = 5{,}6\ cm$, $AC = 8{,}9\ cm$ va $BC = 3{,}3\ cm$ ekani ma’lum. A, B, C nuqtalarning qaysi biri qolgan ikkitasining o‘rtasida yotadi?

33. C nuqta A va B nuqtalar orasida yotadi. Agar: a) $AC = 2{,}5\ cm$, $CB = 3{,}4\ cm$; b) $AC = 5{,}3\ dm$, $CB = 4{,}2\ dm$ bo‘lsa, AB kesma uzunligini toping.

34*. 37-rasmda AB, BC, CD va DE kesmalar o‘zaro teng. Agar birlik kesma sifatida: a) AB; b) AC; c) AE kesma olingan bo‘lsa, AD kesmaning uzunligini toping.

35. 38-rasmda Katta Ayiq yulduz turkumi tasvirlangan. Agar bu yulduzlarni kesmalar bilan tutashtirsak, cho‘michga o‘xshash shakl hosil bo‘ladi. “Cho‘mich”ning oxirgi ikki yulduzi hosil qilgan AB kesmani B nuqtadan boshlab AB nur bo‘yicha 5 marta qo‘yib chiqilsa, Qutb yulduzining yaqiniga boriladi. Rasmdan Qutb yulduzi qayerda joylashganini aniqlang.

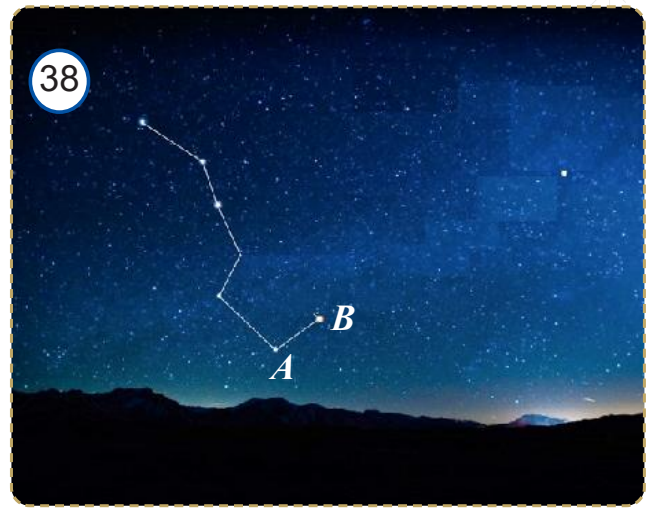
36. 39-rasmda dala sharoitida to‘g‘ri chiziq tortish jarayoni tasvirlangan. Undan bog‘da yoki tomorqada ko‘chatlarni bir tekis ekishda foydalaniladi. Bu amaliy ish to‘g‘ri chiziqning qaysi geometrik xossasiga asoslanganligini izohlang.

37. 40-rasmda bo‘yoqchining shiftga to‘g‘ri chiziq chizish jarayoni tasvirlangan. Buning uchun ip bo‘r bilan oq rangga bo‘yaladi. Ipning bir uchi mixga bog‘lanadi, ikkinchi uchi esa tegishli nuqtaga qo‘yiladi. So‘ng uning o‘rtasidan ushlab, tarang tortiladi va qo‘yib yuboriladi. Shunda ipga ilashgan bo‘r zarralari shiftga yopishib, to‘g‘ri chiziqni hosil qiladi. Xuddi shunday usulda devorga ham to‘g‘ri chiziq chizish mumkin. Bu amaliy ish to‘g‘ri chiziqning qaysi geometrik xossasiga asoslanganligini izohlang.

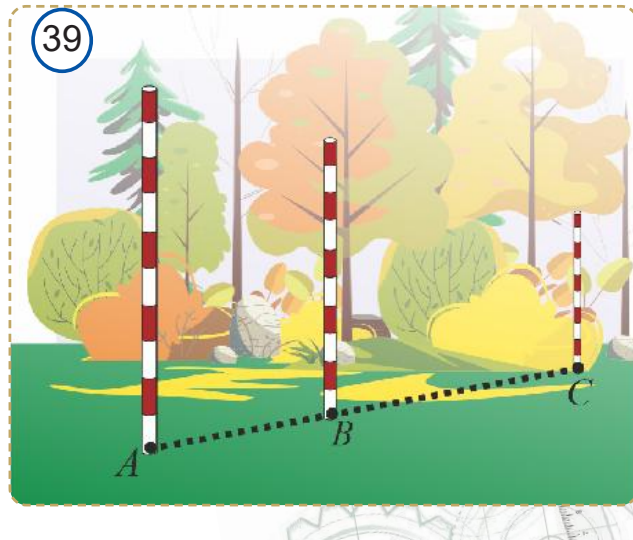
37



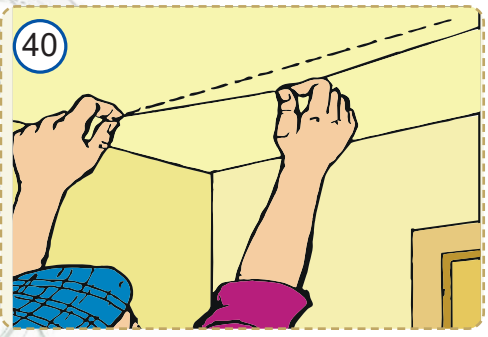
38



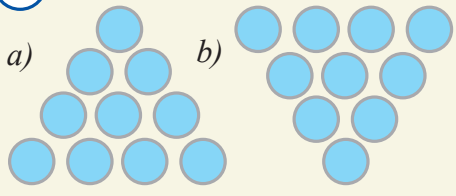
39



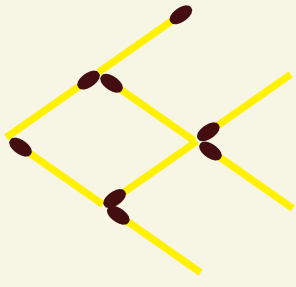
40



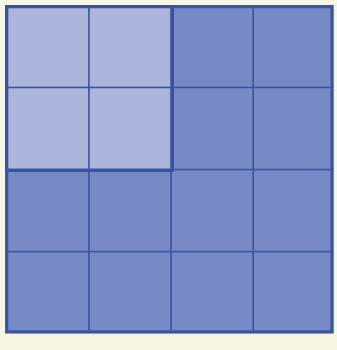
41



42



43

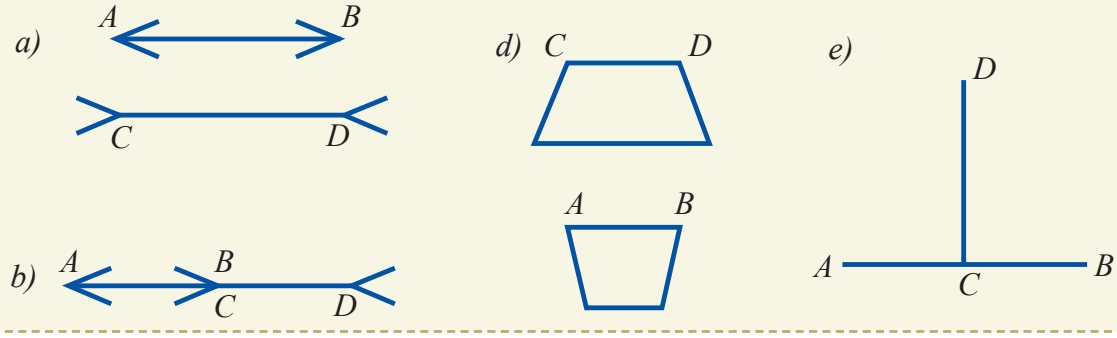


Geometrik boshqotirmalar

1. 10 ta bir xil tanga 41a-rasmdagidek qilib terilgan. Faqat 3 ta tanganing joyini o‘zgartirib, ularni 41b-rasmdagi ko‘rinishga keltiring.
2. 42-rasmdagi 3 ta cho‘pning joyini o‘zgartirib, “baliq”ni orqaga qaytaring.
3. Dehqon buvaning kvadrat shakldagi tomorqasi bor edi. U tomorqaning chorak qismini 43-rasmda ko‘rsatilgandek qilib o‘zi uchun qoldirdi. Qolgan qismini esa bir xil shakldagi teng bo‘laklarga bo‘lib, to‘rt o‘g‘liga taqsimlab berdi. Chol bu ishni qanday amalga oshirgan?
4. 44-rasmda tasvirlangan AB va CD kesmalarni ko‘z bilan chamalab o‘zaro taqqoslang. So‘ng bu ishni chizg‘ich yoki shaffof plyonka yordamida bajaring.

Xulosa. *Geometriyada o‘lchash va taqqoslash ishlarini aniq bajarish kerak. Ko‘z aldashi mumkin!*

44



3 BURCHAK. BURCHAKLARNI TAQQOSLASH VA O‘LCHASH

3.1. Burchak. Burchaklarni taqqoslash

Bir nuqtadan chiquvchi ikki nurdan iborat shakl *burchak* deb ataladi.

Burchakni tashkil etgan nurlar *burchakning tomonlari,* ularning umumiy uchi esa *burchakning uchi deb* ataladi *(1-rasm)*. Bu burchak $\angle AOB$ yoki $\angle BOA$ tarzida belgilanadi va AOB *burchak* yoki BOA *burchak* deb o‘qiladi. Bunday yozuvda burchakning uchi har doim o‘rtada yoziladi. Shuningdek, bu burchak qisqacha $\angle O$ tarzida ham yozilib, O *burchak* deb o‘qilishi mumkin. Chizmada burchakni ajratib ko‘rsatish uchun ba’zida uning ikki tomoni 1-rasmda ko‘rsatilgandek qilib yoysimon chiziq bilan tutashtirib qo‘yiladi.

Yoyiq burchak deb tomonlari bir-birini to‘ldiruvchi nurlardan iborat burchakka aytiladi.

2-rasmda yoyiq burchaklar tasvirlangan.

Yoyiq bo‘lmagan O burchak berilgan bo‘lsin. Uchlari shu burchakning tomonlarida bo‘lgan biror AB kesmani qaraymiz *(3-rasm)*.

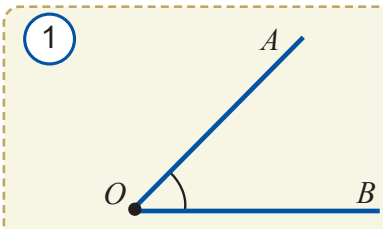
Agar burchakning uchidan chiquvchi OC nur *(3-rasm)* AB kesmani kesib o‘tsa, bu nur *burchak tomonlari orasidan o‘tadi*. Ravshanki, burchak tomonlari orasidan o‘tuvchi nur burchakni ikkita burchakka ajratadi.

Har qanday burchak tekislikni ikki bo‘lakka ajratadi *(4-rasm)*. Tekislikning burchak tomonlari orasidan o‘tuvchi biror nur yotgan bo‘lagi *burchakning ichki sohasi*, ikkinchi bo‘lagi esa *tashqi sohasi* deb ataladi.

Ixtiyoriy OB nur va yoyiq bo‘lmagan A burchak berilgan bo‘lsin*(5a-rasm)*. Ma’lumki, OB to‘g‘ri chiziq tekislikni ikkita yarimtekislikka ajratadi. A burchakni bir tomoni OB nur bilan ustma-ust tushadigan qilib qo‘yish mumkin. Bu amal burchak qaysi yarimtekislikka yotishiga qarab ikki usulda bajariladi (*5b, 5d-rasmlar)* va *burchakni nurdan yarimtekislikka qo‘yish* deb yuritiladi.

Istalgan nur va undan o‘tuvchi to‘g‘ri chiziq hosil qilgan yarimtekisliklardan biri berilgan bo‘lsin.

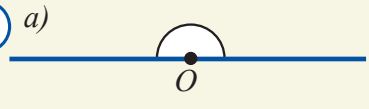
1



$\angle AOB$ – AOB burchak
O – burchakning uchi
OA, OB – burchakning tomonlari

2

a)

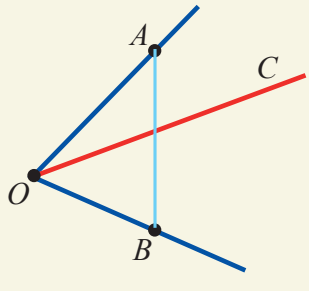


b)



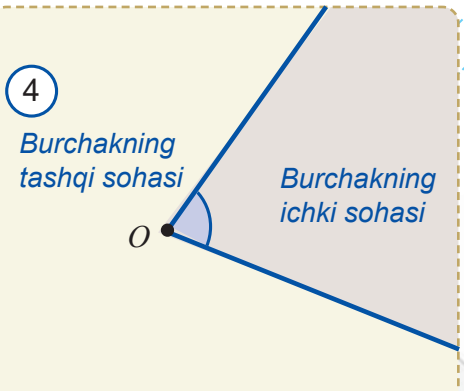
$\angle O$ – yoyiq burchak

3

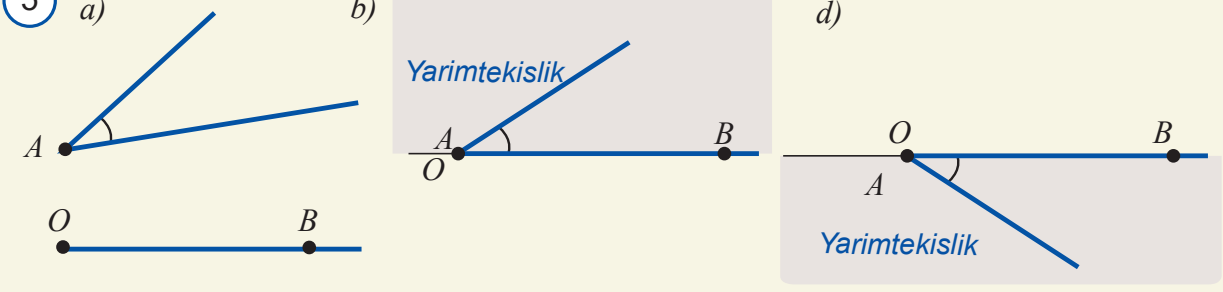


OC – burchak tomonlari orasidan o‘tuvchi nur

4



5 a) b) d)

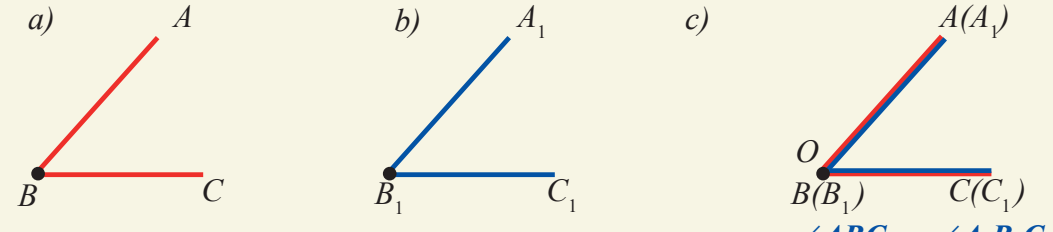


Ixtiyoriy nurdan tayin yarimtekislikka berilgan yoyiq bo'lmagan burchakka teng yagona burchakni qo'yish mumkin.

Ikki burchakni biror nurdan tayin yarimtekislikka qo'yish orqali o'zaro taqqoslash mumkin. Berilgan ABC va $A_1B_1C_1$ burchaklarni O nurdan tayin yarimtekislikka qo'yganimizda:

1-hol (6-rasm). BA tomon B_1A_1 tomon bilan, BC tomon esa B_1C_1 tomon bilan ustma-ust tushsa, ABC va $A_1B_1C_1$ burchaklar *teng* deb ataladi va bu $\angle ABC=\angle A_1B_1C_1$ tarzda yoziladi (shaklda burchaklarning tengligi bir xil sondagi yoychalar bilan belgilanadi).

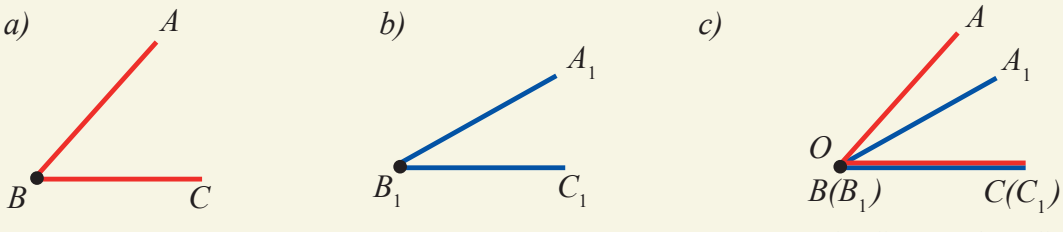
6 a) b) c)



$\angle ABC = \angle A_1B_1C_1$

2-hol (7-rasm). A_1B_1 tomon ABC burchak ichki sohasida yotsa, ABC burchak $A_1B_1C_1$ burchakdan *katta* deyiladi va bu $\angle ABC>\angle A_1B_1C_1$ tarzda ifodalanadi.

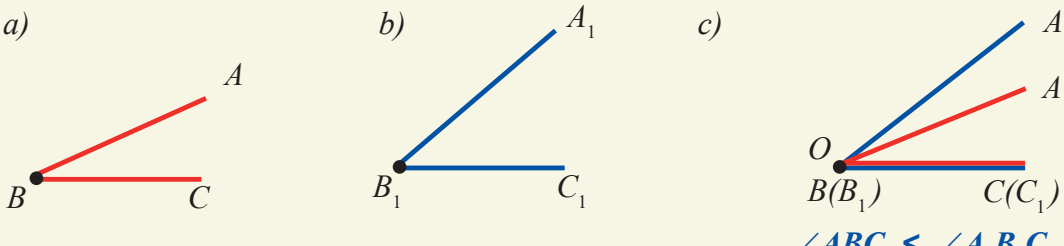
7 a) b) c)



$\angle ABC > \angle A_1B_1C_1$

3-hol (8-rasm). AB tomon $A_1B_1C_1$ burchak ichki sohasida yotsa, ABC burchak $A_1B_1C_1$ burchakdan *kichik* deyiladi va bu $\angle ABC<\angle A_1B_1C_1$ tarzda yoziladi.

8 a) b) c)

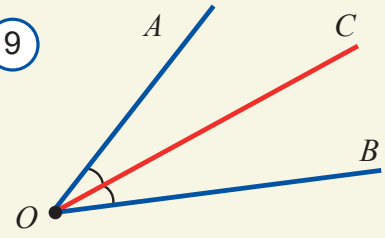


$\angle ABC < \angle A_1B_1C_1$

Burchak uchidan chiqib, uni teng ikki burchakka ajratuvchi nur *burchak bissektrisasi* deb ataladi.

9-rasmda AOB burchakning OC bissektrisasi tasvirlangan.

9



$\angle AOC = \angle COB$

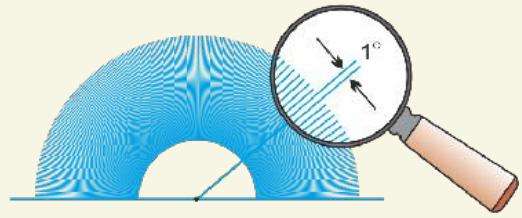
OC – AOB burchak bissektrisasi

3.2. Burchaklarni o‘lchash

Yoyiq burchak o‘zining tomonlari orasidan o‘tuvchi va uchidan chiquvchi nurlar bilan 180 ta teng burchakka bo‘lingan bo‘lsin *(10-rasm)*. Bu burchaklardan birini o‘lchov birligi – *birlik burchak* sifatida olish qabul qilingan. Uning burchak kattaligi *bir gradus* deb ataladi va 1° deb belgilanadi. Istalgan burchakning gradus o‘lchovini shu birlik asosida aniqlash mumkin. *Burchakning gradus o‘lchovi* burchak ichki sohasiga nechta birlik burchak va uning qismlari joylashishini ko‘rsatadi.

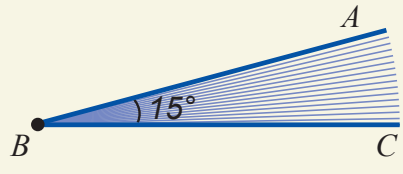
11-rasmda tasvirlangan ABC burchak 15° ga teng. Chunki uning ichki sohasiga 15 ta birlik burchak joylashyapti.

10



1 gradusli burchak

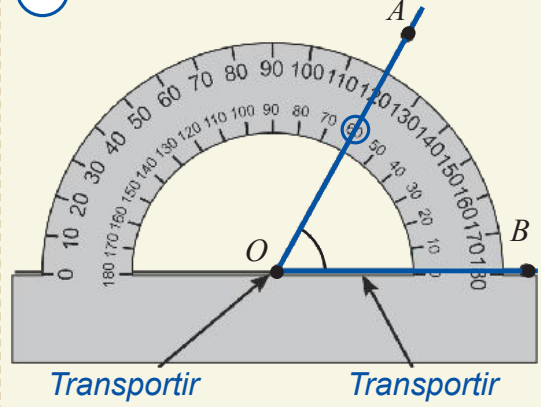
11



Har qanday burchak tayin gradus o‘lchoviga ega bo‘lib, uning qiymati musbat son bilan ifodalanadi. Yoyiq burchakning gradus o‘lchovi 180° ga teng.

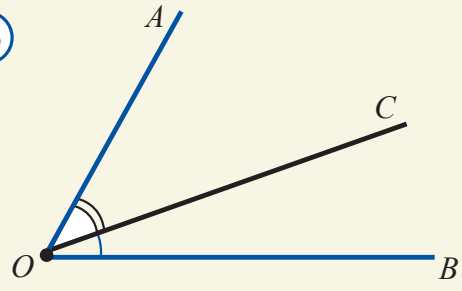
Burchaklarning gradus o‘lchovi *transportir* deb ataladigan asbob yordamida o‘lchanadi. Transportir bilan quyi sinflarda tanishgansiz. Uning shkalali yoysimon qismi chiziqchalar bilan 180 ta teng bo‘lakka bo‘lingan bo‘lib, har bir bo‘lak bir gradusni anglatadi. 12-rasmda transportir yordamida burchakni o‘lchash jarayoni tasvirlangan. Rasmda ko‘rib turganingizdek, AOB burchakning kattaligi 60 gradusga teng va bu $\angle AOB = 60°$ tarzida yoziladi. Ravshanki, bir xil gradus o‘lchoviga ega burchaklar o‘zaro teng bo‘ladi va aksincha, o‘zaro teng burchaklarning gradus o‘lchovlari ham teng bo‘ladi. Katta burchakning gradus o‘lchovi ham katta bo‘ladi va aksincha.

12



Burchaklarni o‘lchashda gradusning ulushlaridan ham foydalaniladi. 1° ning $\frac{1}{60}$ bo‘lagi *minut*, $\frac{1}{3600}$ bo‘lagi *sekund* deb nomlanadi va mos ravishda “′” va “″” kabi belgilanadi. Masalan, kattaligi 45 gradus 38 minut 59 sekundga teng burchak gradus o‘lchovi 45°38′59″ kabi yoziladi. Ravshanki, 1° = 60′, 1′ = 60″.

13



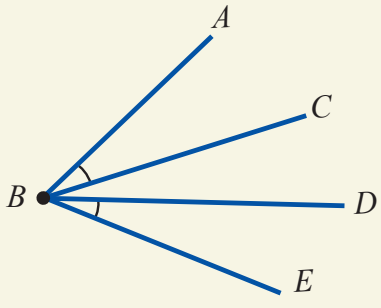
$\angle AOB = \angle AOC + \angle COB$

Aytaylik, AOB burchak berilgan bo‘lib, uning tomonlari orasidan o‘tuvchi ixtiyoriy OC nur uni AOC va COB burchaklarga ajratsin *(13-rasm)*. U holda:

$$\angle AOB = \angle AOC + \angle COB.$$

Bu xossani quyidagicha ifodalash mumkin:

14



Burchakning gradus o‘lchovi burchak tomonlari orasidan o‘tuvchi ixtiyoriy nur ajratgan burchaklar gradus o‘lchovlari yig‘indisiga teng.

1-masala. Agar $\angle ABC = \angle DBE$ bo‘lsa, $\angle ABD = \angle CBE$ ekanini ko‘rsating *(14-rasm)*.

Yechish. Berilgan $\angle ABC = \angle DBE$ tenglikning har ikkala tomoniga $\angle CBD$ ni qo‘shamiz:

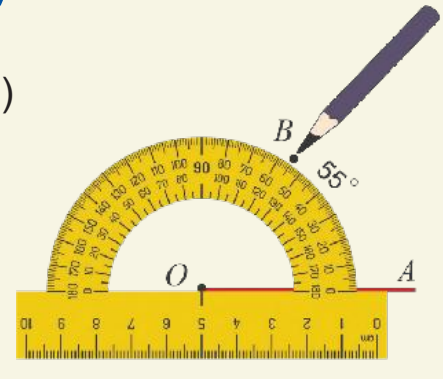
$$\angle ABC + \angle CBD = \angle CBD + \angle DBE.$$

Lekin $\angle ABC + \angle CBD = \angle ABD$ va $\angle CBD + \angle DBE = \angle CBE$.

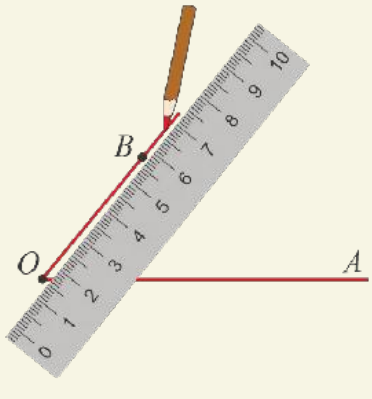
Demak, $\angle ABD = \angle CBE$.

15

a)



b)



Nurga berilgan gradus o‘lchovli burchakni qo‘yishning amaliy yo‘rig‘i:

1. Ixtiyoriy OA nur chizib olinadi.
2. Transportirning asosini berilgan OA nur ustiga, markazini esa O nuqtaga 15*a*-rasmda ko‘rsatilgandek qilib qo‘yiladi.
3. Transportir shkalasidan burchakning berilgan gradus o‘lchovini ko‘rsatuvchi bo‘linmasi topiladi va uning to‘g‘risiga B nuqta qo‘yiladi.
4. O va B nuqtalar orqali nur o‘tkaziladi *(15b-rasm)*. Natijada berilgan gradus o‘lchovli AOB burchak hosil bo‘ladi.

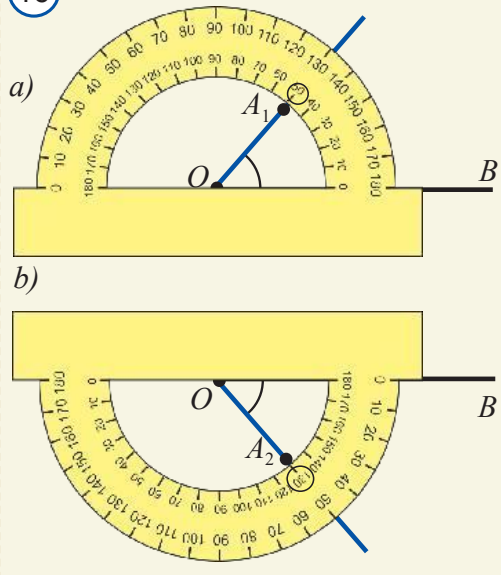
Bu ishni yana qanday yo‘l bilan amalga oshirish mumkinligi haqida o‘ylab ko‘ring.

2-masala. Berilgan OB nurga 50° li burchakni qo‘ying.

Yechish. OB to‘g‘ri chiziq tekislikni ikkita yarimtekislikka ajratishi ma’lum. Transportirning asosini OB nur ustiga, markazini esa O nuqtaga 2 xil usulda qo‘yib, uning shkalasida 50° ga mos keluvchi bo‘linma topiladi va burchaklar yasaladi *(16-rasm)*.

Demak, berilgan nurdan har bir yarimtekislikka bittadan 50° li burchak qo‘yish mumkin ekan: $\angle A_1OB = \angle A_2OB = 50°$.

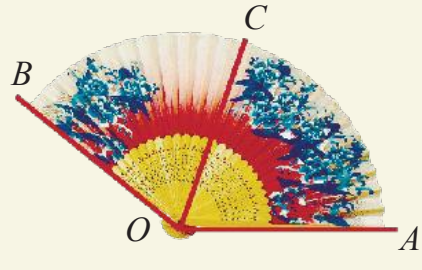
16



Mavzu bo‘yicha savollar

1. Burchak nima va u qanday belgilanadi?
2. Yoyiq burchak nima?
3. Burchak tekislikni qanday bo‘laklarga ajratadi?
4. Burchakni nurdan tayin yarimtekislikka qo‘yish deganda nimani tushunasiz?
5. Qachon burchaklar o‘zaro teng bo‘ladi?
6. Qachon bir burchak ikkinchisidan katta yoki kichik bo‘ladi?
7. Burchak bissektrisasiga ta’rif bering.
8. Burchakning gradus o‘lchovi deb nimaga aytiladi?
9. Yoyiq burchak necha gradus?
10. 1° ga teng burchak deganda qanday burchakni tushunasiz?
11. Ikki burchakning gradus o‘lchovlari teng bo‘lsa, ular teng bo‘ladimi?
12. 17-rasmda burchak gradus o‘lchovining qaysi xossasi keltirilgan?

17

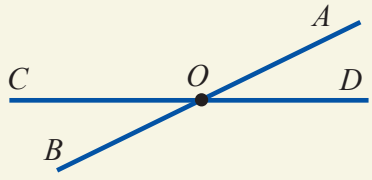




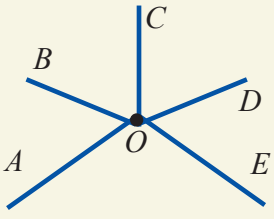
Amaliy mashq va tatbiq

18

a)



b)



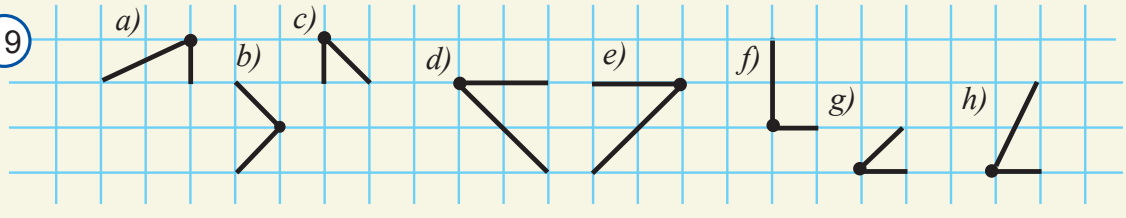
1. a) $\angle MNL$; b) $\angle ABO$ burchakni chizing. Uning uchi va tomonlarini yozing.

2. Ixtiyoriy A, B, C va D nuqtalarni belgilab, AB, AC va DA to‘gri chiziqlarni chizing. Hosil bo‘lgan burchaklarni, ularning uchlari va tomonlarini yozing.

3. 18-rasmda tasvirlangan barcha burchaklarni aniqlang va ularni yozing.

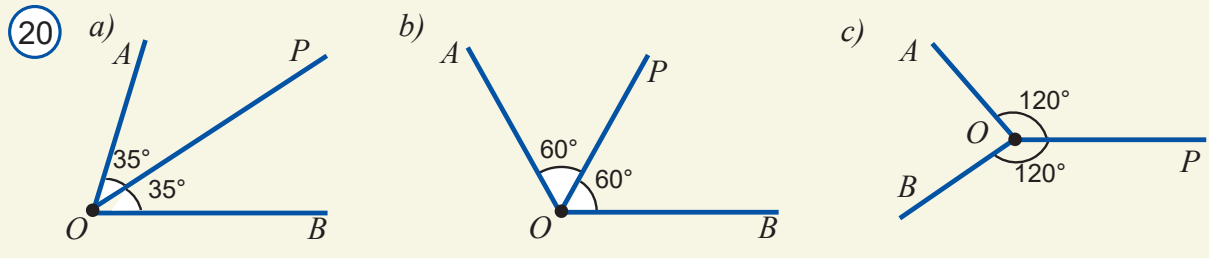
4. 19-rasmda tasvirlangan burchaklar orasidan teng burchaklarni aniqlang.

19



5. 20-rasmda tasvirlangan OP nur AOB burchak bissektrisasi bo‘ladimi?

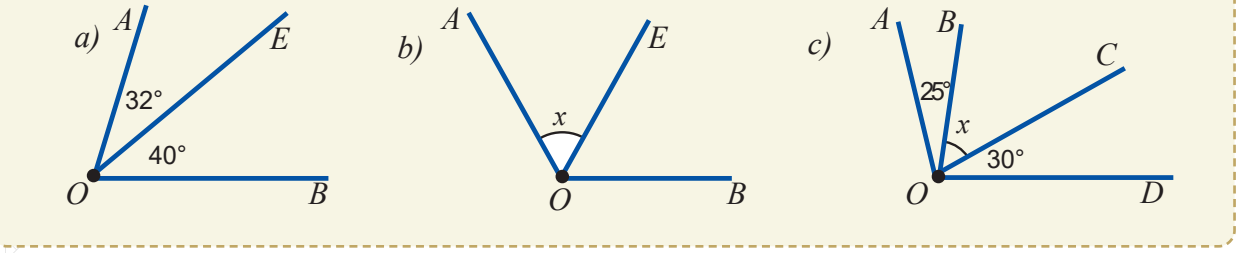
20



6. 21-rasmdan foydalanib quyidagilarni aniqlang:

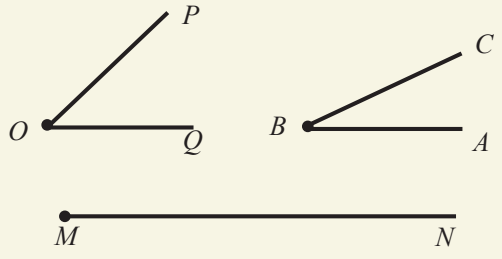
a) $\angle AOB = ?$; b) $\angle AOB = 120°38'$, $x = ?$; c) $\angle AOD = 105°45'$, $x = ?$

21

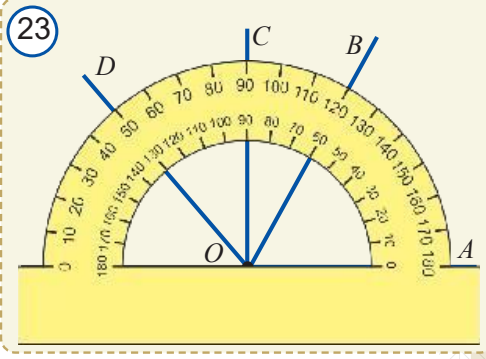


7. 22-rasmdagi POQ va ABC burchaklarni MN nurga qo‘yish orqali taqqoslang.

22



23



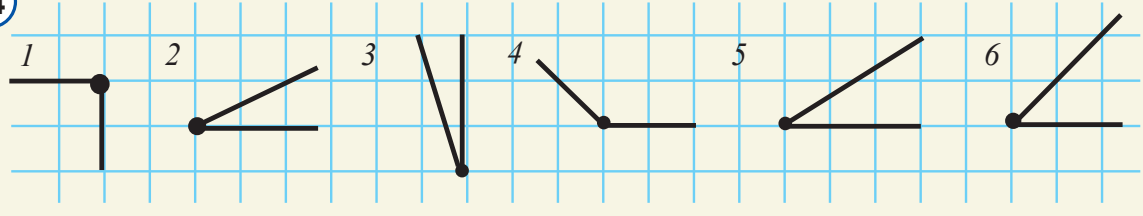
8. 23-rasmdan foydalanib AOB, AOC, AOD, BOC, BOD va COD burchaklarning gradus o‘lchovini aniqlang hamda transportir yordamida bissektrisalarini yasang.

9. Quyidagi tengliklar ma’noga egami? a) $\angle AOB = \angle BOA$; b) $\angle AOB = \angle ABO$.

10. Transportir bilan 60° va 176° li burchaklar, ularning bissektrisalarini yasang.

11. Transportir yordamida 49°, 79° va 142° li burchaklar hamda ularning bissektrisalarini yasang.

24



12*. 24-rasmda tasvirlangan burchaklar raqamlarini ularning gradus o‘lchovlari o‘sish tartibida yozing.

13. Hisoblang:

a) 34°18′+ 53°38′; b) 15°8′38″+113°21′9″; c) 115°8′38″-113°21′9″.

14. Transportir yordamida 10°, 30°, 74°45′, 113°30′ va 165° li burchaklarni yasang.

15. $\angle A$ va $\angle B$ berilgan $\angle A > \angle B$ bo‘lsa, $\angle A + \angle B$ va $\angle A - \angle B$ burchaklarni yasang.

16. Berilgan AB nurga 150° li OAB burchakni qo‘ying.

17*. Agar; a) $\angle AOE = 20°$, $\angle EOB = 40°$, $AOB = 60°$; b) $\angle AOE = 80°$, $EOB = 120°$; c) $\angle AOE > \angle AOB$ bo‘lsa, OE nur $\angle AOB$ tomonlari orasidan o‘tadimi?

18*. OC nur AOB burchak tomonlari orasidan o‘tadi. Agar $\angle AOB = 108°$, $\angle BOC = 68°$ bo‘lsa, $\angle AOC$ ni toping va bu burchaklarni transportir yordamida yasang.

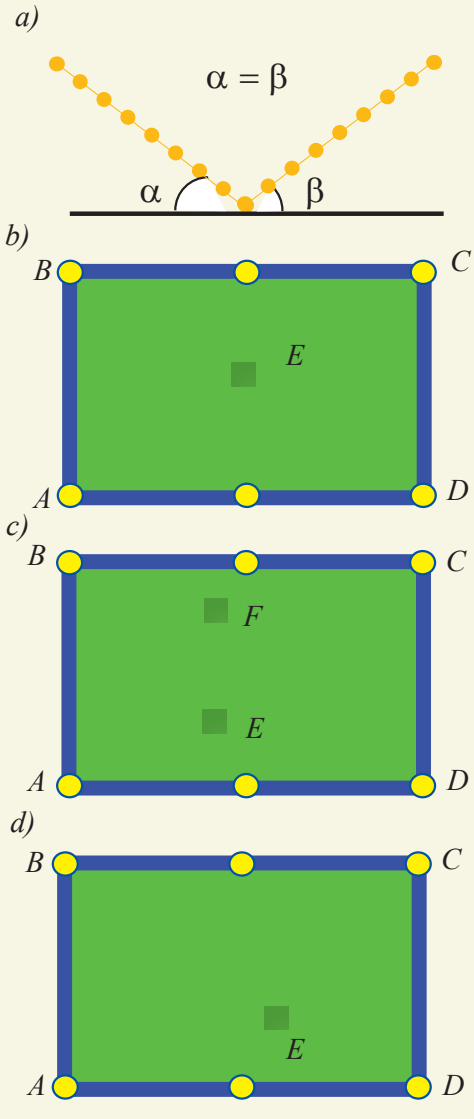
19*. OT nur ROS burchak tomonlari orasidan o‘tadi. Agar $\angle ROT = 37°$, $\angle ROS = 98°$ bo‘lsa, $\angle TOS$ ni toping va bu burchaklarni transportir yordamida yasang.

20*. Strelkali soatda: a) 3:00; b) 6:00 bo‘lganda, soat va minut millari hosil qilgan burchak necha gradusga teng bo‘lishini aniqlang.

21. Daftaringizga nur chizing va unga ko‘zingiz bilan chamalab oddiy chizg‘ich yordamida 15°, 30°, 45°, 60°, 75°, 90°, 120° va 150° li burchaklarni qo‘ying. So‘ngra ularni transportir yordamida o‘lchang va qanchalik to‘g‘ri chizganingizni tekshiring. Mashqni takrorlang.

25

7



Amaliy mashq va tatbiq

1. Qog‘oz varag‘iga burchak chizing. Varaqni buklash orqali chizilgan burchakdan: a) 2 marta katta; b) 2 marta kichik; c) uni to‘g‘ri burchakka to‘ldiruvchi burchakni hosil qiling.
2. Sinf doskasida ikkita burchak tasvirlangan. Qo‘lingizda bo‘r va ip bor. Ular yordamida burchaklarni qanday taqqoslash mumkin?
3. Yerga to‘g‘ri burchak chizilgan. Uning to‘g‘ri burchak ekanini ip bilan qanday tekshirish mumkin?
4. Yerda berilgan burchakka teng burchakni faqat ip yordamida qanday yasash mumkin?
5. Tanga yordamida aylana chizing. Bu aylananing markazini qanday aniqlasa bo‘ladi?
6. Bilyard stoli ustidagi sharning harakatini kuzatganmisiz? U har gal stol tomoniga urilganda, qanday burchak ostida kelib urilgan bo‘lsa, shunday burchak ostida tomondan qaytadi *(25a-rasm)*:

   a) stol markazida turgan sharni biror yo‘nalishda harakatlantiring va uning harakat trayektoriyasini chizing *(25b-rasm)*;

   b) E nuqtada turgan sharning AD va AB tomonlarga urilib, F nuqtada turgan sharga tegadigan trayektoriyasini chizib ko‘rsating *(25c-rasm)*;

   d) E nuqtada turgan sharning AD, AB va BC tomonlariga urilib, D teshikka tushadigan trayektoriyasini chizib ko‘rsating *(25d-rasm)*.

26



Tarixiy lavhalar

Usturlab (astrolyabiya) – burchak o‘lchaydigan asbob bo‘lib, uni qadimgi yunon astronomi Gipparx miloddan oldingi II asrda kashf qilgan (26-rasm). Ko‘rinishi juda sodda bo‘lgan bu asbobda o‘nlab o‘lchash ishlarini bajarish mumkin bo‘lgan.

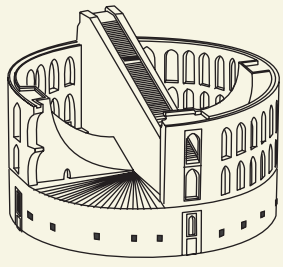
Samarqanddagi Ulug‘bek astronomik rasadxonasida ham burchak o‘lchash ishlari olib borilgan. Radiusi 42 m bo‘lgan ulkan silindr shaklidagi uch qavatli rasadxonada (27a-rasm) ko‘plab qurilma va asboblar bo‘lgan (27b-rasm). 27c-rasmda uning yer ostida saqlanib, shu kungacha yetib kelgan qismi tasvirlangan. Ulug‘bek bu qurilma yordamida 1018 ta yulduzning koinotdagi o‘rnini hayratomuz aniqlikda o‘lchab, o‘zining “Ziji jadidi Ko‘ragoniy” asarida keltirgan.

27

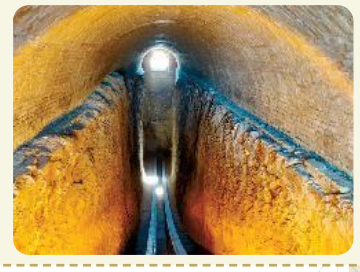
a)



b)

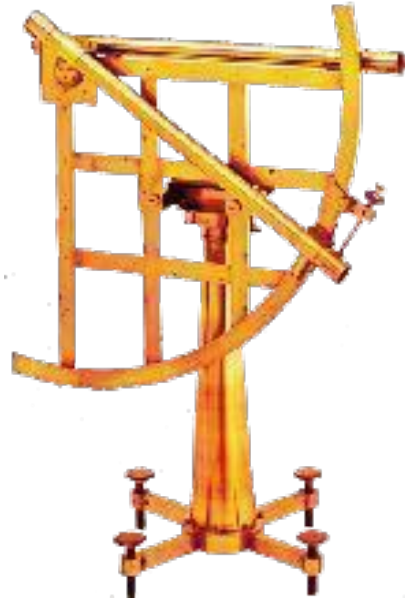


c)



28-rasmda yevropalik olimlar teleskop ixtiro qilinishidan avval foydalangan kvadrant tasvirlangan. U Ulug‘bek kvadrantidan ancha kichik, albatta.

28



*Hozir yer o‘lchash ishlarida yuqori aniqlikka ega **elektron teodolit** (29a-rasm) degan asbob qo‘llanadi. U yordamida Yerning istalgan nuqtasida sun’iy yo‘ldosh orqali bog‘lanib, o‘lchash, natijalarni taqqoslash va uzatish mumkin (29b-rasm).*

29

a)



b)



4 AMALIY MASHQ VA TATBIQ

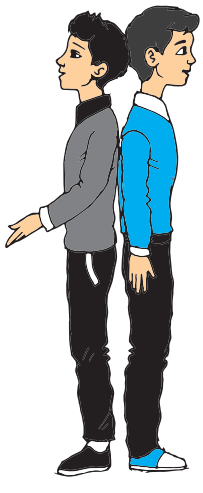
1. Qo‘lingizdagi darslikning bo‘yi, eni va qalinligini chizg‘ich yordamida o‘lchang.
2. Qo‘lingizdagi darslikning bir varag‘i qalinligini qanday o‘lchash mumkin? Chizg‘ich yordamida g‘ishtning diagonalini o‘lchay olasizmi?
3. Duradgor ustalar ikkita taxta bo’lagi uzunligini qay yo’sinda taqqoslashadi? *(30-rasm)*
4. Sinfdoshlaringizning bo‘yini chamalab o‘lchang va taqqoslang. Bo‘yi eng uzun sinfdoshingizni aniqlang. Ikki sinfdoshingiz bo‘yini o‘zaro taqqoslashning amaliy usulini tushuntiring *(31-rasm)*.
5. Qarichingizni chizg‘ich yordamida santimetrlarda o‘lchang. So‘ng bir nechta predmetning o‘lchamlarini (partaning eni, uzunligi va balandligini, deraza va doskaning bo‘yi hamda enini) qarichlab o‘lchang va natijalarni santimetrlarda ifodalang.

30



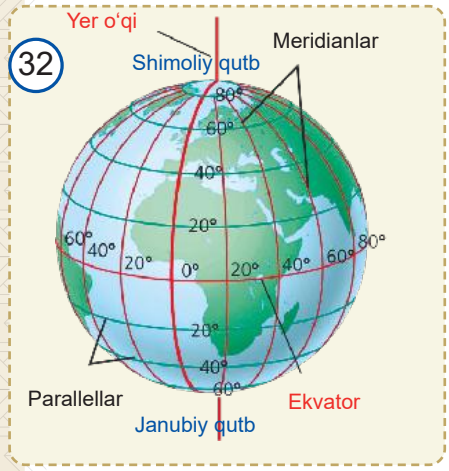
Qarichingiz va qadamingiz uzunligini o‘lchab, eslab qoling. Ularni bilish sizga kundalik turmushda ko‘p asqatadi!

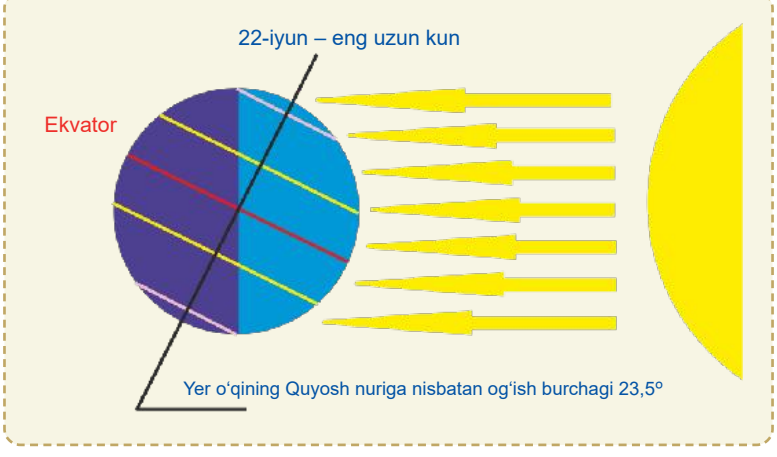
31



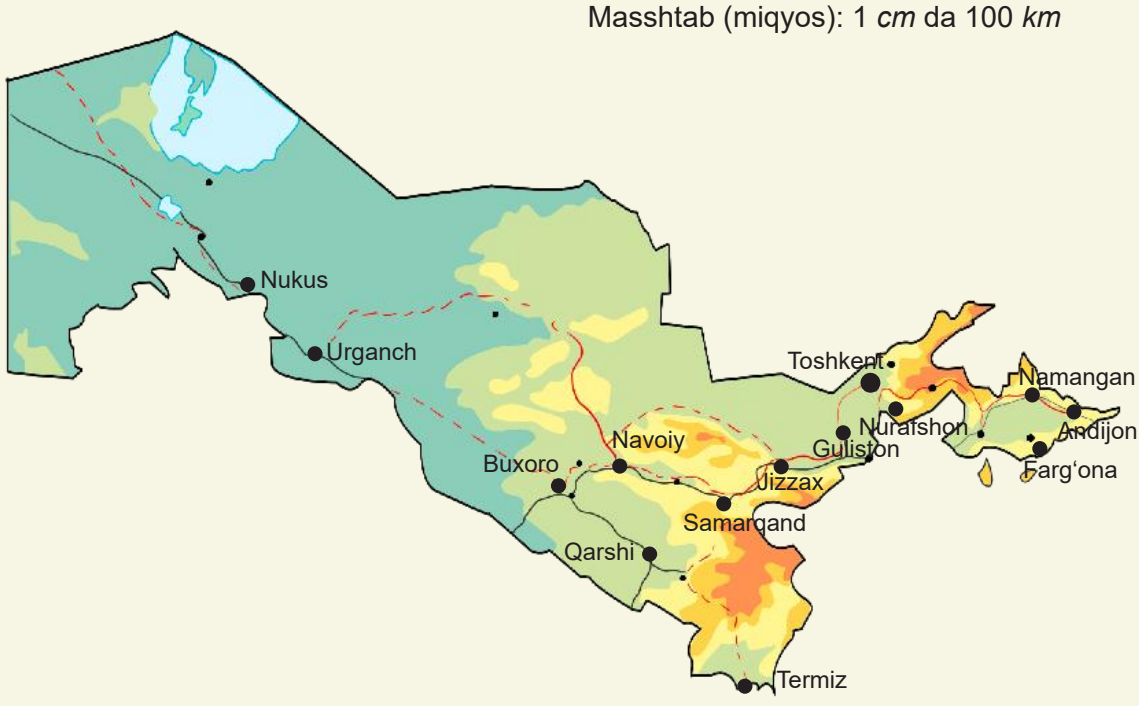
6. 30 santimetrlik chizg‘ich yordamida 1 metrlik kesmani qanday yasash mumkin?
7. Qadamingiz uzunligini o‘lchang. Maktab binosining bo‘yi va enini, sport maydonchasining uzunligi va enini qadamlab o‘lchang va metrlarda ifodalang.
8. Yer shari o‘qi Quyoshdan tushayotgan nurga nisbatan 23,5° ga og‘gan holatda aylanadi *(32-rasm)*. Yer 8 soatda o‘z o‘qi atrofida necha gradusga aylanadi? Yer necha soatda o‘z o‘qi atrofida 90° ga buriladi?
9. Agar bir chaqirim 900 m ekani ma’lum bo‘lsa, Buxoro va Samarqand shaharlari orasidagi masofani chaqirimlarda ifodalang.
10. O‘zbekiston xaritasidan berilgan masshtabga ko‘ra turli shaharlar orasidagi masofalarni toping *(33-rasm)*.

32





33



Namuna: *Toshkent va Buxoro shaharlari o'rtasidagi masofani topish.* Xaritada shaharlar orasidagi masofani chizg'ich yordamida o'lchab, 4,38 cm ga teng ekanini topamiz. Masshtabga ko'ra, 4,38 · 100 *km* = 438 *km* ekanini aniqlaymiz.

Javob: 438 *km*.

Ko'pgina davlatlarda xalqaro o'lchov birliklaridan tashqari, quyidagi uzunlik o'lchov birliklari ham ishlatiladi: 1 *dyuym* = 2,54 *cm*, 1 *mil* = 1,609 *km*.

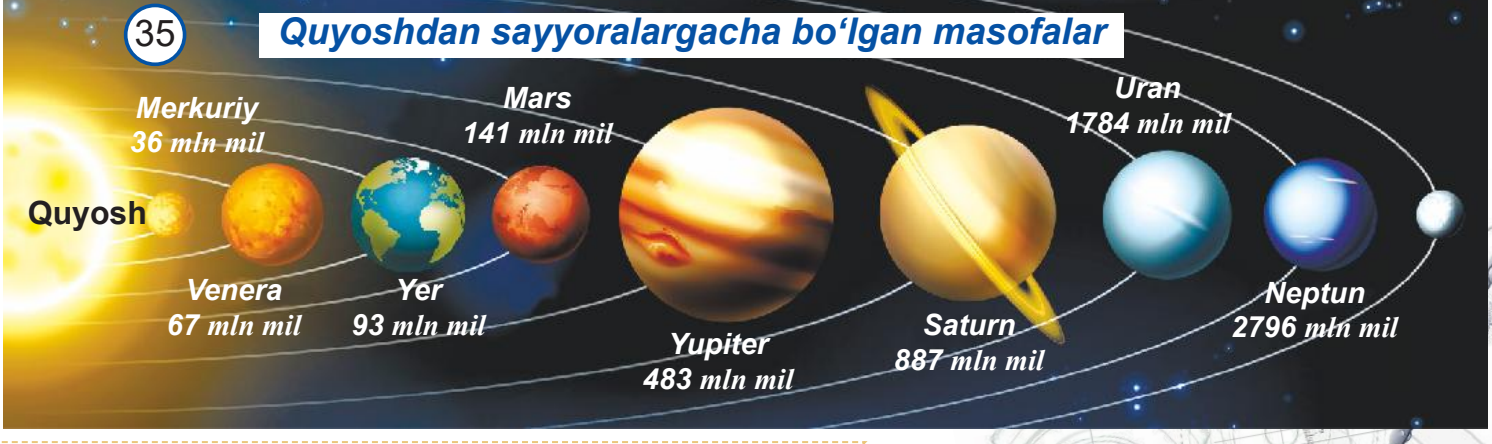
11. Televizor va kompyuter monitorining diagonali *(34-rasm)* dyuymlarda o'lchanadi. 15, 17 va 19 *dyuym*li monitor diagonalini santimetrlarda ifodalang.

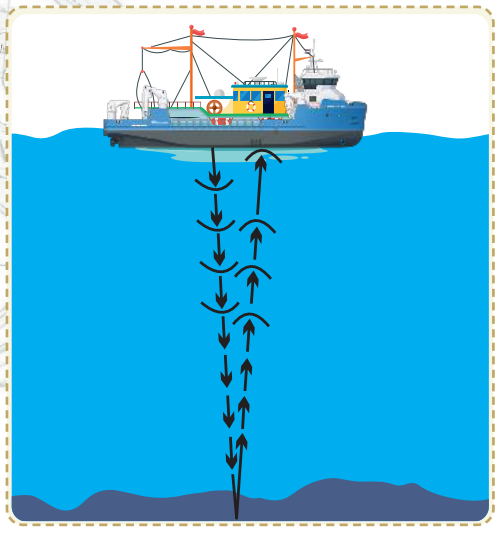
12. 35-rasmda berilgan ma'lumotlardan foydalanib, Yerdan Quyoshgacha va boshqa sayyoralargacha bo'lgan masofani toping va km larda ifodalang.

34



35





13. *Masofani tovush bilan o‘lchash.* Dengizda suzayotgan kemani boshqarish uchun dengiz chuqurligini bilish juda muhimdir. Buning uchun dengiz tubiga ultratovush signali yuboriladi va uning dengiz tubiga urilib, qancha vaqtda qaytib kelgani o‘lchanadi.

Bu vaqtning yarmini tovushning suvdagi tezligi – 1490 *m/s* ga ko‘paytirib dengiz tubining chuqurligi aniqlanadi.

Agar bu vaqt: a) 3; b) 10,5; c) 14,6 sekundni tashkil qilgani ma’lum bo‘lsa, dengiz chuqurligini toping.

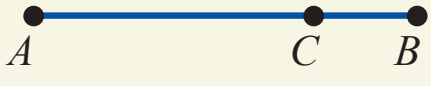


Geometriyada AKT

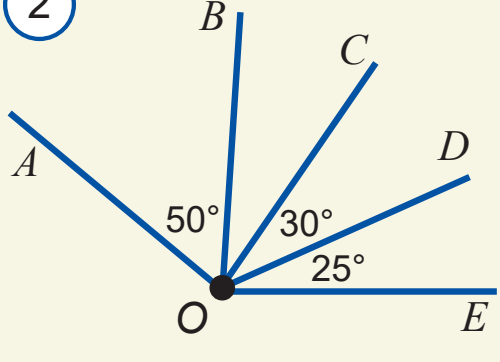
Mobil telefonlar uchun qiyalikni o‘lchaydigan dasturiy ilovalar ishlab chiqilgan bo‘lib, ular yordamida qiyalik burchagini avtomatik tarzda o‘lchash mumkin. Rasmda Italiyadagi mashhur Piza minorasining qiyaligini telefondagi shunday dastur yordamida o‘lchash tasvirlangan.

1-nazorat ishi namunasi

1



2



Namunaviy nazorat ishi ikki qismdan iborat bo‘ladi:

I. Nazariy qism. Shu paytgacha o‘rganilgan geometrik shakllarni sanang. Ularga ta’rif bering va xossalarini yozing.

II. Amaliy qism. Quyidagi masalalarni yeching (4-masala “a’lo” baho olmoqchi bo‘lgan o‘quvchilarga mo‘ljallangan):

1. Bir to‘g‘ri chiziqda yotuvchi A, B va C nuqtalar uchun $AB = 9\ cm$, $AC = 12\ cm$ bo‘lsa, BC kesmaning uzunligi nimaga teng?

2. $AB = 48$, $AC = 3BC$, $BC = ?$ *(1-rasm)*

3. Agar 2-rasmda $\angle AOE = 140°$ bo‘lsa, BOC burchakning gradus o‘lchovini toping.

4*. Soat 5:00 bo‘lganda soat va minut millari (strelkalari) hosil qilgan burchak necha gradus bo‘ladi?

Matematik masalalar xazinasi

Ma’lumki, keyingi paytlarda axborot kommunikatsiya texnologiyalari juda tez rivojlanmoqda va jadal sur’atlar bilan ta’lim tizimiga ham kirib bormoqda. Hozirgi kunda internet tarmog‘iga shunchalik ko‘p axborot manbalari joylashtirilganki, bu xazinadan foydalanish har bir yosh avlod uchun juda zarur va foydali. Mazkur vebsahifalardan siz o‘zbek, rus, ingliz va boshqa tillarda matematika olamidagi eng oxirgi yangiliklarni, elektron kutubxonalar omborida saqlanayotgan ko‘plab elektron darsliklarni, raqamli resurslarni topishingiz mumkin. Shuningdek, ular orqali turli-tuman nazariy materiallar, uslubiy tavsiyalar, son-sanoqsiz masalalar, misollar va ularning yechimlari, turli davlatlarda o‘tkazilayotgan matematik ko‘rik-tanlov va olimpiadalar to‘g‘risidagi ma’lumotlar va ularda taqdim etilgan qiziqarli matematik masalalar hamda ularning yechimlari bilan tanishishingiz mumkin.

Quyida bir qator axborot-resurs manbalarining internet manzillari berilmoqda. Ulardan geometriyaga oid o‘zingizni qiziqtirgan turli ma’lumotlarni olib ko‘rish, matematikani mustaqil o‘rganish imkoniyatlaridan foydalanishni tavsiya etamiz:

uzedu.uz – Xalq ta’limi vazirligining rasmiy sayti, axborot-ta’lim portali;

ziyonet.uz – “Ziyonet” ijtimoiy ta’lim portali;

dr.rtm.uz – yangi darsliklar, o‘qituvchilar uchun uslubiy qo‘llanmalarning elektron shakllari, taqdimotlar, multimedia ilovalari, videodarslar va raqamli resurslar platformasi;

maktab.uz – 1–11-sinflar uchun onlayn maktab hamda maktab o‘quv dasturi bo‘yicha videodarslar va boshqa materiallar platformasi;

stesting.uz – o‘quvchilar uchun “Xalqaro baholash tadqiqotlariga tayyorlanish” elektron platformasi (o‘zbek tilida);

masofa.uz – A. Avloniy nomidagi pedagoglarni kasbiy rivojlantirish va yangi metodikalarga o‘rgatish milliy-tadqiqot instituti masofadan o‘qitish portali;

onlinedu.uz – A. Avloniy nomidagi pedagoglarni kasbiy rivojlantirish va yangi metodikalarga o‘rgatish milliy-tadqiqot instituti “Uzluksiz kasbiy ta’lim” elektron platformasi;

skillsgrover.uz – Finlandiyaning matematikani o‘rganish va o‘quvchilar bilimlarini baholash platformasi (o‘zbek tilida);

khanakademy.org – “Xon akademiyasi” masofaviy ta’lim sayti (ingliz tilida);

xanakademiyasi.uz – matematika, informatika, kimyo, fizika, iqtisodiyot, biologiya va astronomiya kabi fanlar bo‘yicha videodarslar platformasi (o‘zbek tilida);

school.edu.ru – umumta’lim portali (rus tilida);

problems.ru – matematikadan masalalar izlash tizimi (rus tilida);

geometry.net – algebra va geometriyadan o‘quv materiallari (ingliz tilida);

mathproblem.narod.ru – matematik to‘garaklar va olimpiadalar (rus tilida);

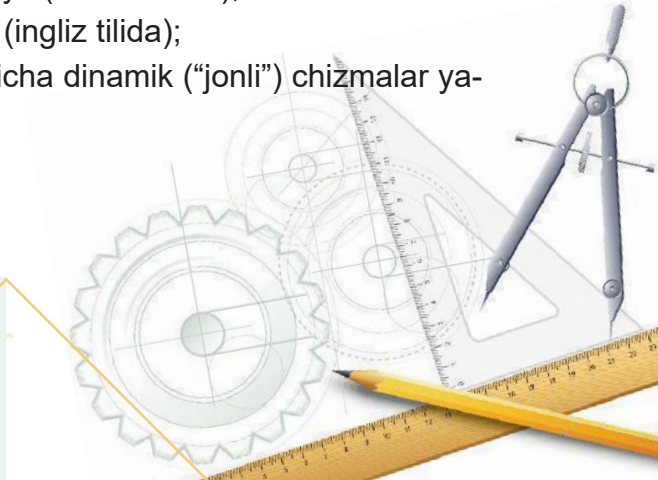
ixl.com – masofadan turib o‘qitish matematika ta’limi portali (ingliz tilida);

mathkang.ru – “Kenguru” xalqaro matematik tanlov sayti (rus tilida);

olimpia.uz – “Kenguru” xalqaro matematik tanlov sayti (o‘zbek tilida);

brilliant.org – matematikadan masofaviy ta’lim sayti (ingliz tilida);

geogebra.com – geometriya va algebra fanlari bo‘yicha dinamik (“jonli”) chizmalar yaratish imkoniyatini beradigan bepul dastur;





yaklass.ru – maktab o‘quvchilari va o‘qituvchilar uchun onlayn ta’lim platformasi;
schulen-ans-netz.de – Germaniya “Internet-maktab” sayti (nemis tilida);
studienkreis.de – Germaniya o‘quv to‘garaklari sayti (nemis tilida);
educasource.education.fr – Fransiya ta’lim sayti (fransuz tilida);
educmath.inrp.fr – Fransiya matematika ta’limi raqamli resurslari (fransuz tilida);
mat-game.narod.ru – matematik gimnastika. Matematik masalalar va boshqotirmalar (rus tilida);
mathproblem.narod.ru – matematik to‘garaklar, maktablar va olimpiadalar (rus tilida);

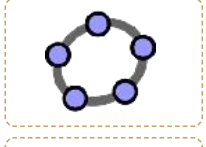
“GeoGebra” – matematikadan “jonli” chizmalar dasturi

GeoGebra bu geometriya, algebra va boshqa yo‘nalishlar bo‘yicha ta’limning turli darajalarida foydalanish uchun dinamik (“jonli”) chizmalar yaratish imkoniyatini beradigan bepul dastur hisoblanadi. U geometrik shakllar, algebraik ifodalar, jadvallar, grafiklar va statistika bilan ishlash uchun keng imkoniyatlarni taqdim etadi va qulaylik uchun barcha funksiyalar bitta paketga kiritilgan. Ushbu ochiq kodli dasturiy ta’minot 2002-yilda avstriyalik matematik ***Markus Xenvarter*** tomonidan Java dasturlash tilida yaratilgan bo‘lib, bir qancha tillarda ishlash imkoniyati mavjud. Hozirgi kunda undan dunyo bo‘ylab millionlab insonlar foydalanib kelmoqda.

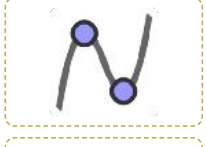
“GeoGebra” dasturining afzalliklari:

- bepul tarqatiladi;
- ko‘p tilli interfeys;
- grafik interfeysning soddaligi va qulayligi;
- turli xil operatsion tizimlarga (hatto planshetlar va smartfonlarga) o‘rnatish imkoniyati va onlayn versiyaning mavjudligi
- foydalanuvchilar materiallar qo‘shishi uchun ochiq bo‘lgan misollar bazasi.

“GeoGebra” dasturining qismlari



Kalkulyatorlar to‘plami
Funksiyalarni tekshirish, tenglamalarni yechish, geometrik shakllar va 3D obyektlarni qurishga mo‘ljallangan.



Grafik kalkulyator
Turli funksiyalar grafiklarini qurish, tenglamalarni tadqiq qilish va ma’lumotlarni tasvirlashga mo‘ljallangan.



3D kalkulyator
Turli chizmalar, 3D (uch o‘lchovli) geometrik shakllar va obyektlarni chizishga mo‘ljallangan.



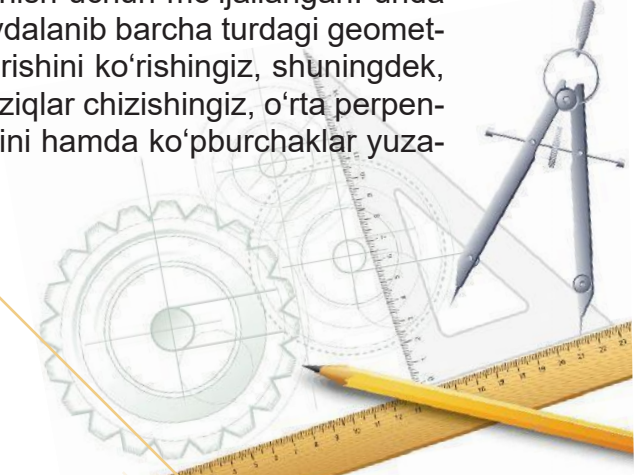
Geometriya
Turli geometrik shakllarni chizish va almashtirishga mo‘ljallangan.



CAS kalkulyator
Turli tenglamalarni yechish, algebraik ifodalar shaklini almashtirish, hosila va integrallarni hisoblashga mo‘ljallangan.

“GeoGebra” geometriya dasturida

GeoGebra dasturi geometriyaga oid masalalarni yechish uchun mo‘ljallangan: unda siz nuqta, kesma, vektor, segment, to‘g‘ri chiziqlardan foydalanib barcha turdagi geometrik shakllarni yaratishingiz, ularni dinamik ravishda o‘zgarishini ko‘rishingiz, shuningdek, ma’lum bir to‘g‘ri chiziqqa perpendikulyar yoki parallel chiziqlar chizishingiz, o‘rta perpendikulyar, burchaklarning bissektrisalari, kesmalar uzunligini hamda ko‘pburchaklar yuzasini aniqlashingiz mumkin.



“GeoGebra” dasturiga doir onlayn resurslar



• ***GeoGebra*** rasmiy vebsayti (GeoGebra.org)dan GeoGebra haqida batafsil ma’lumot oling va uni shu yerdan yuklab oling (bepul dastur).

• Youtube kanalida ***GeoGebra*** (youtube.com GeoGebraChannel)ni o‘rganish uchun videodarslar va undan foydalanishga oid misollar keltirilgan.

• ***GeoGebra***ni saqlash uchun sayt (tube.geogebra.org)da foydalanuvchilar o‘zlari yaratgan ilova va dars rejalari bilan o‘rtoqlashadilar.

“GeoGebra” dasturini o‘rnatish

GeoGebrani kompyuterga o‘rnatish yuzasidan ko‘rsatmalar:

1. Google Chrome brauzerini ishga tushiring va ***GeoGebra***ning rasmiy saytiga o‘ting: geogebra.org. Yuklab olish bo‘limi – “App Downloads”ga o‘ting.
2. Kompyuteringizga mos ilovalardan birini tanlang:

a) ***GeoGebra*** vebilovasi – Chrome brauzeri ushbu ilovadan foydalanish uchun kompyuterdan administratorlikni talab qilmaydi, faqat ilovadan foydalanish jarayonida kompyuter internetga ulangan bo‘lishi kerak;

b) ***GeoGebra*** ilovasi – Windows, Mac OS X, Linux va boshqa operatsion tizimlarda internetga ulanmasdan foydalanish mumkin bo‘lgan ilova.

GeoGebrani planshetga o‘rnatish yuzasidan ko‘rsatmalar:

1. Android tizimidagi qurilmalarda Google Play Market yoki iOS tizimidagi qurilmalarda Apple Store ilovasiga kiring.
2. ***GeoGebra*** ilovasini qidiruvga bering.
3. Ilovani qurilmangizga yuklab oling.
4. Muqobil variant sifatida ***GeoGebra***ning rasmiy saytiga o‘ting va yuklab olish bo‘limi – **“App Downloads”**dan planshet uchun ilovani yuklab oling.

“GeoGebra”da amaliy topshiriqlar bajarish

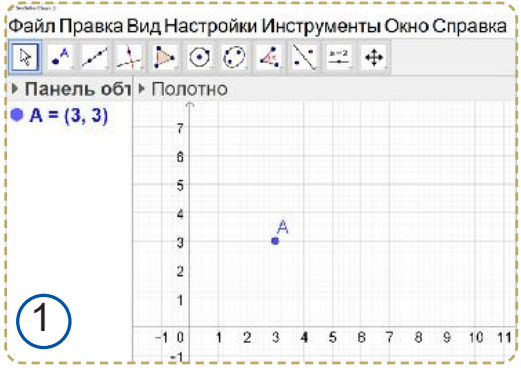
1. Nuqta yasash

Nuqta eng sodda va eng kichkina geometrik shakldir. Uning tasvirini hosil qilish uchun:

1) “Точка” (“Nuqta”) uskunasi ustiga kursorni olib borib, sichqonchaning chap tugmasini bosamiz;

2) ishchi maydonning qayeriga nuqtani qo‘ymoqchi bo‘lsak, o‘sha joyga sichqonchaning chap tugmasini bosamiz;

3) ekranda nuqta hosil bo‘ladi *(1-rasm)*.



Nuqta tasviri ustiga sichqonchaning chap tugmasini bosib, uning o‘rnini almashtirish, ya’ni siljitish ham mumkin.

Nuqtaning rangi va stilini aniqlash hamda uni harf bilan belgilash ham mumkin. Buning uchun ekranning chap qismida joylashgan obyektlar panelidan foydalanish kerak. Sichqonchaning o‘ng tugmasini “Точка” (“Nuqta”) uskunasi ustiga bosib, qo‘shimcha darcha ochiladi. Undagi “Свойство” (“Xossa”) qatoriga sichqonchaning chap tugmasi bosiladi va tegishli rang yoki stil tanlanadi.

5 BURCHAKNING TURLARI

5.1. To‘g‘ri, o‘tkir va o‘tmas burchaklar

Burchaklar kattaligiga qarab turlarga ajratiladi. Agar burchakning gradus o‘lchovi:

90° dan kichik bo‘lsa (*1a-rasm*), *o‘tkir burchak*;

90° ga teng bo‘lsa (*1b-rasm*), *to‘g‘ri burchak*;

90° dan katta va 180° dan kichik bo‘lsa (*1c-rasm*), *o‘tmas burchak* deb ataladi.

Chizmada burchakning to‘g‘ri burchak ekanligi alohida, 1*b*-rasmdagidek belgilanadi.

Masala. Agar $\angle AOD = 135°$,

$\angle AOB = \angle BOC = \angle COD$ bo‘lsa *(2a-rasm)*,

a) chizmada nechta o‘tkir, o‘tmas va to‘g‘ri burchak borligini aniqlang;

b) AOB va COD burchaklarning bissektrisalari orasidagi burchakni toping.

Yechish. a) $\angle AOB = \angle BOC = \angle COD = \alpha$ bo‘lsin. U holda, burchaklarni o‘lchashning asosiy xossasiga ko‘ra, $\angle AOD = \alpha+\alpha+\alpha = 135°$. Bundan $\alpha = 45°$. Demak, $\angle AOC = 2\alpha = 90°$, $\angle BOD = 2\alpha = 90°$. Shunday qilib, chizmada 3 ta o‘tkir, 2 ta to‘g‘ri va 1 ta o‘tmas burchak bor.

b) OO_1 va OO_2 mos bissektrisalar bo‘lsin *(2b-rasm)*. $\angle AOB = \angle COD = 45°$ bo‘lgani uchun burchak bissektrisasining ta’rifiga ko‘ra:

$$\angle O_1OB = \angle O_2OC = \frac{\alpha}{2} = 22{,}5°.$$

Izlanayotgan burchakni topamiz:

$$\angle O_1OO_2 = \angle O_1OB + \angle BOC + \angle COO_2 =$$
$$= \frac{\alpha}{2} + \alpha + \frac{\alpha}{2} = 2\alpha = 90°,$$

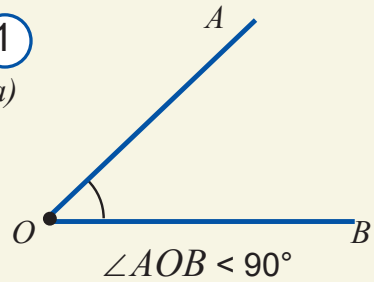
ya’ni O_1OO_2 – to‘g‘ri burchak.

Javob: AOB va COD burchaklarning bissektrisalari orasidagi burchak 90° ga teng.

Eslatma. Odatda burchak o‘lchovi grek alifbosining kichik α (alfa), β (beta), γ (gamma) ... kabi harflari bilan belgilanadi.

1

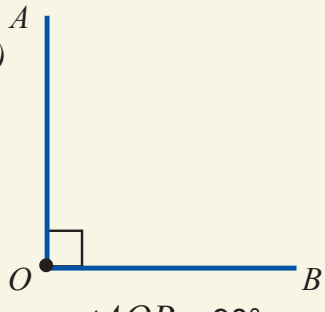
a)



$\angle AOB < 90°$

AOB o‘tkir burchak

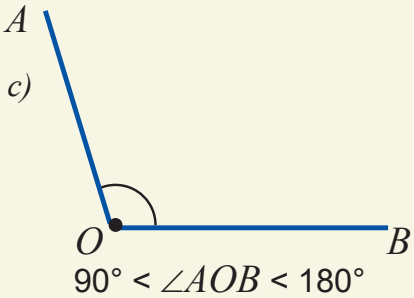
b)



$\angle AOB = 90°$

AOB to‘g‘ri burchak

c)

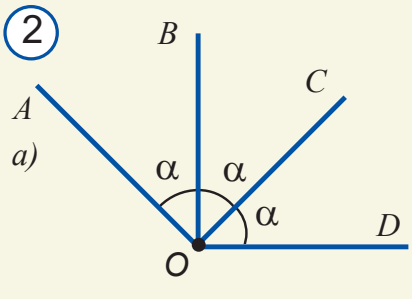


$90° < \angle AOB < 180°$

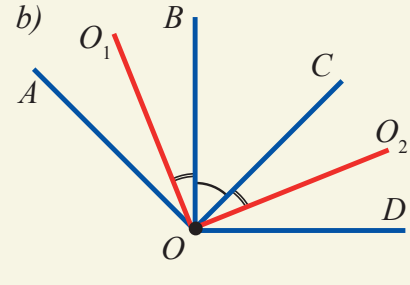
AOB o‘tmas burchak

2

a)



b)



5.2. Qo‘shni va vertikal burchaklar

Bittadan tomoni ustma-ust tushib, qolgan tomonlari bir-birini to‘ldiruvchi nurlardan iborat bo‘lgan ikkita burchak *qo‘shni burchaklar* deyiladi.

3-rasmda AOB va BOC qo‘shni burchaklar tasvirlangan. Ularda OB tomon umumiy, OC va OA nurlar esa bir to‘g‘ri chiziqda yotadi va bir-birini to‘ldiradi.

Bu AOC burchakning yoyiq burchak ekanligidan dalolat beradi. Ikkinchi tomondan, ta’rifga ko‘ra, AOC burchak AOB va BOC qo‘shni burchaklar yig‘indisidan iborat. Mulohazalarimiz quyidagi xossaning o‘rinli ekanini ko‘rsatadi:

Xossa: **qo‘shni burchaklar yig‘indisi 180° ga teng.**

Bu xossadan bevosita quyidagi natijalar ham kelib chiqadi:

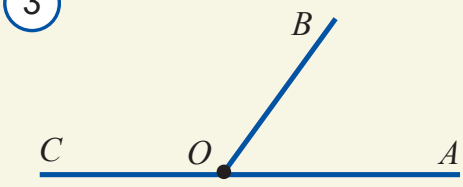
1-natija. **Agar qo‘shni burchaklar teng bo‘lsa, ular to‘g‘ri burchak bo‘ladi.**

2-natija. **To‘g‘ri burchakka qo‘shni burchak ham to‘g‘ri burchak bo‘ladi.**

3-natija. **Qo‘shni burchaklarning biri o‘tkir (o‘tmas) bo‘lsa, ikkinchisi o‘tmas (o‘tkir) bo‘ladi.**

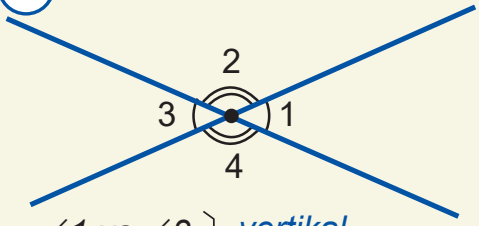
Vertikal burchaklar deb birining tomonlari ikkinchisining tomonlari davomidan iborat nurlardan tashkil topgan burchaklarga aytiladi. 4-rasmda $\angle 1$ va $\angle 3$ vertikal burchaklardir. Shuningdek, $\angle 2$ va $\angle 4$ ham vertikal burchaklar juftini hosil qiladi. Endi vertikal burchaklarning quyidagi xossasini isbotlaymiz.

3



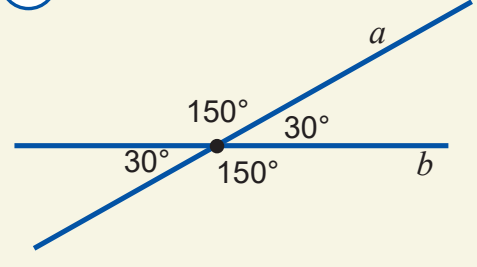
$\angle AOB$, $\angle BOC$ — *qo‘shni burchaklar*

4



$\angle 1$ va $\angle 3$, $\angle 2$ va $\angle 4$ — *vertikal burchaklar*

5



Xossa: **vertikal burchaklar o‘zaro teng.**

Aytaylik, $\angle 1$ va $\angle 3$ vertikal burchaklar berilgan bo‘lsin *(4-rasm)*. $\angle 1 = \angle 3$ bo‘lishini isbotlaymiz.

Isbot: $\angle 1 + \angle 2 = 180°$, chunki $\angle 1$ va $\angle 2$ qo‘shni burchaklardir.

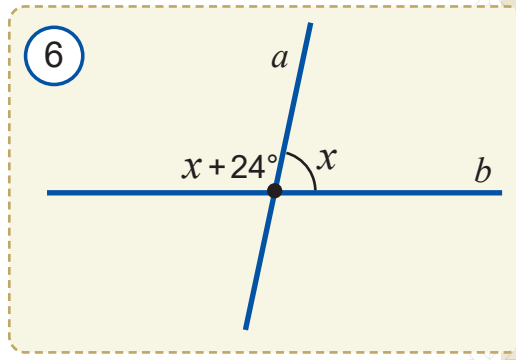
$\angle 2 + \angle 3 = 180°$, chunki $\angle 2$ va $\angle 3$ lar ham qo‘shni burchaklardir.

Bu ikki tenglikdan $\angle 1 + \cancel{\angle 2} = \cancel{\angle 2} + \angle 3$, ya’ni $\angle 1 = \angle 3$ ekanini hosil qilamiz.

Xossa isbotlandi.

Shunday qilib, ikki to‘g‘ri chiziq kesishganda vertikal va qo‘shni burchaklar hosil bo‘ladi. Ma’lumki, qo‘shni burchaklar jufti o‘zaro yoyiq burchakni tashkil qiladi. Ularning biri 90° dan katta bo‘lsa, ikkinchisi 90° dan kichik bo‘ladi. Qo‘shni burchaklardan kichigining gradus o‘lchovini *to‘g‘ri chiziqlar orasidagi burchak* deb atash qabul qilingan. 5-rasmdagi to‘g‘ri chiziqlar orasidagi burchak 30° ni tashkil etadi. Buni *“to‘g‘ri chiziqlar 30° li burchak ostida kesishadi”*, deb ham aytamiz.

Masala. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklardan biri ikkinchisidan 24° katta bo‘lsa, bu burchaklarni toping.



Yechish. Ma’lumki, ikki a va b to‘g‘ri chiziq kesishishidan hosil bo‘lgan burchaklar qo‘shni yoki vertikal burchaklar bo‘ladi *(6-rasm)*. Lekin vertikal burchaklar o‘zaro teng bo‘ladi.

Demak, masala shartida berilgan burchaklar qo‘shni burchaklar ekan. Ularning birini (kichigini) x bilan belgilasak, ikkinchisi $x+24°$ ga teng bo‘ladi. Qo‘shni burchaklar xossasiga ko‘ra, $x+x+24°=180°$. Bundan $x = 78°$ va $x+24°=102°$ ekanini aniqlaymiz.

Javob: a va b to‘g‘ri chiziqlar kesishganda 78°, 102°, 78° va 102° li burchaklar hosil bo‘ladi.

5.3. Geometriyani o‘rganishda fikrlar ketma-ketligi va bog‘liqligi

Shu paytgacha qator geometrik shakllar va ularning xossalari bilan tanishib chiqdik. Masalan, o‘tgan mavzuda vertikal burchaklar bilan tanishdik va ularning o‘zaro teng bo‘lishini ko‘rsatdik. Eslasangiz, bu xossa bilan shunchaki tanishmasdan, uni isbotladik. “Vertikal burchaklar teng” degan tasdiqning to‘g‘riligini mulohaza yuritish orqali asosladik. Bu “isbot” tushunchasi bilan ilk tanishishimiz bo‘ldi. Geometriyaga birinchi bo‘lib “isbot” tushunchasini olib kirgan matematik miloddan avvalgi 625–527-yillarda yashagan miletlik yunon olimi Fales hisoblanadi.

Biror tasdiqning to‘g‘riligini mantiqiy mulohazalar yordamida keltirib chiqarish *isbot* deb ataladi. To‘g‘riligi isbotlash yo‘li bilan asoslanadigan tasdiq esa *teorema* deb ataladi. Teorema odatda shart va xulosa qismlardan iborat bo‘ladi. Teoremaning birinchi – shart qismida nimalar berilgani bayon qilinadi. Ikkinchi – xulosa qismida esa nimani isbotlash lozimligi ifodalanadi. Masalan, quyidagi teoremani olib qaraylik.

Teorema. **Agar qo‘shni burchaklar o‘zaro teng bo‘lsa, ularning har ikkisi ham to‘g‘ri burchak bo‘ladi.**

Bu teoremaning *shart qismi* – “o‘zaro qo‘shni burchaklarning teng”ligi bo‘lsa, *xulosa qismi* – “ularning har ikkalasi ham to‘g‘ri burchak” bo‘lishidan iborat.

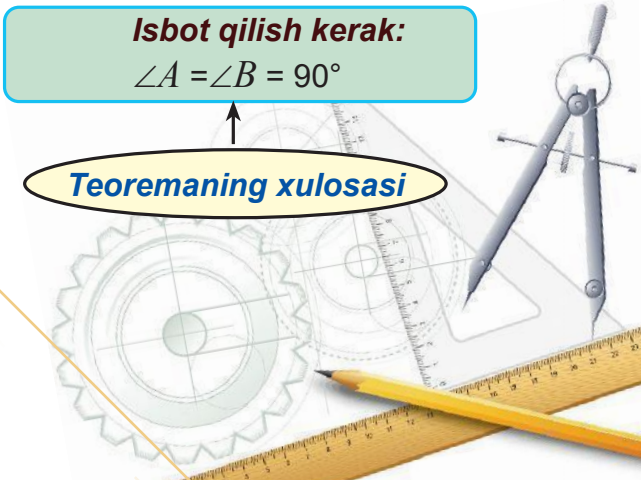
Teoremani isbotlash – uning shartidan foydalanib, bungacha isbotlangan va qabul qilingan xossalarga tayanib, mulohaza yuritib, xulosa qismida ifodalangan jumlaning to‘g‘riligini keltirib chiqarishdir. Teoremaning shart va xulosa qismlarini aniqlashtirib olish teoremani oydinlashtiradi, uni tushunish va isbotlash jarayonini yengillashtiradi. Shu bois teoremani isbotlashdan oldin uni shart va xulosa qismlarga ajratib, qayta yozib olish maqsadga muvofiq bo‘ladi. Masalan, yuqorida keltirilgan teoremani quyidagi ko‘rinishda qayta yozib olish mumkin:

Berilgan: $\angle A$ va $\angle B$ qo‘shni burchaklar, $\angle A = \angle B$

Teoremaning sharti

Isbot qilish kerak: $\angle A = \angle B = 90°$

Teoremaning xulosasi



Umuman olganda, teoremani shart va xulosa qismlarga ajratib, quyidagi sxema ko‘rinishida tasvirlash mumkin:

Agar **A jumla** o‘rinli bo‘lsa, **B jumla** o‘rinli bo‘ladi.

Teoremaning sharti — ***Teoremaning xulosasi***

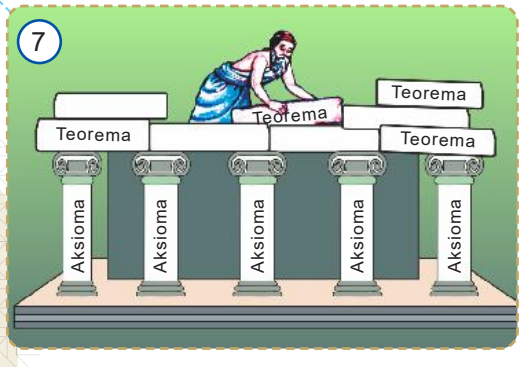
Boshlang‘ich tushuncha va aksiomalar. Geometriyada tushunchalar ma’lum uzviy izchillik va mantiqiy ketma-ketlikda kiritiladi. Nuqta, to‘g‘ri chiziq va tekislik kabi tushunchalar geometriyaning boshlang‘ich tushunchalari hisoblanadi. Ularga ta’rif bermadik. *Geometriyaning boshlang‘ich tushunchalari* ta’rifsiz to‘g‘ridan to‘g‘ri kiritiladigan tushunchalardir. Geometriyani bir bino deb olsak, bu tushunchalar uning poydevoridir. Boshlang‘ich tushunchalardan foydalanib boshqa yangi tushunchalar aniqlanadi, ya’ni ularga *ta’rif* beriladi.

Shuningdek, shu paytgacha nuqta, to‘g‘ri chiziq va tekislikning ba’zi xossalarini ham isbotsiz, to‘g‘ridan to‘g‘ri qabul qildik. Bunday xossalar *aksiomalar* deb ataladi. Agar e’tibor bergan bo‘lsangiz, darslikda barcha aksiomalarni asosiy matndan alohida ajratib, maxsus belgi ostida berib keldik. Shu paytgacha tanishib chiqqan aksiomalarimizga misollar keltiramiz (qolganlarini darslik sahifalaridan topib, yozib chiqing):

1. Tekislikda qanday to‘g‘ri chiziq olinmasin, unda bu to‘g‘ri chiziqqa tegishli bo‘lgan nuqtalar ham, tegishli bo‘lmagan nuqtalar ham mavjud.

2. Har qanday ikki nuqtadan faqat bitta to‘g‘ri chiziq o‘tkazish mumkin.

3. To‘g‘ri chiziqda olingan istalgan uchta nuqtadan faqat bittasi qolgan ikkitasining orasida yotadi.



Yunon olimi Platon geometriyada ajoyib bir qonuniyatni payqagan: avval o‘rganilgan, to‘g‘riligi isbotlangan xossalardan mantiqiy fikrlash, mushohada yuritish orqali yangi xossalarni keltirib chiqarsa bo‘lar ekan.Bunday ajoyib imkoniyatdan foydalanib, qolgan xossalar teoremalar ko‘rinishida ifodalanadi va aksiomalar hamda bu paytgacha to‘g‘riligi isbotlangan xossalarga asoslanib, mantiqiy mulohazalar yuritish orqali isbotlanadi.

Mulohaza yuritish jarayonida isbotlanmagan xossalardan (garchi ularning to‘g‘riligi ochiq-oydin ko‘rinib turgan bo‘lsa ham) foydalanish taqiqlanadi.

Bunday mashg‘ulot o‘quvchilarning fikrlash qobiliyatini o‘stirgani uchun geometriya maktablarda asosiy fanga aylangan.

Shunday qilib, geometriyani bir bino deb qaraydigan bo‘lsak *(7-rasm)*, boshlang‘ich tushunchalar va aksiomalar uning poydevorini tashkil qiladi. Bu poydevor ustiga terilgan g‘ishtlar ta’riflangan yangi tushunchalardan va teoremalar ko‘rinishida isbotlangan xossalardan iborat bo‘ladi.

Geometriyada analogiyalar. Ba'zida kesma va burchaklar xossalari haqidagi masalalarni yechishda bir xil usul yoki yondashuvlardan foydalaniladi. Bunga sabab ushbu geometrik shakllar ba'zi xossalarining bir-biriga o'xshashligidir. Bunday o'xshashlik fanda analogiya deb ataladi. Analogiya tushunchasini quyidagi misolda tushuntirishga harakat qilamiz.

1-masala. Uzunligi 32 cm bo'lgan AB kesmada C nuqta olingan. AC va CB kesmalar o'rtalari orasidagi masofani toping *(8-rasm)*.

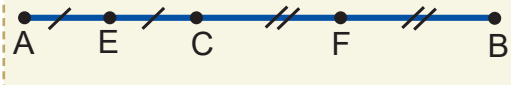
2-masala. 120° li AOB burchakni OC nur ikki burchakka ajratadi. AOC va COB burchaklar bissektrisalari orasidagi burchakni toping *(9-rasm)*.

Bir qarashda bu masalalar bir-biriga hech o'xshamaydi. Chunki ularning birida kesma haqida, ikkinchisida esa burchaklar haqida so'z boradi.

Shunday bo'lsa-da, ularning umumiy tomonlari ham bor. Har ikki masalada bir butun narsa ikkiga bo'lingan. Ikkinchi tomondan, kesmaning o'rtasi kesmani, burchakning bissektrisasi esa burchakni teng ikkiga bo'ladi.

Yechish.

8



E va F nuqtalar mos ravishda AC va CB kesmalar o'rtalari bo'lsin. U holda,

$$EC = \frac{AC}{2} \quad \text{va} \quad CF = \frac{CB}{2}.$$

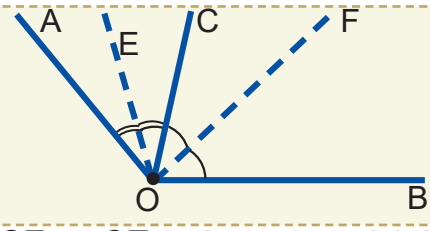
Bundan:

$$EF = EC + CF = \frac{AC}{2} + \frac{CB}{2} = \frac{AB}{2}$$

$EF = 32 : 2 = 16$ (cm).

Yechish.

9



OE va OF nurlar mos ravishda $\angle AOC$ va $\angle COB$ burchaklar bissektrisalari bo'lsin. U holda,

$$\angle EOC = \frac{\angle AOC}{2} \quad \text{va} \quad \angle COF = \frac{\angle COB}{2}.$$

Bundan:

$$\angle EOF = \angle EOC + \angle COF = \frac{\angle AOC}{2} + \frac{\angle COB}{2} = \frac{\angle AOB}{2}$$

$\angle EOF = 120^\circ : 2 = 60^\circ$.

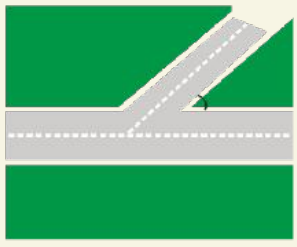
Ko'rib turganingizdek, har ikki masalani yechish g'oyasi ham bir-biriga o'xshab ketadi. Shuning uchun birinchi masalani yechishda qo'llagan mulohazalarimizni ikkinchi masalani yechishga ham bosqichma-bosqich qo'llashimiz mumkin bo'ldi. Bunday hollarda "ikkinchi masala yechimi birinchi masala yechimiga o'xshash" deb aytiladi.

Mavzu bo'yicha savollar

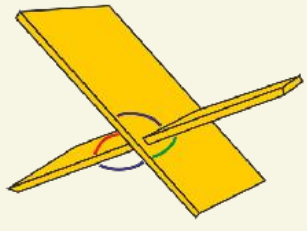
1. Burchaklar kattaligiga qarab qanday turlarga ajratiladi?
2. Tevarak-atrofdan o'tkir, o'tmas va to'g'ri burchaklarga misollar keltiring.
3. Qanday burchaklar qo'shni burchaklar deyiladi?
4. Qo'shni burchaklarning yig'indisi nimaga teng? Javobingizni asoslang.
5. Qo'shni burchaklar o'zaro teng bo'lishi mumkinmi? Qachon?

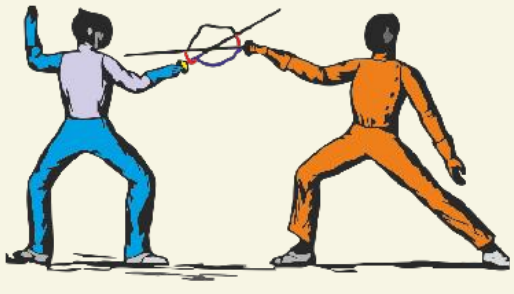
10





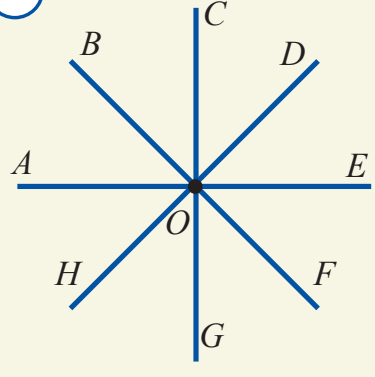




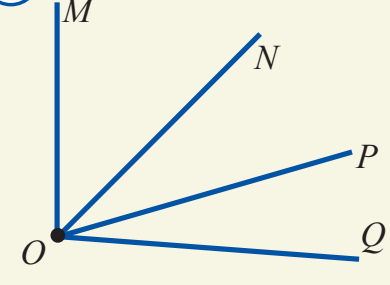


6. Qanday burchaklar vertikal burchaklar deb ataladi?
7. Vertikal burchaklarning asosiy xossasini izohlang.
8. 10-rasmda qanday burchaklarni ko‘ryapsiz?
9. *Qo‘shni burchaklarning ikkalasi ham: a) o‘tkir; b) to‘g‘ri; c) o‘tmas burchaklar bo‘la oladimi?*
10. *Agar ikki burchak teng bo‘lsa, ularga qo‘shni bo‘lgan burchaklar ham teng bo‘ladimi?*
11. *Ta’rif nima? Qaysi tushunchalar ta’rifsiz qabul qilinadi?*
12. *Teorema nima? U qanday qismlardan iborat?*
13. *Teoremalar qanday isbotlanadi? Isbot deganda nimani tushunasiz?*
14. *Aksioma nima?*

11



12



Amaliy mashq va tatbiq

1. Uchta burchak chizing. Ularni mos ravishda $\angle AOB$, $\angle MNL$, $\angle PQR$ tarzda belgilang. Transportirda ularni o‘lchang va turlarini aniqlang.
2. OA nur chizing. Transportir yordamida gradus o‘lchovi mos ravishda 25°, 72° va 146° bo‘lgan AOB, AOC va AOD burchaklarni yasang.
3. To‘g‘ri burchakning bissektrisasi uning bitta tomoni bilan qanday burchak hosil qiladi?
4. OC nur AOB burchakning bissektrisasi. OD esa AOC burchakning bissektrisasi. AOB burchak DOC burchakdan necha marta katta?
4. 11-rasmda nechta: a) o‘tkir; b) o‘tmas; c) to‘g‘ri; d) yoyiq burchak bor?
5. 12-rasmda nechta o‘tkir va nechta o‘tmas burchak bor?
6. Qog‘oz varag‘ini buklab to‘g‘ri burchak hosil qila olasizmi?
7. Soatning soat va minut millari to‘g‘ri burchak hosil qiladigan vaqtlardan bir nechtasini ayting.

8. Soatning minut va soat millari hosil qilgan burchaklarni toping *(13-rasm)*.

9*. Soatning soat mili: a) 1 soatda; b) 6 soatda; c) 2 minutda necha gradusga buriladi?

10. a) 20°; b) 90°; c) 145°; d) 9° li burchakka qo‘shni bo‘lgan burchak necha gradusli bo‘ladi?

11. a) 34°; b) 109°; c) 5°; d) 167° li burchakka qo‘shni bo‘lgan burchak necha gradusli bo‘ladi?

12. Agar qo‘shni burchaklarning biri ikkinchisidan uch marta katta bo‘lsa, ularni toping.

13. Agar qo‘shni burchaklarning biri ikkinchisidan to‘rt marta kichik bo‘lsa, ularni toping.

14. 14-rasmdagi noma’lum x burchakni toping.

15. Agar qo‘shni burchaklar gradus o‘lchovlari nisbati a) 2:7; b) 11:25; c) 1:9 bo‘lsa, ularni toping.

16. Agar ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklardan biri 40° bo‘lsa, qolgan burchaklarni toping.

17. $\angle 1$ va $\angle 2$ qo‘shni burchaklar. Quyidagi jadvalni to‘ldiring.

$\angle 1$	34°			19°	175°
$\angle 2$		118°	132°		

18. $\angle 1$ va $\angle 2$ qo‘shni burchaklar. Quyidagi jadvalni to‘ldiring.

$\angle 1$	12°		120°		45°
$\angle 2$		18°		165°	

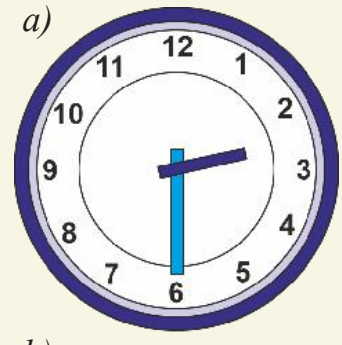
19*. $\angle ABC$ =30°, $\angle CBD$ =80°. $\angle ABD$ ni toping. Hamma hollarni ko‘ring.

20*. $\angle MON$ =45°, $\angle NOL$ =104°. $\angle MOL$ ni toping. Hamma hollarni ko‘ring.

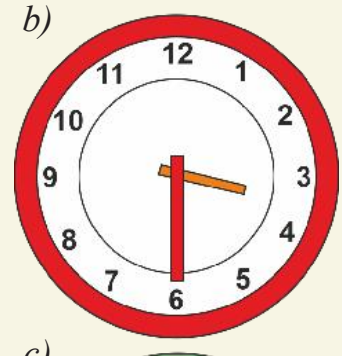
21. 15-rasmdagi noma’lum x burchakni toping.

13

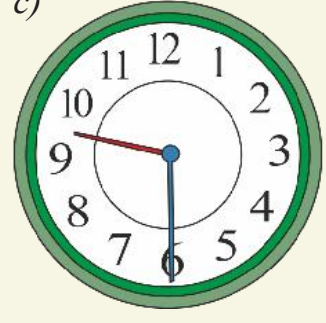
a)



b)

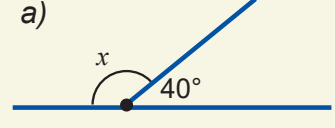


c)

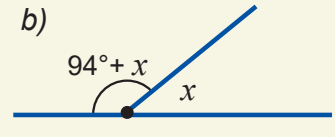


14

a)

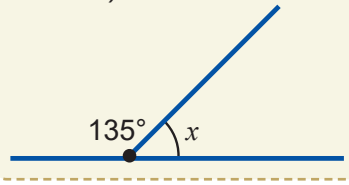


b)

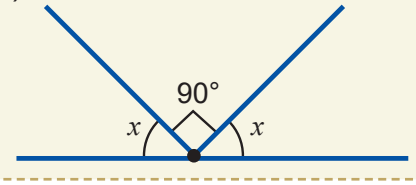


15

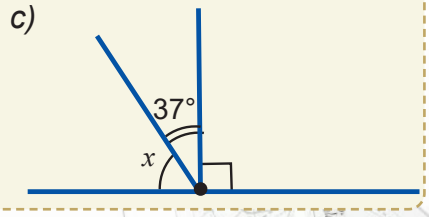
a)



b)



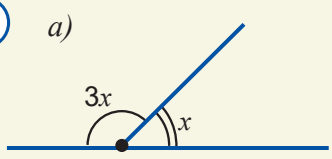
c)



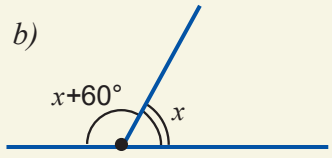
22*. 16-rasmdagi noma'lum x burchaklarni toping.

16

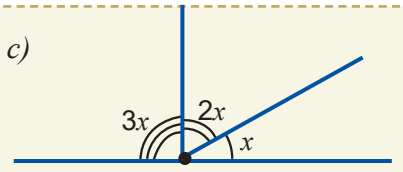
a)



b)



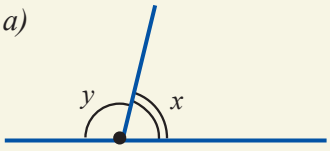
c)



23*. 17-rasmga qarab masala tuzing va uni yeching.

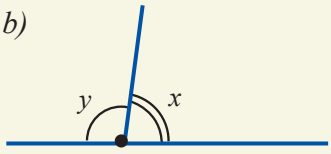
17

a)



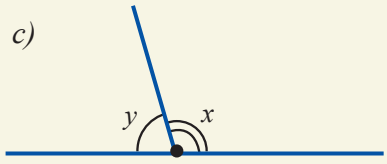
$y - x = 30°$

b)



$x : y = 4 : 5$

c)

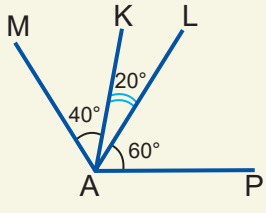


$2x = 3y$

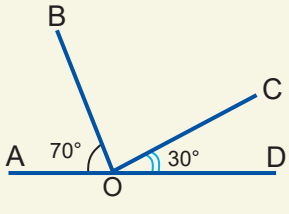
24. 18-rasmdagi burchaklarning gradus o'lchovlarini toping.

18

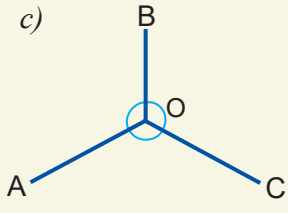
a)



b)



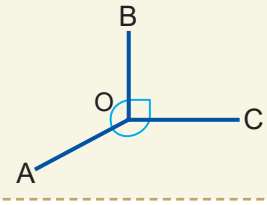
c)



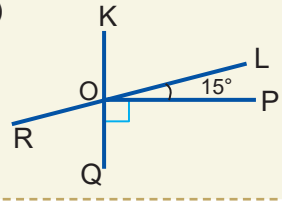
25. 19-rasmdagi burchaklarning gradus o'lchovlarini toping.

19

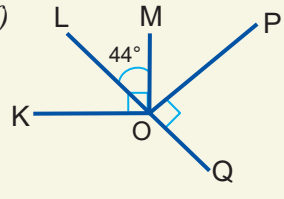
d)



e)



f)

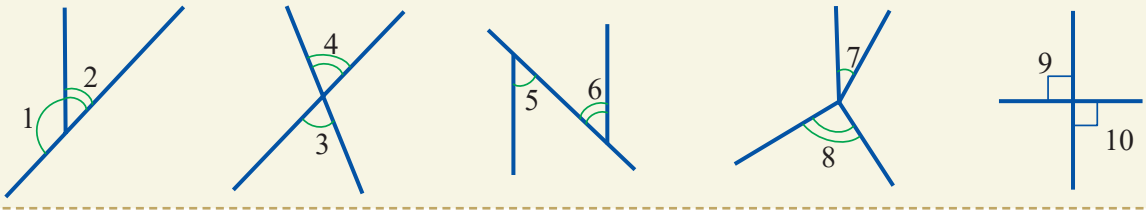


26*. Transportir yordamida $\angle PQR = 45°$ burchakni yasang. Bu burchakka qo'shni va QR tomon umumiy bo'lgan qo'shni burchakni yasang va uning gradus o'lchovini toping.

27*. Transportir yordamida $\angle MNL = 120°$ burchakni yasang. Bu burchakka qo'shni va MN tomon umumiy bo'lgan qo'shni burchakni yasang va uning gradus o'lchovini toping.

28. 20-rasmdan: a) vertikal; b) qo'shni burchaklar juftini toping.

20



29. AOB burchak OC, OD va OE nurlar bilan to'rtta teng burchakka bo'lingan. Bu nurlar qaysi burchaklarning bissektrisasi bo'ladi?

30*. To'g'ri chiziqda A, B va C nuqtalar berilgan. Agar $AB = 42\ cm$, $AC = 3\ dm\ 2\ cm$ va $BC = 74\ cm$ bo'lsa, bu nuqtalarning qaysi biri qolganlarining orasida yotadi? Javobingizni asoslang.

6 PERPENDIKULYAR TO‘G‘RI CHIZIQLAR

6.1. Perpendikulyar to‘g‘ri chiziqlar

Faollashtiruvchi mashq

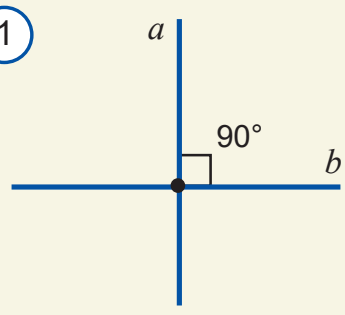
Ikki to‘g‘ri chiziq kesishganda hosil bo‘lgan burchaklarning bittasi to‘g‘ri burchak bo‘lsa *(1-rasm)*, qolgan burchaklar haqida nima deyish mumkin?

To‘g‘ri (90° li) burchak ostida kesishuvchi to‘g‘ri chiziqlar *perpendikulyar to‘g‘ri chiziqlar* deb ataladi.

Qishda tarnovdan yerga tik (perpendikulyar) o‘sib tushgan sumalaklarga*(2-rasm)* ko‘zingiz tushgan bo‘lsa kerak. 1-rasmda bir-biriga perpendikulyar a va b to‘g‘ri chiziqlar tasvirlangan. Bu to‘g‘ri chiziqlarning perpendikulyarligi maxsus belgi yordamida $a \perp b$ tarzda yoziladi va “a to‘g‘ri chiziq b to‘g‘ri chiziqqa perpendikulyar” deb o‘qiladi.

Perpendikulyar to‘g‘ri chiziqlar kesishishidan to‘rtta to‘g‘ri burchak hosil bo‘ladi.

1



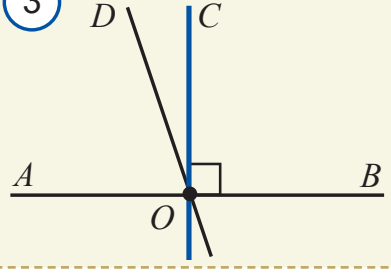
$a \perp b$ – a to‘g‘ri chiziq b to‘g‘ri chiziqqa perpendikulyar

2



Teorema. **To‘g‘ri chiziqning ixtiyoriy nuqtasidan shu to‘g‘ri chiziqqa yagona perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin.**

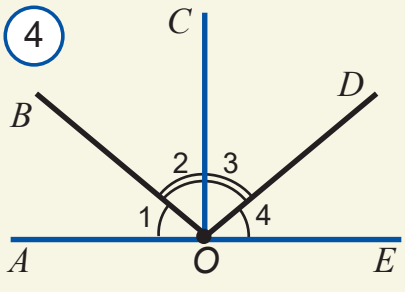
3



Isbot. Aytaylik, AB to‘g‘ri chiziq va undagi O nuqta berilgan bo‘lsin *(3-rasm)*. Ma’lumki, OB nurga uchi O nuqtada bo‘lgan 90° li COB burchak qo‘yish mumkin. Unda CO to‘g‘ri chiziq AB to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziq bo‘ladi.

Endi bu to‘g‘ri chiziqning yagona ekanini isbotlaylik. Teskarisini faraz qilamiz: O nuqtadan o‘tuvchi, berilgan AB to‘g‘ri chiziqqa perpendikulyar yagona bo‘lmasin, ya’ni yana bitta perpendikulyar DO to‘g‘ri chiziq mavjud bo‘lsin. U holda DOB va COB burchaklarning har biri 90° li bo‘lib, OB nurga qo‘yilgan burchaklar bo‘lib qoladi. Lekin bu OB nurga muayyan gradus o‘lchovga ega yagona burchak qo‘yish mumkinligi haqidagi aksiomaga zid, ya’ni bunday bo‘lishi mumkin emas.

4



Demak, AB to‘g‘ri chiziqqa uning O nuqtasidan faqat bitta perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin ekan. Teorema isbotlandi.

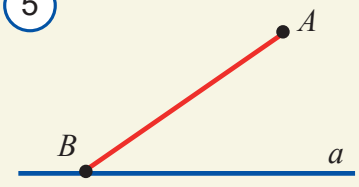
Masala. Agar 4-rasmda $\angle 1=\angle 4$, $\angle 2=\angle 3$ bo‘lsa, $CO \perp AE$ bo‘lishini ko‘rsating.

Yechish. Aytaylik, $\angle 1=\angle 4=\alpha$, $\angle 2=\angle 3=\beta$ bo‘lsin. Burchaklarni o‘lchashning xossasiga ko‘ra:

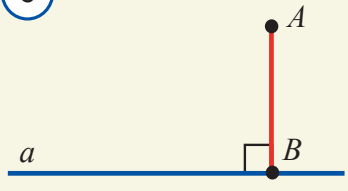
$\angle AOE=\angle 1+\angle 2+\angle 3+\angle 4=\alpha+\beta+\alpha+\beta=2\alpha+2\beta=180°$, $2(\alpha+\beta)=180°$, ya’ni $\alpha+\beta=90°$ bo‘ladi. Unda:

$\angle AOC=\angle 1+\angle 2=\alpha+\beta=90°$, ya’ni $CO \perp AE$ bo‘ladi.

5



6



a to‘g‘ri chiziq va unda yotmaydigan A nuqta berilgan bo‘lsin. A nuqtani a to‘g‘ri chiziqning biror B nuqtasi bilan tutashtiramiz *(5-rasm)*. Agar AB kesma a to‘g‘ri chiziqqa perpendikulyar bo‘lmasa, AB kesma *og‘ma* deb ataladi.

Agar AB kesma a to‘g‘ri chiziqqa perpendikulyar bo‘lsa, u holda AB kesma *A nuqtadan a to‘g‘ri chiziqqa tushirilgan perpendikulyar* deyiladi.

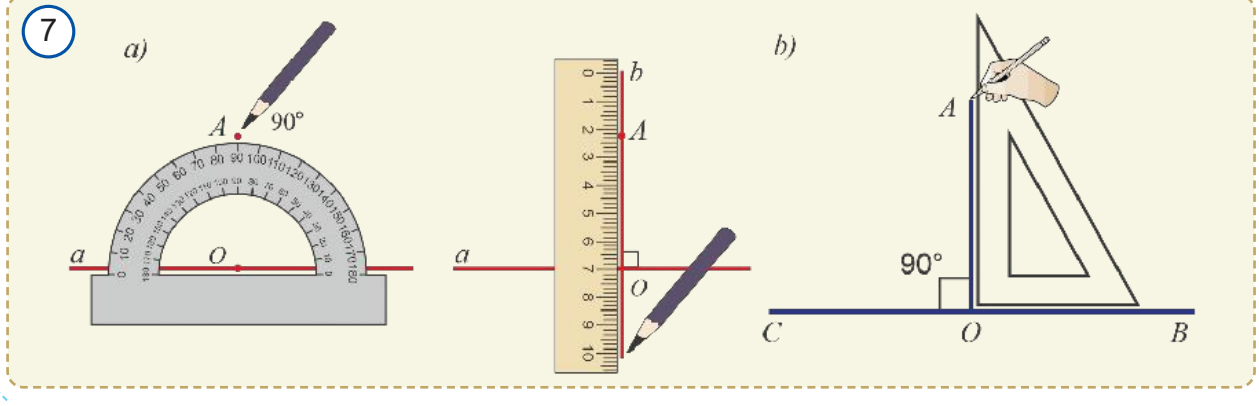
6-rasmda A nuqtadan a to‘g‘ri chiziqqa tushirilgan perpendikulyar tasvirlangan. B nuqta perpendikulyarning *asosi* deb nomlanadi.

To‘g‘ri chiziqning O nuqtasiga perpendikulyar tushirishning amaliy yo‘riqlari:

1-usul. Transportir yordamida *(7a-rasm)*.

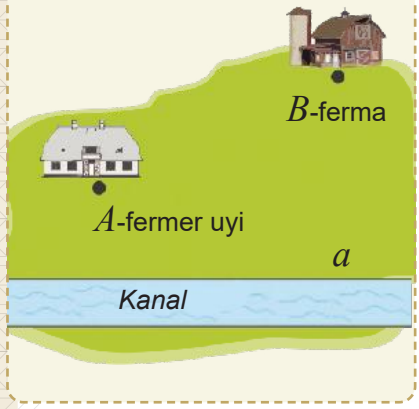
2-usul. To‘g‘ri burchakli chizg‘ich (go‘niya) yordamida *(7b-rasm)*.

7



Biror to‘g‘ri chiziq chizing. Unda yotmaydigan biror nuqtadan to‘g‘ri chiziqqa perpendikulyar va bir nechta og‘malar o‘tkazing. Perpendikulyar va og‘malarning uzunliklarini o‘lchab, o‘zaro taqqoslang. Qaysi kesmaning uzunligi eng kichik bo‘ladi? Javobingizni faraz (gipoteza) ko‘rinishida ifodalang. Bu farazning to‘g‘riligini isbotsiz qabul qilsa bo‘ladimi yoki uni albatta isbotlash kerakmi?

8



Geometrik tadqiqot

Mashq. Dehqon-fermer xo‘jaligining xaritasi 8-rasmda berilgan.

1. Fermer uyidan fermaga olib boruvchi yo‘l qurmoqchi. Unga yo‘lni qaysi chiziq bo‘yicha qurishni maslahat berasiz? Nega? Chizmada bu yo‘lni chizib ko‘rsating.

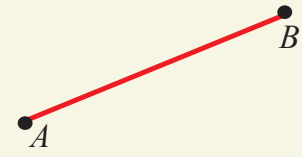
2. Fermer fermasidan kanalga olib boruvchi yo‘l qurmoqchi. Unga yo‘lni qaysi chiziq bo‘yicha qurishni maslahat berasiz? Nega? Chizmada bu yo‘lni chizib ko‘rsating.

Ma’lumki, 9-rasmda tasvirlangan A va B nuqtalarni tutashtiruvchi eng qisqa “yo‘l” bu – AB kesmadir. Shu bois quyi sinflarda AB kesma uzunligini *A va B nuqtalar orasidagi masofa* deb qabul qilgan edik. Shunga o‘xshash, *A nuqtadan a to‘g‘ri chiziqqacha bo‘lgan masofa* deb A nuqtadan a to‘g‘ri chiziqqa tushirilgan AB perpendikulyarning uzunligini qabul qilamiz. Ravshanki, bu masofa A nuqtadan a to‘g‘ri chiziqqa tushirilgan barcha og‘malar uzunligidan kichik bo‘ladi *(10-rasm)*. Bu tasdiqning isbotiga keyin to‘xtalamiz.

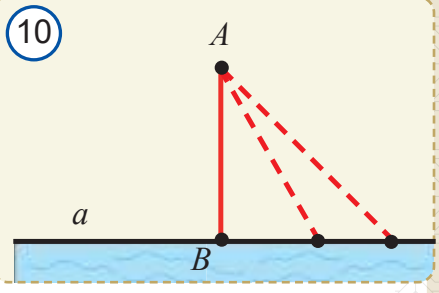
Nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa sportda ham ishlatiladi. Masalan, futbolda 11 metrlik jarima zarbasi darvoza chizig‘idan 11 metr uzoqlikda yotgan nuqtadan beriladi *(11-rasm)*.

Qurilishda devorlar va ustunlarning tikligi (polga nisbatan perpendikulyarligi) *shoqul* degan asbob yordamida tekshiriladi. Hozir quruvchilarimiz lazerli va elektron shoqullardan ham foydalanmoqda *(12-rasm)*.

9



10



11

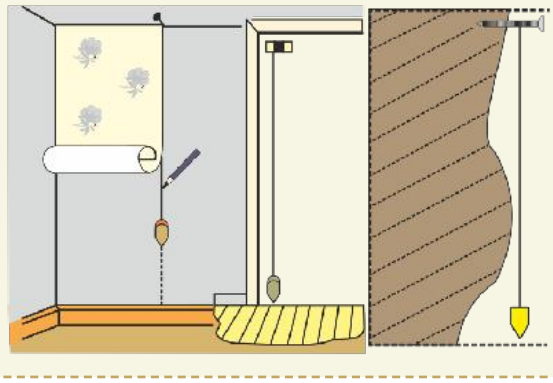


12









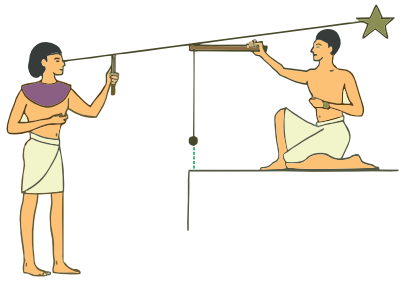




13-rasmda qadimgi Misrda burchaklarni o‘lchash jarayoni keltirilgan. Unga qarab bu ishlar qanday amalga oshirilganini ayting.

Dalada to‘g‘ri chiziqlarni o‘tkazish uchun *eker* asbobidan foydalaniladi. 14-rasmga qarab undan qanday foydalanish mumkinligini tushunib olsa bo‘ladi.

13



14



6.2. Teskarisini faraz qilib isbotlash usuli

Bu usulga ko‘ra, teoremaning sharti bajarilgan bo‘lsa-da, uning xulosasi o‘rinli bo‘lmasin deb teoremada keltirilgan tasdiqning teskarisi faraz qilinadi: AB to‘g‘ri chiziqning O nuqtasiga tushirilgan perpendikulyar yagona bo‘lmasin, ya’ni yana bitta perpendikulyar DO to‘g‘ri chiziq mavjud bo‘lsin deb olinadi *(3-rasm)*.

U holda, DOB va COB burchaklarning har biri 90° li bo‘lib, OB nurga qo‘yilgan to‘g‘ri burchaklar bo‘lib qoladi. Lekin bu OB nurga muayyan gradus o‘lchovga ega yagona burchak qo‘yish mumkinligi haqidagi aksiomaga zid. Bu farazimizning noto‘g‘ri ekanini ko‘rsatadi.

Demak, AB to‘g‘ri chiziqqa uning O nuqtasidan yagona perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin.

Teskarisini faraz qilib isbotlash usulini qo‘llash sxemasi

Teorema (to‘g‘ri tasdiq)	***Agar A o‘rinli bo‘lsa, B o‘rinli bo‘ladi.***
Isbot:	
Teskarisini faraz qilamiz	Teoremada keltirilgan tasdiqning teskarisini faraz qilamiz, ya’ni teoremaning sharti bajarilsin-u, lekin xulosasi o‘rinli bo‘lmasin: ***agar A o‘rinli bo‘lsa, B o‘rinli bo‘lmaydi.***
Mulohaza yuritamiz	To‘g‘riligi oldin isbotlangan teorema yoki qabul qilingan aksiomalarga tayanib mantiqiy mulohaza yuritamiz.
Ziddiyatga kelamiz	To‘g‘riligi oldin isbotlangan teorema yoki qabul qilingan aksiomalarning biriga zid bo‘lgan tasdiqqa duch kelib qolamiz.
Xulosa chiqaramiz	Demak, farazimiz noto‘g‘ri, ya’ni berilgan teorema to‘g‘ri ekan.
Teorema isbotlandi.	

Teskarisini faraz qilib isbotlash usulidan hayotiy vaziyatlarda ham ko‘p foydalaniladi. Masalan, shifokor bemorning gripp bilan og‘rimaganligini quyidagicha mushohada yuritib aniqlashi mumkin. Faraz qilaylik, bemor gripp bilan kasallangan bo‘lsin. Unda uning boshi og‘rishi va isitmasi yuqori bo‘lishi kerak. Lekin bemorda bunday alomatlar yo‘q. Demak, u gripp emas.

Faollashtiruvchi mashq

Quyida berilgan tasdiqqa teskari bo‘lgan tasdiqni tuzing:
a) CD kesma a to‘g‘ri chiziqni kesib o‘tadi;
b) A va B nuqtalar a to‘g‘ri chiziqning bir tomonida yotadi;
d) CD kesmaning uzunligi 15 ga teng;
e) AOB burchak to‘g‘ri burchak emas;
f) $\angle ABC > \angle MNL$;
g) AB og‘ma AC perpendikulyardan uzun.

Teorema. **Bitta to‘g‘ri chiziqqa perpendikulyar bo‘lgan ikki to‘g‘ri chiziq o‘zaro kesishmaydi.**

AA_1, BB_1 va CD to‘g‘ri chiziqlar, $AA_1 \perp CD$ va $BB_1 \perp CD$ (*15-rasm*) ⇒ AA_1 va BB_1 to‘g‘ri chiziqlar o‘zaro kesishmaydi

Isbot. Bu teoremani isbotlash uchun teskarisini faraz qilib isbotlash usulini qo‘llaymiz. Faraz qilamiz: teoremaning sharti bajarilgan bo‘lsa-da, uning xulosasi o‘rinli bo‘lmasin, ya’ni AA_1 va BB_1 to‘g‘ri chiziqlar qandaydir M nuqtada kesishsin.

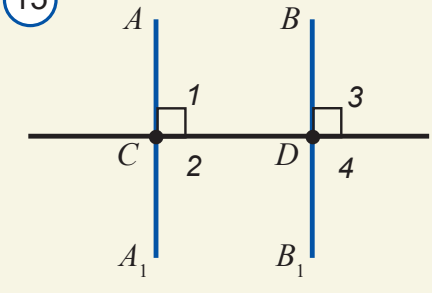
Xayolan 15-rasmni CD to‘g‘ri chiziq bo‘ylab buklaymiz, ya’ni yuqori yarimtekislikni “ag‘darib”, pastki yarimtekislikka ustma-ust qo‘yamiz. Bunda 1- va 2-to‘g‘ri burchaklar teng bo‘lgani uchun CA va CA_1 nurlar ustma-ust tushadi. Xuddi shunga o‘xshab, DB va DB_1 nurlar ham ustma-ust tushadi.

U holda, AA_1 va BB_1 to‘g‘ri chiziqlarning kesishish nuqtasi bo‘lgan M nuqta pastki yarimtekislikdagi qandaydir M_1 nuqta bilan ustma-ust tushadi. Bu esa M_1 nuqta ham AA_1 va BB_1 to‘g‘ri chiziqlarning kesishish nuqtasi ekanini anglatadi.

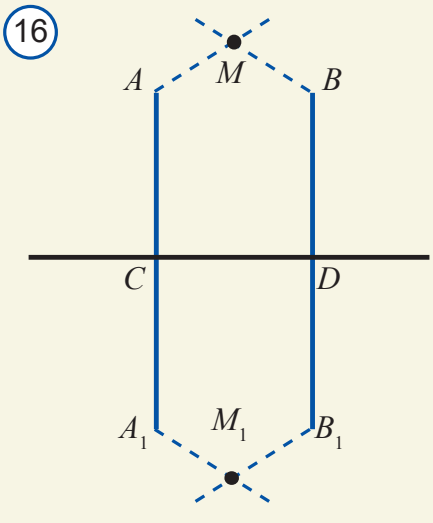
Natijada M va M_1 nuqtalardan ikkita – AA_1 va BB_1 to‘g‘ri chiziq o‘tib qoladi. Lekin bu “har qanday ikki nuqtadan faqat bitta to‘g‘ri chiziq o‘tadi” degan aksiomaga zid. Demak, qilgan farazimiz noto‘g‘ri: bitta to‘g‘ri chiziqqa perpendikulyar AA_1 va BB_1 to‘g‘ri chiziqlar o‘zaro kesishmaydi.

Teorema isbotlandi.

15



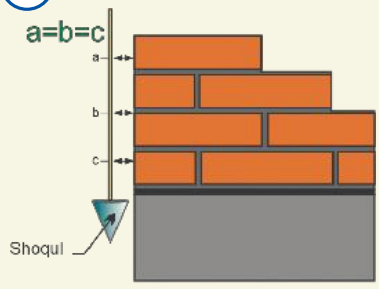
16



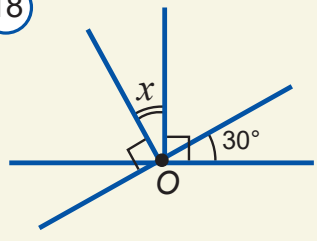
Teorema. **To‘g‘ri chiziqda yotmagan nuqtadan shu to‘g‘ri chiziqqa bittadan ortiq perpendikulyar to‘g‘ri chiziq tushirish mumkin emas.**

Bu xossani teskarisini faraz qilib isbotlash usuli yordamida mustaqil isbotlang.

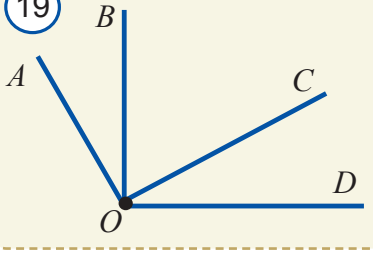
17



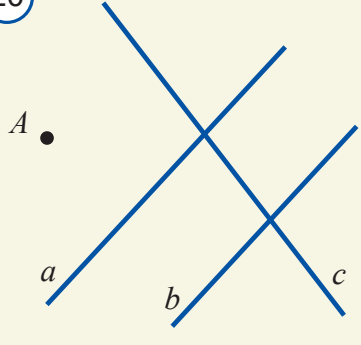
18



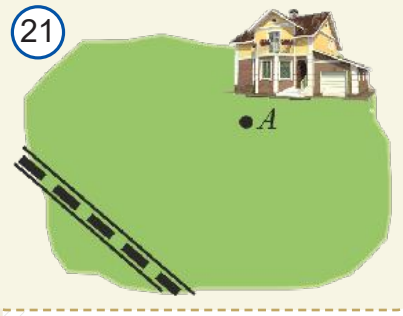
19



20



21



Mavzu bo‘yicha savollar

1. Qachon to‘g‘ri chiziqlar perpendikulyar deyiladi? Javobingizni chizmada sharhlang.
2. Berilgan to‘g‘ri chiziqda yotuvchi nuqtadan unga nechta perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin? Javobingizni izohlang.
3. Berilgan nuqtadan to‘g‘ri chiziqqa tushirilgan: a) perpendikulyar; b) og‘ma deb nimaga aytiladi?
4. 17-rasmga qarab qurilishda shoqul asbobining ishlatilishini izohlang.
5. Berilgan A nuqtadan to‘g‘ri chiziqqa nechta og‘ma tushirish mumkin?
6. Teskarisini faraz qilib isbotlash usuli qanday qoidaga asoslangan?
7. To‘g‘ri burchakka vertikal bo‘lgan burchak necha gradus?
8. a to‘g‘ri chiziq A burchakning tomonlarini B va C nuqtalarda kesib o‘tadi. AB va AC to‘g‘ri chiziqlar a to‘g‘ri chiziqqa perpendikulyar bo‘la oladimi?
9. Ikki to‘g‘ri chiziqning kesishishi natijasida 4 ta teng burchak hosil bo‘ldi. Bu to‘g‘ri chiziqlar perpendikulyar bo‘ladimi?
10. Nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa nima?

Amaliy mashq va tatbiq

1. Chizg‘ich va go‘niya yordamida berilgan to‘g‘ri chiziqqa unda yotuvchi nuqtadan o‘tuvchi perpendikulyar to‘g‘ri chiziq yasang.

2. a to‘g‘ri chiziqda A, B, C nuqtalarni belgilang va transportir yordamida bu nuqtalarning har biri orqali a to‘g‘ri chiziqqa perpendikulyar bo‘lgan to‘g‘ri chiziqlarni o‘tkazing.

3. 18-rasmdagi noma’lum x burchakni toping.

4. 19-rasmda agar $OB \perp OD$ va $OA \perp OC$ bo‘lsa, $\angle AOB = \angle COD$ bo‘lishini ko‘rsating.

5. Go‘niya yordamida A nuqtadan a, b va c to‘g‘ri chiziqlargacha bo‘lgan masofalarni toping *(20-rasm)*.

6. Transportir va oddiy chizg‘ich yordamida 21-rasmda tasvirlangan oromgohdan temiryo‘lgacha bo‘lgan eng qisqa masofani aniqlang. Masshtab (miqyos): 1 : 10 000.

7. A, B, C nuqtalar bir to‘g‘ri chiziqda yotsa va: a) $AB=3,6$; $BC=5,4$; $AC=9$; b) $AB=2,4$; $BC=4,2$; $AC=1,8$ bo‘lsa, C nuqtaning A va B nuqtalar orasida yotmasligini isbotlang. Bu nuqtalardan qaysi biri qolgan ikkitasi orasida yotadi?

8*. Qo‘shni burchaklar bissektrisalari orasidagi burchakni toping.

9*. Vertikal burchaklar tengligini teskarisini faraz qilib isbotlash usuli bilan isbotlang.

10*. Agar $\angle AOB=58°$; $\angle BOC=17°$ va $\angle AOC=41°$ bo‘lsa, OA, OB va OC nurlardan qaysi biri qolgan ikkitasining orasida yotadi?

11. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklardan ikkitasining yig‘indisi 120°. Bu burchaklarni toping.

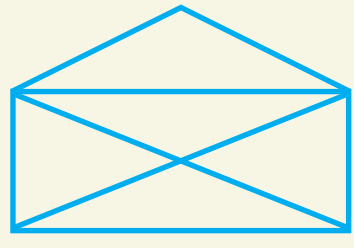
12. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklardan ikkitasining ayirmasi 20°. Bu burchaklarni toping.

13*. Vertikal burchaklarning bissektrisalari bir to‘g‘ri chiziqda yotishini isbotlang.

14*. Tekislikda uchta – A, B, C nuqta berilgan: $AB=2,6$; $AC=8,3$; $BC=6,7$. Bu nuqtalarning bir to‘g‘ri chiziqda yotmasligini isbotlang.

15*. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklardan ikkitasining yig‘indisi 180° ga teng emas. Bu burchaklarning vertikal burchaklar ekanini isbotlang.

22



Geometrik boshqotirmalar

1. a) 10 ta; b) 11 ta bir xil cho‘pdan 3 ta teng kvadrat tuzing.

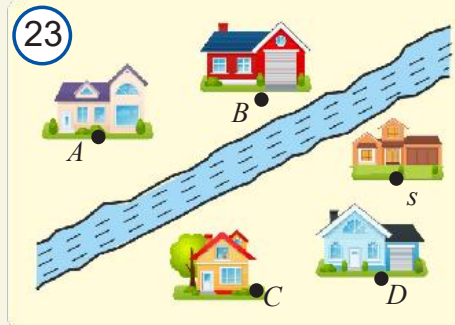
2. 12 ta bir xil cho‘pdan, ularni sindirmasdan

a) 4 ta; b) 6 ta teng kvadrat yasay olasizmi?

3. 22-rasmda ko‘rsatilgan shaklni qalamni qog‘ozdan uzmasdan va bir kesma ustidan ikki marta yurgizmasdan chizib ko‘ring.

4. Daryo bo‘yida beshta qishloq bo‘lib, ulardan uchtasi daryoning bir tomonida, qolgan ikkitasi esa daryoning ikkinchi tomonida joylashgan *(23-rasm)*. Agar har bir qishloq qolgan qishloqlar bilan bevosita yo‘llar bilan bog‘langan bo‘lsa, bu yo‘llarning nechtasi daryoni kesib o‘tadi?

23



Geometriyada AKT

Quruvchi ustalar uchun ishlab chiqilgan, “iHandy Carpenter” deb nomlangan mobil telefon dasturiy ta’minoti ixtiyoriy bino yoki inshootning ufqqa (gorizontga) nisbatan qanchalik tik ekanini aniqlab beradi. Buning uchun telefonda mazkur dasturni ishga tushirib, bino yoki inshootga qaratish kifoya *(24-rasm)*.

24



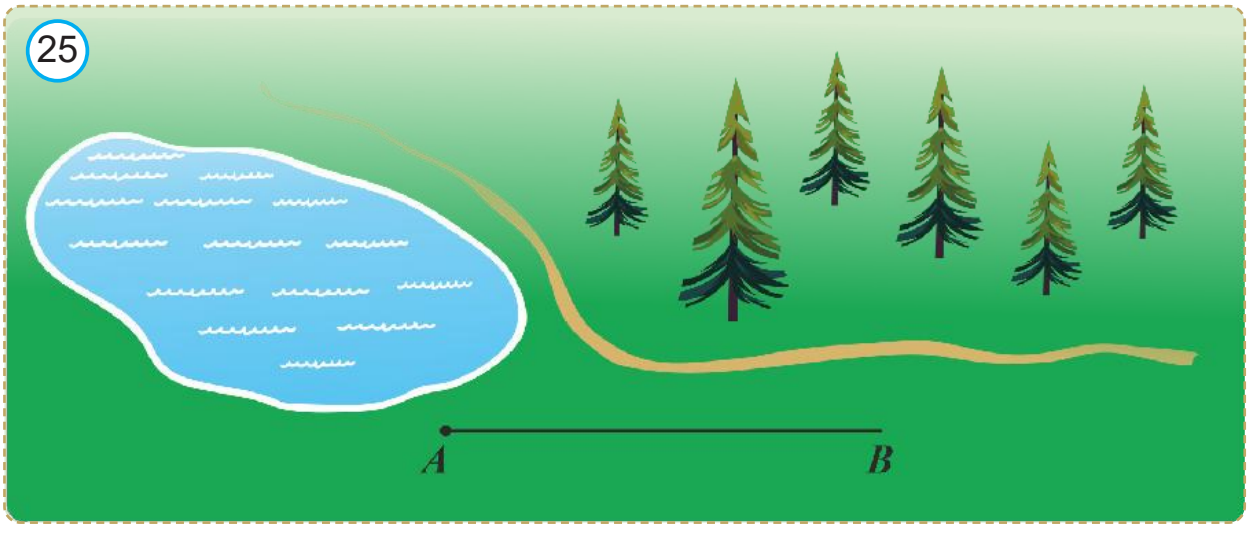
Amaliy mashq va tatbiq

1. Xazinani toping

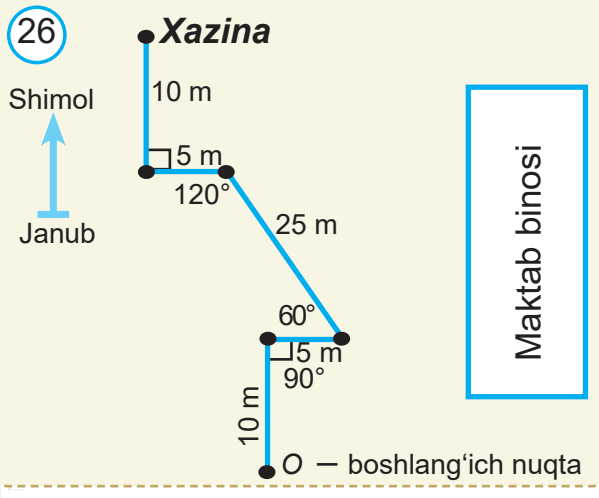
25-rasmda xarita va AB nur tasvirlangan. Bu nurga ko‘l joylashgan yarimtekislikda yotuvchi 60° li burchak qo‘ying. Yasalgan burchakning AB dan farqli tomoni bo‘ylab 60 m yuring. C nuqtaga kelasiz. CA nurga yana o‘sha ko‘l joylashgan yarimtekislikda yotuvchi 120° li burchak qo‘ying. Bu burchakning CA nurdan farqli tomoni bo‘ylab 120 m yuring. Shu yerda – baland qarag‘ay ostida xazina ko‘milgan.

Xarita masshtabi (miqyosi): 1:2000. Xaritaning o‘lchamlarini o‘zgartirmagan holda uni daftaringizga chizib oling. Xazina yashirilgan nuqtani toping.

25



26



2. Ochiq havoda geometrik musobaqa

Musobaqada ikki yoki undan ortiq guruhlar qatnashishi mumkin. Har bir guruhga ruletka va katta transportirdan foydalanishga ruxsat beriladi.

Sinf guruhlarga bo‘linib, maktab maydonining turli burchaklarida ish olib boradi. “Xazina” (masalan, pufakcha, konvertdagi xat, ...) oldindan maydonning biror joyiga ko‘mib qo‘yiladi.

Xazinaga olib boruvchi xaritalar o‘qituvchi tomonidan oldindan tuziladi va guruhlarga tarqatiladi. (Xarita namunasi 26-rasmda ko‘rsatilgan.) Guruhlar o‘z xaritalari asosida xazinani topishga kirishadi. Birinchi bo‘lib xaritada ko‘rsatilgan siniq chiziq bo‘ylab hamma nuqtalarni aniqlab, xazinani topgan guruh g‘olib deb topiladi.

Uyga vazifa. Uyingizdan maktabga keladigan yo‘lning 26-rasmdagi kabi xaritasini tuzing. Chamalab bu yo‘lning uzunligini aniqlang.

7 AMALIY MASHQ VA TATBIQ. BILIMINGIZNI SINAB KO‘RING

1. Quyidagi jumlalar to‘g‘rimi? ”Ha” yoki ”Yo‘q” deb javob bering.

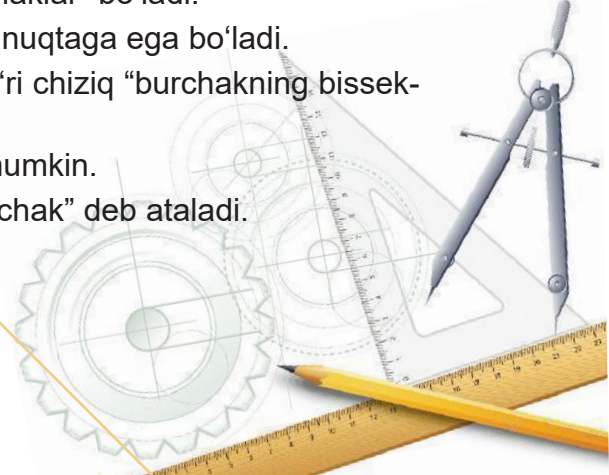
1. Har qanday burchak uchun faqat bitta vertikal burchak yasash mumkin.
2. Har qanday burchak uchun faqat bitta qo‘shni burchak yasash mumkin.
3. Agar burchaklar vertikal bo‘lsa, ular teng bo‘ladi.
4. Agar burchaklar teng bo‘lmasa, ular vertikal burchak bo‘lmaydi.
5. Agar burchaklar vertikal bo‘lmasa, ular teng bo‘lmaydi.
6. Agar ikki burchak qo‘shni bo‘lsa, unda ularning biri o‘tmas, ikkinchisi o‘tkir burchak bo‘ladi.
7. Agar ikki burchak qo‘shni bo‘lsa, unda ularning biri ikkinchisidan katta bo‘ladi.
8. Agar ikki burchak yig‘indisi 180° ga teng bo‘lsa, ular qo‘shni burchaklar bo‘ladi.
9. Agar ikki burchak yig‘indisi 180° ga teng bo‘lmasa, ular qo‘shni burchaklar bo‘lmaydi.
10. Agar ikki burchak teng bo‘lsa, ularga qo‘shni bo‘lgan burchaklar ham teng bo‘ladi.
11. Agar qo‘shni burchaklar teng bo‘lsa, ular to‘g‘ri burchaklar bo‘ladi.
12. Agar ikki burchak umumiy uchga ega bo‘lsa, ular vertikal burchaklar bo‘ladi.
13. Agar ikki burchak umumiy tomonga ega bo‘lsa, ular qo‘shni burchaklar bo‘ladi.

2. Jumlalarni mohiyatidan kelib chiqib to‘ldiring.

1. Nuqta va uchlari shu nuqtada bo‘lgan ... iborat shakl “burchak” deb ataladi.
2. Tekislikda ikki nuqta orqali ... to‘g‘ri chiziq o‘tkazish mumkin.
3. Yoyiq burchakning gradus o‘lchovi ... teng.
4. Ikkita to‘g‘ri chiziq faqat ... kesishadi.
5. Burchakning uchidan chiqib, uni ... “burchak bissektrisasi” deb ataladi.
6. To‘g‘ri chiziqning biror nuqtasi va undan bir tomonda yotgan nuqtalardan iborat qismi ... deb ataladi.
7. Umumiy tomonga ega bo‘lib, qolgan ikkita tomoni to‘g‘ri chiziq hosil qiluvchi burchaklar ... deb ataladi.
8. To‘g‘ri chiziq tekislikni ... ajratadi.
9. Vertikal burchaklarning bissektrisalari ... hosil qiladi.
10. Kesmani teng ... shu “kesmaning o‘rtasi” deb ataladi.
11. Agar qo‘shni burchaklar ..., ular to‘g‘ri burchaklar bo‘ladi.
12. Teng kesmalarning ... ham teng bo‘ladi.

3. Quyida keltirilgan jumlalarda xato bo‘lsa, uni toping va tuzating.

1. Yig‘indisi 180° ga teng bo‘lgan burchaklar “qo‘shni burchaklar” bo‘ladi.
2. Tekislikdagi ixtiyoriy ikki to‘g‘ri chiziq faqat bitta umumiy nuqtaga ega bo‘ladi.
3. Burchakning uchidan o‘tib, uni teng ikkiga bo‘luvchi to‘g‘ri chiziq “burchakning bissektrisasi” deb ataladi.
4. Ixtiyoriy nuqta orqali faqat ikkita to‘g‘ri chiziq o‘tkazish mumkin.
5. Ikkala tomoni ham nurlarda yotuvchi burchak “yoyiq burchak” deb ataladi.



6. Tekislikdagi ikki to‘g‘ri chiziq uni ikkita yarimtekislikka ajratadi.
7. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklar “vertikal burchaklar” deb ataladi.
8. Kesmani ikkiga bo‘luvchi nuqta “kesmaning o‘rtasi” deb ataladi.
9. Berilgan nurdan yarimtekislikka faqat bitta to‘g‘ri burchak qo‘yish mumkin.
10. Tekislikdagi ixtiyoriy A, B, C nuqtalar uchun $AB + BC = AC$ tenglik o‘rinli.
11. Vertikal burchaklarning yig‘indisi 180° ga teng.

4. Berilgan xossaga ega bo‘lgan geometrik shaklni o‘ng ustundagi mos qatorga yozing.

1	Yig‘indisi 180° ga teng	
2	Tomonlari nurlardan iborat	
3	Kattaligi 180° ga teng	
4	Tayin uzunlikka ega	
5	Kesmani teng ikkiga bo‘ladi	
6	Isbotsiz to‘g‘ri deb qabul qilingan xossa	
7	Burchakni teng ikkiga bo‘ladi	
8	To‘g‘ri chiziqlar kesishganda hosil bo‘ladi	
9	To‘g‘riligini isbotlash zarur	
10	O‘lchovga ega emas	

5. Birinchi ustunda berilgan geometrik tushunchalarga tegishli xossa yoki talqinlarni ikkinchi ustundan topib, moslab qo‘ying:

Geometrik tushunchalar	*Xossasi yoki talqini*
1. Nuqta 2. To‘g‘ri chiziq 3. Yer o‘lchash 4. Kesma 5. Nur 6. Kesma uzunligi 7. Teng shakllar 8. Yarimtekislik 9. Planimetriya 10. Burchak 11. 1 gradus 12. Yoyiq burchak gradus o‘lchovi 13. Vertikal burchaklar 14. Qo‘shni burchaklar 15. Teorema 16. Aksioma 17. Bissektrisa	(A) “Geometriya” so‘zining ma’nosi. (B) yig‘indisi 180° ga teng. (C) o‘zaro teng burchaklar. (D) to‘g‘ri chiziqdagi nuqta va undan bir tomonda yotgan nuqtalar. (E) 180°. (F) umumiy uchga ega bo‘lgan ikki nur. (G) uzunligini o‘lchab bo‘lmaydi. (H) to‘g‘ri burchakning 90 dan 1 qismi. (I) isbotsiz qabul qilinadigan tasdiq. (J) isbotlanishi lozim bo‘lgan tasdiq. (K) to‘g‘ri chiziqning ikki nuqtasi va ular orasidagi nuqtalar. (L) tekislikdagi geometrik shakllarning xossalarini o‘rganadi. (M) burchakni teng ikkiga bo‘ladi. (N) tekislikning to‘g‘ri chiziq ajratgan qismlaridan biri. (O) qismlarga ega emas. (P) musbat son. (Q) aynan ustma-ust tushadigan qilib qo‘yish mumkin.

6. Testlar.

1. Ta'rifsiz qabul qilingan asosiy geometrik tushunchalarni ko'rsating: a) tekislik; b) nuqta; c) kesma; d) nur; e) to'g'ri chiziq; f) yarimtekislik.

A) a; b; c B) b; c; e C) a; b; c; e D) a; b; e

2. Ikki qo'shni burchakning ayirmasi 24° ga teng bo'lsa, ulardan kichigini toping:

A) 72° B) 76° C) 78° D) 82°

3. Geometriya fan sifatida ilk bor qaysi mamlakatda shakllangan?

A) Qadimgi Misr B) Bobil C) Yunoniston D) Xitoy

4. Ikki to'g'ri chiziqning kesishishidan hosil bo'lgan burchaklardan uchtasining yig'indisi 200° ga teng. Burchaklardan kichigini toping:

A) 20° B) 40° C) 60° D) 80°

5. Hech qanday uchtasi bir to'g'ri chiziqda yotmaydigan 4 ta nuqta berilgan. Shu nuqtalarning har bir jufti orqali to'g'ri chiziqlar o'tkazildi. Ularning sonini toping.

A) 1 B) 4 C) 5 D) 6

6. Burchak bissektrisasi uning tomoni bilan 60° li burchak hosil qiladi. Berilgan burchakka qo'shni bo'lgan burchakni toping:

A) 30° B) 60° C) 90° D) 120°

7. AB kesmani 2 ta to'g'ri chiziq kesib o'tsa, ko'pi bilan nechta AB kesmada yotgan kesma hosil bo'ladi?

A) 3 B) 4 C) 5 D) 6

8. Soat 4 bo'lganda soat va minut millari orasidagi burchak necha gradus bo'ladi?

A) 60° B) 75° C) 105° D) 120°

9. $AB = 6,\ C \in AB,\ AC = 3BC,\ BC$ = ?

A) 1 B) 1,5 C) 2 D) 3

10. Soat mili 30 minutda necha gradusga buriladi?

A) 180° B) 15° C) 60° D) 30°

11. $AB = 18,\ C \in AB,\ AC - BC = 4,\ BC$ = ?

A) 7 B) 8 C) 10 D) 11

12. Vertikal burchaklarning yig'indisi 180° ga teng. Bu burchaklarni toping:

A) 60° va 120° B) 45° va 135°

C) 90° va 90° D) 45° va 45°

13. Uchta to'g'ri chiziq tekislikni eng ko'pi bilan nechta qismga ajratishi mumkin?

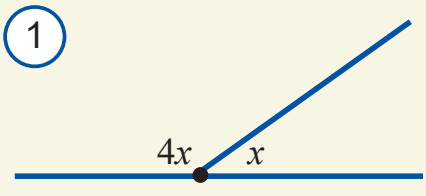
A) 4 B) 5 C) 6 D) 7

14. 1-rasmdagi x ni toping.

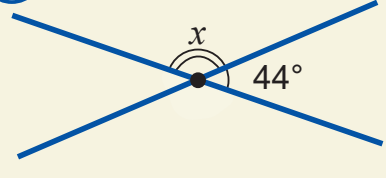
A) 30° B) 36° C) 45° D) 60°

15. 2-rasmdagi x ni toping.

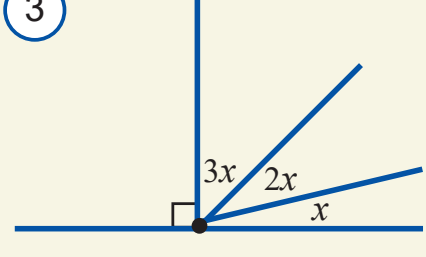
1



2



3



A) 136° B) 72° C) 56° D) 96°

16. 3-rasmdagi x ni toping.

A) 15° B) 30° C) 45° D) 60°

17. Quyidagi mulohazalardan to‘g‘risini toping.

A) Tekislikda berilgan nuqtadan faqat bitta to‘g‘ri chiziq o‘tkazish mumkin.

B) To‘g‘ri chiziqning biror nuqtasi va undan bir tomonda yotgan nuqtalaridan iborat qismi “nur” deb ataladi.

C) To‘g‘ri chiziqning ikki nuqtasi orasida yotgan nuqtalaridan iborat qismi “kesma” deb ataladi.

D) Har qanday nurdan tayin yarimtekislikka faqat bitta burchak qo‘yish mumkin.

18. Quyidagi mulohazalardan to‘g‘risini toping.

A) Qo‘shni burchaklar yoyiq burchak bo‘ladi.

B) Agar $AB = 5\ cm$, $BC = 6\ cm$ bo‘lsa, $AC = 11\ cm$ bo‘ladi.

C) Agar burchaklar teng bo‘lsa, ular vertikal burchaklar bo‘ladi.

D) Agar ikkita burchak teng bo‘lsa, ularga qo‘shni bo‘lgan burchaklar ham teng bo‘ladi.

7. Masalalar.

1. Transportir yordamida bir tomoni umumiy bo‘lgan 10°, 20°, 40°, 60°, 90°, 130°, 170° li burchaklarni yasang.

2. Yoyiq burchakning bissektrisasi uning tomonlari bilan qanday burchak hosil qiladi?

3. Burchakning bissektrisasi uning tomoni bilan 30° li burchak hosil qilgan bo‘lsa, burchakning o‘zi necha gradus?

4. Burchakning bissektrisasi uning tomonlari bilan o‘tmas burchak tashkil qilishi mumkinmi?

5. $\angle AOB$=50°, $\angle BOC$=80° bo‘lsa, AOB va BOC burchaklarning bissektrisalari orasidagi burchakni toping. Masala nechta yechimga ega?

6. 15° li burchakka 10 marta kattalashtiruvchi lupa (ko‘zgu) orqali qaralganda, u necha gradusli burchak bo‘lib ko‘rinadi?

7. a) 90°; b) 60°; c) 50°; d) 20° li burchakning bissektrisasini transportir yordamida yasang.

8*. $\angle AOB$=120° bo‘lgan burchakning OK bissektrisasini transportir yordamida yasang. So‘ng hosil bo‘lgan AOK va KOB burchaklarning bissektrisalarini yasang va bu bissektrisalar orasidagi burchakni toping.

9*. Agar $AB = 1{,}8\ m$, $AC = 1{,}3\ m$ va $BC = 3\ m$ bo‘lsa, A, B va C nuqtalar bir to‘g‘ri chiziqda yotadimi?

10. A, B va C nuqtalar bir to‘g‘ri chiziqda yotadi. Agar $AB = 2{,}7\ m$, $AC = 3{,}2\ m$ bo‘lsa, BC kesmaning uzunligini toping. Masala nechta yechimga ega?

11. Uzunligi 15 m bo‘lgan AB kesmada C nuqta belgilangan. Agar:

a) AC kesma BC kesmadan 3 m uzun;

b) C nuqta AB kesmaning o‘rtasi bo‘lsa;

c) AC va BC kesmalarning uzunliklari 2:3 nisbatda bo‘lsa, AC va BC kesmalar uzunliklarini toping.

12. A, B, C, D nuqtalar bir to‘g‘ri chiziqda yotadi. Agar B nuqta AC kesmaning, C nuqta esa BD kesmaning o‘rtasi bo‘lsa, $AB = BC = CD$ ekanini ko‘rsating.

13. Hech bir uchtasi bir to‘g‘ri chiziqda yotmaydigan: a) 6; b) 7; c) 10 ta nuqtaning har ikkitasi orqali to‘g‘ri chiziq o‘tkazilgan. Jami nechta to‘g‘ri chiziq o‘tkazilgan?

14. OA va OB nurlar qachon ustma-ust tushadi?

15. AB nurda C nuqta, BA nurda D nuqta shunday olinganki, AC = 0,7 va BD = 2,1. Agar AB = 1,5 bo‘lsa, CD ni toping.

16. 4-rasmda nechta vertikal burchaklar juftligi tasvirlangan?

17*. Agar soat va minut millari orasidagi burchak 45° bo‘lib, minut mili 6 da turgan bo‘lsa, soat qaysi vaqtni ko‘rsatayotgan bo‘ladi?

18. To‘g‘ri chiziqqa unda yotmagan O nuqtadan OA og‘ma va OB perpendikulyar o‘tkazilgan. Ularning uzunliklari yig‘indisi 13, ayirmasi esa 1 ga teng bo‘lsa, O nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofani toping.

19. AOB va BOC qo‘shni burchaklar ekanligi ma’lum. Agar:

a) AOB burchak BOC burchakdan 40° katta;

b) AOB burchak BOC burchakdan 4 marta kichik;

c) $\angle AOB = \angle BOC + 44°$;

d) $\angle AOB = 5 \cdot \angle BOC$ bo‘lsa, bu burchaklarni toping.

20. Ikki to‘g‘ri chiziq kesishishidan to‘rtta burchak hosil bo‘ldi. Ulardan ikkitasining gradus o‘lchovlari yig‘indisi 100° ga teng bo‘lsa, bu to‘rtta burchakning gradus o‘lchovlarini toping.

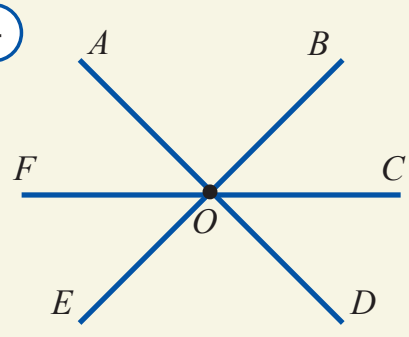
21*. A, B va C nuqtalar tekislikda shunday joylashganki, a) $AC+CB=AB$; b) $AB+AC=BC$. Qaysi nuqta qolgan ikkitasining orasida yotadi?

22. 5-rasmdagi burchaklarning tomonlariga ularning A va B nuqtalari orqali perpendikulyar to‘g‘ri chiziqlar o‘tkazing. Bu to‘g‘ri chiziqlar kesishish nuqtasida qanday burchaklar hosil qiladi?

23*. Burchakning uchidan chiqib, ichki sohasidan o‘tuvchi: a) 2 ta; b) 3 ta; c) n ta nur o‘tkazilgan. Bunda jami berilgan burchak bilan birga jami nechta burchak hosil bo‘ladi?

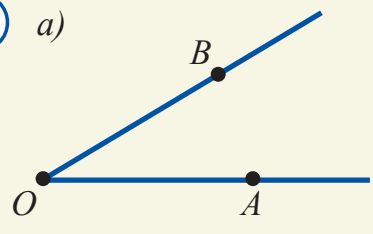
24*. Gazeta varag‘i qalinligi 0,1 mm. U teng qilib ikki buklandi. Hosil bo‘lgan ikki qavat gazeta bo‘lagi yana buklandi va hokazo. Bu ish shu tariqa 50 marta qaytarildi. Natijada qanday qalinlikdagi gazeta qatlami hosil bo‘ladi?

4

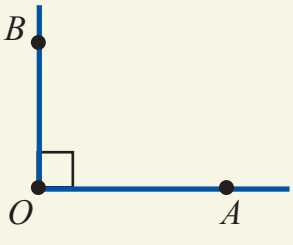


5

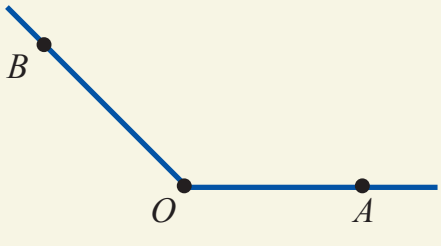
a)



b)



c)



Amaliy mashq va tatbiq

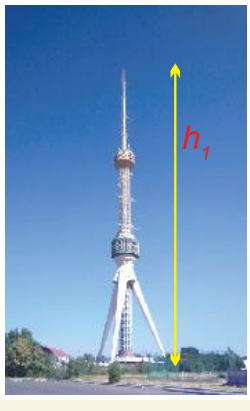
1. 7-sinf “Geometriya” darsligining bir varag‘i qalinligini aniqlash yo‘lini o‘ylab toping va o‘lchang.

2. Biror jismning balandligi uning eng baland nuqtasidan asosi yotgan tekislikka tushirilgan perpendikulyar uzunligi bilan aniqlanadi. Agar bunday perpendikulyarni tushirish imkoni bo‘lmasa, unga teng bo‘lgan kesma balandlik sifatida qaraladi *(6-rasm)*. Masalan, bino, piramida, minora balandligi yoki quduq chuqurligi va hokazo. Ba’zida tekislikdagi yassi shakllar balandligi ham shunday aniqlanadi.

a) Choynak, piyola, kosa, guldon, qozon kabi turli uy jihozlarining balandliklarini o‘lchash yo‘lini o‘ylab toping va ularning balandliklarini o‘lchang.

b) To‘g‘ri burchakli parallelepiped, uchburchakli piramida, konus va shar kabi geometrik shakl (jism) modellarining balandliklarini o‘lchang.

6

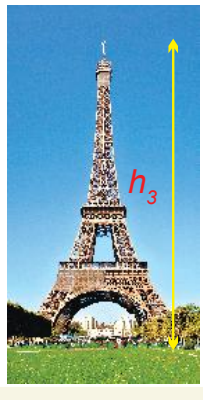


$h_1 = 375$ m

$h_2 = 146{,}6$ m



$h_3 = 300$ m



2-nazorat ishi namunasi

Namunaviy nazorat ishi ikki qismdan iborat bo‘lib, birinchi qismga yuqorida keltirilgan testlarga o‘xshash beshta test kiritiladi. Ikkinchi qismda esa quyidagi masalalarga o‘xshash 4 ta masala beriladi (4-masala “a’lo” baho olmoqchi bo‘lgan o‘quvchilar uchun qo‘shimcha).

1. MN va KL to‘g‘ri chiziqlarning kesishishidan hosil bo‘lgan MOL va KON vertikal burchaklarning yig‘indisi 148° ga teng. MOK burchakni toping.

2. Qo‘shni burchaklarning ayirmasi 60° ga teng. Bu burchaklarning kichigini toping.

3. Burchak bissektrisasi shu burchakning tomoni bilan 66° li burchak hosil qiladi. Bu burchakka qo‘shni bo‘lgan burchakni toping.

4*. Qo‘shni burchaklar bissektrisalari to‘g‘ri burchak ostida kesishishini isbotlang.

“GeoGebra”dan foydalanish asoslari

“GeoGebra” interfeysi

GeoGebra ishga tushirilgandan so‘ng quyidagi ko‘rinishdagi oyna hosil bo‘ladi *(1-rasm)*:



Главное меню (asosiy menyu) – ***GeoGebra*** dasturi tomonidan taqdim etilgan funksiyalarga kirish menyusi.

Панель инструментов (asboblar paneli) – dastur tomonidan taqdim etilgan vosita (uskuna)lar majmui orqali turli chizmalarni yaratish mumkin.



Графическое окно (grafik ko‘rinish maydoni) – geometrik chizmalarni chizish va tasvirlash maydoni.

Панель объектов (algebraik ko‘rinish maydoni) – geometrik chizmalarga oid tegishli koordinata va tenglamalarni tasvirlash maydoni.

Строка ввода (kiritish satri) – nuqta o‘rnini, algebraik tenglama yoki buyruqlarni kiritish maydoni.

Asboblar panelida taqdim etilgan geometriya vositalaridan foydalanib, sichqoncha yordamida turli xil geometrik shakllarni yaratishingiz mumkin. Shu bilan bir vaqtda algebraik ko‘rinish oynasida chizmaga oid tegishli koordinata va tenglamalar tasvirlanadi. Boshqa tomondan, siz to‘g‘ridan to‘g‘ri klaviatura yordamida **Ввод...** maydoniga algebraik ma’lumotlar, buyruqlar va funksiyalarni kiritishingiz mumkin. Barcha chizmalarning grafik ko‘rinishi **grafik ko‘rinish maydoni**da aks ettirilsa, ularning algebraik sonli ko‘rinishi **algebraik ko‘rinish oynasi**da aks ettiriladi. ***GeoGebra***da geometriya va algebra bir-biri bilan bog‘liq holda ishlaydi.

“GeoGebra” interfeysi moslashuvchan bo‘lib, uni o‘zingizga moslashtirishingiz

mumkin. ***GeoGebra***da, shuningdek, algebraik ko‘rinish hamda grafik ko‘rinish oynalari ham mavjud.

Ushbu turli xil ko‘rinishlar **“Вид”** menyusi yordamida ko‘rsatilishi yoki yashirin holatga o‘tkazilishi mumkin.

- Tegishli tugmani bosish orqali unga mos funksiyani faollashtiring.
- Tugmani pastki qismiga bosish orqali qo‘shimcha menyuni oching va undan kerakli funksiyani tanlang.
- Uskunalar panelining o‘ng tomonidagi ⓘ belgini bosib, ayni vaqtda faol bo‘lgan vosita (uskuna) haqida ma’lumot oling.

“GeoGebra”da fayllarni saqlash

- **“Файл”** menyusini oching va **Сохранить** buyrug‘ini tanlang.
- Ochilgan muloqot oynasidan kerakli papkani tanlang.
- Fayl nomini kiriting.
- **“Сохранить”** tugmasini bosing va jarayonni yakunlang.

Eslatma. *“.ggb” kengaytmali fayl yaratiladi va uni faqat GeoGebra dasturi orqali ochish mumkin.*

“GeoGebra”da fayllarni ochish

GeoGebra dasturida yangi oyna (**“Файл” – “Новый окно”**) oching.

Ochilgan yangi oynada kompyuter xotirasidagi faylni oching. Buning uchun **“Файл”** – **“Открыть”** buyrug‘ini tanlang.

Fayl saqlangan papkani oching va **.ggb** kengaytmali faylni tanlang, “**Открыть”** tugmasini bosing.

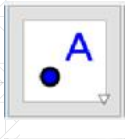
“GeoGebra”ning uskunalar panelidan foydalanish bo‘yicha ko‘rsatmalar

GeoGebra vositalarining ba’zilari haqida nazariy ma’lumotlarga ega bo‘lmasangiz ham, ularning tashqi ko‘rinishi orqali osongina tushunib olish mumkin. Ushbu vositalardan foydalanish yuzasidan o‘rgatuvchi o‘quv fayllari mavjud bo‘lib, har bir vosita uchun alohida ***GeoGebra*** (.ggb) fayli kiritilgan.

“GeoGebra” uskunalar panelidagi asosiy uskunalarning vazifasi



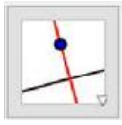
Перемещать (Siljitish) – turli obyektlar (nuqta, to‘g‘ri chiziq, ko‘pburchak va boshqalar)ni siljitish



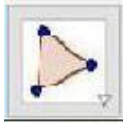
Точка (Nuqta) – tekislikda nuqta yasash.



Прямая (To‘g‘ri chiziq) – tekislikda berilgan ikkita nuqta orqali to‘g‘ri chiziq yasash.



Перпендикулярная прямая (Perpendikulyar to‘g‘ri chiziq) – berilgan nuqta orqali to‘g‘ri chiziqqa perpendikulyar o‘tkazish.



Многоугольник (Ko‘pburchak) – ixtiyoriy turdagi ko‘pburchakni yasash.



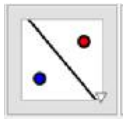
Окружность по центру и точке (Aylana markazi va nuqtasiga ko‘ra) – aylana markazini belgilash orqali aylana yasash.



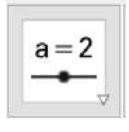
Эллипс (Ellips) – ellips fokuslarini va undagi nuqtani berish orqali ellips yasash.



Угол (Burchak) – uchta nuqta yoki ikkita to‘g‘ri chiziq orqali burchak yasash.



Отражение относительно прямой (To‘g‘ri chiziqqa nisbatan akslantirish) – kerakli obyektni va to‘g‘ri chiziqni belgilash orqali obyektni to‘g‘ri chiziqqa nisbatan aks ettirish.



Ползунок (Surgich) – uzunlik va burchaklar kattaligini o‘zgaruvchi sifatida biror oraliqda uzluksiz o‘zgartirish imkoniyatini beruvchi animatsiya uskunasi.

1. To‘g‘ri chiziq yasash

To‘g‘ri chiziq eng sodda va cheksiz geometrik shakldir. Uning tasvirini hosil qilish uchun:

1) uskunalar panelida “Прямые линии” (“To‘g‘ri chiziqlar”) guruhini ochamiz *(2-rasm).*

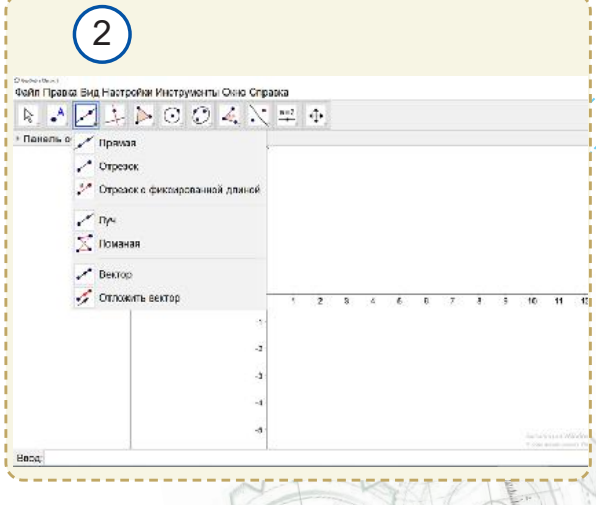
2) “Прямая” (“To‘g‘ri chiziq”) uskunasi ustiga sichqonchaning chap tugmasini bosamiz;

3) sichqonchaning chap tugmasini bosib, ikkita A va B nuqtalarni belgilaymiz;

4) ekranda shu ikkita nuqtadan o‘tuvchi to‘g‘ri chiziq paydo bo‘ladi *(3-rasm).*

To‘g‘ri chiziq tasvirining ustiga sichqonchaning chap tugmasini bosib uning o‘rnini almashtirish, ya’ni uni siljitish mumkin. Shuningdek, uning rangi va stilini aniqlash hamda uni ixtiyoriy harf bilan belgilash imkoni ham bor.

2

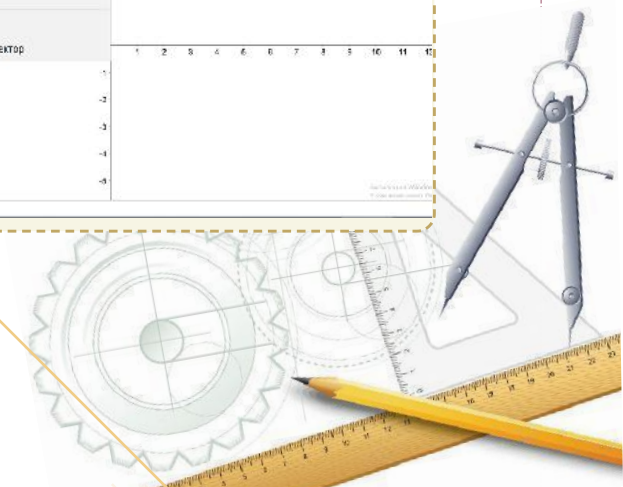


2. Nur yasash algoritmini tuzish va yasash

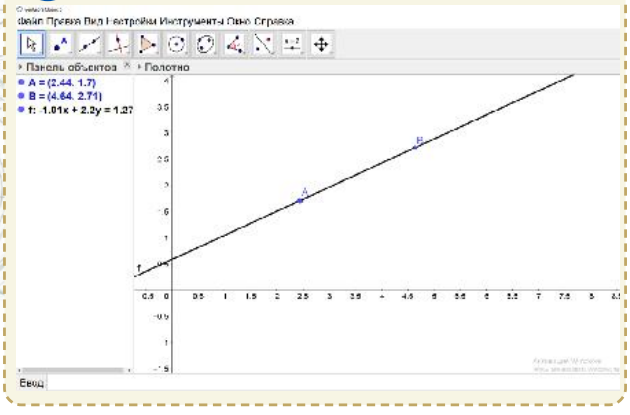
Yuqoridagi algoritmga o‘xshash nur yasash algoritmini tuzing va nur yasang.

3. Kesma yasash algoritmini tuzish va yasash

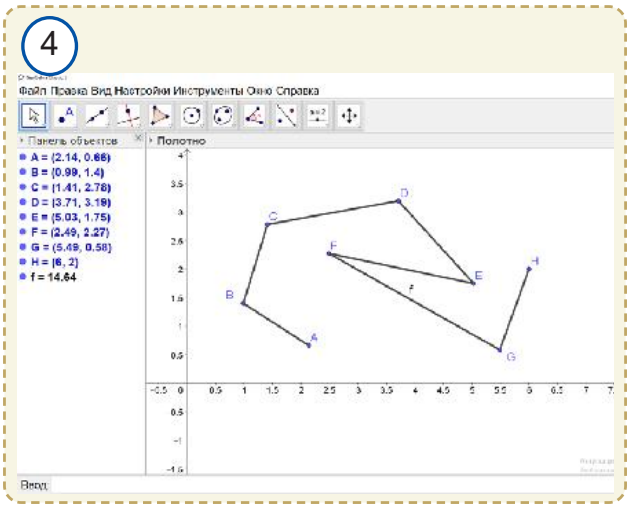
Yuqoridagi algoritmga o‘xshash kesma yasash algoritmini tuzing va kesma yasang.



3



4



4. Siniq chiziq yasash

Siniq chiziq – nuqtalar va ularni birlashtirib turuvchi kesmalardan iborat shakl. Uning tasvirini hosil qilish uchun:

1) uskunalar panelida “Прямые линии” (“To‘g‘ri chiziqlar”) guruhini ochamiz;

2) menyudan “Ломанная” (“Siniq chiziq”) ustiga sichqonchaning chap tugmasini bosib tanlaymiz;

3) sichqonchaning chap tugmasini bosib, siniq chiziqning ketma-ket uchlarini belgilaymiz;

4) ekranda uchlari shu nuqtalardan iborat bo‘lgan siniq chiziq paydo bo‘ladi *(4-rasm)*.

To‘g‘ri chiziq tasviri ustiga sichqonchaning chap tugmasini bosib, uning o‘rnini almashtirish, ya’ni uni siljitish mumkin. Shuningdek, uning rangi va stilini aniqlash hamda uni harf bilan belgilash imkoni ham bor.

5. Perpendikulyar to‘g‘ri chiziqlarni yasash

Ikki to‘g‘ri chiziq to‘g‘ri burchak ostida kesishsa, ular “perpendikulyar to‘g‘ri chiziqlar” deb ataladi. Ularning tasvirini hosil qilish uchun:

1) uskunalar panelida “Специальные прямые” (“Maxsus to‘g‘ri chiziqlar”) guruhini ochamiz;

2) menyudan “Перпендикуляр” (“Perpendikulyar”) ustiga sichqonchaning chap tugmasini bosib tanlaymiz;

3) kesma (to‘g‘ri chiziq) yasaymiz;

4) nuqta belgilaymiz (nuqta kesma yoki to‘g‘ri chiziqda ham yotishi mumkin);

5) sichqonchaning chap tugmasini bosib, ishchi sohada perpendikulyar to‘g‘ri chiziqlarni hosil qilamiz.

6. Parallel to‘g‘ri chiziqlarni yasash algoritmini tuzish va uni yasash

Yuqoridagi algoritmga o‘xshash parallel to‘g‘ri chiziqlarni yasash algoritmini tuzing va uni yasang.



II BOB

UCHBURCHAKLAR

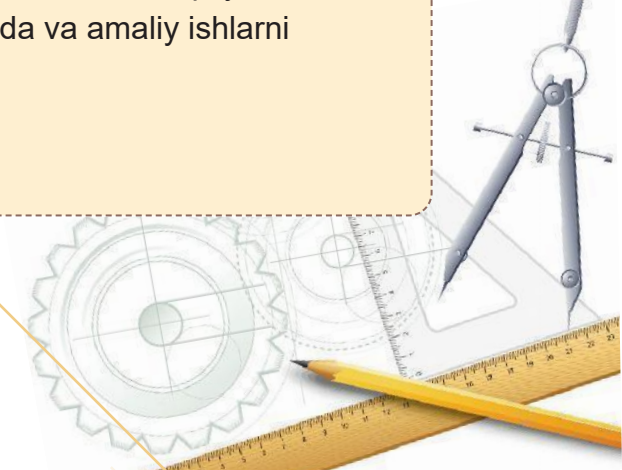
Bu bobni o‘rganib chiqqach, quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilim:

– ko‘pburchak haqida tasavvurga ega bo‘lish;
– uchburchak va uning asosiy elementlarini bilish hamda ular bo‘yicha uchburchaklarni turlarga ajratish;
– uchburchak medianasi, bissektrisasi va balandliklarini bilish va farqlay olish;
– uchburchaklar tengligining alomatlari;
– teng yonli uchburchak va uning xossalari;
– teng tomonli uchburchak va uning xossalari;
– kesma o‘rta perpendikulyarining xossasi.

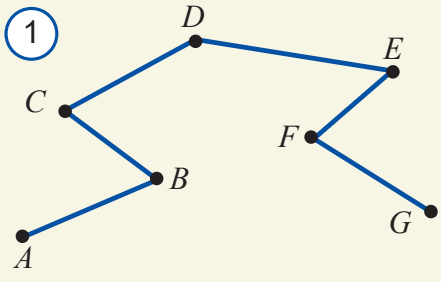
Amaliy ko‘nikmalar:

– uchburchaklar tengligi alomatlariga ko‘ra teng uchburchaklarni aniqlay olish;
– uchburchaklar tengligi alomatlarini masalalar yechishda va amaliy ishlarni bajarishda qo‘llay olish.



8 UCHBURCHAK, UNING TURLARI VA ELEMENTLARI

1



$ABCDEFG$ – *siniq chiziq*
A, B, C, D, E, F, G – *uchlari*
AB, BC, CD, DE, EF, FG – *bo‘g‘inlari (tomonlari)*

8.1. Siniq chiziq. Ko‘pburchak

Ketma-ket kelgan har ikkitasi bir to‘g‘ri chiziqda yotmagan A_1A_2, A_2A_3,..., $A_{n-1}A_n$ kesmalardan tashkil topgan shakl *siniq chiziq* deyiladi. $A_1, A_2,..., A_n$ nuqtalar *siniq chiziqning uchlari*, A_1A_2, A_2A_3,..., $A_{n-1}A_n$ kesmalar esa *siniq chiziqning bo‘g‘inlari* yoki *tomonlari* deb ataladi. 1-rasmda $ABCDEFG$ siniq chiziq tasvirlangan.

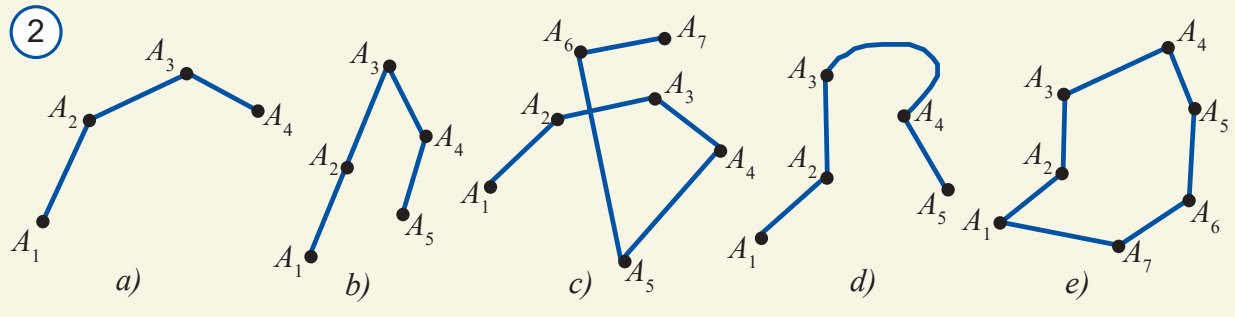
Boshlang‘ich va oxirgi uchlari ustma-ust tushadigan siniq chiziqni *yopiq siniq chiziq* deb ataymiz. O‘z-o‘zini kesmaydigan yopiq siniq chiziq *ko‘pburchak* deb ataladi.

Faollashtiruvchi mashq.

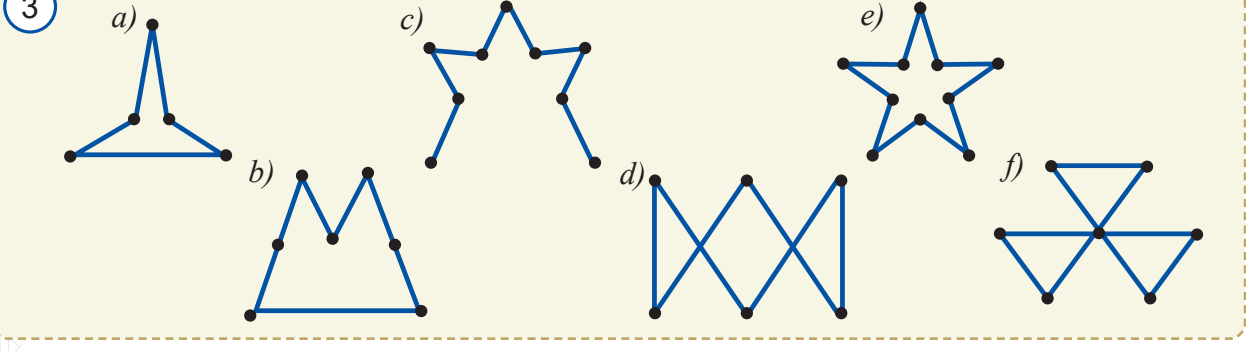
1. 2-rasmda tasvirlangan chiziqlarning qaysilari siniq chiziq bo‘lishini aniqlang va javobingizni izohlang.

2. Ko‘pburchakning ta’rifidan kelib chiqadigan xususiyatlarini sanang va 3-rasmdagi shakllarning qaysilari ko‘pburchak bo‘lishini aniqlang va izohlang.

2



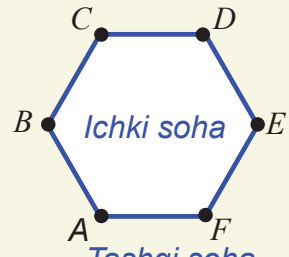
3



Tomonlarining soniga qarab, ko‘pburchaklar uchburchak, to‘rtburchak, beshburchak, oltiburchak, umumiy holda *n-burchak* deb nomlanadi. Siz ba’zi ko‘pburchaklar bilan quyi sinflarda tanishgansiz.

Har qanday ko‘pburchak tekislikni ikki sohaga ajratadi. Ko‘pburchak bilan chegaralangan chekli soha ko‘pburchakning *ichki sohasi*, ikkinchi – cheksiz soha esa ko‘pburchakning *tashqi sohasi* deb ataladi. 4-rasmda $ABCDEF$ oltiburchakning ichki va tashqi sohalari ko‘rsatilgan.

4



8.2. Uchburchak va uning turlari

Bir to‘g‘ri chiziqda yotmagan uchta nuqta va ularni tutashtiruvchi kesmalardan iborat shaklni *uchburchak* deb ataymiz.

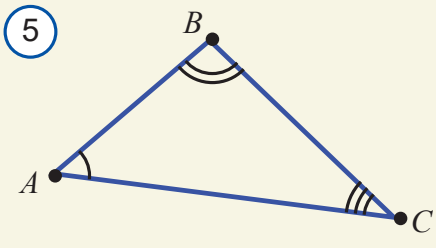
Bir to‘g‘ri chiziqda yotmagan uchta – A, B va C nuqtalarni belgilaymiz *(5-rasm)*. Ularni o‘zaro kesmalar bilan tutashtirib chiqsak, uchburchak hosil bo‘ladi. Belgilangan uchta nuqta uchburchakning *uchlari*, ularni tutashtiruvchi kesmalar esa uchburchakning *tomonlari* deyiladi.

Odatda “uchburchak” so‘zi o‘rniga Δ belgisi ishlatiladi. ΔABC yozuvi ABC uchburchak deb o‘qiladi. $\angle BAC, \angle ABC, \angle ACB$ uchburchakning *burchaklari* deb yuritiladi *(5-rasm)*. Ularni ba’zida aniqlik uchun *ichki burchaklar* deb ham atashadi.

Uchburchak burchaklarini $\angle A$, $\angle B$ va $\angle C$ tarzda ham belgilash mumkin.

Uchburchakning tomonlari va burchaklari uning *asosiy elementlari* deb ataladi. Uchburchakning uchala tomoni uzunliklari yig‘indisi uning *perimetri (P)* deyiladi.

5



ΔABC – ABC uchburchak
A, B, C nuqtalar – uchlari
AB, BC, AC
kesmalar – tomonlari
$\angle A, \angle B, \angle C$ – burchaklari
$P = AB + BC + AC$ – perimetri

Shuningdek:

BAC burchak uchburchakning AB va AC tomonlari orasida yotuvchi burchagi;

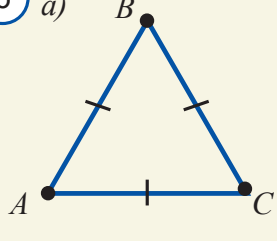
AB va AC tomonlar BAC burchakka yopishgan;

BC tomon BAC burchak qarshisida yotibdi kabi iboralar qo‘llanadi.

Uchburchaklar tomonlari uzunliklariga qarab *teng tomonli, teng yonli va turli tomonli* turlarga ajratiladi. Teng tomonli uchburchakning uchala tomonlari uzunliklari ham teng, teng yonli uchburchakning esa ikkita tomoni uzunliklari teng va turli tomonli uchburchakning hamma tomonlari uzunliklari turlicha bo‘ladi.

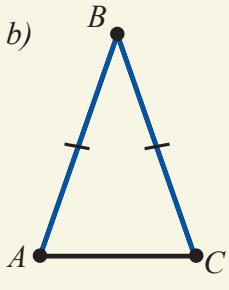
6

a)



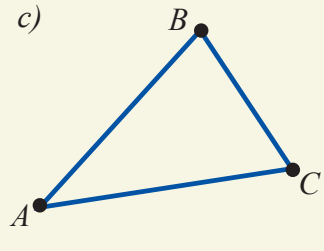
Teng tomonli uchburchak

b)



Teng yonli uchburchak

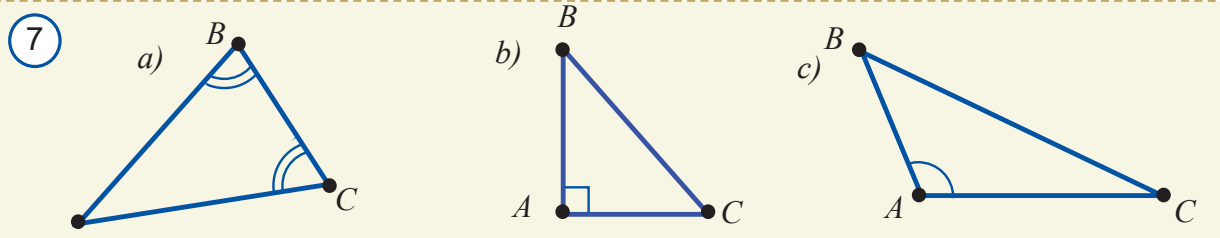
c)



Turli tomonli uchburchak

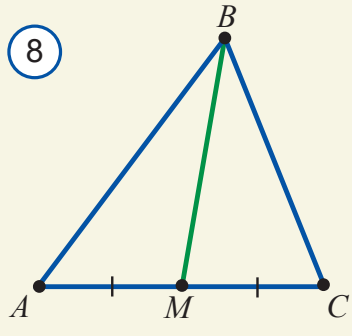
Uchburchaklar burchaklariga ko‘ra *o‘tkir burchakli, to‘g‘ri burchakli va o‘tmas burchakli* turlarga ajratiladi. O‘tkir burchakli uchburchakning uchala burchagi ham o‘tkir, to‘g‘ri burchakli uchburchakning esa bitta burchagi to‘g‘ri va o‘tmas burchakli uchburchakning bitta burchagi o‘tmas bo‘ladi.

7



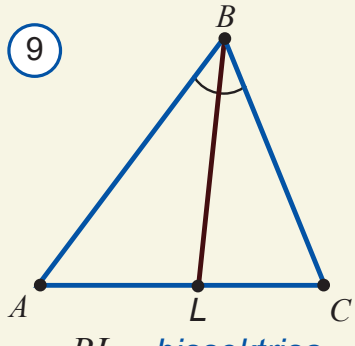
O‘tkir burchakli — To‘g‘ri burchakli — O‘tmas burchakli

8



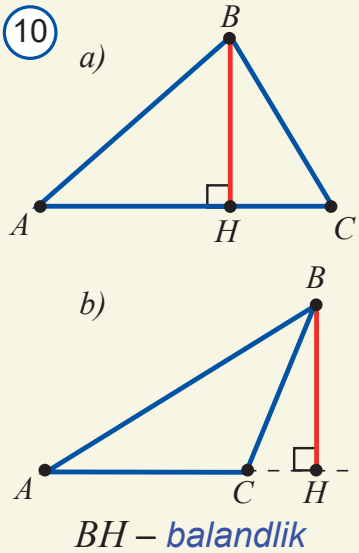
BM – *mediana*

9



BL – *bissektrisa*

10



BH – *balandlik*

8.3. Uchburchakning mediana, balandlik va bissektrisasi

Uchburchak uchini shu uch qarshisidagi tomonning o‘rtasi bilan tutashtiruvchi kesma *uchburchakning medianasi* deb ataladi.

ABC uchburchakning B uchini uning qarshisida yotuvchi AC tomonning o‘rtasi – M nuqta bilan tutashtiramiz *(8-rasm)*. Hosil bo‘lgan BM kesma ABC uchburchakning medianasi bo‘ladi. Bu mediana haqida “B uchdan chiqqan” yoki “AC tomonga tushgan” deb ham aytiladi.Uchburchak burchagi bissektrisasining uchburchak ichki sohasida yotgan qismi (kesmasi) *uchburchakning bissektrisasi* deyiladi.

ABC uchburchak B burchagining bissektrisasini o‘tkazamiz *(9-rasm)*. Uning AC tomon bilan kesishgan nuqtasini L bilan belgilaymiz. Hosil bo‘lgan BL kesma ABC uchburchak bissektrisasi bo‘ladi. Uchburchak uchidan shu uch qarshisidagi tomon yotgan to‘g‘ri chiziqqa tushirilgan perpendikulyar *uchburchakning balandligi* deb ataladi.

ABC uchburchakning B uchidan AC tomon yotgan to‘g‘ri chiziqqa perpendikulyar tushiramiz *(10-rasm)*. Rasmda bo‘lishi mumkin bo‘lgan ikki hol keltirilgan. Perpendikulyar asosini H bilan belgilaymiz. Hosil bo‘lgan BH kesma ABC *uchburchakning balandligi* bo‘ladi.

Uchburchakning uchta uchi bo‘lgani sababli u uchtadan mediana, balandlik va bissektrisaga ega. 11–13-rasmlarda ABC uchburchakning AM_1, BM_2 va CM_3 medianalari, AL_1, BL_2 va CL_3 bissektrisalari va AH_1, BH_2 va CH_3 balandliklari chizib ko‘rsatilgan. Uchburchakning bu muhim elementlari xossalari bilan keyingi darslarda tanishamiz.

Geometrik tadqiqotlar

1. Ixtiyoriy uchburchak chizing. Uning hamma medianalarini o‘tkazing *(11-rasm)*. Nimani payqadingiz? Tajribani yana ikkita uchburchak uchun bajarib ko‘ring va aniqlangan xossani faraz ko‘rinishida ifodalang.

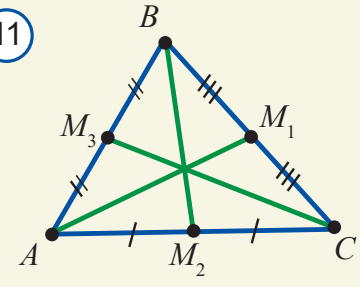
2. Ixtiyoriy uchburchak chizing. Uning hamma balandliklarini o‘tkazing *(12-rasm)*. Nimani payqadingiz? Tajribani yana ikkita uchburchak uchun bajarib ko‘ring va aniqlangan xossani faraz ko‘rinishida ifodalang.

3. Ixtiyoriy uchburchak chizing. Uning hamma bissektrisalarini o‘tkazing *(13a-rasm)*. Nimani payqadingiz? Tajribani yana ikkita uchburchak uchun bajarib ko‘ring va aniqlangan xossani faraz ko‘rinishida ifodalang. O‘tkazilgan tadqiqotlar asosida aniqlangan xossalarni teorema deb hisoblasak bo‘ladimi? Nega?

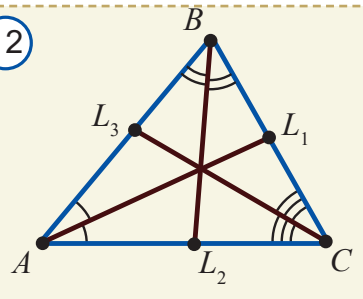
Mashq. O‘tmas burchakli uchburchakning balandliklarini o‘tkazing.

Bajarish. Uchburchakning, xususan, o‘tmas burchakli uchburchakning ham uchta balandligi bor. O‘tmas burchakli ABC uchburchakka qaraymiz (*13b-rasm)*. O‘tmas burchagi uchidan tushirilgan BH_2 balandlik uchburchakning ichki sohasida yotadi. O‘tkir burchagi A uchidan balandlik tushirish uchun shu burchak qarshisidagi BC tomonni davom ettiramiz va BC tomon davomiga A nuqtadan AH_1 perpendikulyar tushiramiz. Hosil bo‘lgan AH_1 kesma ABC uchburchakning A uchidan tushirilgan balandligi bo‘ladi. Xuddi shunga o‘xshab, AB tomon davomiga CH_3 balandlikni tushirish mumkin.

11

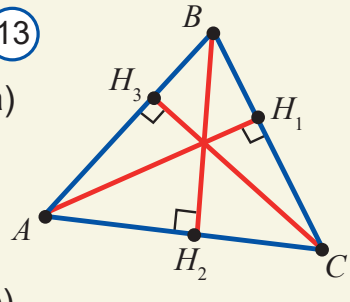


12

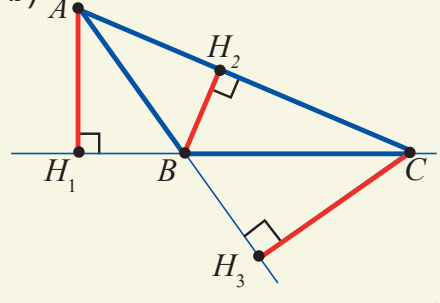


13

a)



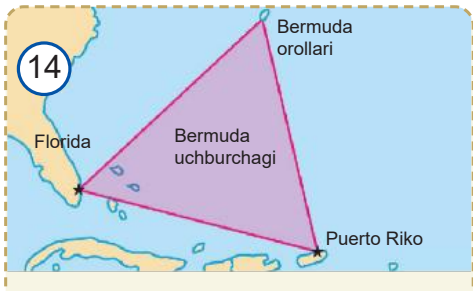
b)

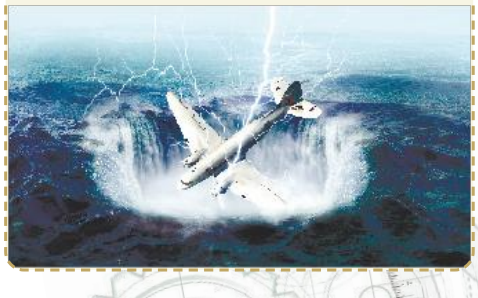


Bermuda uchburchagi

Atlantika okeanida uchlari Florida, Bermuda orollari va Puerto Rikoda bo‘lgan uchburchak shaklidagi “Bermuda uchburchagi” deb ataladigan hudud bor (14-rasm). Bu joy o‘zining sirliligi va tez-tez falokatlar sodir bo‘lib turishi bilan nom qozongan. Gap shundaki, bu hududdan o‘tayotib, ko‘plab dengiz va havo kemalari sirli ravishda halokatga uchrab, nom-nishonsiz g‘oyib bo‘lib turadi.

14





Mavzuga doir savollar

1. *Siniq chiziq chizing, uning uchlarini belgilang va bo‘g‘inlarini chizmada ko‘rsating.*
2. *Ko‘pburchak nima? Misollar keltiring.*
3. *Qanday shakl uchburchak deb ataladi?*
4. *Uchburchakning uchi, tomoni va burchagi deb nimaga aytiladi?*
5. *Uchburchakning perimetri nima?*
6. *PQR uchburchakda: a) $\angle P$ qarshisida qaysi tomon yotadi; b) PQ tomonga qaysi burchaklar yopishgan; c) PQ va QR tomonlar orasida qaysi burchak joylashgan? d) PR tomon qaysi burchak qarshisida yotadi? Bu savollarga shaklga qaramay javob berishga harakat qiling.*
7. *Uchburchakning: a) medianasi; b) balandligi; d) bissektrisasiga ta’rif bering.*
8. *Burchak bissektrisasi bilan uchburchakning bissektrisasi o‘rtasidagi umumiylik va farqlarni ayting?*

Amaliy topshiriqlar va tatbiq

1. Yetti bo‘g‘inli siniq chiziq chizing. Uning uchlari va bo‘g‘inlarini yozing.

2. Har ikki qo‘shni bo‘g‘ini bir-biriga perpendikulyar bo‘lgan besh bo‘g‘inli siniq chiziq chizing. Bunday siniq chiziq yopiq bo‘lishi mumkinmi?

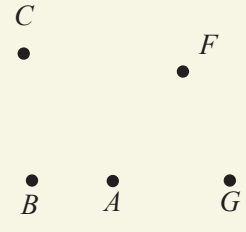
3. Uchburchakning uchlari M, N va L harflari bilan belgilandi. Uning tomonlari va burchaklarini yozing.

4. CDE uchburchakning uchlari, burchaklari va tomonlarini yozing.

5. 15-rasmdagi: a) C, F va A; b) A, B va G; c) F, B va G nuqtalar uchburchakning uchlari bo‘la oladimi? Nega?

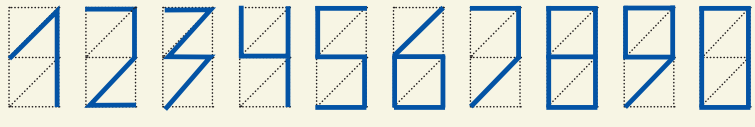
6. 16-rasmda tasvirlangan raqam belgilari siniq chiziq bo‘ladimi?

15

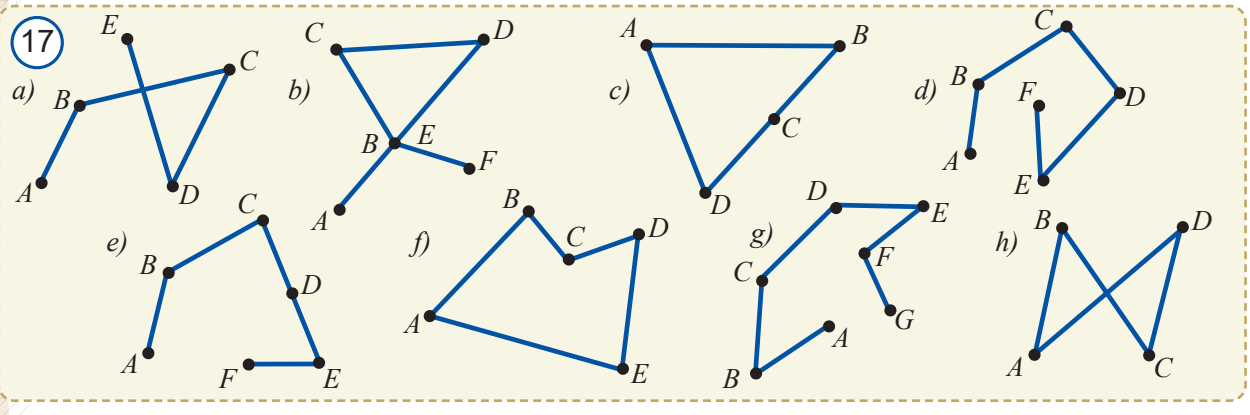


7*. 17-rasmda tasvirlangan shakllarning qaysilari: a) siniq chiziq; b) yopiq siniq chiziq; c) ko‘pburchak bo‘lishini aniqlang.

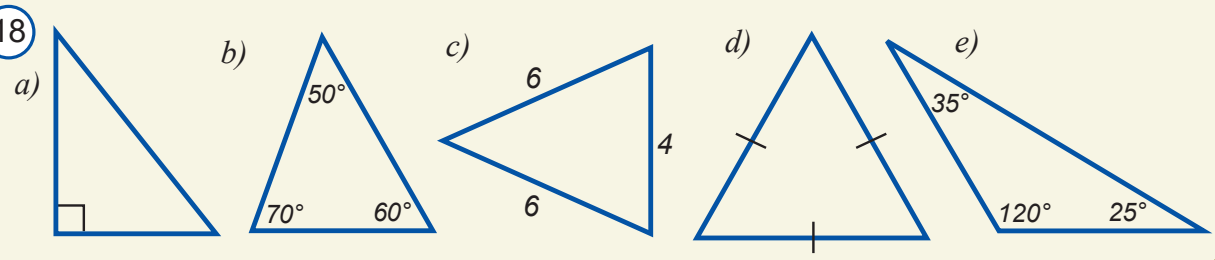
16



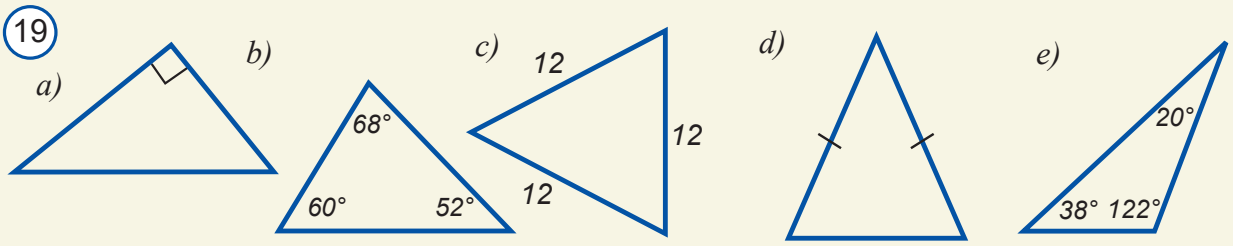
17



18



19



8. Uchburchakning qanday turlari bor? Har bir uchburchak turidan bitta uchburchak chizing. Ularni belgilang. Uchburchak turlarining ta'rifidan kelib chiqib, ularning o'ziga xos xususiyatlari nimadan iborat ekanini ayting.

9. 18-rasmdagi uchburchaklarning turlarini aniqlang.

10. 19-rasmdagi uchburchaklarning turlarini aniqlang.

11. Tomonlari: a) 2,3 dm; 4,6 dm va 5,3 dm; b) 32,3 m; 54,8 m va 25,3 m bo'lgan uchburchakning perimetrini hisoblang.

12. Tomonlari: a) 25 mm; 64,6 mm va 5 cm; b) 7,3 dm; 5,8 dm va 350 mm bo'lgan uchburchakning perimetrini hisoblang.

13. Ko'z bilan chamalab, teng tomonli uchburchak yasang. So'ng tomonlarini chizg'ich bilan o'lchab, natijalarni taqqoslang.

14. Teng tomonli uchburchak chizib, burchaklarini o'lchang va xulosa chiqaring.

15. 20-rasmda bir uchi: *a*) A nuqtada; b) B nuqtada; c) C nuqtada bo'lgan nechta uchburchak bor?

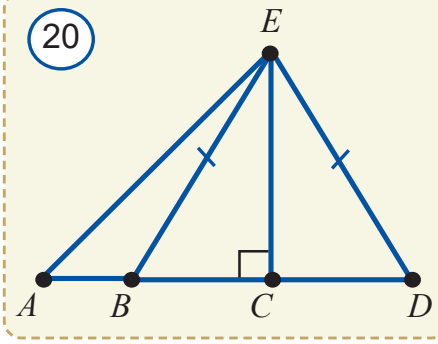
16. 20-rasmda uchburchakning qanday turlarini ko'r-yapsiz? Ularni turlari bo'yicha yozib chiqing.

17. O'tkir burchakli uchburchak chizing. Chizg'ich yordamida uning tomonlarini o'lchang va perimetrini toping.

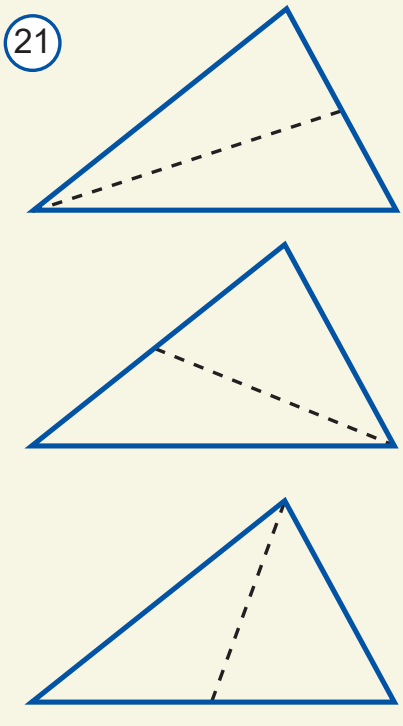
18. O'tmas burchakli uchburchak chizing. Chizg'ich yordamida uning tomonlarini o'lchang va perimetrini toping.

19. (Amaliy mashq). Uchta bir xil uchburchakni turli medianalari bo'ylab qirqing *(21-rasm)*. Hosil bo'lgan 6 ta uchburchakdan bitta uchburchak yasang.

20



21



20. Uchburchakning qaysi elementlari har doim uchburchakning ichki sohasida yotadi?

21*. Qanday uchburchakning uchta balandligi ham uchburchakning bir uchida kesishadi?

22*. Uchburchakning balandligi uning uchala tomonidan ham kichik bo‘lishi mumkinmi?

23. Perimetri 36 ga teng bo‘lgan uchburchakning balandligi uni perimetrlari 18 va 24 ga teng bo‘lgan uchburchaklarga ajratadi. Berilgan uchburchakning balandligini toping.

24. Perimetri 36 ga teng bo‘lgan uchburchakning bissektrisasi uni perimetrlari 24 va 30 ga teng bo‘lgan uchburchaklarga ajratadi. Berilgan uchburchakning shu bissektrisasini toping.

25. ABC uchburchakda $AB = BC$ va BD mediana 4 cm. Agar ABD uchburchak perimetri 12 cm bo‘lsa, ABC uchburchak perimetrini toping.

26. Chizg‘ich va transportir yordamida shunday ABC uchburchak yasangki, unda AB = 8 cm, AC= 5 cm va $\angle A$ = 60° bo‘lsin.

27. Chizg‘ich va transportir yordamida shunday KLM uchburchak yasangki, unda KL = 4 cm, KM= 3 cm va $\angle K$ = 90° bo‘lsin. Bu qanday uchburchak? Uning uchinchi tomonini o‘lchang va yozing.

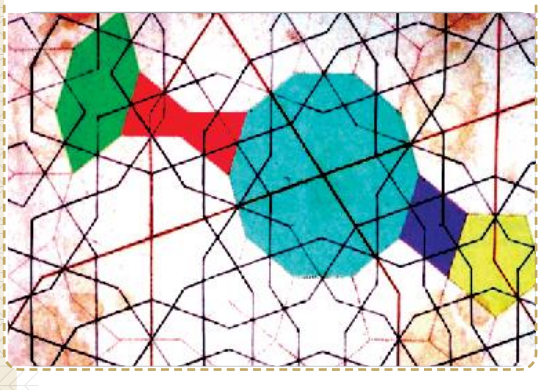
28*. Uchburchak perimetri 72 cm. Agar uning tomonlari uzunliklari nisbati 3:4:5 kabi bo‘lsa, bu tomonlarni toping. Uning burchaklarini o‘lchang. Bu qanday uchburchak?

29*. Uchburchak perimetri 126 mm. Agar uning tomonlari uzunliklari nisbati 3:4:5 kabi bo‘lsa, bu tomonlarni toping.

Tarixiy lavha

Handasa ilmida o‘z davridan besh asr o‘zib ketgan me’mor ustalarimiz





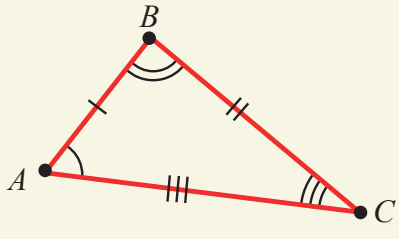
2007-yil fevral oyida Amerikada bosilgan o‘rta asr me’morchiligi haqidagi ilmiy maqola shov-shuvga sabab bo‘ldi. Gap shundaki, 2005-yilda Samarqanddagi Abdullaxon madrasasi gumbazidagi koshin-naqshlarni ko‘zdan kechirgan Harvard universitetining aspiranti Piter Lu hayratdan yoqasini ushlab qoldi. Uning ko‘z o‘ngida 1970-yillarda kashf qilingan deb hisoblangan, Penrouz naqshlari deb nomlangan murakkab geometrik shakllar turar edi. Bundan chiqdi, bizning me’mor ajdodlarimiz aql-u zakovatda o‘z davridan besh asr ilgarilab ketib, fanga yaqindagina kiritilgan murakkab geometrik shakllarni bilibgina qolmasdan, ulardan o‘z ishlarida ijodiy foydalanishgan ekan-da! Ha, darhaqiqat, shunday bo‘lib chiqdi. rasmda me’morchilik obidasidagi naqsh tasvirlangan. Ushbu rasm o‘rta asr qo‘lyozmalaridan olingan bo‘lib, unda mazkur naqsh asosini tashkil qiluvchi ko‘pburchaklar tasvirlangan.

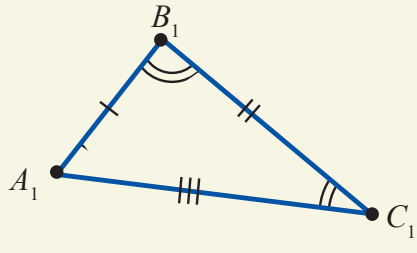
9 UCHBURCHAKLAR TENGLIGINING BIRINCHI ALOMATI

Geometrik shakllarning tengligi ta'rifiga ko'ra, ikkita uchburchakdan birini ikkinchisiga aynan ustma-ust tushadigan qilib qo'yish mumkin bo'lsa, ular *teng* bo'ladi. 1-rasmda ABC va $A_1B_1C_1$ teng uchburchaklar tasvirlangan. Ulardan ixtiyoriy bittasini ikkinchisiga ustma-ust qo'yish mumkin. Bunda bir uchburchakning uchta uchi – A, B, C va uchta tomoni – AB, BC, CA mos ravishda ikkinchi uchburchakning uchta uchi – A_1, B_1, C_1 va uchta tomoni – A_1B_1, B_1C_1, C_1A_1 bilan ustma-ust tushadi. Ravshanki, bunda uchburchaklarning mos burchaklari ham mos ravishda ustma-ust tushadi.

ABC va $A_1B_1C_1$ uchburchaklarning tengligi $\Delta ABC = \Delta A_1B_1C_1$ tarzda ifodalanadi. Chizmada teng burchaklar bir xil yoychalar bilan, teng tomonlar esa bir xil chiziqchalar bilan 1-rasmda tasvirlanganidek ajratib ko'rsatiladi.

1





$\Delta ABC = \Delta A_1B_1C_1$:

$AB = A_1B_1$,
$BC = B_1C_1$,
$CA = C_1A_1$

uchta mos tomonlar teng

$\angle BAC = \angle B_1A_1C_1$,
$\angle ABC = \angle A_1B_1C_1$,
$\angle ACB = \angle A_1C_1B_1$

uchta mos burchaklar teng

Faollashtiruvchi mashq

Uchburchak shaklidagi ikkita yer maydonining o'zaro tengligini amalda qanday tekshirish mumkin? Axir ulardan birini ikkinchisining ustiga qo'yib bo'lmaydi-ku? Ikki uchburchakning o'zaro teng yoki teng emasligini aniqlash uchun har doim ham ularni ustma-ust qo'yish shartmi?

Bunga hojat yo'q ekan. Bu masalani uchburchaklarning ba'zi elementlarini solishtirib hal etish mumkin ekan. "Uchburchaklarning tenglik alomatlari" deb nom olgan teoremalar shu haqda. Bu teoremalarning "alomat" deb yuritilishiga sabab shuki, ulardan foydalanib uchburchaklarning teng yoki teng emasligi haqida xulosa chiqarish mumkin.

Umuman olganda, geometriyada "alomat" shaklning biror xususiyatini aniqlashga yordam beruvchi shartlar haqidagi teoremadan iborat bo'ladi.

Quyidagi teorema "Uchburchaklarning ikki tomoni va ular orasidagi burchagi bo'yicha tengligi haqidagi teorema" deb ataladi. Biz uni qisqacha "Uchburchaklar tengligining TBT alomati" deb yuritamiz.

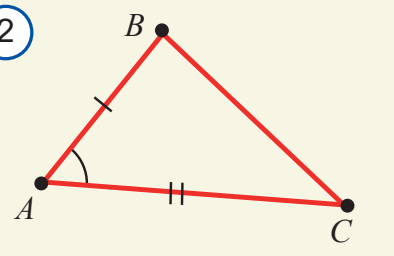
(TBT yozuvi "tomon", "burchak", "tomon" so'zlarining bosh harflaridan tuzilgan bo'lib, burchakning ikki tomon orasida yotganligini ham ko'rsatib turibdi.)

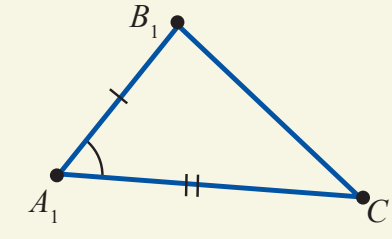
Teorema. (Uchburchaklar tengligining TBT alomati) **Agar bir uchburchakning ikki tomoni va ular orasidagi burchagi ikkinchi uchburchakning ikki tomoni va ular orasidagi burchagiga mos ravishda teng bo‘lsa, bunday uchburchaklar o‘zaro teng bo‘ladi** *(2-rasm).*

ΔABC va $\Delta A_1B_1C_1$
$AB = A_1B_1$, $AC = A_1C_1$, $\angle BAC = \angle B_1A_1C_1$

$\Rightarrow$ $\Delta ABC = \Delta A_1B_1C_1$

2



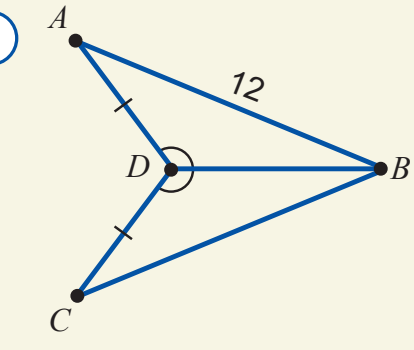


Isbot. $\angle BAC = \angle B_1A_1C_1$ bo‘lgani uchun ABC uchburchakni $A_1B_1C_1$ uchburchakka shunday qo‘yish mumkinki, unda A uch A_1 uchga, AB va AC nurlar esa mos ravishda A_1B_1 va A_1C_1 nurlar ustiga tushadi. $AB = A_1B_1$ va $AC = A_1C_1$ bo‘lgani uchun AB tomon A_1B_1 tomon bilan, AC tomon esa A_1C_1 tomon bilan ustma-ust tushadi. Xususan, B nuqta B_1 nuqta bilan, C nuqta esa C_1 nuqta bilan ustma-ust tushadi. Unda B_1C_1 va BC tomonlar ham ustma-ust tushadi. Natijada ABC uchburchakning uchta uchi (tomoni) $A_1B_1C_1$ uchburchakning uchta uchi (tomoni) bilan mos ravishda ustma-ust tushadi.

Demak, ABC va $A_1B_1C_1$ uchburchaklar o‘zaro teng.

Teorema isbotlandi.

3



Masala. 3-rasmda berilganlardan foydalanib BC kesmani toping.

Yechish. ADB va CDB uchburchaklarga qaraymiz. Shartga ko‘ra, bu uchburchaklar uchun $AD = DC$, $\angle ADB = \angle CDB$. BD esa umumiy tomon.

Demak, ADB uchburchakning ikki tomoni va ular orasidagi burchagi CDB uchburchakning ikki tomoni va ular orasidagi burchagiga mos ravishda teng ekan. Unda uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta ADB = \Delta CDB$.

Xususan, $CB = AB = 12$ bo‘ladi. ***Javob:*** 12.

Mavzuga doir savollar

1. *Qanday uchburchaklar teng deyiladi?*
2. *TBT alomatga ko‘ra, uchburchaklar tengligi qanday elementlarini taqqoslash orqali aniqlanadi?*
3. *Uchburchaklar tengligining TBT alomatini izohlang.*

Amaliy topshiriqlar va tatbiq

1. $\Delta ABC = \Delta DEF$. Bu uchburchaklarning mos uchlari, tomonlari va burchaklarini aniqlab yozing.

2. MNL va RST uchburchaklar teng. Bu uchburchaklarning mos uchlari, tomonlari va burchaklarini aniqlab yozing.

3. Agar $\Delta ABC = \Delta BAC$ ekanligi ma'lum bo'lsa, ABC uchburchakning qaysi tomonlari teng bo'ladi?

4. $\Delta MNL = \Delta LMN$. MNL uchburchakning qaysi burchaklari teng bo'ladi?

5. $\Delta ABC = \Delta DEF$. Agar $\angle A = 52°$, $\angle E = 80°$ va $\angle C = 48°$ bo'lsa, bu uchburchaklarning qolgan burchaklarini toping.

6. $\Delta ABC = \Delta LMN$. Agar $AB = 5$, $MN = 8$ va $AC = 9$ bo'lsa, bu uchburchaklarning qolgan tomonlarini toping.

7. 4-rasmdan noma'lum kesma – x ni toping.

8. Agar 5-rasmda $\angle CAB = \angle ABD$ bo'lsa, $AD = BC$ ekanini isbotlang.

9. 6-rasmda $\angle BAE = \angle BCD$ ekanini isbotlang.

10. 7-rasmda $\Delta ABC = \Delta CDA$ ekanini isbotlang.

11. 8-rasmda $\Delta ABC = \Delta ABD$ bo'lishini isbotlang.

12. AD va BC kesmalar O nuqtada kesishadi va bu nuqtada teng ikkiga bo'linadi *(9-rasm)*.

a) $\Delta AOB = \Delta DOC$; b) $BD = AC$; c) $\Delta ABD = \Delta DCA$;

d) $\angle OAB = 35°$, $\angle OBA = 62°$ bo'lsa, $\angle ODC$ va $\angle DCO$ ni toping.

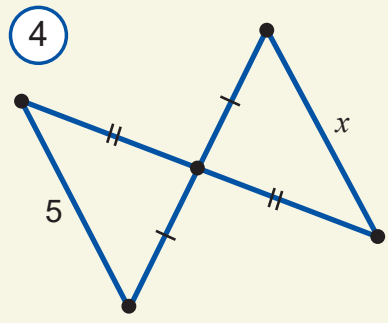
13. 10-rasmdagi noma'lum burchak – x ni toping.

14. Bir uchburchak perimetri ikkinchi uchburchak perimetriga teng. Bu uchburchaklar teng bo'ladimi?

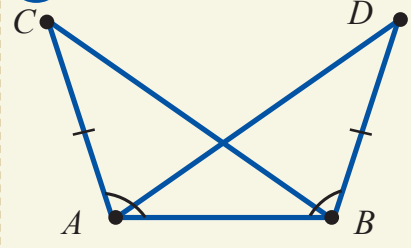
15. Bir uchburchak perimetri ikkinchi uchburchak perimetridan katta. Bu uchburchaklar teng bo'ladimi?

16. ABC uchburchakning AB tomonida D nuqta, $A_1B_1C_1$ uchburchakning A_1B_1 tomonida D_1 nuqta olingan. $\Delta ADC = \Delta A_1D_1C_1$ va $BD = B_1D_1$ tengliklar ma'lum. ABC va $A_1B_1C_1$ uchburchaklar tengligini isbotlang.

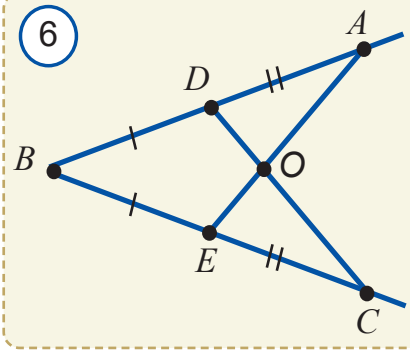
4



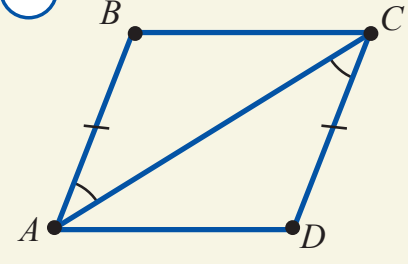
5



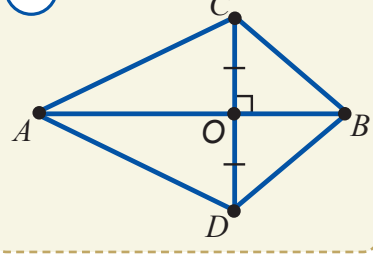
6



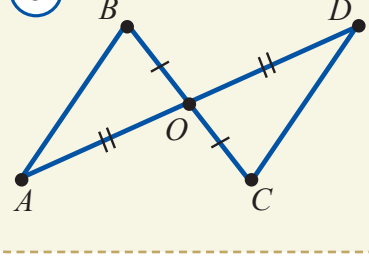
7



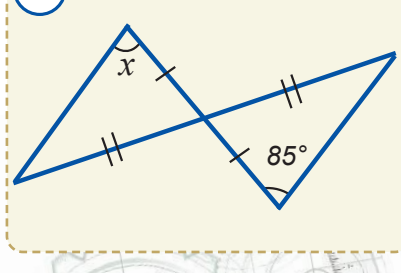
8



9

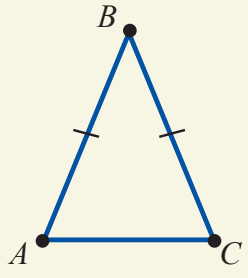


10



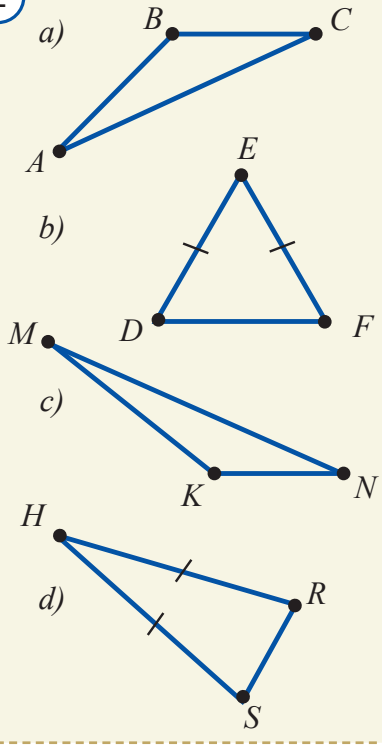
10 TENG YONLI UCHBURCHAKNING XOSSALARI

1

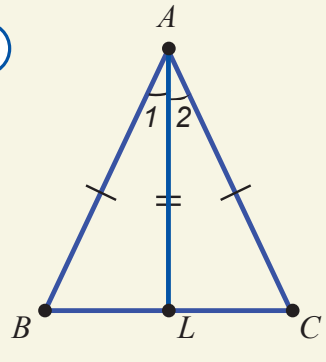


ABC – *teng yonli uchburchak*
AB, BC – *yon tomonlari*
AC – *asosi*
B – *uchi*

2



3



Ikkita tomoni teng bo‘lgan uchburchakni *teng yonli uchburchak* deb atagan edik *(1-rasm)*. Teng yonli uchburchakning teng tomonlari uning *yon tomonlari*, uchinchi tomoni esa *asosi,* asosi qarshisida yotgan uchi esa teng yonli uchburchakning *uchi* deb ataladi.

Faollashtiruvchi mashq

2-rasmdagi uchburchaklarning qaysilari teng yonli? Ularning uchi, asosi va yon tomonlarini ayting.

Geometrik tadqiqot

Chizg‘ich yordamida ixtiyoriy teng yonli uchburchak chizing. Uning asosiga yopishgan burchaklarini o‘lchang va solishtiring. Bu ishni yana 2–3 ta boshqa teng yonli uchburchaklar uchun qaytaring va o‘z taxminingizni tasdiq ko‘rinishida ifodalang. “Tajriba natijasida topilgan bu xossa barcha teng yonli uchburchaklar uchun ham o‘rinli bo‘ladi” deb aytish mumkinmi?

Teorema. **Teng yonli uchburchakning asosidagi burchaklari teng.**

$\Delta ABC,\ AB = AC$ *(3-rasm)* ⇒ $\angle B = \angle C$

Isbot. ABC uchburchakning A uchidan AL bissektrisani tushiramiz *(3-rasm)*.

ABL va ACL uchburchaklarga qaraymiz. Ularda AL tomon umumiy. Ikkinchidan, teorema shartiga ko‘ra, $AB = AC$ (ΔABC – teng yonli). Uchinchidan esa $\angle 1 = \angle 2$, chunki yasashga ko‘ra AL – bissektrisa. Demak, ABL uchburchakning ikki tomoni va ular orasidagi burchagi ACL uchburchakning ikki tomoni va ular orasidagi burchagiga mos ravishda teng ekan. Unda uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta ABL = \Delta ACL$ bo‘ladi.

Xususan, bu uchburchaklarning mos burchaklari sifatida $\angle B = \angle C$ bo‘ladi.

Teorema isbotlandi.

Geometrik tadqiqot

Teng yonli uchburchak chizing. Uning uchidan chiqqan bissektrisasini o‘tkazing. Bu bissektrisa uchburchak asosini ikki bo‘lakka bo‘ladi. Shu bo‘laklar uzunligini o‘lchab taqqoslang. Bundan qanday xulosa chiqadi? So‘ng bissektrisa bilan asos hosil qilgan burchaklarni transportirda o‘lchang va taqqoslang. Bundan qanday xulosa chiqadi? Bu xulosalarni tasdiq ko‘rinishida ifodalang. “Bu xulosalar barcha teng yonli uchburchaklar uchun ham o‘rinli” deb aytish mumkinmi?

Teorema. **Teng yonli uchburchak asosiga tushirilgan bissektrisa uning ham medianasi, ham balandligi bo‘ladi** *(4-rasm).*

ΔABC, $AB = AC$, AL – bissektrisa ⇨ AL – mediana va balandlik

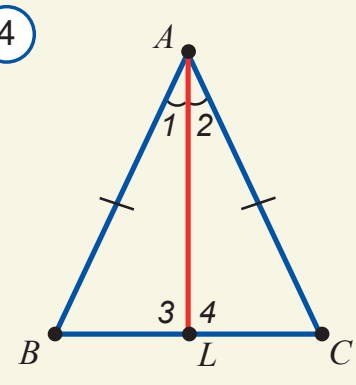
Isbot. 1. AL kesma ABC uchburchakning bissektrisasi bo‘lsa *(4-rasm)*, yuqorida isbotlangan teoremaga ko‘ra, $\Delta ABL = \Delta ACL$ bo‘ladi. Bu uchburchaklar tengligidan $BL = LC$ va $\angle 3 = \angle 4$ ekanini topamiz.

Demak, L nuqta – BC tomonning o‘rtasi, AL esa ABC uchburchakning medianasi ekan.

2. $\angle 3$ va $\angle 4$ o‘zaro teng va qo‘shni burchaklar bo‘lgani uchun ular to‘g‘ri burchaklardir.

Demak, AL kesma ABC uchburchakning balandligi ham bo‘lar ekan. ***Teorema isbotlandi.***

4



Natija. **Teng yonli uchburchakning uchidan chiqarilgan bissektrisasi, medianasi va balandligi ustma-ust tushadi.**

Masala. Teng yonli ABC uchburchakning yon tomonlariga AD va CF medianalar tushirilgan. $\Delta ADC = \Delta CFA$ va $\Delta ADB = \Delta CFB$ ekanini isbotlang.

ΔABC, $AB = BC$, AD va CF – medianalar *(5-rasm)* ⇨ $\Delta ADC = \Delta CFA$, $\Delta ADB = \Delta CFB$

Isbot. $AB = BC$ bo‘lgani uchun bu tomonlardan AD va CF medianalar ajratgan kesmalar o‘zaro teng bo‘ladi:

$AF = FB = BD = CD$. (1)

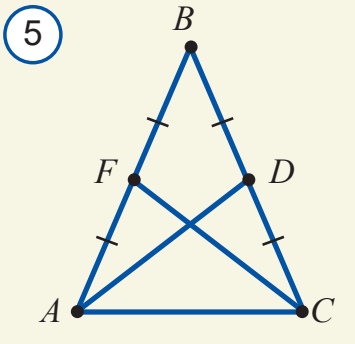
1) ADC va CFA uchburchaklarni qaraymiz. Ularda:

1. $\angle ACD = \angle FAC$, chunki ΔABC teng yonli;
2. AC tomon umumiy;
3. $AF = CD$ – (1) tenglikka ko‘ra.

Demak, uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta ADC = \Delta CFA$.

2) $\Delta ADB = \Delta CFB$ ekanini mustaqil isbotlang.

5

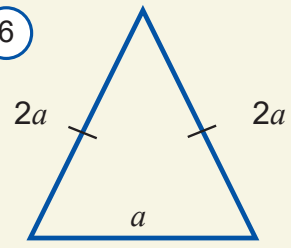


Mavzuga doir savollar

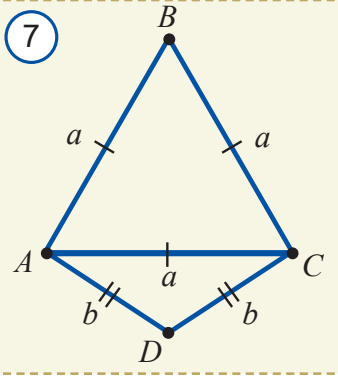
1. Teng yonli uchburchak ta’rifi va xossalarini ayting.
3. Teng yonli uchburchak asosiga tushirilgan mediana uning bissektrisasi bo‘ladimi?

Amaliy mashq va tatbiq

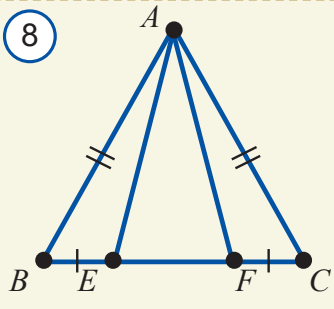
6



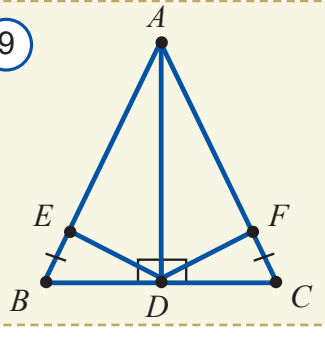
7



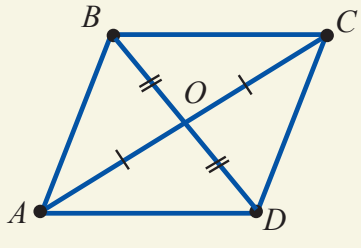
8



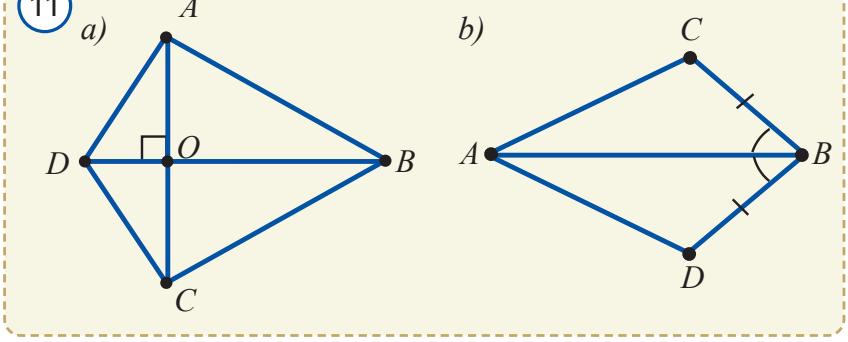
9



10



11



1. 6-rasmda $P = 50\ cm$ bo‘lsa, $a = ?$

2. 7-rasmda $P_{ABC} = 36$ va $P_{ADC} = 28$ bo‘lsa, $a = ?$, $b = ?$

3. Teng yonli uchburchakning yon tomonlariga tushirilgan medianalari teng bo‘lishini isbotlang.

4. 8-rasmda $AB=AC,\ BE=FC.$ a) $\Delta ABE=\Delta ACF$; b) $AE=AF$; c) $\Delta ABF=\Delta ACE$ ekanini isbotlang.

5. 9-rasmda $AB=AC,\ BE=CF.$ a) $\Delta AED=\Delta AFD$; b) $\Delta BED=\Delta CFD$ tengliklarni isbotlang.

6. Teng tomonli uchburchakning barcha burchaklari teng ekanini isbotlang.

7*. Ikkita teng yonli uchburchaklarning asoslari va shu asosga tushirilgan balandliklar tengligiga ko‘ra, ularning tengligini isbotlang.

8. Teng yonli uchburchakning asosi yon tomonidan $3\ cm$ katta, lekin yon tomonlarining yig‘indisidan $5\ cm$ kichik. Uchburchakning tomonlarini toping.

9*. Teng yonli uchburchak tomonlarining o‘rtalari tutashtirilsa, teng yonli uchburchak hosil bo‘lishini isbotlang.

10*. Bir uchburchakning ikki tomoni va bitta burchagi ikkinchi uchburchakning ikki tomoni va bitta burchagiga teng. Bu uchburchaklar teng bo‘ladimi?

11*. Shunday ikkita uchburchak chizingki, ulardan birining ikki tomoni va bitta burchagi ikkinchisining ikki tomoni va bitta burchagiga teng bo‘lsin, lekin ular teng bo‘lmasin.

12. 10-rasmda: $AO=OC$ va $BO=OD$. $AB=CD$ va $BC=AD$ ekanini toping.

13*. 11-rasmda a) $\Delta AOD=\Delta COD,\ \Delta ABD=\Delta CBD$; b) $\Delta ABD = \Delta ABC$ ekanini isbotlang.

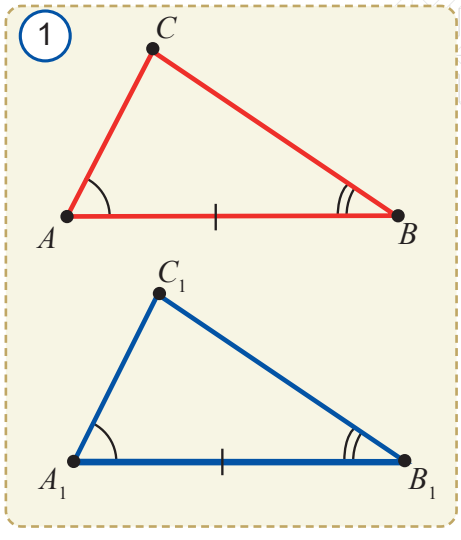
11 UCHBURCHAKLAR TENGLIGINING IKKINCHI ALOMATI

Endi uchburchaklarning bir tomoni va unga yopishgan burchaklari bo‘yicha tenglik alomatini ko‘ramiz. Kelgusida uni “uchburchaklar tengligining BTB alomati” deb yuritamiz.

Teorema. ***(Uchburchaklar tengligining BTB alomati)*** **Agar bir uchburchakning bir tomoni va unga yopishgan ikki burchagi ikkinchi uchburchakning bir tomoni va unga yopishgan ikki burchagiga mos ravishda teng bo‘lsa, bunday uchburchaklar o‘zaro teng bo‘ladi.**

ΔABC va $\Delta A_1B_1C_1$ *(1-rasm)*,
$AB = A_1B_1$, $\angle A = \angle A_1$, $\angle B = \angle B_1$

$\Rightarrow$ $\Delta ABC = \Delta A_1B_1C_1$



Isbot. ABC uchburchakni $A_1B_1C_1$ uchburchak ustiga shunday qo‘yaylikki, A uch A_1 uch bilan, AB tomon A_1B_1 tomon bilan ustma-ust tushsin, C va C_1 uchlar A_1B_1 to‘g‘ri chiziqning bir tomonida yotsin.

U holda $\angle A = \angle A_1$ bo‘lgani uchun AC tomon A_1C_1 nurda yotadi, $\angle B = \angle B_1$ bo‘lgani uchun BC tomon B_1C_1 nurda yotadi.

Shu sababli C nuqta AC va BC nurlarning umumiy nuqtasi sifatida A_1C_1 va B_1C_1 nurlarning har ikkalasida ham yotadi.

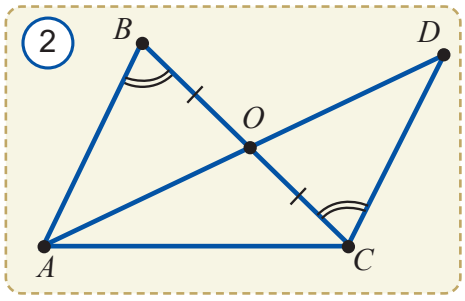
Unda C nuqta A_1C_1 va B_1C_1 nurlarning umumiy nuqtasi – C_1 bilan ustma-ust tushadi.

Natijada AC va A_1C_1, BC va B_1C_1 tomonlar ham o‘zaro ustma-ust tushadi.

Demak, ABC va $A_1B_1C_1$ uchburchaklar aynan ustma-ust tushadi. Bu esa ular teng deganidir.

Teorema isbotlandi.

Masala. 2-rasmda berilganlardan foydalanib, $\Delta AOB = \Delta DOC$ ekanini isbotlang.

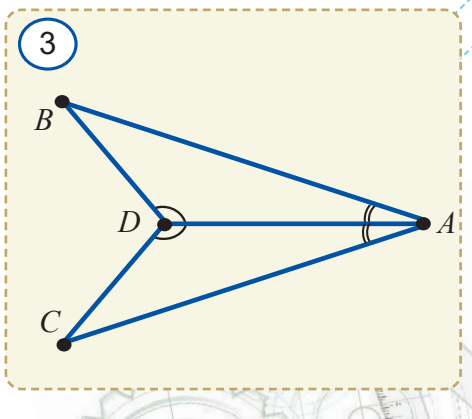


Isbot. $\angle AOB$ va $\angle DOC$ vertikal burchaklar bo‘lgani uchun ular teng bo‘ladi.

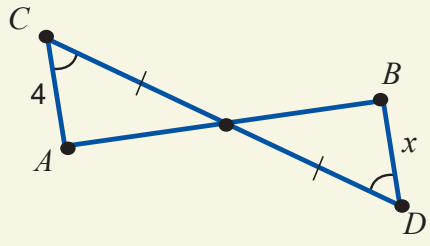
Natijada $BO = OC$, $\angle ABO = \angle DCO$ va $\angle AOB = \angle DOC$ tengliklarga egamiz.

Demak, AOB uchburchakning bir tomoni va unga yopishgan ikki burchagi DOC uchburchakning bir tomoni va unga yopishgan ikki burchagiga mos ravishda teng ekan.

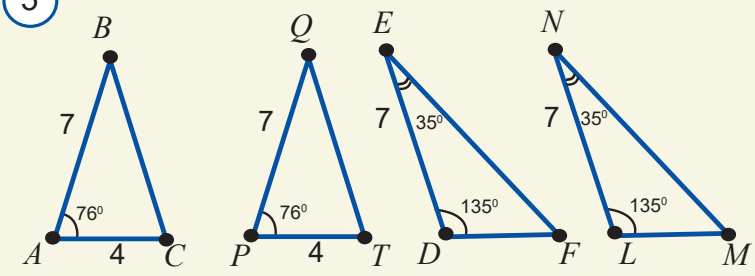
Unda uchburchaklar tengligining BTB alomatiga ko‘ra, AOB va DOC uchburchaklar teng bo‘ladi.



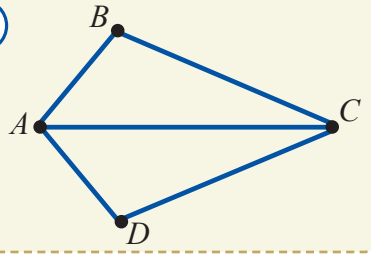
4



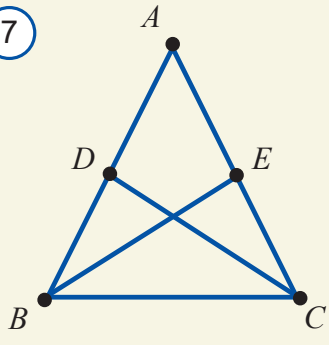
5



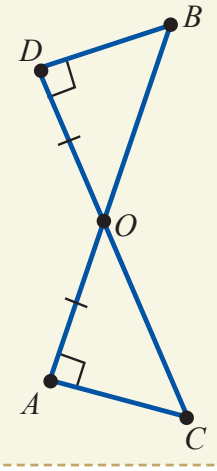
6



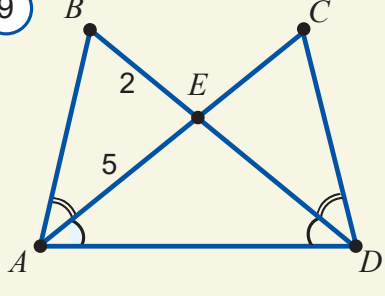
7



8



9



Mavzuga oid savollar

1. BTB alomatiga ko‘ra, uchburchaklar tengligi qanday elementlar bo‘yicha aniqlanadi?
2. Uchburchaklar tengligining BTB alomatini izohlang.
3. Uchburchaklar tengligining BTB alomati nega shunday nomlangan?

Amaliy mashq va tatbiq

1. 3-rasmdagi ABD va ACD uchburchaklarning tengligini isbotlang.

2. 4-rasmdagi noma’lum – x ni toping.

3. 5-rasmdagi uchburchaklarning qaysilari bir-biriga teng? Nega?

4. 6-rasmda AC kesma BAD va BCD burchaklarning bissektrisasi bo‘lsa, $\Delta ABC=\Delta ADC$ ekanini isbotlang.

5. ABC va $A_1B_1C_1$ uchburchaklarda $AB=A_1B_1$, $BC=B_1C_1$ va $\angle B=\angle B_1$ ekanligi ma’lum. AB va A_1B_1 tomonlarda mos ravishda D va D_1 nuqtalar $\angle ACD=\angle A_1C_1D_1$ bo‘ladigan qilib olingan. Unda $\Delta BCD=\Delta B_1C_1D_1$ ekanini isbotlang.

6. AB va CD kesmalar O nuqtada kesishadi. Agar $BO=CO$ va $\angle ACO=\angle DBO$ bo‘lsa, ACO va DBO uchburchaklar teng ekanini isbotlang.

7. 7-rasmdagi ABC uchburchakda $AB=AC$. BE va CD bissektrisalarning teng ekanini isbotlang.

8. $\Delta OAC=\Delta ODB$ bo‘lishini isbotlang *(8-rasm)*.

9. ABC va ADC uchburchaklar teng. B va D nuqtalar AC to‘g‘ri chiziqning turli tomonida yotadi. ABD va BCD uchburchaklar teng yonli ekanini isbotlang.

10*. 9-rasmdagi ma’lumotlar asosida AC va BD kesmalarni toping.

12 UCHBURCHAKLAR TENGLIGINING UCHINCHI ALOMATI

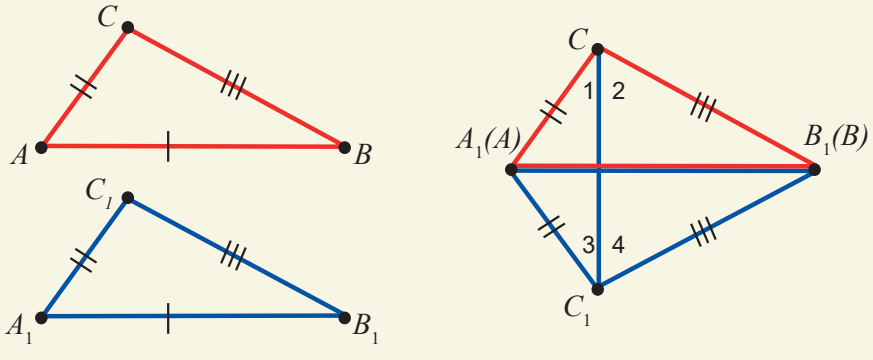
Endi uchburchaklarning uchta tomoni bo‘yicha tenglik alomati bilan tanishamiz. Kelgusida uni “uchburchaklar tengligining TTT alomati” deb yuritamiz.

Teorema. ***(Uchburchaklar tengligining TTT alomati)*** **Agar bir uchburchakning uchta tomoni ikkinchi uchburchakning uchta tomoniga mos ravishda teng bo‘lsa, bunday uchburchaklar o‘zaro teng bo‘ladi.**

Berilgan: ΔABC va $\Delta A_1B_1C_1$;
$AB = A_1B_1$, $AC = A_1C_1$, $BC = B_1C_1$.

$\Delta ABC = \Delta A_1B_1C_1$

1



Isbot. Aytaylik, ABC uchburchakning eng katta tomoni AB bo‘lsin. ABC uchburchakni $A_1B_1C_1$ uchburchak ustiga shunday qo‘yaylikki, AB tomon A_1B_1 tomon bilan ustma-ust tushsin hamda C va C_1 uchlar A_1B_1 to‘g‘ri chiziqning turli tomonlarida yotsin *(1-rasm)*.

U holda $AC = A_1C_1$ va $BC = B_1C_1$ bo‘lgani uchun A_1C_1C va B_1C_1C uchburchaklar teng yonli bo‘ladi.

Unda teng yonli uchburchak xossasiga ko‘ra, $\angle 1 = \angle 3$ va $\angle 2 = \angle 4$ bo‘ladi. Shuning uchun $\angle ACB = \angle A_1CB_1 = \angle A_1C_1B_1$ bo‘ladi.

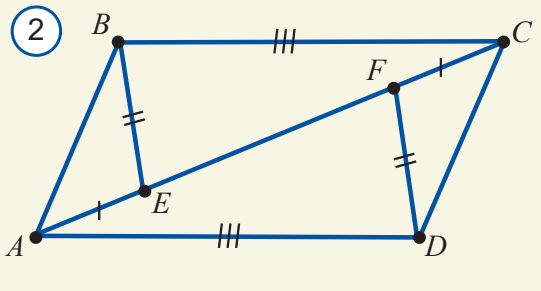
Demak, ABC va $A_1B_1C_1$ uchburchaklarda $AC = A_1C_1$, $BC = B_1C_1$ va $\angle ACB = \angle A_1C_1B_1$. Uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta ABC = \Delta A_1B_1C_1$.

Teorema isbotlandi.

Xulosa. Agar ikki uchburchakning har uchala tomoni mos ravishda teng bo‘lsa, ularning mos burchaklari ham o‘zaro teng bo‘ladi.

Masala. 2-rasmda berilganlardan foydalanib:
a) $\Delta AFD = \Delta CEB$;
b) $\Delta AEB = \Delta CFD$ ekanini isbotlang.

2



Isbot. 2-rasmda berilganlarga ko‘ra $AE = FC$, $BE = FD$ va $AD = BC$.

1) $AF = AE + EF$ bo‘lgani uchun
$EC = EF + FC = EF + AE = AF$.

Demak, ΔAFD va ΔCEB ning mos tomonlari o‘zaro teng va uchburchaklar tengligining TTT alomatiga ko‘ra, $\Delta AFD = \Delta CEB$.

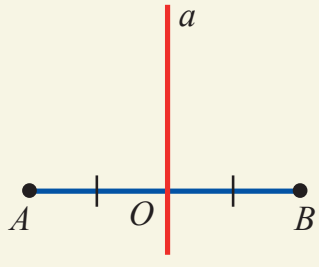
2) $\Delta AFD = \Delta CEB$ bo‘lgani uchun $\angle BEF = \angle EFD$. U holda, BEF va AEB, EFD va CFD burchaklar qo‘shni burchaklar bo‘lgani sababli $\angle AEB = \angle CFD$ bo‘ladi.

AEB va CFD uchburchaklarda: **1.** $AE = FC$; **2.** $BE = FD$; **3.** $\angle AEB = \angle CFD$.

Demak, uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta AEB = \Delta CFD$ bo‘ladi.

Kesma o‘rta perpendikulyarining xossasi

3



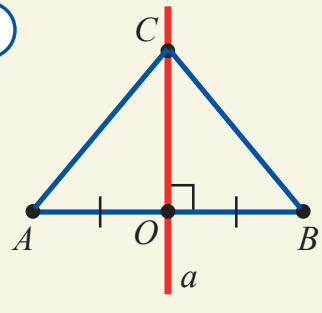
Endi uchburchaklar tenglik alomatlarining teoremalarni isbotlashda qo‘llanishini ko‘rib chiqamiz.

AB kesma berilgan bo‘lsin. Uning o‘rtasi bo‘lgan O nuqtadan AB kesmaga perpendikulyar a to‘g‘ri chiziqni o‘tkazamiz *(3-rasm)*. Bu to‘g‘ri chiziq AB kesmaning *o‘rta perpendikulyari* deb ataladi.

Teorema. **Kesma o‘rta perpendikulyarining ixtiyoriy nuqtasi kesma uchlaridan teng uzoqlikda joylashgan bo‘ladi.**

AB kesma, C – AB kesma o‘rta perpendikulyarining ixtiyoriy nuqtasi (4-rasm) ⇒ $AC = BC$

4



Isbot. ACO va BCO uchburchaklarda *(4-rasm)*:

1) OC – umumiy tomon;

2) $AO = BO$ – shartga ko‘ra;

3) $\angle AOC = \angle BOC = 90°$ – shartga ko‘ra.

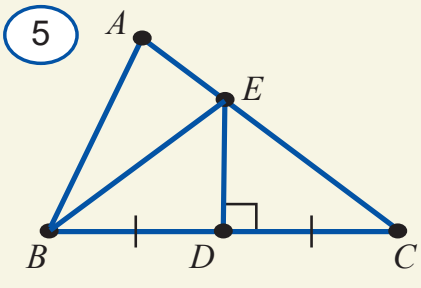
Demak, uchburchaklar tengligining *TBT* alomatiga ko‘ra, $\Delta AOC = \Delta BOC$.

Xususan, $AC = BC$.

Teorema isbotlandi.

Masala. ABC uchburchakning BC tomoniga o‘tkazilgan o‘rta perpendikulyar AC tomonni E nuqtada kesib o‘tadi. Agar BE = 6 cm, AC = 8,4 cm bo‘lsa, AE va CE kesmani toping.

5



Yechish. ABC uchburchak BC tomonining o‘rta perpendikulyari DE bo‘lsin *(5-rasm)*.

Kesma o‘rta perpendikulyarining xossasiga ko‘ra, $CE = BE$ = 6 cm.

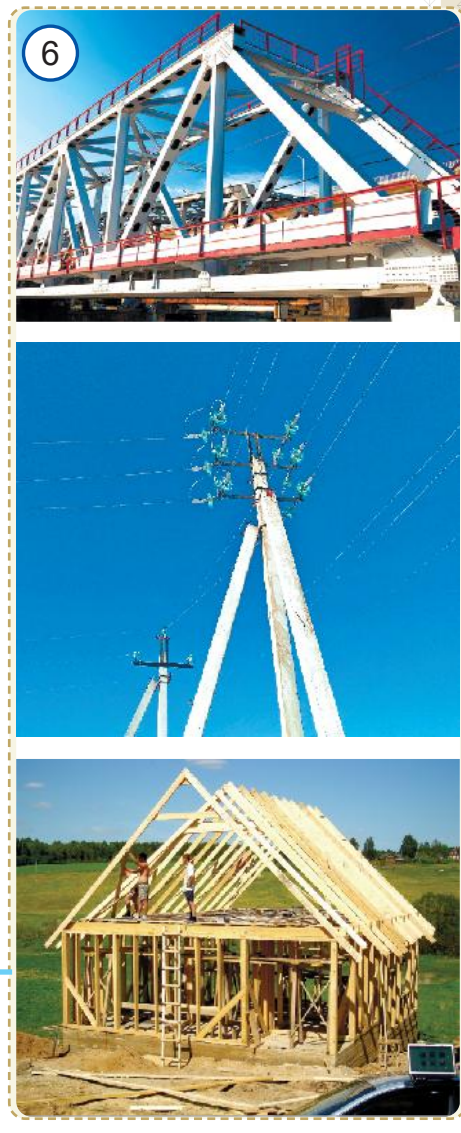
$AE + EC = AC$ bo‘lgani uchun,

$AE = AC - EC$ = 8,4 – 6 = 2,4 (cm).

Javob: AE = 2,4 cm, CE = 6 cm.

Mavzuga doir savollar

1. *6-rasmdan uchburchak shaklidagi qurilmalarni toping. Ular nima sababdan aynan uchburchak shaklda?*
2. *Uchburchaklar tengligining TTT alomatida uchburchaklar tengligi qanday elementlar bo‘yicha taqqoslanib aniqlanadi?*
2. *Uchburchaklar tengligining TTT alomatini izohlang.*
3. *Kesmaning o‘rta perpendikulyari nima?*
4. *Kesma o‘rta perpendikulyarining xossasini izohlang.*
5. *Biror uchburchak chizing va uning har bir tomoniga o‘rta perpendikulyar o‘tkazing. Nimani payqadingiz? Chizmangizni sinfdoshingiz chizmasi bilan solishtiring va aniqlangan xossani faraz sifatida ifodalang.*
6. *Qanday uchburchakda uchburchak tomoniga tushirilgan o‘rta perpendikulyar shu tomonga tushirilgan balandlik bilan ustma-ust tushadi?*



Amaliy mashq va tatbiq

1. 7-rasmda berilganlarga ko‘ra, $\Delta ABC = \Delta CDA$ ekanini isbotlang.

2. 8-rasmda: a) $\Delta ABC = \Delta ABD$; b) $\Delta BOC = \Delta BOD$ ekanini isbotlang.

3. 8-rasmda: a) $\Delta AOC = \Delta AOD$; b) $AB \perp CD$ ekanini isbotlang.

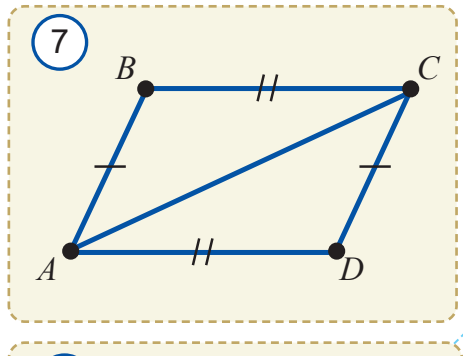
4. ABC va ABD asoslari AB bo‘lgan teng yonli uchburchaklar bo‘lsa, $\Delta ACD = \Delta BCD$ ekanini isbotlang.

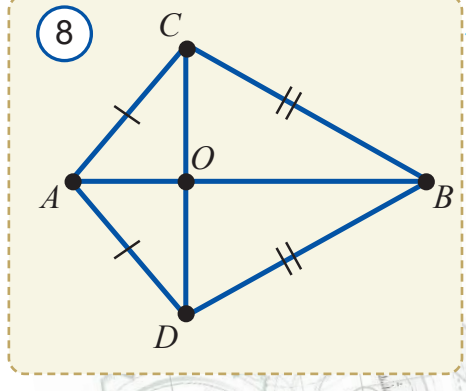
5. Agar 9-rasmda $BA = AK$, $AC = AN$, $\angle BAC = \angle NAK$ bo‘lsa, uchlari A, B, C, K va N nuqtalarda bo‘lgan barcha teng uchburchaklar juftligini aniqlang.

6. ABC va $A_1B_1C_1$ uchburchaklarda $AB = A_1B_1$ va $BC = B_1C_1$ bo‘lib, ularning perimetrlari teng bo‘lsa, $\Delta ABC = \Delta A_1B_1C_1$ ekanini ko‘rsating.

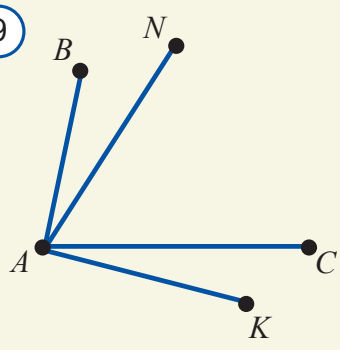
7.* AB va CD kesmalar kesishish nuqtasida teng ikkiga bo‘linadi. $\Delta ACD = \Delta BDC$ ekanini isbotlang.

8. 10-rasmda nechta o‘zaro teng uchburchaklar jufti borligini aniqlang.

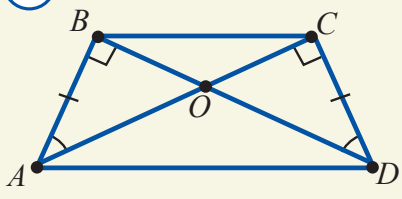




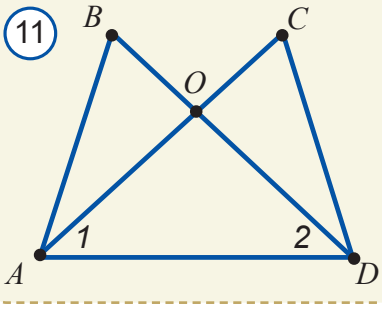
9



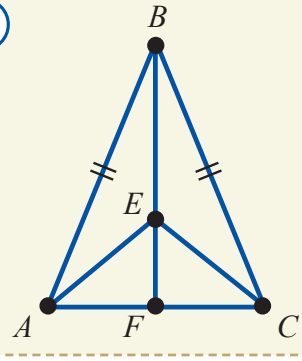
10



11



12



9*. Agar 11-rasmda: a) $\angle 1 = \angle 2$, $AC = BD$; b) $\angle 1 = \angle 2$, $BO = OC$, $AB = CD$ bo‘lsa, $\Delta ABD = \Delta DCA$ ekanini ko‘rsating.

10. ABC uchburchakning BC tomoniga o‘tkazilgan o‘rta perpendikulyar AC tomonni D nuqtada kesib o‘tadi. Agar $BD = 7{,}2\ cm$, $AD = 3{,}2\ cm$ bo‘lsa, AC nimaga teng?

11. ABC va ABD teng yonli uchburchaklar umumiy AB asosga ega. CD to‘g‘ri chiziq AB kesmaning o‘rta perpendikulyari bo‘lishini isbotlang.

12*. ABC teng yonli uchburchakning AB yon tomoniga o‘tkazilgan o‘rta perpendikulyar BC tomonni D nuqtada kesib o‘tadi. Agar ADC uchburchakning perimetri 24 cm ga teng va AB = 16 cm bo‘lsa, AC asosni toping.

13*. Uchburchakning tomonlariga tushirilgan o‘rta perpendikulyarlar bir nuqtada kesishishini isbotlang.

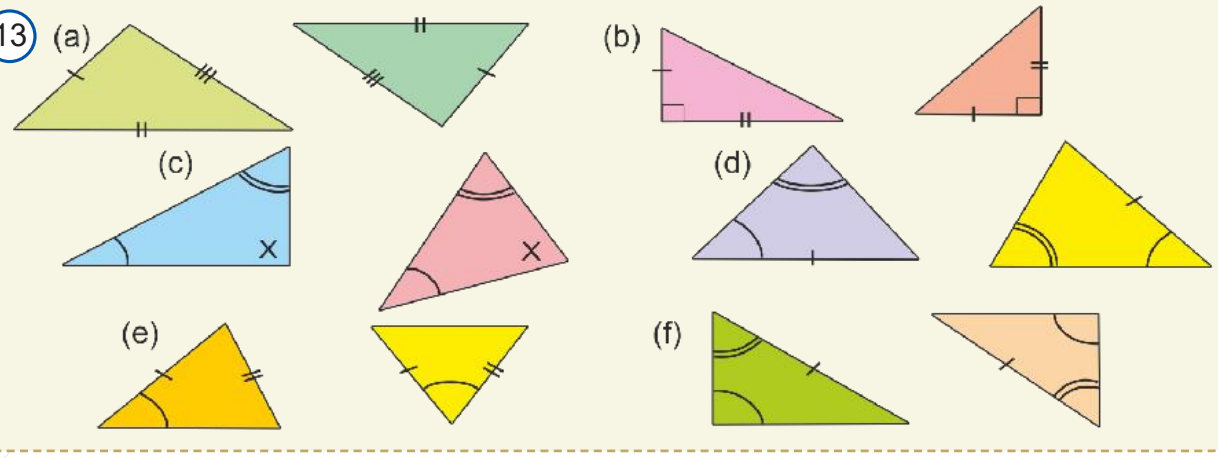
14. Teng yonli ABC uchburchakning asosiga tushirilgan BF bissektrisasida E nuqta olingan (*12-rasm*). $\Delta ABE = \Delta CBE$ tenglikni TTT alomatdan: a) foydalanib; b) foydalanmasdan isbotlang.

15. ABC va $A_1C_1B_1$ uchburchaklarda $AB = A_1B_1$, $BC = B_1C_1$, $CA = C_1A_1$, $\angle A = 30°$, $\angle B = 60°$ va $\angle C_1 = 90°$ ekanligi ma’lum. Bu uchburchaklarning qolgan burchaklarini toping.

16. ABC va DEF teng yonli uchburchaklar o‘zaro teng. ABC uchburchakda $AC = BC$ va AB = 2 cm. Agar $DE = 4\ cm$ bo‘lsa, har bir uchburchak perimetrini toping.

17. 13-rasmdagi uchburchaklar juftlaridan qaysi biri o‘zaro teng bo‘ladi? Nega?

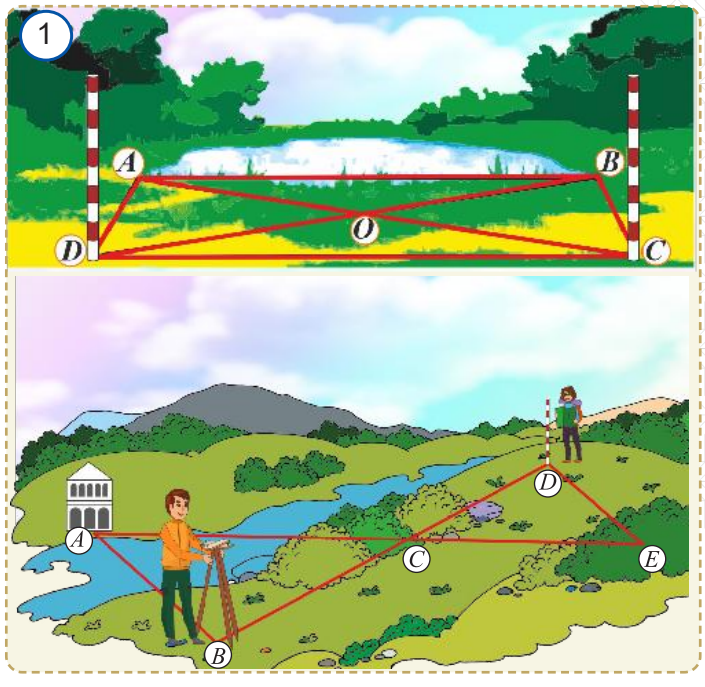
13



13 AMALIY MASHQ VA TATBIQ. BILIMINGIZNI SINAB KO‘RING

Ko‘lning kengligini o‘lchash

Aytaylik, A va B nuqtalar ko‘lning chetki nuqtalari bo‘lsin *(1-rasm)*. Ravshanki, AB kesmani bevosita o‘lchab bo‘lmaydi. Quruqlikda qanday yasash ishlarini bajarib bu masofani o‘lchash mumkin?



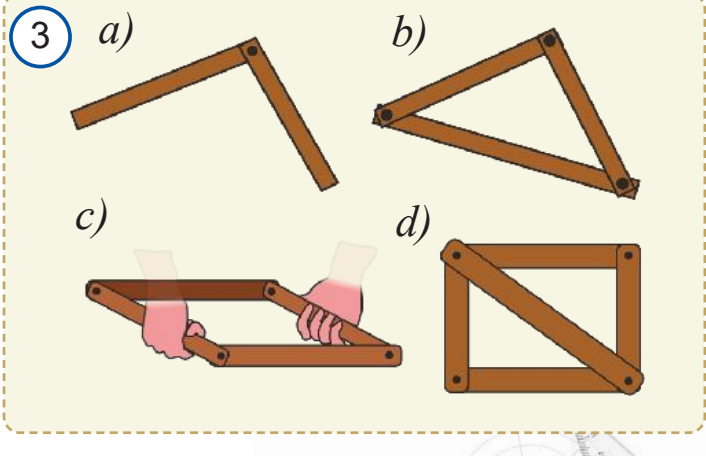
Yechish. Quruqlikda shunday O nuqtani tanlaymizki, OA va OB kesmalar bo‘ylab quruqlikdan A va B nuqtalarga borib bo‘lsin. ABC uchburchak yasaymiz. AO va BO tomonlarni davom ettirib, $OC = AO$ va $OD = BO$ kesmalarni qo‘yamiz. C va D nuqtalarni tutashtiramiz. Natijada uchburchaklar tengligining TBT alomatiga ko‘ra, $\Delta AOB = \Delta COD$ bo‘ladi. Xususan, $AB = DC$ ekani kelib chiqadi.

Demak, yasalgan DC kesmaning uzunligini o‘lchab, AB kesmaning ham uzunligini topgan bo‘lamiz. 2-rasmda tasvirlangan holatni mustaqil izohlang.

Uchburchakning “mustahkam” shakl ekanini asoslash

Ikki yog‘och taxtacha (reyka)larning uchlarini bir-biriga 3a-rasmda ko‘rsatilgandek qilib, mix bilan birlashtiramiz. Hosil bo‘lgan shakl “mustahkam” bo‘lmaydi, chunki uning erkin uchlarini turli tomonga burib, tomonlari orasidagi burchakni istalgancha o‘zgartirish mumkin.

Endi bu reykalarning erkin uchlariga uchinchi reykani, 3b-rasmda ko‘rsatilgandek qilib, mix qoqib birlashtiramiz. Hosil bo‘lgan uchburchak “mustahkam” shakl bo‘ladi. Chunki har qancha urinmang, uning tomonlarini burib, burchaklarini o‘zgartira olmaysiz.



1. Uchburchakning “mustahkam” shakl ekanini uchburchaklar tengligining TTT alomati yordamida izohlang.
2. 3 c, d-rasmdagi to‘rtburchakli moslama nima evaziga mustahkam bo‘ldi?
3. Uchburchakning mustahkamlik xususiyatidan qurilishda keng foydalaniladi. 89-betdagi 6-rasmda tasvirlangan inshoot va qurulishlarda nima sababdan uchburchak shakldagi qurilmalardan foydalanilayotganini izohlang.

13.2. Bilimingizni sinab ko‘ring

1. Bo‘sh qoldirilgan joylarni mantiqan to‘g‘ri so‘zlar bilan to‘ldiring.

1. Agar uchburchakning ikkita tomoni teng bo‘lsa, u ... bo‘ladi.
2. Teng yonli uchburchakning ... uning ham medianasi, ham balandligi bo‘ladi.
3. O‘z-o‘zini kesmaydigan yopiq siniq chiziqdan iborat shakl ... deyiladi.
4. Hamma tomonlari o‘zaro teng bo‘lgan uchburchakning ... teng bo‘ladi.
5. ... uchburchakning medianalari, bissektrisalari va balandliklari o‘zaro teng.
6. ... asosiga yopishgan burchaklari teng.
7. Teng tomonli uchburchak ... uchburchak ham bo‘ladi.

2. Quyida keltirilgan jumlalardagi xatoni toping va tuzating.

1. Teng yonli uchburchakning burchaklari teng.
2. Agar ikki uchburchakning burchaklari mos ravishda teng bo‘lsa, bu uchburchaklar teng bo‘ladi.
3. Teng yonli uchburchakning medianasi uning ham bissektrisasi, ham balandligi bo‘ladi.
4. Uchburchakning burchagidan chiqib, shu burchakni teng ikkiga bo‘luvchi nur “uchburchak bissektrisasi” deyiladi.
5. Mediana – uchburchak tomonini teng ikkiga bo‘luvchi chiziq.
6. Agar ikki uchburchakning bir tomoni va ikkita burchagi mos ravishda teng bo‘lsa, bu uchburchaklar teng bo‘ladi.
7. Bir uchburchakning ikki tomoni va bir burchagi ikkinchi uchburchakning ikki tomoni va bir burchagiga mos ravishda teng bo‘lsa, bu uchburchaklar teng bo‘ladi.
8. Perimetri teng bo‘lgan uchburchaklar o‘zaro teng bo‘ladi.
9. Uchburchakning balandligi uning tomoni o‘rtasiga tushirilgan perpendikulyardir.
10. Agar bir uchburchakning uchta burchagi ikkinchi uchburchakning uchta burchagiga mos ravishda teng bo‘lsa, bunday uchburchaklar o‘zaro teng bo‘ladi.

3. Berilgan xossaga ega bo‘lgan geometrik shakl nomini o‘ng ustundagi mos qatorga yozing.

1	Hamma medianalari teng	
2	Uchburchakning bir uchi va shu uchining qarshisidagi tomon o‘rtasini tutashtiruvchi kesma	
3	Uchburchakning bir uchidan shu uchining qarshisidagi tomonga tushirilgan perpendikulyar	
4	Uchburchak tomonlarining yig‘indisi	
5	O‘z-o‘zini kesmaydigan yopiq siniq chiziq	

4. Birinchi ustunda berilgan geometrik tushunchaga ikkinchi ustundan tegishli xossa yoki talqinni toping.

Geometrik tushunchalar	Xossasi yoki talqini
1. Siniq chiziq 2. Ko‘pburchak 3. Uchburchak perimetri 4. O‘tkir burchakli uchburchak 5. Teng yonli uchburchak 6. To‘g‘ri burchakli uchburchak 7. Uchburchak medianasi 8. Uchburchak bissektrisasi 9. Uchburchak balandligi 10. Kesmaning o‘rta perpendikulyari	(A) bitta burchagi to‘g‘ri burchak. (B) uchburchak uchini shu uch qarshisidagi tomon o‘rtasi bilan tutashtiradi. (C) ikki tomoni teng. (D) o‘z-o‘zini kesmaydigan yopiq siniq chiziq. (E) ketma-ket kelgan ikkitasi bir to‘g‘ri chiziqda yotmagan A_1A_2, A_2A_3, … , $A_{n-1}A_n$ kesmalardan tashkil topgan. (F) uchala tomoni yig‘indisi. (G) hamma burchaklari o‘tkir. (H) uchburchak burchagi bissektrisasining uchburchak uchki sohasida yotgan qismi. (I) uchburchak uchidan shu uch qarshisidagi tomon yotgan to‘g‘ri chiziqqa tushirilgan perpendikulyar. (J) kesma o‘rtasiga tushirilgan perpendikulyar.

5. Testlar.

1. Teng yonli uchburchakning ikki tomoni 8 va 3 ga teng. Uning uchinchi tomonini toping.

A) 5 B) 8 C) 11 D) 9

2. Teng yonli uchburchakning bir tomoni ikkinchisidan 3 cm qisqa va perimetri 36 cm bo‘lsa, uning yon tomonini toping.

A) 11 B) 12 C) 14 D) 18

3. Teng yonli uchburchakning perimetri 48, yon tomoni 18 ga teng. Uning asosini toping.

A) 18 B) 12 C) 16 D) 14

4. Teng yonli uchburchakning perimetri 48 ga teng. Uning tomonlaridan biri 12 ga teng bo‘lsa, qolgan tomonlarini toping.

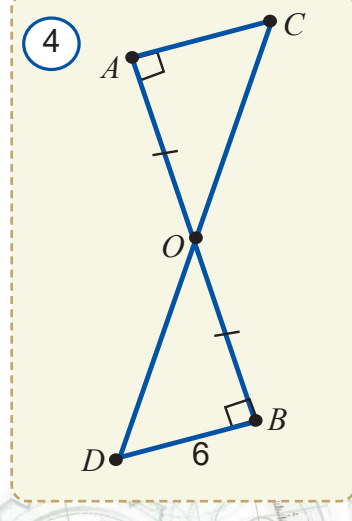
A) 12; 12 B) 16; 16 C) 18; 24 D) 18; 18

5. Teng yonli uchburchakning perimetri 36 ga, tomonlaridan biri esa 16 ga teng. Uchburchakning qolgan ikki tomonining uzunliklarini toping.

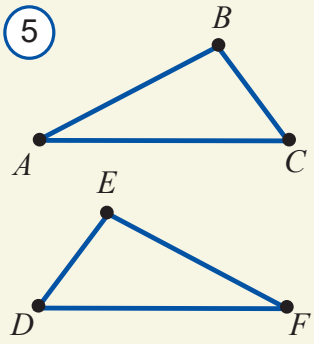
A) 16 va 4 B) 10 va 10 C) 10 va 10 yoki 16 va 4
D) Bunday uchburchak mavjud emas.

6. AC kesmaning uzunligini toping *(4-rasm).*

A) 6 B) 8 C) 12 D) 10,5

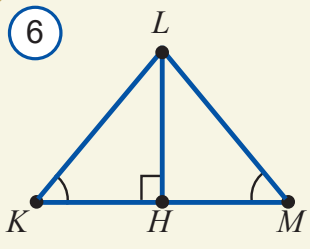


5

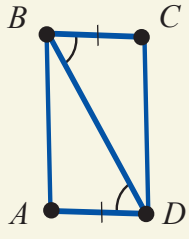


$AC = DF,\ \angle A = \angle F,$
$AB = FE$

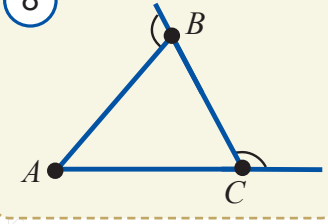
6



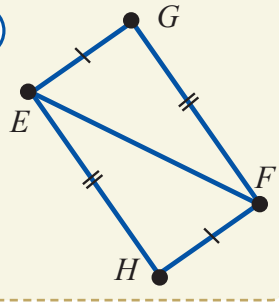
7



8



9



7. Uchburchakning nechta medianasi bor?

A) bitta B) ikkita C) uchta D) oltita

8. Uchburchak bissektrisasi qanday shakl?

A) kesma B) nur C) to‘g‘ri chiziq D) nuqta

9. Uchburchakning qaysi elementi uning tashqi sohasida yotishi mumkin?

A) medianasi B) balandligi
C) bissektrisasi D) diagonali

10. “Agar uchburchakning ikki burchagi teng bo‘lsa, bu uchburchak teng yonli uchburchak bo‘ladi” degan tasdiqni qanday atash mumkin?

A) ta’rif B) xossa
C) alomat D) aksioma

11. 5-rasmdagi ABC va FED uchburchaklar tengmi?

A) ha B) yo‘q

12. 6-rasmdagi qaysi uchburchaklar o‘zaro teng?

A) $\Delta KLM = \Delta LMH$ B) $\Delta KLH = \Delta MLH$
C) $\Delta KLM = \Delta KLH$ D) Hech qaysi

13. 7-rasmdagi ABD va CDB uchburchaklar qaysi alomatga asosan teng bo‘ladi?

A) Uchburchaklar tengligining TBT alomatiga ko‘ra
B) Uchburchaklar tengligining BTB alomatiga ko‘ra
C) Uchburchaklar tengligining TTT alomatiga ko‘ra
D) Bu uchburchaklar teng emas

14. 8-rasmga qarab uchburchak turini aniqlang.

A) teng tomonli B) teng yonli
C) o‘tmas burchakli D) hech narsa aytib bo‘lmaydi

15. 9-rasmdagi ma’lumotlarga ko‘ra, quyidagi tengliklardan noto‘g‘risini toping.

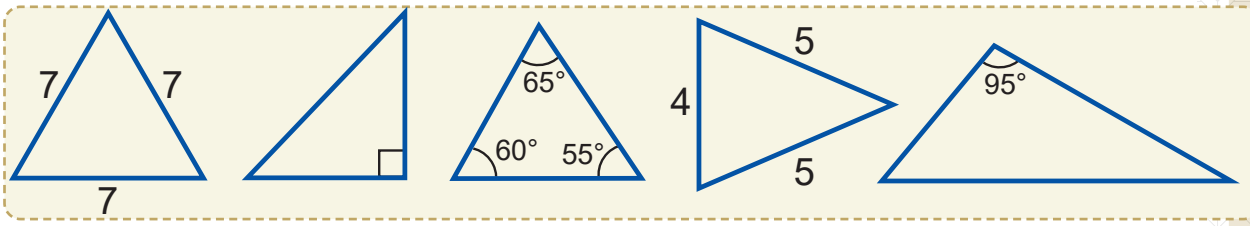
A) $\angle GEF = \angle HFE$ B) $\angle EGF = \angle FHE$
C) $\angle EHF = \angle FEG$ D) $\angle EFH = \angle GEF$

16. Perimetri $12\ cm$ bo‘lgan uchburchakning balandligi uni perimetrlari $7\ cm$ va $9\ cm$ bo‘lgan uchburchaklarga ajratadi. Balandlik uzunligini toping.

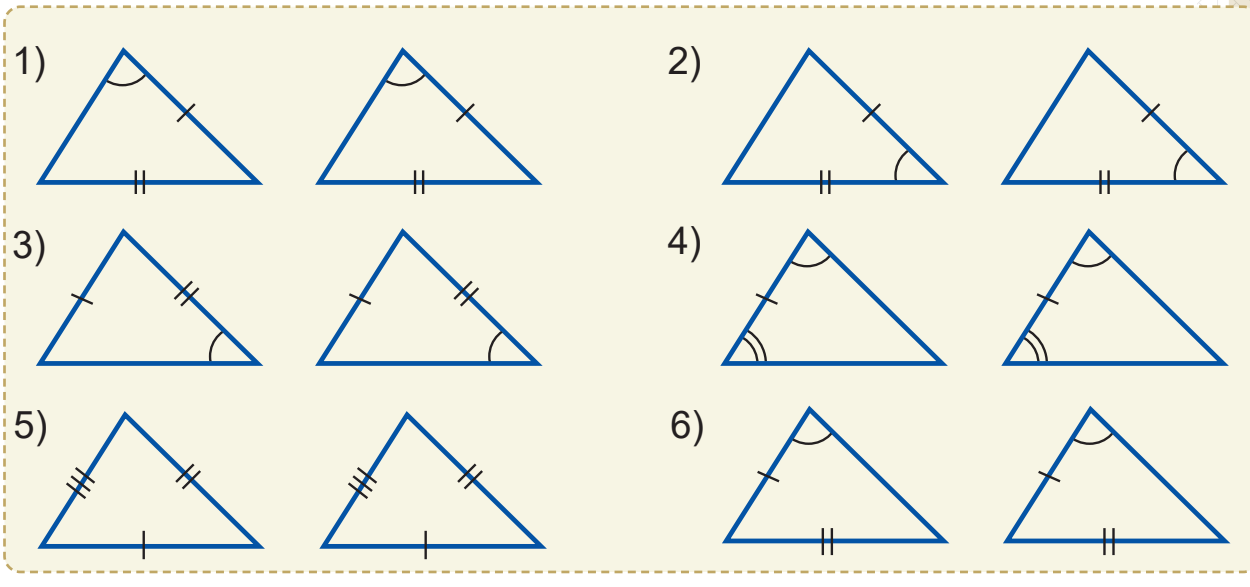
A) $2\ cm$ B) $3\ cm$ C) $1\ cm$ D) $4\ cm.$

6. Masalalar.

1. Rasmda berilgan ma'lumotlar asosida uchburchaklarning turlarini aniqlang.



2. Quyida keltirilgan uchburchaklar juftlaridan qaysilari o'zaro teng bo'ladi? Qaysi alomatga ko'ra?



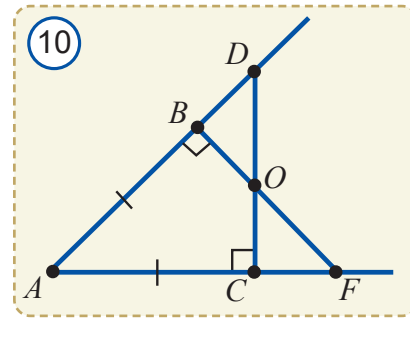
3. 10-rasmda $\Delta ACD = \Delta ABF$ ekanini isbotlang.

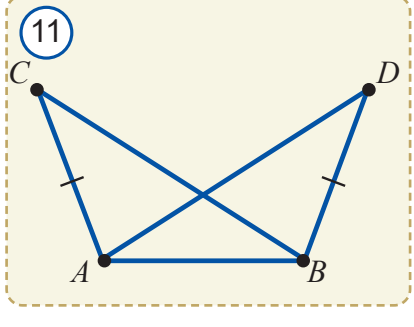
4. Agar 11-rasmda $\angle CAB = \angle ABD$ bo'lsa, $AD = BC$ ekanini ko'rsating.

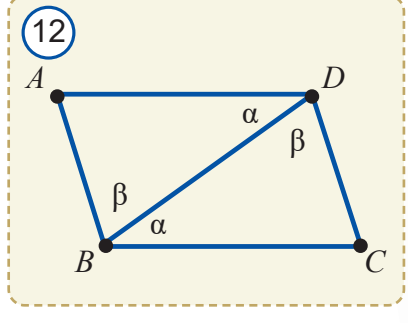
5. 12-rasmda $\Delta ABD = \Delta BCD$ bo'lishini isbotlang.

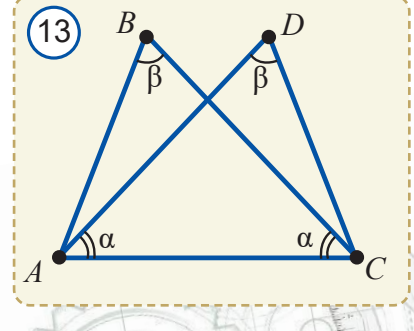
6. 13-rasmda $\Delta ABC = \Delta ADC$ bo'lishini isbotlang.

7. Agar ΔABC va ΔPQR da $AB = PQ$, $AC = PR$ va $BC = QR$ bo'lsa, ΔABC va ΔPQR teng bo'ladimi?

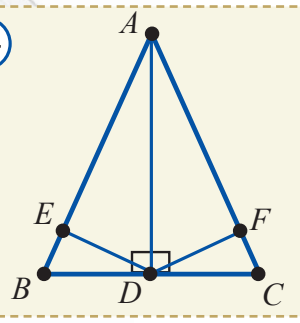








14



8. Agar 14-rasmda $BD=DC$, $AE=AF$ bo‘lsa, barcha teng uchburchaklar juftlarini aniqlang va asoslang.

9. 15-rasmda $\Delta ABC=\Delta EFD$ bo‘lishini isbotlang.

10. 16-rasmda $AD=CE$ ekanini isbotlang.

11. 17-rasmdagi ma’lumotlarga ko‘ra x ni toping.

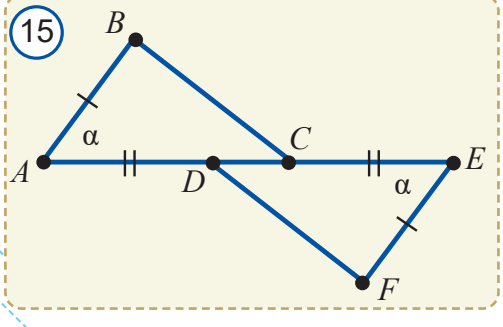
12. AE va BD kesmalar C nuqtada kesishadi. Agar $DC=DE$, $AB=BC$ va $\angle BAC=48°$ bo‘lsa, $\angle CED$ ni toping.

13. ABC uchburchak ichida D nuqta olingan. Agar $AC=AB$, $CD=BD$ va $\angle BDA=120°$ bo‘lsa, $\angle ADC$ ni toping.

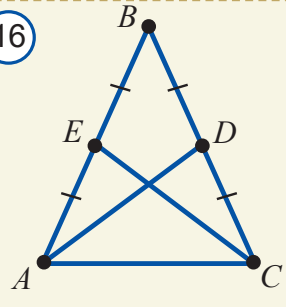
14. Agar uchburchakning biror bissektrisasi uning balandligi ham bo‘lsa, bu uchburchak teng yonli uchburchak bo‘lishini isbotlang.

15. Agar uchburchakning biror balandligi uning bissektrisasi ham bo‘lsa, bu uchburchak teng yonli uchburchak bo‘lishini isbotlang.

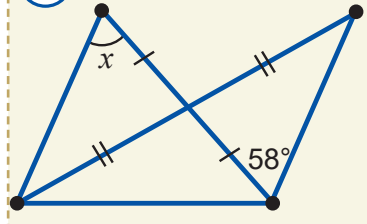
15



16

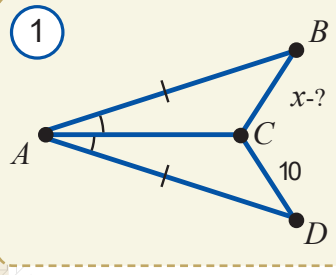


17



3-nazorat ishi namunasi

1



Nazorat ishi ikki qismdan iborat bo‘ladi:

1) 93–94-betlardagi test savollariga o‘xshash 5 ta test;

2) Quyidagi masalalarga o‘xshash 3 ta masala (4-masala “a’lo” baho olmoqchi bo‘lgan o‘quvchilar uchun qo‘shimcha).

1. 1-rasmda berilgan ma’lumotlar bo‘yicha noma’lum kesmani toping.

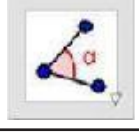
2. AB va CD kesmalar O nuqtada kesishadi. Agar $\angle CAB=\angle ABD$ va $AO=BO$ bo‘lsa, $\angle ACO=\angle BDO$ ekanini isbotlang.

3. Teng yonli uchburchakning perimetri 18,4 m ga teng, asosi esa yon tomonidan 3,6 m ga qisqa. Bu uchburchakning tomonlarini toping.

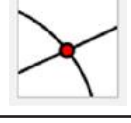
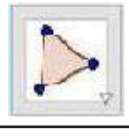
4*. Uchburchaklar tengligini ikki tomonlari va shu tomonlarining biriga tushirilgan medianalari tengligiga ko‘ra isbotlang.

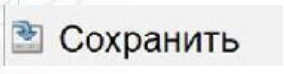
“GeoGebra”da amaliy topshiriqlar bajarish

- ***Kerakli komponentlar***
- Yasashni boshlashdan oldin teng tomonli uchburchakning xossalarini esga oling.
- ***GeoGebra***da yangi oyna oching.
- ***GeoGebra*** interfeysini ***Настройки*** – ***Геометрия*** ko‘rinishiga o‘tkazing.
- ***“GeoGebra” uskunalar panelidagi kerakli uskunalar***

	**Окружность с центром через точку** **Eslatma.** Sichqonchani bir marta bosish orqali aylana markazini, ikkinchi marta bosish orqali aylana radiusini belgilang.
	**Показывать объект** **Eslatma.** Yashirin holatga o‘tkazish kerak bo‘lgan obyektni tanlang.
	**Угол** **Eslatma.** Uchta nuqtani soat strelkasiga qarshi yo‘nalishda belgilang va kerakli burchakni hosil qiling.

Teng tomonli uchburchak yasash algoritmi

1		***AB*** kesma yasang.
2		Markazi ***B*** nuqtada bo‘lgan aylana chizing.
3		Markazi ***A*** nuqtada bo‘lgan ***B*** nuqtadan o‘tuvchi aylana chizing.
4		Ikkala aylana kesishish nuqtalarini belgilang.
5		***ABC*** ko‘pburchakni soat strelkasiga qarshi yo‘nalish bo‘ylab yasang.
6		Ikkala aylanani yashirin holatga o‘tkazing.

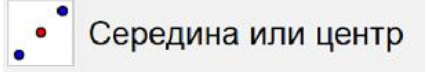
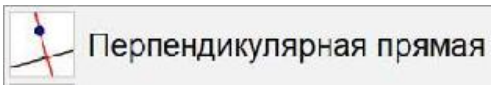
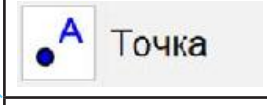
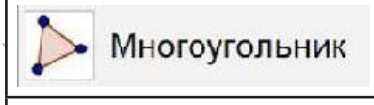
7		Uchburchakning ichki burchaklarini uning tomonlariga bosish orqali ko‘rsating. **Eslatma.** Sichqonchani soat strelkasi bo‘ylab harakatlantirish orqali uchburchakning ichki burchaklarini toping.
8		O‘zgarishlarni saqlang.
9		***Перемещать*** vositasidan foydalanib teng tomonli uchburchak to‘g‘ri yasalganini tekshiring.

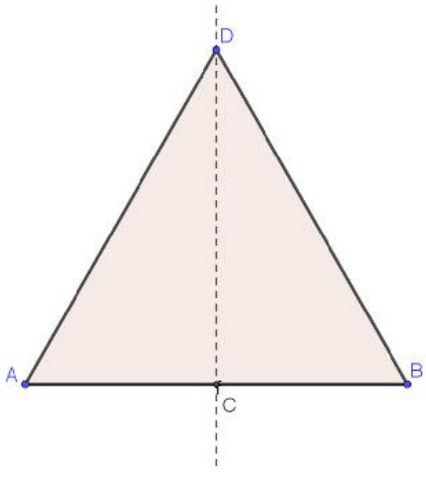
Mustaqil bajarish uchun topshiriq

Teng yonli uchburchak yasash

Asosi va balandligi uzunligini sichqoncha bilan sudrab o‘zgartirish mumkin bo‘lgan teng yonli uchburchak yasang.

Ushbu vazifani bajarish uchun sizga quyidagi vositalar kerak bo‘ladi:

	Kesma yasash
	Kesma yoki ikki nuqta o‘rtasini topish
	Berilgan nuqta orqali to‘g‘ri chiziq yoki kesmaga perpendikulyar o‘tkazish
	Tekislikda nuqta yasash
	Ko‘pburchak yasash
	Grafik obyektni sichqoncha bilan sudrab, dinamik ravishda o‘zgartirish





III BOB

PARALLEL TO‘G‘RI CHIZIQLAR

Bu bobni o‘rganib chiqqach, quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilim:

– parallel to‘g‘ri chiziqlarning ta’rifi va xossalari;
– ikki to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘ladigan burchaklarning turlarini bilish va ularni chizmada farqlay olish;
– ikki to‘g‘ri chiziqning parallellik alomatlari;
– berilgan teoremaga teskari bo‘lgan teoremani ifodalay olish.

Amaliy ko‘nikmalar:

– uchburchakli va oddiy chizg‘ich yordamida parallel to‘g‘ri chiziqlarni yasay olish;
– ikki to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘ladigan burchaklarni chizmada ko‘rsatib bera olish.
– parallel to‘g‘ri chiziqlarning xossalari va ikki to‘g‘ri chiziqning parallellik alomatlaridan masalalar yechishda foydalanish.

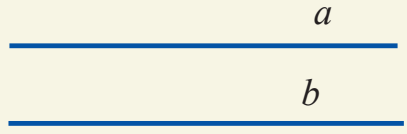
14 PARALLEL TO‘G‘RI CHIZIQLAR

14.1. To‘g‘ri chiziqlarning parallelligi

Faollashtiruvchi mashq

Agar ikki to‘g‘ri chiziq bitta to‘g‘ri chiziqqa perpendikulyar bo‘lsa, ular o‘zaro kesishishi mumkinmi? Javobingizni asoslang.

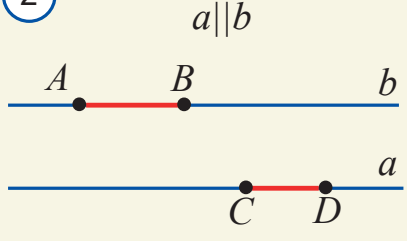
1



Bir tekislikda yotgan va o‘zaro kesishmaydigan to‘g‘ri chiziqlar *parallel to‘g‘ri chiziqlar* deb ataladi. 1-rasmda parallel to‘g‘ri chiziqlar tasvirlangan. a va b to‘g‘ri chiziqlarning parallelligi $a||b$ tarzda yoziladi va qisqacha “*a to‘g‘ri chiziq b to‘g‘ri chiziqqa parallel*” deb o‘qiladi.

Parallel to‘g‘ri chiziqlarda yotgan kesmalar (nurlar) *parallel kesmalar* (nurlar) deb yuritiladi. 2-rasmda parallel kesmalar tasvirlangan.

2



Parallel kesmalarni hayotda ko‘p uchratgansiz. Misol uchun, temiryo‘l relslari *(3-rasm)*, to‘g‘ri to‘rtburchak shaklidagi stolning qarama-qarshi qirralari, katakli daftar varag‘idagi gorizontal yoki vertikal chiziqlar va hokazo.

Shunday qilib, ta’rifga ko‘ra, to‘g‘ri chiziqlar parallel bo‘lishi uchun ular bir tekislikda yotishi va umumiy nuqtaga ega bo‘lmasligi, ya’ni kesishmasligi kerak.

53-betda oldin isbotlangan teoremani endi quyidagicha ifodalash mumkin:

3

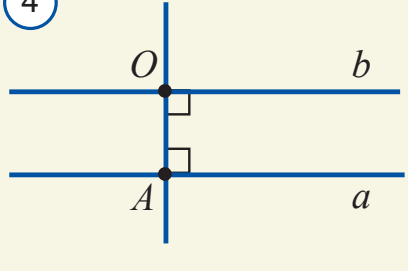


Teorema. Bitta to‘g‘ri chiziqqa perpendikulyar bo‘lgan ikki to‘g‘ri chiziq o‘zaro parallel.

Masala. a to‘g‘ri chiziqqa tegishli bo‘lmagan O nuqtadan unga parallel to‘g‘ri chiziq o‘tkazish mumkinligini ko‘rsating.

Yechish. 57-betda isbotlangan teoremaga ko‘ra, O nuqtadan a to‘g‘ri chiziqqa perpendikulyar OA to‘g‘ri chiziq o‘tkazamiz *(4-rasm)*. So‘ng O nuqtadan OA to‘g‘ri chiziqqa, yana o‘sha teoremaga ko‘ra, perpendikulyar b to‘g‘ri chiziqni o‘tkazamiz. Natijada $a \perp OA$ va $OA \perp b$, ya’ni OA to‘g‘ri chiziqqa perpendikulyar bo‘lgan ikkita a va b to‘g‘ri chiziqlarga ega bo‘lamiz. Unda yuqoridagi teoremaga ko‘ra, a va b to‘g‘ri chiziqlar o‘zaro parallel bo‘ladi, ya’ni b izlangan to‘g‘ri chiziqdir.

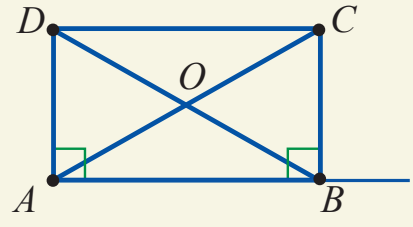
4



Geometrik tadqiqot

A nur olib, unga AB kesma qo‘ying *(5-rasm)*. A va B to‘g‘ri burchaklarni A nurdan boshlab tayin yarimtekislikka qo‘ying. Bu burchaklar tomonlariga bir-biriga teng bo‘lgan AD va BC kesmalarni qo‘ying. So‘ng DC kesmani o‘tkazing. Natijada “*Sakkeri to‘rtburchagi*” deb nomlangan to‘rtburchakka ega bo‘lasiz. Bu to‘rtburchak qanday xossalarga ega? Uning to‘g‘ri to‘rtburchak ekanini isbotlashga urinib ko‘ring va xulosa chiqaring.

5



Ma’lum bo‘lishicha, Sakkeri to‘rtburchagida: a) diagonallar o‘zaro teng: $AC=BD$; b) $\angle ADC$ va $\angle DCB$ ham o‘zaro teng. Bu xossalarni mustaqil isbotlab, ishonch hosil qiling.

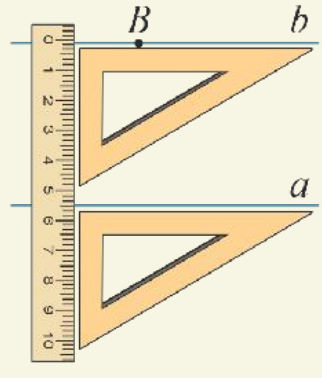
Lekin $\angle ADC$ va $\angle BCD$ larning $90°$ ga tengligini isbotlashning hech iloji yo‘q ekan. Buni isbotlash uchun “parallellik aksiomasi” deb nomlangan xossa zarur bo‘lar ekan. To‘g‘ri chiziqqa unda yotmagan nuqtadan nechta parallel to‘g‘ri chiziq o‘tkazish mumkin? Bu savolga ham parallellik aksiomasi javob beradi. Quyida shu aksioma haqida to‘xtalamiz.

Tekislikdagi to‘g‘ri chiziqqa unda yotmagan nuqtadan faqat bitta parallel to‘g‘ri chiziq o‘tkazish mumkin.

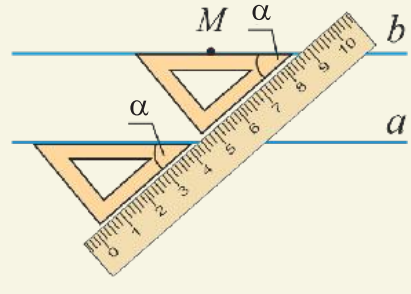
Bu tasdiqni aksioma sifatida isbotsiz qabul qilamiz va unga keyingi bandlarda yana atroflicha to‘xtalamiz. Parallel to‘g‘ri chiziqlarni amaliyotda oddiy va uchburchakli chizg‘ichlar yordamida 6-rasmda tasvirlangan tartibda chizish mumkin.

6

a)



b)



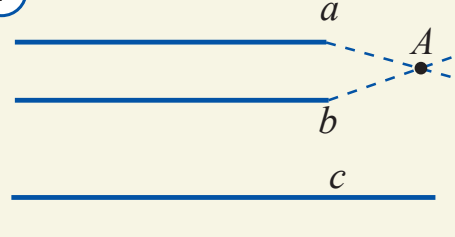
Teorema. **Bir to‘g‘ri chiziqqa parallel bo‘lgan ikki to‘g‘ri chiziq o‘zaro paralleldir.**

a, b va c to‘g‘ri chiziqlar $a||c$, $b||c$. ⇒ $a||b$

Isbot. Faraz qilaylik, $a||c$ va $b||c$ bo‘lsa-da, a va b to‘g‘ri chiziqlar parallel bo‘lmasin. U holda ular biror A nuqtada kesishadi *(7-rasm)* va A nuqtadan c to‘g‘ri chiziqqa ikkita a va b parallel to‘g‘ri chiziq o‘tkazilgan bo‘lib qoladi. Bu esa parallellik aksiomasiga zid. Demak, farazimiz noto‘g‘ri: a va b to‘g‘ri chiziqlar o‘zaro parallel ekan.

Teorema isbotlandi.

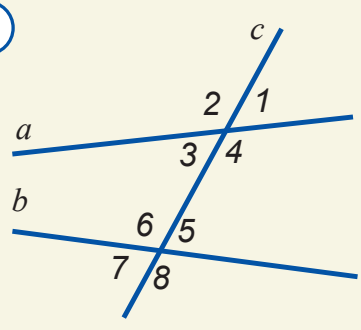
7



14.2. Ikki to‘g‘ri chiziq va kesuvchi hosil qilgan burchaklar

Tekislikda berilgan ikkita – A va b to‘g‘ri chiziq uchinchi – c to‘g‘ri chiziq bilan kesilganda, 8 ta burchak hosil bo‘ladi. Ularni 8-rasmda ko‘rsatilgandek raqamlar bilan belgilaylik. Bu burchaklarning quyidagi juftlarini alohida nomlar bilan ataymiz:

8



∠3 va ∠5, ∠4 va ∠6 — **almashinuvchi burchaklar**

∠4 va ∠5, ∠3 va ∠6 — **bir tomonli burchaklar**

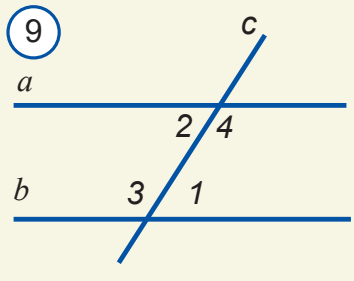
∠1 va ∠5, ∠2 va ∠6, ∠3 va ∠7, ∠4 va ∠8 — **mos burchaklar**

Bu burchaklarning quyidagi xossalarini keltiramiz:

1-xossa. **Agar ikki to‘g‘ri chiziq va kesuvchi hosil qilgan bir juft almashinuvchi burchaklar o‘zaro teng bo‘lsa, ikkinchi juft almashinuvchi burchaklar ham o‘zaro teng bo‘ladi.**

a, b to‘g‘ri chiziqlar va c kesuvchi: $\angle 1 = \angle 2$ *(9-rasm)* ⇒ $\angle 3 = \angle 4$

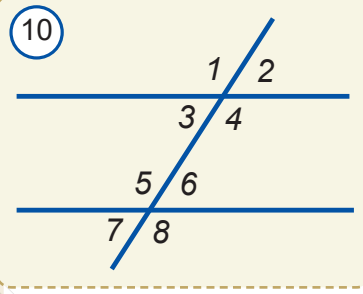
9



Isbot. $\angle 2$ va $\angle 4$ qo‘shni burchaklar bo‘lgani uchun:
$\angle 2 + \angle 4 = 180^\circ$. Bundan $\angle 4 = 180^\circ - \angle 2$.
$\angle 1$ va $\angle 3$ ham qo‘shni burchaklar bo‘lgani uchun:
$\angle 1 + \angle 3 = 180^\circ$. Bundan $\angle 3 = 180^\circ - \angle 1$.
Shartga ko‘ra, $\angle 1 = \angle 2$ ekanini hisobga olsak:
$\angle 3 = 180^\circ - \angle 1 = 180^\circ - \angle 2 = \angle 4$.
Demak, $\angle 3 = \angle 4$.
Xossa isbotlandi.

2-xossa. **Agar mos burchaklar teng bo‘lsa, bir tomonli burchaklar yig‘indisi 180° ga teng bo‘ladi.**

10



Isbot. Mos burchaklardan biror jufti, masalan, $\angle 2 = \angle 6$ bo‘lsin *(10-rasm)*. $\angle 6 + \angle 4 = 180^\circ$ ekanini isbotlaymiz. $\angle 2$ va $\angle 4$ qo‘shni burchaklar bo‘lgani uchun $\angle 2 + \angle 4 = 180^\circ$ bo‘ladi. U holda $\angle 2 = \angle 6$ bo‘lgani uchun $\angle 6 + \angle 4 = 180^\circ$ ekani kelib chiqadi.

Boshqa bir tomonli burchaklar yig‘indisi ham 180° ga tengligi shu tariqa isbotlanadi.

Xossa isbotlandi.

3-xossa. **Agar almashinuvchi burchaklar o‘zaro teng bo‘lsa, u holda mos burchaklar ham o‘zaro teng bo‘ladi.**

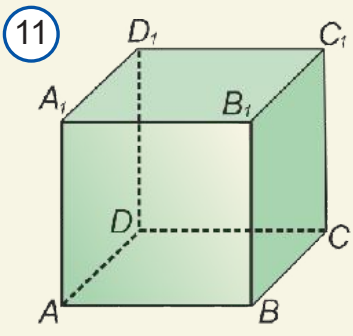
Isbot. $\angle 3$ va $\angle 6$ almashinuvchi burchaklar bo‘lib, $\angle 3 = \angle 6$ bo‘lsin (*10-rasm*). U holda

$\angle 3$ va $\angle 2$ vertikal burchaklar bo‘lgani uchun $\angle 3 = \angle 2$ bo‘ladi.

Demak, mos burchaklar $\angle 6$ va $\angle 2$ teng ekan. Boshqa mos burchaklar juftlari tengligi ham shunga o‘xshash isbotlanadi.

Xossa isbotlandi.

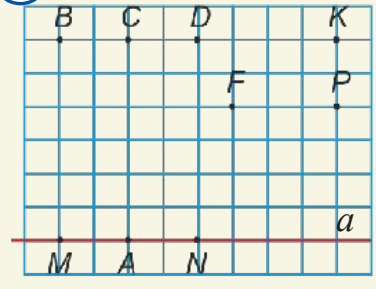
11



Mavzuga doir savollar

1. *Qanday to‘g‘ri chiziqlar parallel bo‘ladi?*
2. *Berilgan to‘g‘ri chiziqda yotmaydigan nuqta orqali shu to‘g‘ri chiziqqa parallel bo‘lgan nechta to‘g‘ri chiziq o‘tkazish mumkin?*
3. *Sinfxonaga nazar soling va parallel kesmalarni aniqlang.*
4. *a va b to‘g‘ri chiziq uchinchi – c to‘g‘ri chiziq bilan kesilganda qanday burchaklar juftlari hosil bo‘ladi?*
5. *Kesishmaydigan ikki kesmani parallel kesmalar desa bo‘ladimi? Kesishmaydigan ikkita nurni-chi?*
6. *Ikki to‘g‘ri chiziq va kesuvchi hosil qilgan burchaklarning qanday xossalarini bilasiz?*
7. *11-rasmdagi kubning o‘zaro parallel qirralarini aniqlang.*
8. *12-rasmda daftar katagidan foydalanib quyidagilarni aniqlang:*
   - *a to‘g‘ri chiziqqa perpendikulyar va A nuqtadan o‘tuvchi to‘g‘ri chiziq yana qaysi nuqtadan o‘tadi?*
   - *a to‘g‘ri chiziqqa parallel va F nuqtadan o‘tuvchi to‘g‘ri chiziq yana qaysi nuqtadan o‘tadi?*
   - *a to‘g‘ri chiziqqa parallel va K nuqtadan o‘tuvchi to‘g‘ri chiziq yana qaysi nuqtalardan o‘tadi?*
9. *12-rasmda daftar katagidan foydalanib quyidagi tasdiqlarning to‘g‘riligini tekshiring: a) $AB \perp a$; b) $BM \perp a$; c) $KP \perp a$; d) $FK \parallel a$; e) $BC \parallel a$; f) $KP \parallel a$.*
10. *Usta duradgorlar 13-rasmda tasvirlangan “shayton” deb nomlanuvchi asbobdan qanday foydalanadi?*

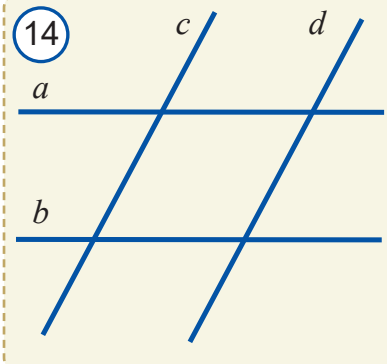
12



13



14



Amaliy mashq va tatbiq

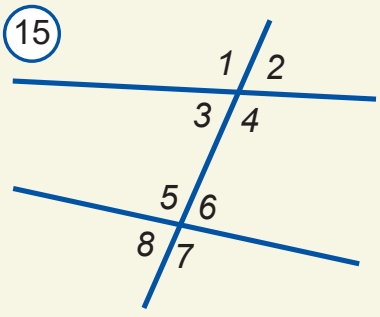
1. To‘g‘ri chiziq chizib, unda A, B va C nuqtalarni belgilang. Chizg‘ich va uchburchakli chizg‘ich yordamida A nuqtadan, B nuqtadan va C nuqtadan o‘tuvchi va bir-biriga parallel bo‘lgan to‘g‘ri chiziqlarni o‘tkazing.

2. 14-rasmdagi parallel to‘g‘ri chiziqlar juftlarini yozing.

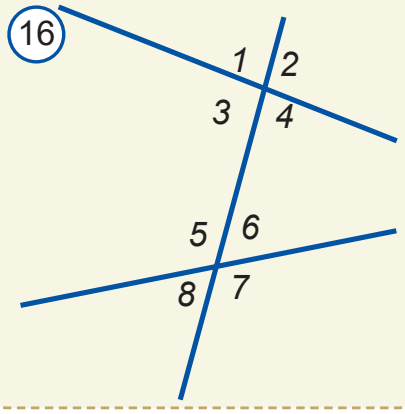
3. Uchinchi to‘g‘ri chiziqqa parallel bo‘lgan ikkita to‘g‘ri chiziqning o‘zaro parallel bo‘lishini ko‘rsating.

4. Agar to‘g‘ri chiziq parallel to‘g‘ri chiziqlarning birini kesib o‘tsa, ikkinchisini ham kesib o‘tadimi? Javobingizni asoslang.

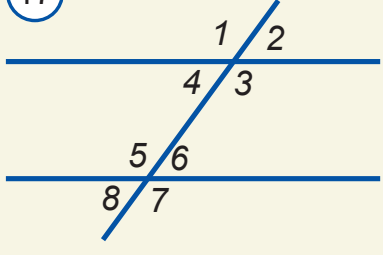
15



16



17



5. To‘g‘ri to‘rtburchakning qarama-qarshi tomonlari o‘zaro parallel ekanini ko‘rsating.

6. Varaqqa ikkita to‘g‘ri chiziq chizildi. Agar varaq bu chiziqlar bo‘ylab qirqilsa, nechta bo‘lak hosil bo‘ladi.

7. Varaqqa ikkita to‘g‘ri chiziqni shunday chizingki, varaq bu chiziqlar bo‘ylab qirqilsa, uchta bo‘lak hosil bo‘lsin.

8. 15-rasmdagi burchaklardan qaysilari vertikal va qaysilari qo‘shni burchak bo‘ladi?

9. Agar 16-rasmda: a) $\angle 4 = \angle 8 = 63°$; b) $\angle 1 = \angle 6 = 63°$ bo‘lsa, qolgan burchaklarni toping.

10. Agar 17-rasmda $\angle 2 = \angle 6 = 63°$ bo‘lsa, qolgan burchaklarni toping.

11. Agar 17-rasmda $\angle 1 = \angle 7 = 122°$ bo‘lsa, qolgan burchaklarni toping.

12. Agar ikki to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq bilan kesganda hosil bo‘lgan burchaklardan biri 82° va yana biri 110° bo‘lsa, qolgan burchaklarni toping.

13*. Agar ikki to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq bilan kesganda hosil bo‘lgan burchaklardan biri 32° va yana biri 134° bo‘lsa, qolgan burchaklarni toping.

14*. 17-rasmda $\angle 3 = \angle 5$ bo‘lsa, $\angle 2 = \angle 8$ va $\angle 4 = \angle 6$ bo‘ladimi? Javobingizni asoslang.

15. 17-rasmda $\angle 1 = \angle 7$ bo‘lsa, $\angle 3 = \angle 5$ va $\angle 4 = \angle 6$ tengliklar bajariladimi? Javobingizni asoslang.

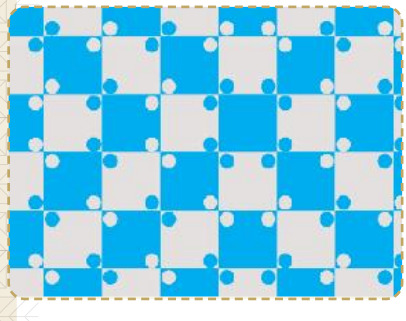
16.* Bir tomonli burchaklar o‘zaro teng bo‘lishi mumkinmi?

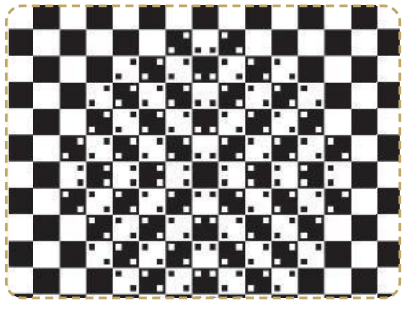
17*. Almashinuvchi burchaklar teng bo‘lsa, bir tomonli burchaklar yig‘indisi 180° ga tengligini ko‘rsating.

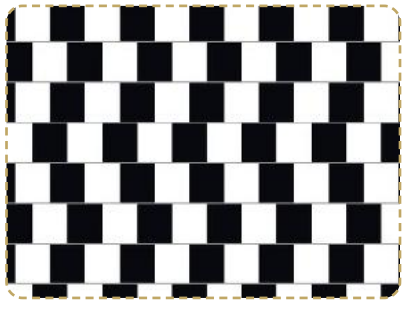
18.* Bir tomonli burchaklar yig‘indisi 180° ga teng bo‘lsa, almashinuvchi burchaklar o‘zaro teng bo‘ladimi?

19. Agar ikki to‘g‘ri chiziq va kesuvchi hosil qilgan bir juft mos burchaklar o‘zaro teng bo‘lsa, ikkinchi juft mos burchaklar ham teng bo‘lishini isbotlang.

Geometrik illyuziya. Quyidagi rasmlardagi gorizontal va vertikal chizilgan chiziqlar parallelmi? Buni chizg‘ich yordamida tekshirsangiz, o‘z ko‘zlaringizga ishonmasdan lol qolasiz!



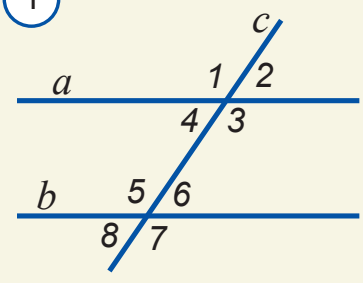




15 IKKI TO‘G‘RI CHIZIQNING PARALLELLIK ALOMATLARI

Faollashtiruvchi mashq

1



1-rasmda a va b parallel to‘g‘ri chiziqlar va c kesuvchi tasvirlangan. Quyidagi topshiriqlarni bajaring va savollarga javob bering.

1. Barcha almashinuvchi burchaklar juftlarini yozing va ularni transportirda o‘lchang. Har bir juft almashinuvchi burchaklarning gradus o‘lchovlari haqida nima deya olasiz?

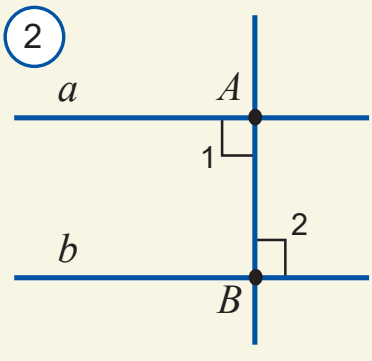
2. Barcha bir tomonli burchaklar juftlarini yozing va ularni transportirda o‘lchang. Har bir juft bir tomonli burchaklar gradus o‘lchovlarining yig‘indisi haqida nima deya olasiz?

3. Barcha mos burchaklar juftlarini yozing va ularni transportirda o‘lchang. Har bir juft mos burchaklarning gradus o‘lchovlari haqida nima deya olasiz?

4. Yuqorida aniqlangan xususiyatlar hamma vaqt ham o‘rinli bo‘ladimi?

Ikki to‘g‘ri chiziqning parallelligini qanday aniqlash mumkin? “Ikki to‘g‘ri chiziqning parallellik alomatlari” deb nomlangan quyidagi teoremalar bu savolga javob beradi.

2



Teorema. **Agar ikki to‘g‘ri chiziq va kesuvchi hosil qilgan almashinuvchi burchaklar teng bo‘lsa, u holda bu ikki to‘g‘ri chiziq parallel bo‘ladi.**

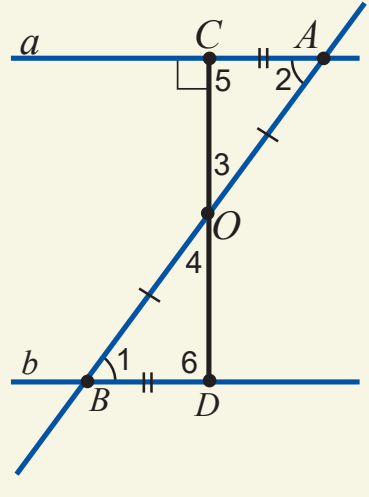
Isbot. 1. Oldin $\angle 1$ va $\angle 2$ to‘g‘ri burchak bo‘lgan holni qaraymiz *(2-rasm)*. Bu holda AB to‘g‘ri chiziq a va b to‘g‘ri chiziqlarga perpendikulyar bo‘ladi. Unda a va b to‘g‘ri chiziqlar bir to‘g‘ri chiziqqa perpendikulyar bo‘lgan ikki to‘g‘ri chiziq haqidagi teoremaga asosan o‘zaro parallel bo‘ladi (100-betdagi teoremaga qarang).

2) Endi $\angle 1$ va $\angle 2$ to‘g‘ri burchak bo‘lmagan holni ko‘ramiz: AB kesmaning o‘rtasi $(AO=BO)$ O nuqtadan a to‘g‘ri chiziqqa OC perpendikulyar tushiramiz *(3-rasm)*. b to‘g‘ri chiziqqa B nuqtadan uzunligi AC ga teng BD kesma qo‘yamiz. AOC va BOD uchburchaklarni qaraymiz.

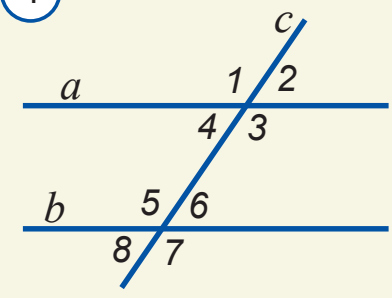
Ularda shartga ko‘ra $\angle 1 = \angle 2$, yasashga ko‘ra $AC=BD$ va $AO=BO$.

Unda uchburchaklar tengligining TBT alomatiga ko‘ra $\Delta AOC = \Delta BOD$ bo‘ladi. Xususan, $\angle 3 = \angle 4$ va $\angle 5 = \angle 6$ bo‘ladi.

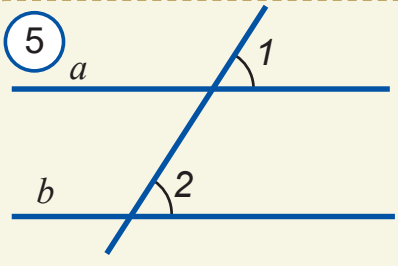
3



4

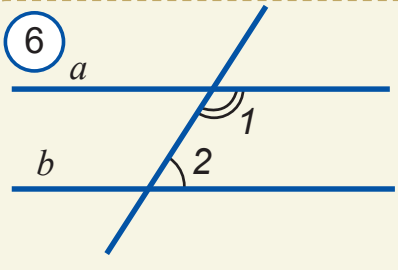


5



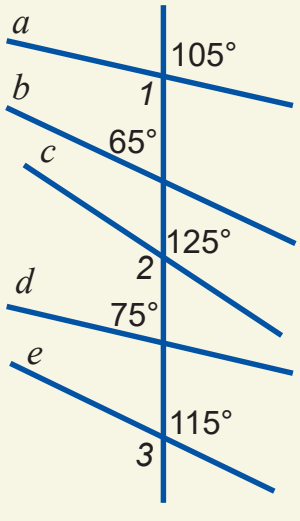
$\angle 1 = \angle 2 \Rightarrow a||b$

6



$\angle 1 + \angle 2 = 180° \Rightarrow a||b$

7



$\angle 3 = \angle 4$ ekanligidan D nuqta CO nurning davomida yotishi, ya'ni C, O va D nuqtalar bitta to'g'ri chiziqda yotishi kelib chiqadi.

$\angle 5 = \angle 6$ ekanligidan $\angle 6$ ham $\angle 5$ kabi to'g'ri burchak ekanligi kelib chiqadi. Shunday qilib, a va b to'g'ri chiziqlar bitta CD to'g'ri chiziqqa perpendikulyar ekan.

Demak, ular o'zaro parallel bo'ladi.

Teorema isbotlandi.

Masala. Agar 4-rasmda $\angle 2 = 55°$ va $\angle 5 = 125°$ bo'lsa, a va b to'g'ri chiziqlar o'zaro parallel bo'ladimi?

Yechish. $\angle 2$ va $\angle 4$ vertikal burchaklar bo'lgani uchun $\angle 4 = \angle 2 = 55°$. $\angle 5$ va $\angle 6$ qo'shni bo'lgani uchun $\angle 6 = 180° - \angle 5 = 180° - 125° = 55°$. Natijada, almashinuvchi burchaklar o'zaro teng ekanini aniqlaymiz: $\angle 4 = \angle 6$.

Demak, yuqorida isbotlangan ikki to'g'ri chiziqning parallellik alomatiga ko'ra, a va b to'g'ri chiziqlar parallel bo'ladi.

Javob: Ha.

Teoremadan to'g'ridan to'g'ri kelib chiqadigan xossa *natija* deyiladi. Oldingi mavzuda *(102-betda)* keltirilgan 1-, 2- va 3-xossalardan quyidagi natijalar kelib chiqadi.

1-natija. ***Agar ikki to'g'ri chiziq va kesuvchi hosil qilgan mos burchaklar teng bo'lsa, u holda bu ikki to'g'ri chiziq parallel bo'ladi*** *(5-rasm).*

2-natija. ***Agar ikki to'g'ri chiziq va kesuvchi hosil qilgan bir tomonli burchaklar yig'indisi 180° ga teng bo'lsa, u holda bu ikki to'g'ri chiziq parallel bo'ladi*** *(6-rasm).*

Masala. 7-rasmdagi to'g'ri chiziqlarning qaysilari parallel?

Yechilishi. Vertikal burchaklar tengligidan $\angle 1 = 105°$, $\angle 2 = 125°$, $\angle 3 = 115°$. a va b to'g'ri chiziqlar parallel emas, chunki $\angle 1 + 65° = 105° + 65° \neq 180°$.

$a||d$ bo'ladi, chunki $\angle 1 + 75° = 105° + 75° = 180°$ (2-natijaga qarang).

Xuddi shunday $b||e$ bo'ladi, chunki

$65° + \angle 3 = 65° + 115° = 180°$.

a, c va e to'g'ri chiziqlar o'zaro parallel emas, chunki ularning mos burchaklari teng emas (1-natijaga qarang).

Xuddi shunday b va d to'g'ri chiziqlar ham parallel emas, chunki mos burchaklar teng emas: $65° \neq 75°$.

Javob: $a||d$, $b||e$.

Masala. 8-rasmda $a||b$ bo‘ladimi?

Yechish. Vertikal burchaklarning xossasiga ko‘ra $x=36°$. Unda $\alpha=4x=4\cdot36°=144°$ bo‘ladi. Bir tomonli burchaklar yig‘indisi $x+\alpha=36°+144°=180°$.

Demak, 2-natijaga ko‘ra $a||b$ bo‘ladi.

Mavzuga doir savollar

1. Ikki to‘g‘ri chiziqning parallelligini qanday aniqlash mumkin?

2. Alomat deb nimaga aytiladi? Misol keltiring.

3. Ikki to‘g‘ri chiziqning parallellik alomatlarini izohlang.

4. Natija deb nimaga aytiladi? Misol keltiring.

Amaliy mashq va tatbiq

1. 9-rasmda a va b to‘g‘ri chiziqlar parallel bo‘lishi uchun noma’lum burchak necha gradus bo‘lishi kerak?

2. 10-rasmda $a||b$ bo‘lishini ko‘rsating.

3. 10-rasmda keltirilgan masalaga o‘xshash masala tuzing va uni yeching.

4. 11-rasmda $a||b$ bo‘lishini ko‘rsating.

5. 12-rasmda $a||b$ bo‘lishini ko‘rsating.

6. 11–12-rasmlarda keltirilgan masalalarga o‘xshash masala tuzing va uni yeching.

7. Agar 1-rasmda: a) $\angle1=132°$, $\angle8=48°$; b) $\angle2=36°$, $\angle5=144°$ bo‘lsa, $a||b$ bo‘ladimi?

8. Agar 1-rasmda: a)$\angle3=113°$, $\angle6=77°$; b) $\angle1+\angle7=180°$ bo‘lsa, $a||b$ bo‘ladimi?

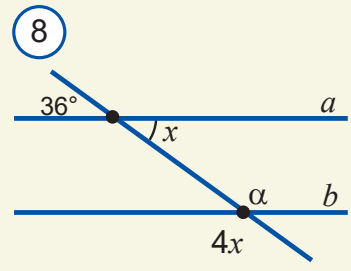
9*. 13-rasmdagi to‘rtburchakning qaysi tomonlari parallel bo‘ladi?

10*. a to‘g‘ri chiziq va unda yotmagan K nuqta berilgan. K nuqta orqali to‘rtta to‘g‘ri chiziq o‘tkazildi. Bu to‘g‘ri chiziqlardan nechtasi a to‘g‘ri chiziq bilan kesishadi?

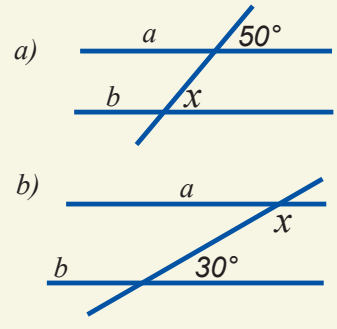
11. Agar 14-rasmda: a) $\angle3=\angle4$, $BD=CE$, $AB=EF$; b) $\angle1=\angle2$, $\angle3=\angle4$, $BD=CE$; c) $AB=EF$, $BD=EC$, $AC=FD$ bo‘lsa, $\Delta ABC=\Delta EFD$ ekanini ko‘rsating.

12. Ikki to‘g‘ri chiziqning kesuvchi bilan kesishishidan hosil bo‘lgan burchaklardan biri: a) 32° va unga mos bo‘lgan burchak esa 33° ga; b) 47° va unga mos bir tomonli bo‘lgan burchak esa 133° ga teng bo‘lsa, bu to‘g‘ri chiziqlar parallel bo‘ladimi?

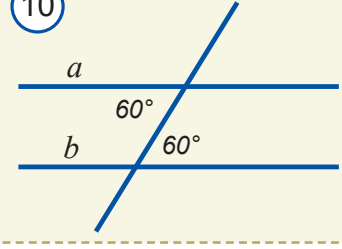
8



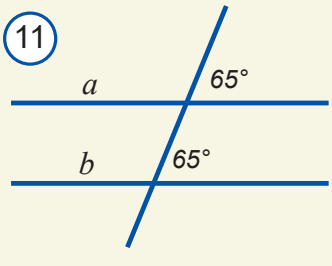
9



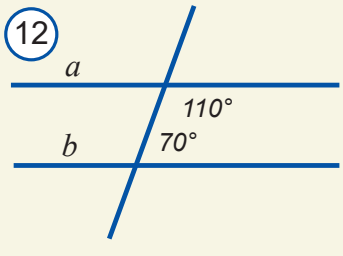
10



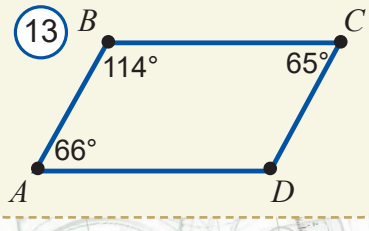
11



12



13



13. 15-rasmdagi noma'lum burchakni toping.

14. Agar 16-rasmda $\angle 1=\angle 5=105°$ bo'lsa, qolgan burchaklarni toping.

15. Agar 17-rasmda $\angle 3=60°$, $\angle 8=120°$ bo'lsa, qolgan burchaklarni toping.

16*. a va b parallel to'g'ri chiziqlarni c to'g'ri chiziq bilan kesishdan hosil bo'lgan almashinuvchi burchaklarning bissektrisalari parallel ekanini ko'rsating *(18-rasm)*.

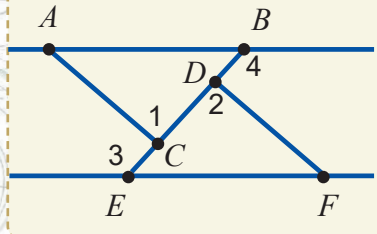
17. 19-rasmlardagi parallel to'g'ri chiziqlarni aniqlang.

18. 20-rasmlardagi parallel to'g'ri chiziqlarni aniqlang.

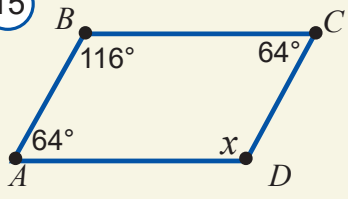
19. 21-rasmlardagi parallel to'g'ri chiziqlarni aniqlang.

20*. To'g'ri chiziqlarning parallellik alomatlariga doir masalalar tuzing va ularni yeching

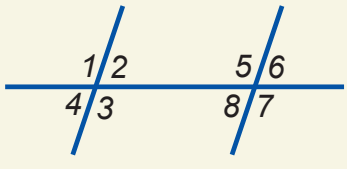
14



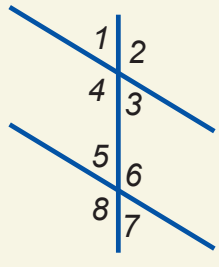
15



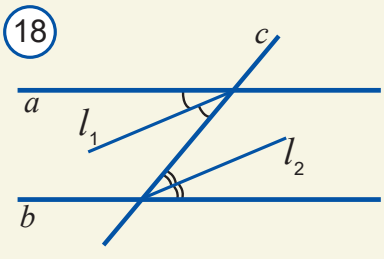
16



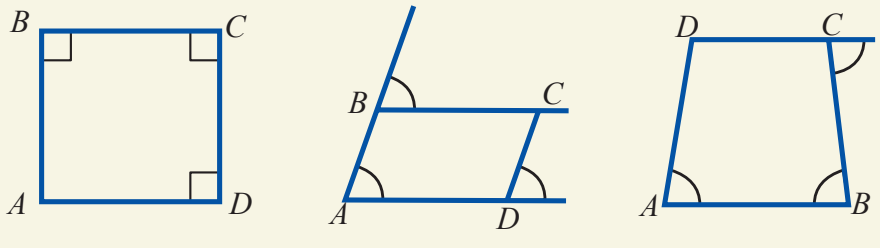
17



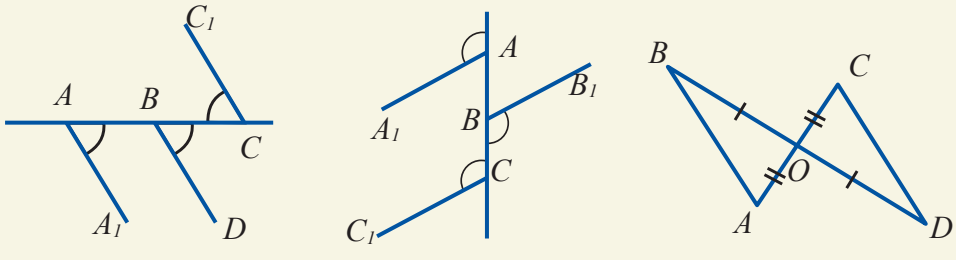
18



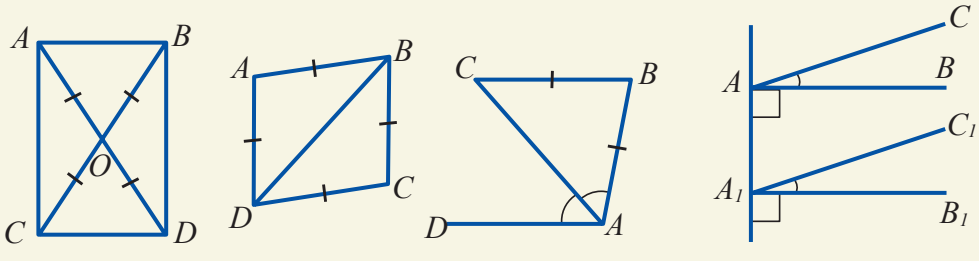
19



20



21



16 IKKI PARALLEL TO‘G‘RI CHIZIQ VA KESUVCHI HOSIL QILGAN BURCHAKLAR

16.1. Teskari teorema

Agar teoremaning sharti va xulosalarining o‘rni almashtirilsa, yangi tasdiq hosil bo‘ladi. Agar bu tasdiq ham to‘g‘ri bo‘lsa (ya’ni uni isbotlab bo‘lsa), u berilgan teoremaga *teskari teorema* deb ataladi. Berilgan teorema esa *to‘g‘ri teorema* deb ham yuritiladi.

To‘g‘ri teorema:

Agar **A jumla** **o‘rinli *bo‘lsa,*** **B jumla** **o‘rinli *bo‘ladi.***

Qisqacha: **A** $\Rightarrow$ **B.**

Teskari teorema:

Agar **B jumla** **o‘rinli *bo‘lsa,*** **A jumla** **o‘rinli *bo‘ladi.***

Qisqacha: **B** $\Rightarrow$ **A.**

Misol. “*Agar uchburchak teng yonli bo‘lsa, uning asosidagi burchaklari teng bo‘ladi*” degan teoremaga teskari teorema quyidagidan iborat: “***Agar uchburchakning ikkita burchagi teng bo‘lsa, u teng yonli uchburchak bo‘ladi***”.

1-mashq. Yuqorida keltirilgan teskari teorema “uchburchakning teng yonli bo‘lish alomati” deb yuritiladi. Uning to‘g‘riligini mustaqil isbotlang.

Berilgan to‘g‘ri teoremaga teskari bo‘lgan tasdiq har doim ham o‘rinli bo‘lavermasligini aytib o‘tish kerak.

Masalan, “Agar burchaklar vertikal bo‘lsa, ular teng bo‘ladi” degan teoremaga teskari “Agar burchaklar teng bo‘lsa, ular vertikal bo‘ladi” degan tasdiq to‘g‘ri emas.

Agar to‘g‘ri va teskari teoremaning har ikkisi ham to‘g‘ri bo‘lsa, bu tasdiqlar *o‘zaro teng kuchli* deb ataladi. Bu qisqacha **A** $\Leftrightarrow$ **B** tarzda yoziladi.

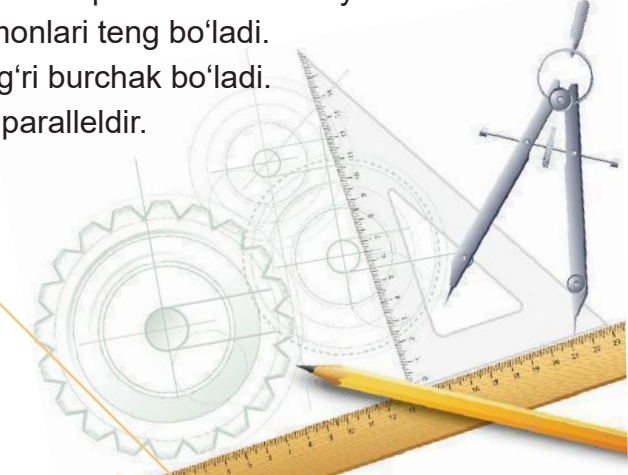
2-mashq.

1. “Agar yomg‘ir yog‘sa, osmonda bulut bo‘ladi” degan tasdiqqa teskari tasdiqni tuzing. Hosil bolgan teskari tasdiqning har doim ham to‘g‘ri bo‘lish yoki bo‘lmasligini izohlang.

2. Quyidagi to‘g‘ri teoremalarga teskari teoremalarni yozib chiqing. Har bir teskari teoremada ifodalangan tasdiqlarning to‘g‘ri yoki noto‘g‘riligini tekshiring.

- Bir to‘g‘ri chiziqqa perpendikulyar bo‘lgan ikki to‘g‘ri chiziq o‘zaro kesishmaydi.
- Agar ikki uchburchak teng bo‘lsa, ularning mos tomonlari teng bo‘ladi.
- Agar qo‘shni burchaklar o‘zaro teng bo‘lsa, ular to‘g‘ri burchak bo‘ladi.
- Bir to‘g‘ri chiziqqa parallel bo‘lgan ikki to‘g‘ri chiziq paralleldir.

3. O‘zaro teng kuchli tasdiqlarga misollar keltiring.



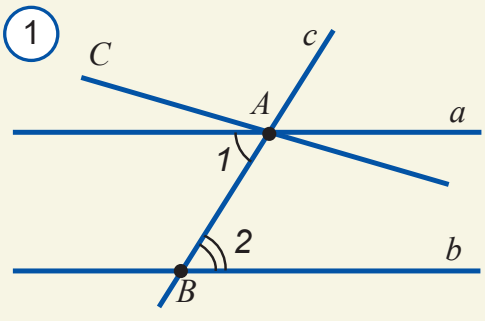
16.2. Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan burchaklar

Quyida ikki to‘g‘ri chiziqning parallellik alomatlariga teskari bo‘lgan teoremalar ustida to‘xtalamiz.

1-teorema. **Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan almashinuvchi burchaklar o‘zaro teng bo‘ladi.**

$a \| b$, c – kesuvchi *(1-rasm)* ⇒ $\angle 1 = \angle 2$

1



Isbot. Teskarisini faraz qilamiz: $\angle 1 \neq \angle 2$ bo‘lsin. AB nurga $\angle 2$ ga teng bo‘lgan CAB burchak qo‘yamiz. U holda CA va b to‘g‘ri chiziqlarni AB kesuvchi bilan kesganda bir biriga teng (yasashga ko‘ra) almashinuvchi $\angle CAB$ va $\angle 2$ burchaklarga ega bo‘lamiz.

Demak, CA va b to‘g‘ri chiziqlar o‘zaro parallel. Shunday qilib, A nuqtadan b to‘g‘ri chiziqqa parallel bo‘lgan ikkita (CA va a) to‘g‘ri chiziqqa egamiz.

Bu esa parallellik aksiomasiga zid. Demak, farazimiz noto‘g‘ri, $\angle 1 = \angle 2$ ekan.

Teorema isbotlandi.

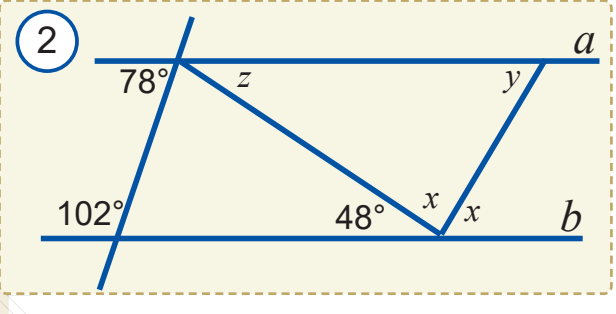
Natija. **Agar to‘g‘ri chiziq parallel to‘g‘ri chiziqlardan biriga perpendikulyar bo‘lsa, ikkinchisiga ham perpendikulyar bo‘ladi.**

Natija tariqasida keltirilgan tasdiqning to‘g‘riligini mustaqil tekshirib ko‘ring.

2-teorema. **Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan mos burchaklar o‘zaro teng bo‘ladi.**

3-teorema. **Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan bir tomonli burchaklar yig‘indisi 180° ga teng bo‘ladi.**
Teoremalarni mustaqil isbotlashga urinib ko‘ring.

2



Masala. 2-rasmdagi noma’lum burchaklarni toping.

Yechilishi. Bir tomonli burchaklar yig‘indisi $78°+102°=180°$ bo‘lgani uchun $a \| b$ bo‘ladi. Demak, 1-teoremaga ko‘ra, $z = 48°$ va $x = y$ bo‘ladi. $x + x + 48° = 180°$ bo‘lgani uchun (yoyiq burchaklar kattaligi) $x = 66°$. Demak, $y = 66°$.

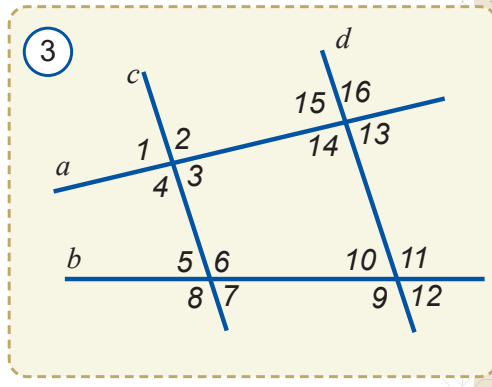
Javob: $x = 66°$; $y = 66°$; $z = 48°$.

Masala. 3-rasmda $a \nparallel b$, $c \| d$. Quyidagi tengliklardan qaysilari to‘g‘ri?

1) $\angle 1 = \angle 15$; 2) $\angle 3 = \angle 13$; 3) $\angle 4 = \angle 16$; 4) $\angle 4 = \angle 8$; 5) $\angle 1 = \angle 12$; 6) $\angle 7 = \angle 10$;

7) $\angle 8=\angle 16$; 8) $\angle 8=\angle 11$; 9) $\angle 4+\angle 13=180°$; 10) $\angle 6+\angle 14=180°$;

11) $\angle 7+\angle 12=180°$; 12) $\angle 8+\angle 9=180°$.



Yechilishi: 3) $\angle 4=\angle 2$ *(vertikal burchaklar xossasiga ko‘ra)*, $\angle 2$ va $\angle 16$ mos burchaklar bo‘lgani uchun $\angle 2=\angle 16$. Demak, $\angle 4=\angle 16$ tenglik to‘g‘ri.

5) $\angle 12=\angle 7$ *(mos burchaklar xossasiga ko‘ra)* va $\angle 7=\angle 5$ *(vertikal burchaklar)*. $\angle 5$ va $\angle 1$ mos burchaklar. $a \nparallel b$, shuning uchun $\angle 1 \neq \angle 5=\angle 7=\angle 12$, ya’ni $\angle 1=\angle 12$ tenglik noto‘g‘ri.

9) $\angle 4=\angle 2$, $\angle 13=\angle 15$ *(vertikal burchaklar)*, $c \parallel d$, $\angle 2$ va $\angle 15$ bir tomonli burchaklar bo‘lgani uchun $\angle 2+\angle 15=180°$. Demak, $\angle 4+\angle 13=180°$ tenglik to‘g‘ri.

11) $c \parallel d$ bo‘lgani uchun $\angle 7=\angle 10$ *(almashinuvchi burchaklar xossasiga ko‘ra)* va $\angle 10=\angle 12$ *(vertikal burchaklar)*. Demak, $\angle 7=\angle 12$.

Shuning uchun $\angle 7+\angle 12=180°$ tenglik faqat $\angle 7=\angle 12=90°$ bo‘lganda o‘rinli bo‘ladi. Qolgan tengliklarni shu tariqa o‘zingiz mustaqil ravishda tekshirib chiqing.

Mavzuga doir savollar

1. *Teskari teorema nima? Uning to‘g‘ri teoremadan qanday farqi bor?*
2. *To‘g‘ri teoremaga teskari bo‘lgan teorema har doim ham o‘rinli bo‘ladimi?*
3. *To‘g‘ri teoremani isbotlab, unga teskari teoremani isbotsiz qabul qilsa bo‘ladimi?*
4. *Berilgan teoremaga teskari teoremaga teskari bo‘lgan teorema nima bo‘ladi?*
5. *Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan almashinuvchi, mos va bir tomonli burchaklar haqida nima deyish mumkin?*

Amaliy mashq va tatbiq

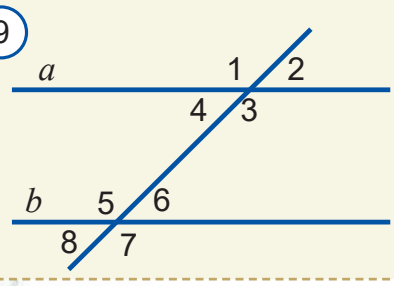
1. Quyidagi tasdiqlarga teskari bo‘lgan tasdiqlarni tuzing, ularni to‘g‘ri bo‘lish yoki bo‘lmasligini aniqlang:

- Agar son tub son bo‘lsa, u natural son bo‘ladi.
- Agar son 9 ga bo‘linsa, u 3 ga ham bo‘linadi.
- Agar uchburchaklar teng bo‘lsa, ularning mos burchaklari ham teng bo‘ladi.
- Agar o‘quvchining harorati yuqori bo‘lsa, u kasal bo‘lib qolgan bo‘ladi.
- Agar futbolchi qizil kartochka olsa, o‘yindan chiqib ketadi.
- Agar uchburchakning tomonlari 3, 4 va 5 ga teng bo‘lsa, uning perimetri 12 ga teng bo‘ladi.
- Agar avtomobilning benzini bo‘lmasa, u yurmaydi.

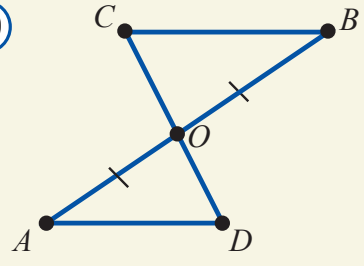
2. Quyidagi teoremalarga teskari teoremalarni tuzing va ularning to‘g‘riligini tekshiring:

- Agar ikki to‘g‘ri chiziqni kesuvchi bilan kesishishidan hosil bo‘lgan mos burchaklar teng bo‘lsa, u holda bu to‘g‘ri chiziqlar parallel bo‘ladi.
- Agar ikki to‘g‘ri chiziq uchinchi to‘g‘ri chiziqqa parallel bo‘lsa, ular parallel bo‘ladi.
- Agar uchburchak teng tomonli bo‘lsa, uning barcha burchaklari o‘zaro teng bo‘ladi.

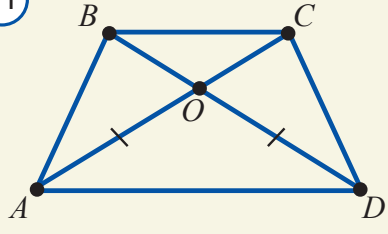
9



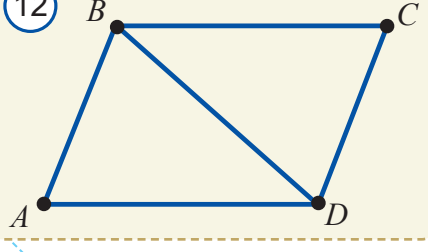
10



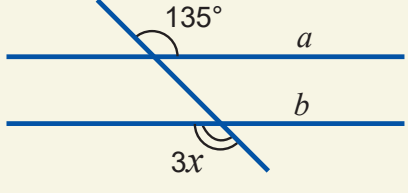
11



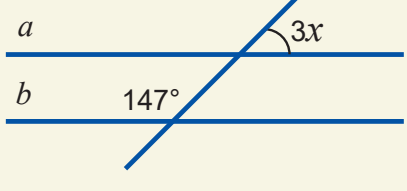
12



13



14



3. Uchburchaklarning tenglik alomatlariga teskari teoremalarni tuzing. Ular to‘g‘rimi?

4. 9-rasmda $a||b$, $\angle 1=135°$. Qolgan burchaklarni toping.

5. 9-rasmda $a||b$, $\angle 2=49°$. Qolgan burchaklarni toping.

6. 10-rasmda $BC||AD$, $AO=OB$ ekanligi ma’lum.
a) $DO=OC$; b) $\Delta AOD=\Delta COB$ tengliklarni isbotlang.

7. 11-rasmda $BC||AD$, $AO=OD$ ekanligi ma’lum.
a) $BO=OC$; b) $AC=BD$; c) $\Delta AOB=\Delta COD$; d) $\Delta ABD=\Delta ACD$ tengliklarni isbotlang.

8. 12-rasmda $BC||AD$ va $AB||CD$ bo‘lsa, $\Delta ABD=\Delta CDB$ ekanini isbotlang.

9. 13-rasmda $a||b$ bo‘lsa, x ni toping.

10. 14-rasmda $a||b$ bo‘lsa, x ni toping.

11*. ABC va $A_1B_1C_1$ o‘tkir burchaklar berilgan. Agar $AB||A_1B_1$ va $BC||B_1C_1$ bo‘lsa, $\angle ABC = \angle A_1B_1C_1$ bo‘lishini isbotlang.

12*. Mos tomonlari parallel to‘g‘ri chiziqlarda yotgan burchaklardan biri o‘tkir, ikkinchisi esa o‘tmas. Bu burchaklar yig‘indisi 180° ga teng bo‘lishini isbotlang.

Eslatma. *12–13-masalalarda keltirilgan teoremalar “mos tomonlari parallel bo‘lgan burchaklarning xossalari” deb yuritiladi.*

13. Agar 15-rasmda $a||b$, $c||d$ va $\angle 1 = 55°$ bo‘lsa, $\angle 2$ va $\angle 3$ ni toping.

14. Agar 15-rasmda $a||b$, $c||d$ va $\angle 3 = 73°$ bo‘lsa, $\angle 1$ va $\angle 2$ ni toping.

15*. Mos tomonlari parallel to‘g‘ri chiziqlarda yotgan burchaklar ayirmasi 40° ga teng. Bu burchaklarni toping.

16*. ABC o‘tkir va $A_1B_1C_1$ o‘tmas burchaklar berilgan. Agar $AB\perp A_1B_1$ va $BC\perp B_1C_1$ bo‘lsa, $\angle ABC + A_1B_1C_1 = 180°$ bo‘lishini isbotlang.

17*. Mos tomonlari perpendikulyar to‘g‘ri chiziqlarda yotgan burchaklardan biri o‘tkir, ikkinchisi esa o‘tmas. Bu burchaklar yig‘indisi 180° ga teng bo‘lishini isbotlang.

Eslatma. *17–18-masalalarda keltirilgan teoremalar “mos tomonlari o‘zaro perpendikulyar bo‘lgan burchaklarning xossalari” deb yuritiladi.*

18*. 16-rasmdagi ABC va ADC burchaklarning mos tomonlari perpendikulyar. Noma'lum burchaklarni toping.

19. ABC burchak 118° ga teng. BCD burchak esa 62° ga teng. AB va CD to'g'ri chiziqlar a) parallel bo'lishi; b) kesishishi mumkinmi?

20. ABC burchak 53° ga teng. BCD burchak ham 53° ga teng. AB va CD to'g'ri chiziqlar a) parallel bo'lishi; b) kesishishi mumkinmi?

21*. 101-betda keltirilgan Sakkeri to'rtburchagining to'g'ri to'rtburchak ekanini isbotlang.

22. 17-rasmdagi x burchaklarni toping.

23. 18-rasmdagi x burchaklarni toping.

24*. Ikki parallel to'g'ri chiziqlarni kesuvchi kesib o'tganda hosil bo'lgan bir tomonli burchaklarning bittasi boshqasidan 24° ga katta bo'lsa, qolgan burchaklarni toping.

25*. Ikki parallel to'g'ri chiziqlarni kesuvchi kesib o'tganda hosil bo'lgan mos burchaklarning yig'indisi 128° ga teng bo'lsa, qolgan burchaklarni toping.

26*. 19-rasmda AB va CD to'g'ri chiziqlar parallel bo'ladimi?

27*. 20-rasmda AB va CD to'g'ri chiziqlar parallel bo'lishi uchun x nimaga teng bo'lishi kerak?

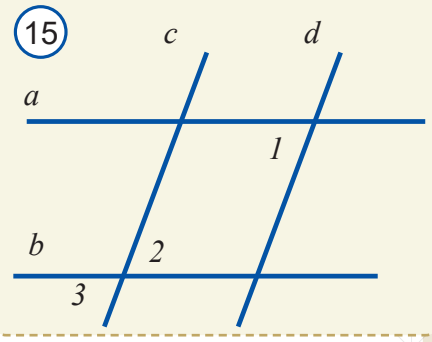
28*. Ikki to'g'ri chiziqni kesuvchi kesib o'tganda hosil bo'lgan burchaklarning beshtasi o'tkir bo'lmasa, ularning barchasini toping.

29*. Ikki to'g'ri chiziqni kesuvchi kesib o'tganda hosil bo'lgan burchaklarning oltitasi yig'indisi 620° ga teng bo'lsa, ularning barchasini toping.

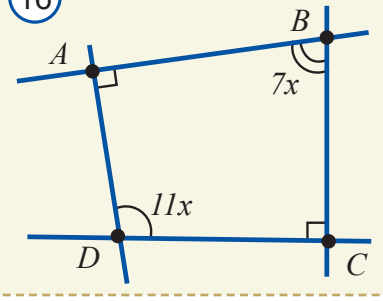
30*. Asosi AC kesma bo'lgan ABC teng yonli uchburchak berilgan. AC tomonga parallel bo'lgan to'g'ri chiziq AB tomonni A_1 va BC tomonni C_1 nuqtada kesib o'tadi. A_1BC_1 uchburchak teng yonli ekanini isbotlang.

31. ABC burchak 98° ga teng. Uning A va C nuqtalaridan o'tib, tomonlariga parallel bo'lgan to'g'ri chiziqlar D nuqtada kesishadi. ADC burchakni toping.

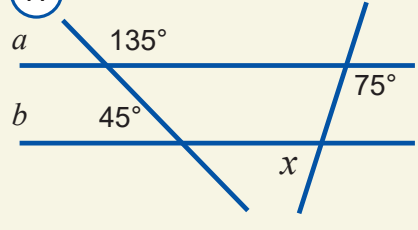
15



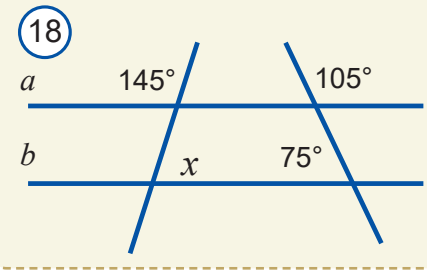
16



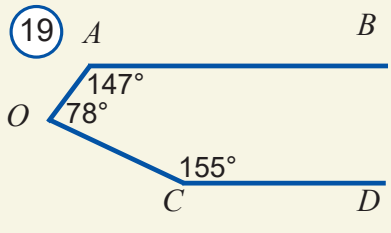
17



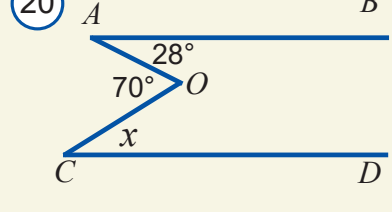
18



19



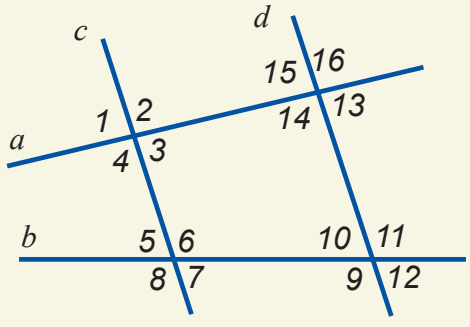
20



17 AMALIY MASHQ VA TATBIQ. BILIMINGIZNI SINAB KO‘RING

17.1. Kichik guruhlarda amaliy mashq

1



1-rasmda $a \nparallel b,\ c \parallel d$ to‘g‘ri chiziqlar berilgan. To‘rtta kichik guruh tuziladi va:

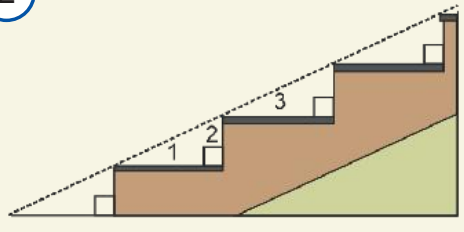
1-guruhga $\angle 1$, $\angle 2$, $\angle 3$ va $\angle 4$;

2-guruhga $\angle 5$, $\angle 6$, $\angle 7$ va $\angle 8$;

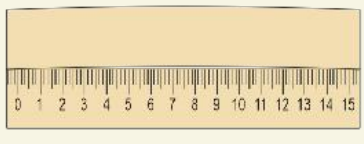
3-guruhga $\angle 9$, $\angle 10$, $\angle 11$ va $\angle 12$;

4-guruhga $\angle 13$, $\angle 14$, $\angle 15$ va $\angle 16$ lar taqsimlab beriladi. Guruhlar navbati bilan o‘z burchaklari kattaliklarini e’lon qiladi. Qolgan guruhlar bu burchaklarga mos o‘z burchaklari kattaliklarini aniqlab aytadi.

2



3



17.2. Amaliy tatbiq

1. Zinapoyaning poygaklari gorizontal bo‘lishi kerak *(2-rasm)*. Shundan kelib chiqib, zinapoya tasvirida belgilangan burchaklar orasida qanday munosabatlar bajarilishini aniqlang va geometriyadan o‘rgangan bilimlaringiz asosida izohlang.

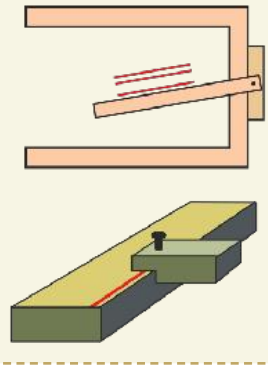
2. Ikkita chizg‘ichning tekisligi 3-rasmdagidek tekshirib ko‘riladi. Bunda to‘g‘ri chiziqlarning qaysi xossasidan foydalaniladi.

3. 4-rasmdan foydalanib, tasvirlangan asboblar yordamida duradgorlar parallel to‘g‘ri chiziqlarni qanday chizishlarini o‘zlashtirgan bilimlaringiz asosida izohlang.

4. 5-rasmda tasvirlangan kitob javonini mustahkam qilib yasashda uchburchakning qaysi xususiyatidan foydalanilgan?

5. A va B nuqtalarda mahkamlangan bloklar orqali o‘tgan ipda P_1 va P_2 jismlar osilgan *(6-rasm)*. P_3 jism esa shu ipning C nuqtasida osilgan bo‘lib, P_1 va P_2 jismlarni muvozanatda saqlab turibdi. $AP_1 \parallel BP_2 \parallel CP_3$ ekanligi ma’lum bo‘lsa, $\angle ACB = \angle A + \angle B$ bo‘lishini isbotlang.

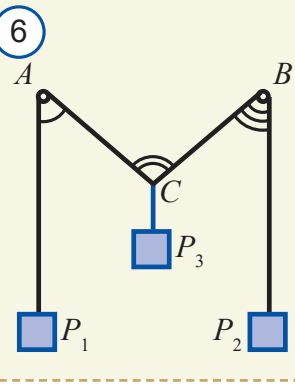
4



5



6



17.3. Bilimingizni sinab ko‘ring

1. Bo‘sh qoldirilgan joylarni mantiqan to‘g‘ri so‘zlar bilan to‘ldiring.

1. To‘g‘ri chiziqda yotuvchi nuqta orqali unga perpendikulyar bo‘lgan ..... o‘tkazish mumkin.

2. Agar ikki to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘lgan .... teng bo‘lsa, bu to‘g‘ri chiziqlar parallel bo‘ladi.

3. Tekislikdagi ikki to‘g‘ri chiziq ..... , ular “parallel to‘g‘ri chiziqlar” deyiladi.

4. Ikki parallel to‘g‘ri chiziqdan birini kesib o‘tgan to‘g‘ri chiziq ..... .

5. To‘g‘ri chiziqda yotmaydigan nuqta orqali unga parallel bo‘lgan ... to‘g‘ri chiziq o‘tkazish mumkin.

6. To‘g‘ri chiziqning ixtiyoriy nuqtasi orqali .... faqat bitta to‘g‘ri chiziq o‘tkazish mumkin.

7. To‘g‘ri burchak ostida kesishuvchi to‘g‘ri chiziqlar ..... deb ataladi.

8. Bitta to‘g‘ri chiziqqa .... ikki to‘g‘ri chiziq o‘zaro parallеldir.

9. Agar ikki to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘lgan bir tomonli burchaklar ..... bu to‘g‘ri chiziqlar parallel bo‘ladi.

10. Parallel to‘g‘ri chiziqlarni kesuvchi bilan kesganda hosil bo‘lgan mos burchaklar ..... bo‘ladi.

11. Parallel to‘g‘ri chiziqlarni kesuvchi bilan kesganda hosil bo‘lgan almashinuvchi burchaklar ..... bo‘ladi.

2. Quyida keltirilgan jumlalardagi xatoni toping va tuzating.

1. To‘g‘ri chiziqning faqat bitta nuqtasidan unga perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin.

2. Berilgan to‘g‘ri chiziqda yotmaydigan faqat bitta nuqtadan shu to‘g‘ri chiziqqa perpendikulyar tushirish mumkin.

3. AB va AK parallel to‘g‘ri chiziqlarning biriga perpendikulyar bo‘lgan to‘g‘ri chiziq ikkinchisiga ham perpendikulyar bo‘ladi.

4. Ikki to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘lgan almashinuvchi burchaklari teng bo‘ladi.

5. Agar ikki kesma kesishmasa, ular “parallel kesmalar” deb ataladi.

6. Mos tomonlari parallel bo‘lgan burchaklar teng bo‘ladi.

7. Agar $a \perp b$, $b \perp c$ bo‘lsa, $a \perp c$ bo‘ladi.

8. Mos tomonlari perpendikulyar bo‘lgan burchaklarning yig‘indisi 180° ga teng.

9. Agar ikki to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘lgan bir tomonli burchaklar teng bo‘lsa, bu to‘g‘ri chiziqlar parallel bo‘ladi.

10. Perpendikulyar to‘g‘ri chiziqlarga parallel bo‘lgan to‘g‘ri chiziqlar o‘zaro parallel bo‘ladi.

11. Agar $a \parallel b$, $b \parallel c$ bo‘lsa, $a \perp c$ bo‘ladi.

12. Agar $a \parallel b$, $b \perp c$ bo‘lsa, $a \parallel c$ bo‘ladi.



3. Jadvalda keltirilgan xossalar va talqinlarga mos keluvchi geometrik tushunchalarni yozing.

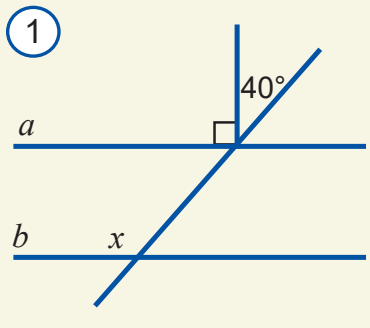
1	Umumiy nuqtaga ega bo‘lmagan to‘g‘ri chiziqlar	
2	To‘g‘ri burchak ostida kesishadi	
3	Nuqtadan to‘g‘ri chiziqqa faqat bitta tushirish mumkin	
4	Nuqtadan to‘g‘ri chiziqqa istagancha tushirish mumkin	
5	Shart va xulosa qismi almashgan	
6	Ikkita to‘g‘ri chiziqni kesuvchi bilan kesganda hosil bo‘ladigan burchaklar	

4. Birinchi ustunda berilgan geometrik tushunchalarga ikkinchi ustundan tegishli xossa yoki talqinni qo‘ying.

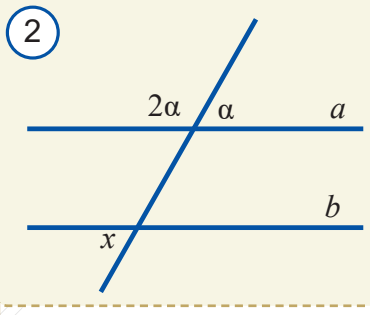
Geometrik tushunchalar	*Xossasi yoki talqini*
1. Parallel to‘g‘ri chiziqlar 2. Perpendikulyar to‘g‘ri chiziqlar 3. Kesuvchi ikki to‘g‘ri chiziqni kesganda 4. Almashinuvchi burchaklar 5. Teskari teorema 6. Bir tomonli burchaklar	(A) har doim ham to‘g‘ri emas. (B) kesishmaydi. (C) kesishganda to‘g‘ri burchaklar hosil bo‘ladi. (D) almashinuvchi, mos va bir tomonli burchaklar hosil bo‘ladi. (E) bitta yarimtekislikda yotadi. (F) teng bo‘lsa, to‘g‘ri chiziqlar parallel bo‘ladi.

5. Testlar.

1



2



1. Berilgan to‘g‘ri chiziqda yotmaydigan nuqta orqali shu to‘g‘ri chiziqqa nechta parallel to‘g‘ri chiziq o‘tkazish mumkin?

A) 1; B) 2; C) 4; D) istalgancha.

2. Agar $a||b$, $b\perp c$, $c\perp d$ bo‘lsa, quyidagi javoblarning qaysi biri to‘g‘ri?

A) $a\perp d$, $b\perp d$; B) $a\perp c$, $b||d$; C) $a||c$, $a\perp d$; D) $a\perp c$, $a\perp d$, $b\perp d$.

3. Tekislikda berilgan to‘g‘ri chiziqda yotmaydigan nuqta orqali shu to‘g‘ri chiziqqa nechta perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin?

A) 1 B) 2 C) 4 D) istalgancha

4. 1-rasmda $a||b$ bo‘lsa, x ni toping.

A) 100° B) 110° C) 130° D) 140°

5. 2-rasmda $a||b$ bo‘lsa, x ni toping.

A) 30° B) 45° C) 60° D) 36°

6. x ni toping *(3-rasm)*.

A) 96° B) 108° C) 112° D) 78°

7. 4-rasmda $a||b$ va $\alpha-\beta=70°$ bo‘lsa, α ni toping.

A) 30° B) 125° C) 75° D) 36°

8. Ikki parallel to‘g‘ri chiziq uchinchi to‘g‘ri chiziq bilan kesishganda nechta teng o‘tmas burchak hosil bo‘lishi mumkin?

A) 3 ta B) 8 ta C) 6 ta D) 4 ta

9. Ikki parallel to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq bilan kesishganda hosil bo‘lgan burchaklardan biri 97° ga teng. Hosil bo‘lgan burchaklardan eng kichigini toping.

A) 97° B) 83° C) 77° D) 7°

10. Ikki to‘g‘ri chiziq uchinchi to‘g‘ri chiziq bilan kesishganda ko‘pi bilan nechta teng o‘tkir burchak hosil bo‘ladi?

A) 3 ta B) 4 ta C) 6 ta D) 5 ta

11. Ikki parallel to‘g‘ri chiziq uchinchi to‘g‘ri chiziq bilan kesishganda ko‘pi bilan nechta to‘g‘ri burchak hosil bo‘ladi?

A) 2 ta B) 6 ta C) 8 ta D) 5 ta

12. Ikki parallel to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq bilan kesganda hosil bo‘lgan uchta almashinuvchi burchaklar yig‘indisi 290° ga teng. To‘rtinchi almashinuvchi burchakni toping.

A) 145° B) 110° C) 36° D) 70°

13. 5-rasmda $a||b$ bo‘lsa, x ni toping.

A) 100° B) 80° C) 110° D) 90°

14. 6-rasmdagi x burchakni toping.

A) 105° B) 95° C) 85° D) 75°

15. 7-rasmda qaysi to‘g‘ri chiziqlar o‘zaro parallel bo‘ladi?

A) $a||b$ B) $a||c$ C) $c||b$ D) $c||d$

16. 8-rasmda $a||b,\ c||d$ va $\angle 1=122°$ bo‘lsa, $\angle 2$ va $\angle 3$ ni toping.

A) $\angle 2 = 122°,\ \angle 3 = 58°$ B) $\angle 2 = 130°,\ \angle 3 = 58°$

C) $\angle 2 = 122°,\ \angle 3 = 68°$ D) $\angle 2 = 130°,\ \angle 3 = 50°$

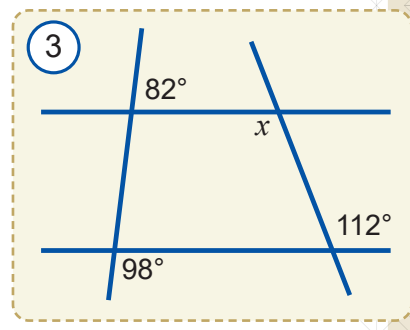
6. Masalalar.

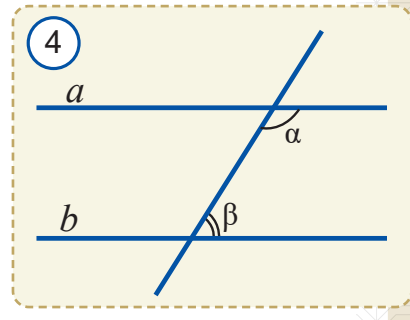
1. 9-rasmda $\angle 4 + \angle 5 = 180°$ bo‘lsa, $a||b$ bo‘ladimi?

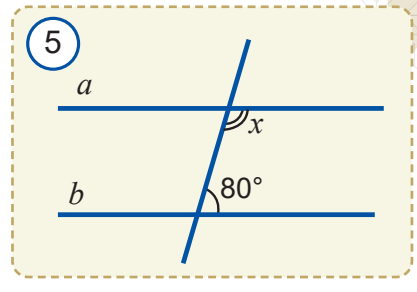
2. 9-rasmda $\angle 2 = \angle 6$ bo‘lsa, $a||b$ bo‘ladimi?

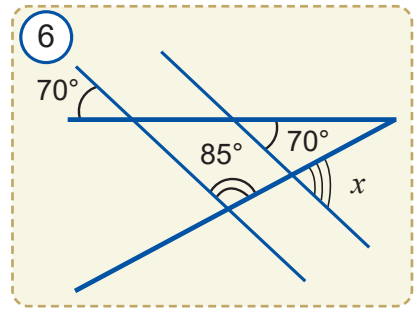
3. 9-rasmda $\angle 1 = \angle 5 = 118°$ bo‘lsa, qolgan burchaklarni toping.

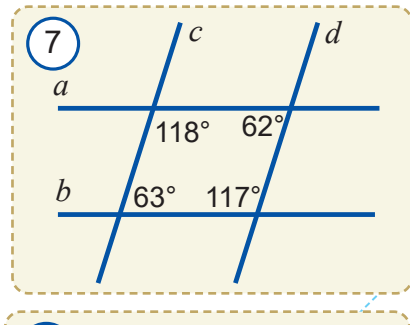
4. 9-rasmda $\angle 2 = 71°$ va $\angle 7 = 119°$ bo‘lsa, $a||b$ bo‘ladimi?

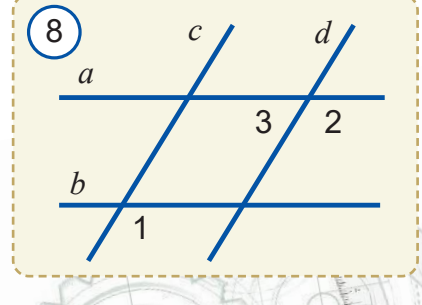




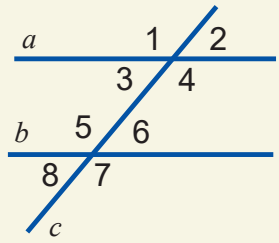




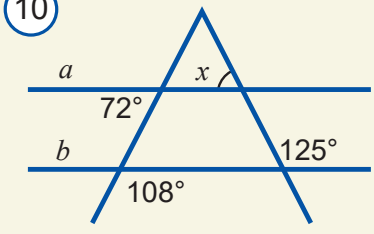




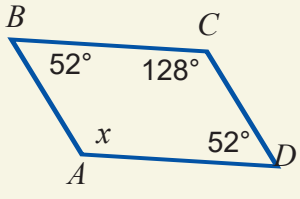
9



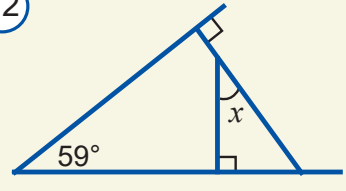
10



11



12



5. 10-rasmdagi x burchakni toping.

6. 11-rasmdagi noma'lum burchaklarni toping.

7. Ikki to'g'ri chiziqni uchinchi to'g'ri chiziq bilan kesganda hosil bo'lgan burchaklardan biri 47° ga teng. Unga mos burchak necha gradus bo'lganda bu ikki to'g'ri chiziq parallel bo'ladi?

8. Ikki parallel to'g'ri chiziqni kesuvchi bilan kesganda hosil bo'lgan almashinuvchi burchaklar yig'indisi 84°. Qolgan burchaklarni toping.

9. Ikki parallel to'g'ri chiziqni kesuvchi bilan kesganda hosil bo'lgan burchaklardan biri ikkinchisidan 8 marta katta. Hosil bo'lgan barcha burchaklarni toping.

10. Ikki parallel to'g'ri chiziqni kesuvchi bilan kesganda hosil bo'lgan bir tomonli burchaklar ayirmasi 30°. Bu burchaklarni toping.

11. 12-rasmdagi noma'lum burchakni toping.

12. Mos tomonlari parallel to'g'ri chiziqlarda yotgan burchaklar ayirmasi 36° ga teng. Bu burchaklarni toping.

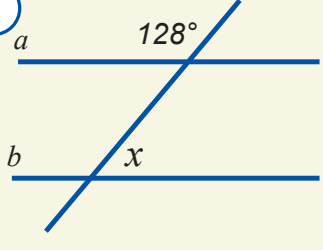
4-nazorat ishi namunasi

Namunaviy nazorat ishi ikki qismdan iborat:

1) 116–117-betdagi testlarga o'xshash 5 ta test;

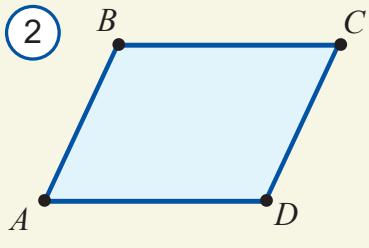
2) quyidagi masalalarga o'xshash 3 ta masala (4-masala "a'lo" baho olmoqchi bo'lgan o'quvchilar uchun qo'shimcha).

1



1. Ikki parallel to'g'ri chiziq kesuvchi bilan kesilganda hosil bo'lgan burchaklardan biri 34° ga teng. Qolgan burchaklarni toping.

2



2. 1-rasmda a va b to'g'ri chiziqlar parallel bo'lishi uchun noma'lum burchak necha gradus bo'lishi kerak?

3. Agar 2-rasmda $BC||AD$ va $AB||CD$ bo'lsa, $AB=CD$ ekanini isbotlang.

4. ABC uchburchakning A uchidan o'tkazilgan bissektrisa BC tomonni D nuqtada kesib o'tadi. D nuqtadan o'tkazilgan to'g'ri chiziq AC tomonni E nuqtada kesib o'tadi. Agar $AE=DE$ bo'lsa, $DE||AB$ ekanini isbotlang.

“GeoGebra”da amaliy topshiriqlar bajarish

1. Berilgan kattalikdagi burchakni yasash

Ikki to‘g‘ri chiziq to‘g‘ri burchak ostida kesishsa, ular “perpendikulyar to‘g‘ri chiziqlar” deb ataladi. Ularning tasvirini hosil qilish uchun:

1) uskunalar panelida “**Прямые линии**” (“To‘g‘ri chiziqlar”) guruhini ochamiz;

2) menyudan “**Прямая**” (“To‘g‘ri chiziq”) ustiga sichqonchaning chap tugmasini bosib tanlaymiz;

3) tekislikda ikkita nuqta belgilaymiz.

4) uskunalar panelida “**Измерения**” (“O‘lchovlar”) guruhini ochamiz;

5) menyudan “**Угол заданной величины**” (“Berilgan kattalikdagi burchak”) ustiga sichqonchaning chap tugmasini bosib tanlaymiz;

6) hosil bo‘lgan darchaga burchak kattaligini kiritamiz va “**Enter**” tugmasini bosamiz. Natijada uchinchi nuqta hosil bo‘ladi;

7) burchak uchi va nuqtadan to‘g‘ri chiziq o‘tkazamiz. Natijada izlanayotgan burchak hosil bo‘ladi.

2. Burchak bissektrisasini yasash algoritmini tuzish va uni yasash

Yuqoridagi algoritmga o‘xshash burchak bissektrisasini yasash algoritmini tuzing va uni yasang.

3. Ko‘pburchak yasash

Ko‘pburchakning tasvirini hosil qilish uchun:

1) uskunalar panelida “**Многоугольник**” (“Ko‘pburchak”) guruhini ochamiz;

2) tekislikda ko‘pburchak uchlarini tartib bilan belgilaymiz va yana birinchi nuqtaga qaytamiz;

3) natijada ko‘pburchak hosil bo‘ladi.

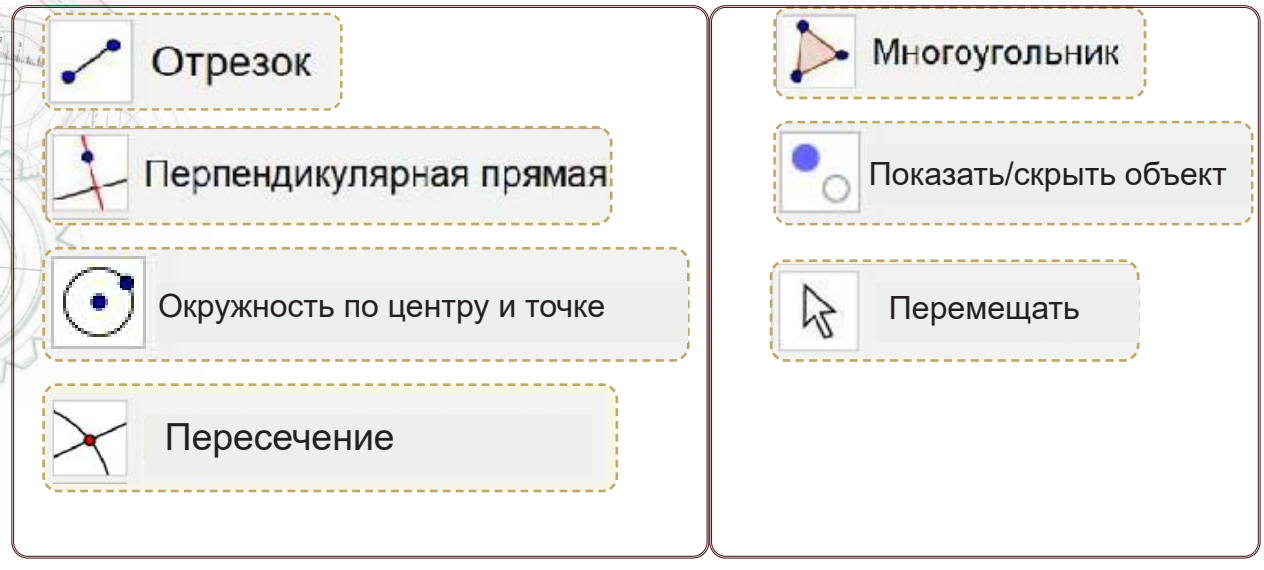
4. Muntazam ko‘pburchak yasash algoritmini tuzish va uni yasash

Yuqoridagi algoritmga o‘xshash muntazam ko‘pburchak yasash algoritmini tuzing va uni yasang.

5. Kvadrat yasash

Kvadrat yasash uchun quyidagi vositalar kerak bo‘ladi. Kvadrat yasashdan oldin dasturning har bir uskunadan foydalanish yo‘riqlarini qaytaring.

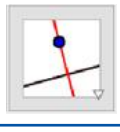
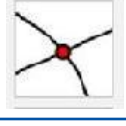
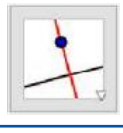


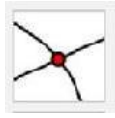
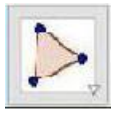
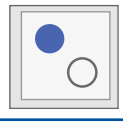


Kerakli komponentlar:

- **GeoGebra**da yangi oyna oching.
- **GeoGebra** interfeysini “**Настройки**” –  “**Геометрия**” ko‘rinishiga o‘tkazing.
- Yangi nuqta uchun sozlamalarni o‘zgartiring.

Kvadratni yasash algoritmi

1		$a=AB$ kesma yasang.
2		AB kesmaning B nuqtasidan o‘tuvchi b perpendikulyar chiziqni yasang.
3		Markazi B nuqtada bo‘lgan va A nuqtadan o‘tuvchi c aylana chizing.
4		b perpendikulyar va c aylananing kesishish nuqtalari – C va D larni yasang.
5		AB kesmaning A nuqtasidan o‘tuvchi d perpendikulyar chiziqni yasang.
6		Markazi A nuqtada bo‘lgan va B nuqtadan o‘tuvchi e aylanani chizing.

7		d perpendikulyar va e aylananing kesishish nuqtalari E va F larni yasang.
8		$ABCE$ ko‘pburchakni yasang.
9		Ikkala aylana va perpendikulyar chiziqlarni yashirin holatga o‘tkazing.
10		Kvadrat to‘g‘ri yasalganini tekshiring.
11		Kvadrat rangi va ko‘rinishini o‘zgartiring.

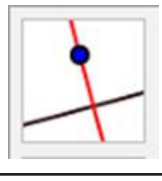
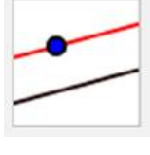
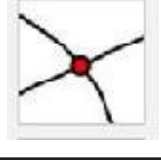
6. Kvadratni boshqa usulda yasash algoritmini tuzing.

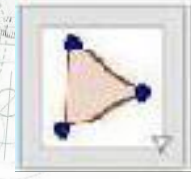
7. To‘g‘ri to‘rtburchak yasash.

Kerakli komponentlar:

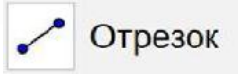
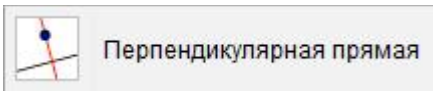
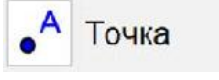
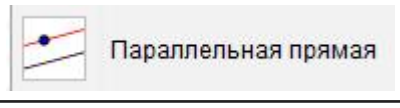
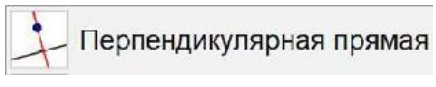
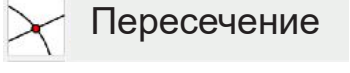
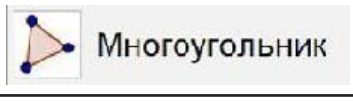
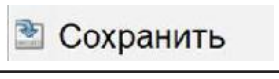
- Yasashni boshlashdan oldin to‘g‘ri to‘rtburchakning xossalarini esga oling.
- Yangi ***GeoGebra*** oynasini oching.
- ***GeoGebra*** interfeysini “**Настройки**” –  “**Геометрия**” ko‘rinishiga o‘tkazing.
- Yangi nuqta uchun sozlamalarni o‘zgartiring.

GeoGebra uskunalar panelidagi kerakli uskunalar

	***Перпендикулярная прямая*** ***Eslatma.*** Nuqta va to‘g‘ri chiziq ustiga bosib, shu nuqta orqali to‘g‘ri chiziqqa perpendikulyar o‘tkazing.
	***Параллельные прямые*** ***Eslatma.*** Nuqta va to‘g‘ri chiziq ustiga bosib, shu nuqta orqali to‘g‘ri chiziqqa parallel chiziq o‘tkazing.
	***Пересечение*** ***Eslatma.*** Ikki to‘g‘ri chiziqning kesishish nuqtasi ustiga bosib, kesishish nuqtasini belgilang yoki ikkita to‘g‘ri chiziq ustiga ketma-ket bosib, ularning kesishish nuqtasini toping.

	***Многоугольник*** ***Eslatma.*** Grafik ko‘rinish ustiga bosing yoki mavjud nuqtalar ustiga bosib, to‘g‘ri to‘rtburchak hosil qiling. Har doim ko‘pburchak yopiq bo‘lishi uchun birinchi va oxirgi nuqtani birlashtiring. Ko‘pburchak yoqlarini soat strelkasiga qarshi yo‘nalishda birlashtiring.

To‘g‘ri to‘rtburchak yasash algoritmi

1		*AB* kesma yasang.
2		*AB* kesmaga *B* nuqta orqali o‘tuvchi perpendikulyar o‘tkazing.
3		Perpendikulyar to‘g‘ri chiziqda yangi *C* nuqtani faollashtiring.
4		*C* nuqta orqali o‘tuvchi va *AB* kesmaga parallel to‘g‘ri chiziq o‘tkazing.
5		*AB* kesmaning *A* nuqtasi orqali o‘tuvchi perpendikulyar o‘tkazing.
6		*D* kesishish nuqtasini belgilang.
7		*ABCD* ko‘pburchakni yasang.
8		O‘zgarishlarni saqlang.
9		**“Перемещать”** vositasidan foydalanib to‘g‘ri to‘rtburchak to‘g‘ri yasalganini tekshiring.



IV BOB

UCHBURCHAK TOMONLARI VA BURCHAKLARI ORASIDAGI MUNOSABATLAR

Bu bobni o‘rganib chiqqach, quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilim:

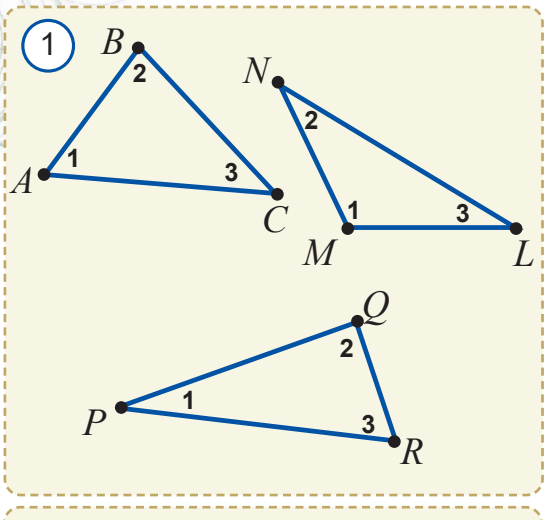
– uchburchak ichki burchaklarining yig‘indisi haqidagi teorema va uni isbotlash;
– uchburchak tashqi burchagi va uning xossasi;
– to‘g‘ri burchakli uchburchakning xossalari;
– to‘g‘ri burchakli uchburchaklarning tenglik alomatlari;
– burchak bissektrisasining xossasi;
– uchburchak burchaklari va tomonlari orasidagi munosabatni ifodalovchi munosabatlar;
– uchburchak tengsizligi.

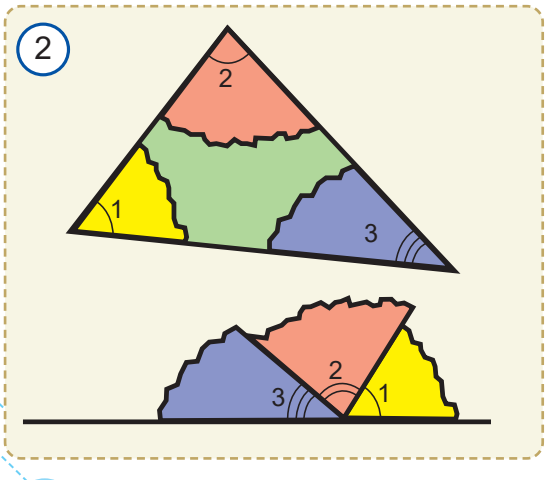
Amaliy ko‘nikmalar:

– uchburchak ichki burchaklari yig‘indisini amaliy usul bilan topa olish;
– o‘zlashtirilgan nazariy bilimlarni, xossalarni masalalar yechishda va amaliy ishlarni bajarishda qo‘llay olish.

18 UCHBURCHAK ICHKI BURCHAKLARI YIG‘INDISI

Faollashtiruvchi mashq





1. 1-rasmda tasvirlangan ABC uchburchakning uchala burchaklarini transportir yordamida o‘lchang va ularning yig‘indisini hisoblang. Xuddi shu ishni MNL va PQR uchburchaklar uchun ham bajaring. Natijalar asosida jadvalni to‘ldiring. Qanday xossani aniqladingiz? Uni bitta jumla bilan ifodalang.

Uchburchaklar	$\angle 1$	$\angle 2$	$\angle 3$	$\angle 1+\angle 2+\angle 3$
ΔABC				
ΔMNL				
ΔPQR				

2. Bir varaq qog‘ozga ixtiyoriy ABC uchburchakni chizing va burchaklarini 1, 2 va 3 raqamlari bilan belgilang. Uning burchaklarini 2-rasmda ko‘rsatilgandek qilib yirtib oling va yonma-yon qo‘ying. Bundan qanday xulosa chiqarish mumkin?

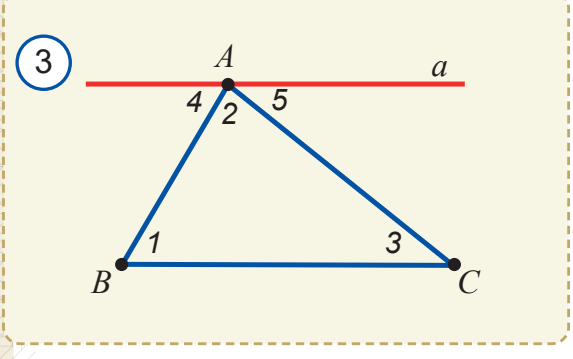
Endi geometriyaning eng muhim teoremalaridan biri – uchburchak ichki burchaklari yig‘indisi haqidagi teoremani isbotlaymiz.

18.1. Uchburchak ichki burchaklarining yig‘indisi

Teorema. **Uchburchak ichki burchaklarining yig‘indisi 180° ga teng.**

ΔABC ⇒ $\angle A + \angle B + \angle C = 180°$

Isbot. ABC uchburchak ichki burchaklarini mos ravishda $\angle 1$, $\angle 2$ va $\angle 3$ bilan belgilaymiz *(3-rasm)*. A uchdan BC tomonga parallel a to‘g‘ri chiziq o‘tkazamiz va $\angle 4$ va $\angle 5$ burchaklarni belgilaymiz.



$\angle 1 = \angle 4$, chunki bu burchaklar a va BC parallel to‘g‘ri chiziqlarni AB kesuvchi bilan kesganda hosil bo‘lgan almashinuvchi burchaklardir.

$\angle 3 = \angle 5$, chunki bu burchaklar a va BC parallel to‘g‘ri chiziqlarni AC kesuvchi bilan kesganda hosil bo‘lgan almashinuvchi burchaklardir.

$\angle 4 + \angle 2 + \angle 5 = 180°$, chunki bu burchaklar umumiy uchga ega va yoyiq burchakni tashkil qiladi. Hosil bo‘lgan bu uchta tenglikdan $\angle 1 + \angle 2 + \angle 3 = 180°$, ya’ni $\angle A + \angle B + \angle C = 180°$ ni hosil qilamiz.

Teorema isbotlandi.

1-natija. **Har qanday uchburchakning kamida ikkita o‘tkir burchagi bor.**

Isbot. Teskarisini faraz qilamiz, ya’ni uchburchakning faqat bitta burchagi o‘tkir bo‘lsin. Unda uning qolgan ikkita burchagi o‘tmas burchak bo‘lib, ularning yig‘indisi 180^o dan katta bo‘ladi. Buning esa yuqorida isbotlangan uchburchak ichki burchaklari yig‘indisi haqidagi teoremaga ko‘ra bo‘lishi mumkin emas.

Demak, farazimiz noto‘g‘ri.

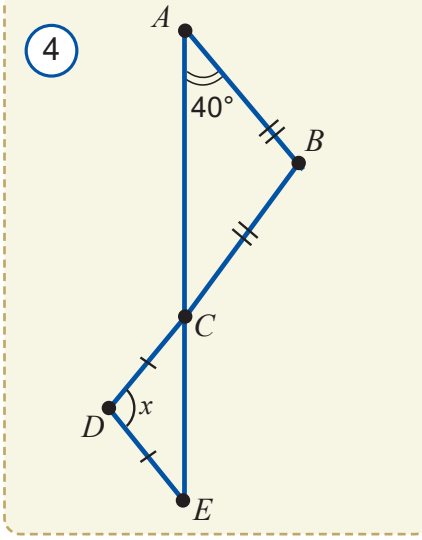
Natija isbotlandi.

2-natija. **Har qanday uchburchakning bittadan ortiq to‘g‘ri yoki o‘tmas burchagi bo‘lishi mumkin emas.**

Bu natijaning isbotini mustaqil bajaring.

1-masala. 4-rasmdagi noma’lum burchak – x ni toping.

4



Yechish. ΔABC teng yonli uchburchak bo‘lgani uchun $\angle ACB = \angle A = 40°$. Vertikal burchaklar xossasiga ko‘ra, $\angle DCE = \angle ACB = 40°$.

Shartga ko‘ra, ΔCED ham teng yonli. Shu bois $\angle DCE = \angle DEC = 40°$. Unda uchburchak burchaklarining yig‘indisi haqidagi teoremaga ko‘ra, ΔCDE da: $40° + 40° + x = 180°$ yoki $x = 100°$.

Javob: 100°.

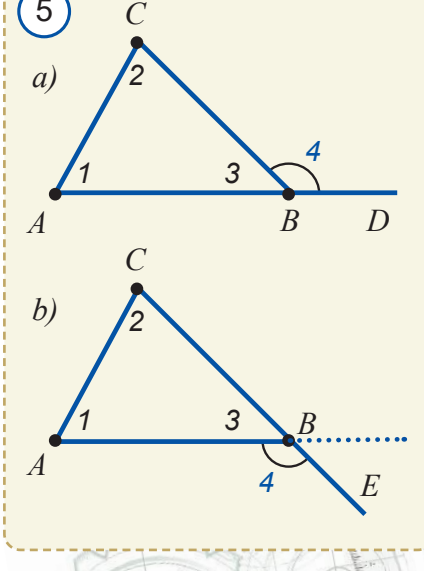
2-masala. Uchburchak ichki burchaklari 2:3:7 kabi nisbatda. Ularning gradus o‘lchovini toping.

Yechish. Shartga ko‘ra, uchburchak ichki burchaklarini $2x$, $3x$ va $7x$ deb belgilaymiz. U holda uchburchak ichki burchaklari yig‘indisi haqidagi teoremaga ko‘ra, $2x+3x+7x=180°$ tenglikka ega bo‘lamiz. Undan $x=15°$ ekanini topamiz:

$2x = 2 \cdot 15° = 30°$, $3x = 3 \cdot 15° = 45°$ va $7x = 7 \cdot 15° = 105°$.

Javob. Uchburchak ichki burchaklari 30°, 45° va 105° ga teng.

5

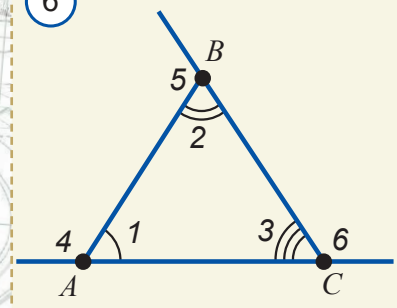


18.2. Uchburchak tashqi burchagining xossasi

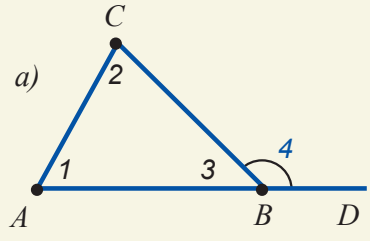
Uchburchakning ichki burchagiga qo‘shni bo‘lgan burchak uchburchakning *tashqi burchagi* deb ataladi.

5-rasmda ABC uchburchakning B burchagiga tashqi bo‘lgan CBD va ABE burchaklar tasvirlangan. Ravshanki, bu burchaklar vertikal bo‘lgani uchun o‘zaro teng bo‘ladi. Qolgan A va C burchaklar tashqi burchaklarini chizib ko‘rsating.

6



7



Uchburchak burchaklarini uning tashqi burchaklaridan farqlash uchun *ichki burchaklar* deb ham ataymiz.

Geometrik tadqiqot

6-rasmdagi ABC uchburchakning hamma ichki va tashqi burchaklarini transportirda o‘lchang va quyidagi burchaklar (har bir tashqi burchak va unga qo‘shni bo‘lmagan ichki burchaklar yig‘indisining) kattaliklarini o‘zaro solishtiring: a) $\angle 4$ va $\angle 2 + \angle 3$; b) $\angle 5$ va $\angle 1 + \angle 3$; c) $\angle 6$ va $\angle 1 + \angle 2$.

Solishtirish natijasida qanday xulosaga keldingiz. Uni taxminiy tasdiq ko‘rinishida ifodalang.

Teorema. **Uchburchak tashqi burchagi uchburchakning unga qo‘shni bo‘lmagan ikki ichki burchaklari yig‘indisiga teng.**

ΔABC, $\angle 4$ – tashqi burchak (7-*rasm*) ⇒ $\angle 1 + \angle 2 = \angle 4$

Isbot. 7-rasmga murojaat qilamiz. Unda qo‘shni burchaklar xossasiga ko‘ra, $\angle 3 + \angle 4 = 180°$.

Uchburchak burchaklari yig‘indisi haqidagi teoremaga ko‘ra, $\angle 1 + \angle 2 + \angle 3 = 180°$.

Bu ikki tenglikdan $\angle 1 + \angle 2 + \cancel{\angle 3} = \cancel{\angle 3} + \angle 4$, ya’ni $\angle 1 + \angle 2 = \angle 4$ tenglikni hosil qilamiz.

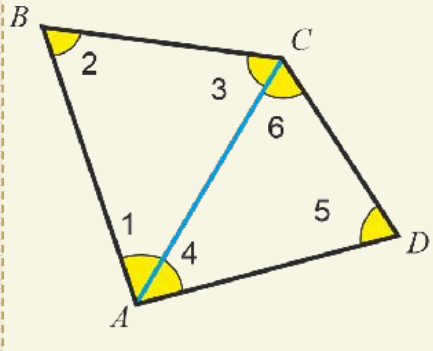
Teorema isbotlandi.

Bu teoremadan quyidagi natija kelib chiqadi.

Natija. ***Uchburchakning tashqi burchagi unga qo‘shni bo‘lmagan ichki burchaklarning har biridan katta.***

Uning to‘g‘riligini mustaqil ravishda tekshiring.

8



Masala. To‘rtburchakning burchaklari yig‘indisi 360° ga teng ekanini isbotlang.

Yechish. Ixtiyoriy $ABCD$ to‘rtburchak chizamiz. A va C nuqtalarni tutashtirib, uni ikkita uchburchakka ajratamiz.

Har bir ABC va ADC uchburchaklar ichki burchaklari yig‘indisi 180° ga teng (*8-rasm*):

$\angle 1+\angle 2+\angle 3=180°$, $\angle 4+\angle 5+\angle 6=180°$.

$\angle A=\angle 1+\angle 4$ va $\angle C=\angle 3+\angle 6$ bo‘lgani uchun $\angle A+\angle B+\angle C+ D=(\angle 1+\angle 4)+\angle 2+(\angle 3+\angle 6)+\angle 5 =$

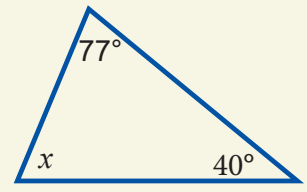
$=(\angle 1+\angle 2+\angle 3)+(\angle 4+\angle 5+\angle 6)=180°+180°= 360°$.

Mavzuga doir savollar

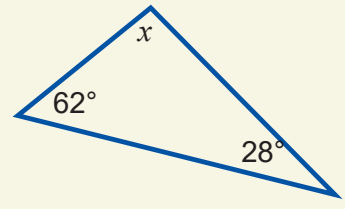
1. *Uchburchak ichki burchaklarining yig‘indisi haqidagi teoremani ayting va rasmda izohlang.*
2. *Uchburchakning ko‘pi bilan nechta burchagi to‘g‘ri bo‘lishi mumkin?*
3. *Uchburchakning ikkita tomoni uchinchi tomonga perpendikulyar bo‘lishi mumkinmi?*
4. *Uchburchakning nechta burchagi o‘tmas bo‘lishi mumkin?*
5. *Burchaklari: a) 5°, 55°, 120°; b) 46°, 150°,4°; c) 100°, 20°, 50°; d) 25°, 35°, 100° bo‘lgan uchburchak mavjudmi?*
6. *Uchburchakning tashqi burchagi nima?*
7. *Uchburchakning o‘tmas tashqi burchaklari: a) 1 ta; b) 2 ta; c) 3 ta bo‘lishi mumkinmi?*
8. *Uchburchakning bir uchidagi ichki va tashqi burchaklari teng bo‘lishi mumkinmi?*
9. *Uchburchakning ko‘pi bilan nechta tashqi burchagi o‘tkir bo‘lishi mumkin?*

9

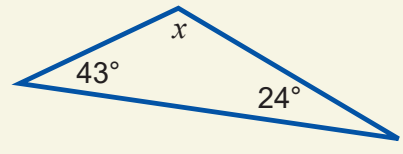
a)



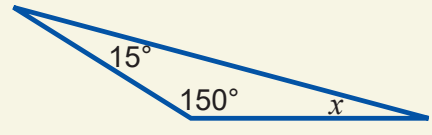
b)



c)



d)



Amaliy mashq va tatbiq

1. Agar uchburchakning ikkita burchagi: a) 60° va 40°; b) 70° va 85°; c) 90° va 45°; d) 105° va 30° bo‘lsa, uning uchinchi burchagini toping.

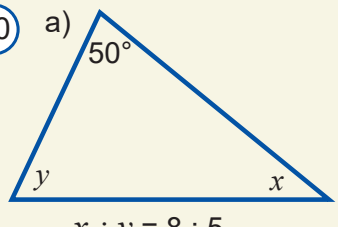
2. 9-rasmdagi noma’lum burchakni toping.

3. Uchburchak ikkita burchagining yig‘indisi 78° ga teng. Uchinchi burchagini toping.

4. 10-rasmdagi noma’lum burchaklarni toping.

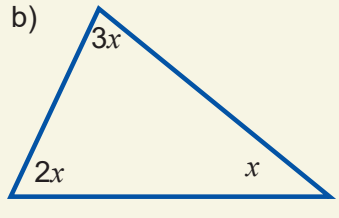
10

a)

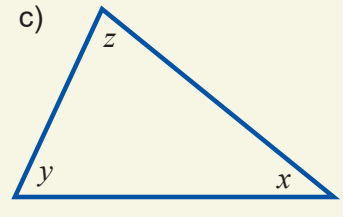


$x : y = 8 : 5$

b)



c)

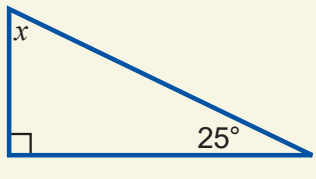


$x : y : z = 5 : 6 : 7$

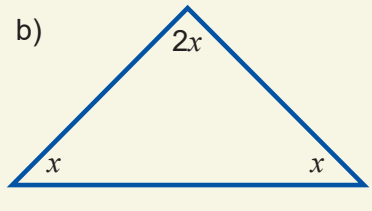
5. 11-rasmdagi noma’lum burchaklarni toping.

11

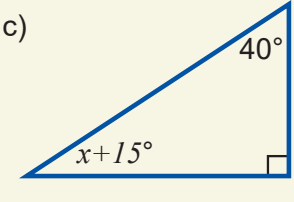
a)



b)

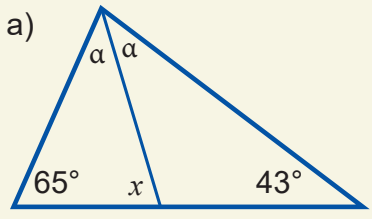


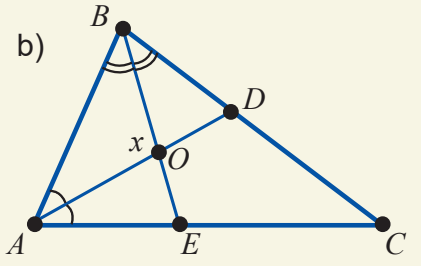
c)



6. 12-rasmdagi noma'lum burchaklarni toping.

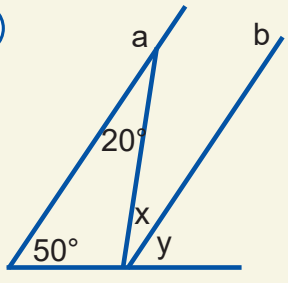
12



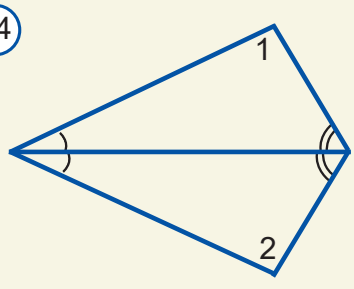


$\angle BAC = 74°$, $\angle ABC = 76°$

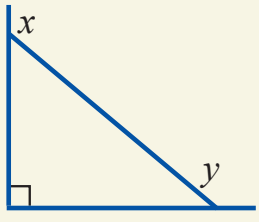
13



14



15



7. 13-rasmda $a||b$. x va y ni toping.

8. 14-rasmda $\angle 1 = \angle 2$ ekanini isbotlang.

9. 15-rasmda $x+y$ ni toping.

10*. Uchburchak burchaklari α, β, γ uchun $\alpha = (\beta+\gamma)/2$ bo'lsa, α ni toping.

11. Teng tomonli uchburchak burchaklarini toping.

12. Teng yonli to'g'ri burchakli uchburchak burchaklarini toping.

13. Agar teng yonli uchburchak burchaklaridan biri a) 50°; b) 60°; c) 105° bo'lsa, uning burchaklarini toping.

14. Uchburchakning ikki tashqi burchagi 120° va 135° bo'lsa, ichki burchaklarini toping.

15. Uchburchakning ichki burchaklaridan biri 30° ga, tashqi burchaklaridan biri 60° ga teng. Uchburchakning qolgan ichki burchaklarini toping.

16. 16-rasmlardagi noma'lum burchakni toping.

17. 17-rasmlardagi noma'lum burchakni toping.

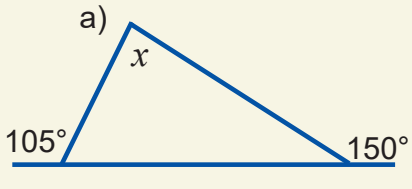
18.* Uchburchak tashqi burchaklarining yig'indisini hisoblang.

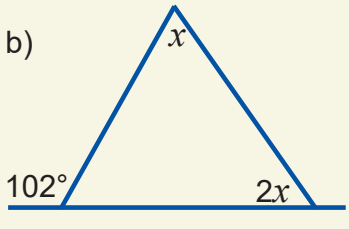
19. Uchburchak ikkita ichki burchagining o'lchovlari nisbati 5:9 kabi, uchinchi ichki burchagi shu burchaklarning kichigidan 10° ga kichik. Uchburchakning ichki burchaklarini toping.

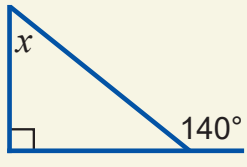
20. Agar 18-rasmlarda $a||b$ bo'lsa, x ni toping.

21. Uchburchakning 108° li tashqi burchagiga qo'shni bo'lmagan ichki burchaklarining

16

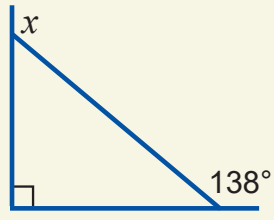




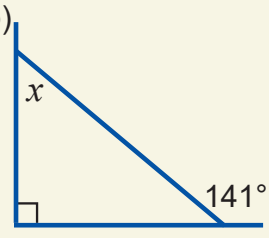


17

a)

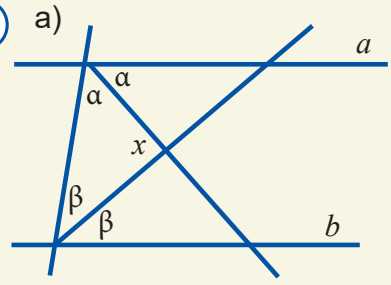


b)

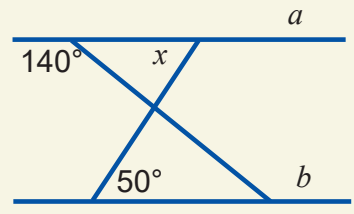


18

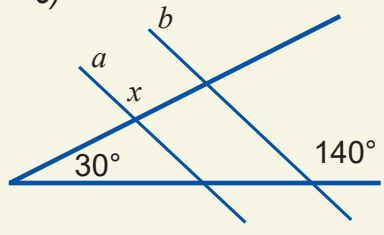
a)



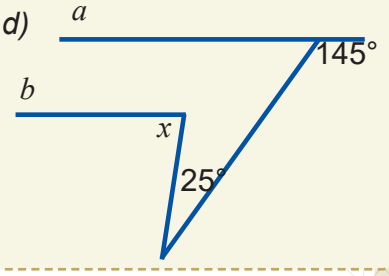
b)



c)



d)



nisbati 5:4 kabi. Shu ichki burchaklarini toping.

22*. 19-rasmda tasvirlangan beshburchak burchaklari yig‘indisini toping.

23. 20-rasmdagi noma’lum burchaklarni toping.

24. Ikkita burchagi teng bo‘lgan uchburchakning teng yonli ekanini ko‘rsating.

25. Teng yonli uchburchakning bir burchagi: a) 120°; b) 70°. Uning qolgan burchaklarini toping.

26. Teng yonli uchburchakning asosidagi burchaklaridan biri a) 15°; b) 75° bo‘lsa, qolgan burchaklari nimaga teng?

27. Ikki uchburchakning barcha mos tomonlari o‘zaro parallel bo‘lsa, ularning mos burchaklari teng bo‘lishini isbotlang.

28. Agar 21-rasmda $AB=BC$, $\angle ABC=50°$, AE va CF bissektrisalar bo‘lsa, AOB va EOC burchaklarni toping.

29. 22-rasmdagi noma’lum – x burchakni toping.

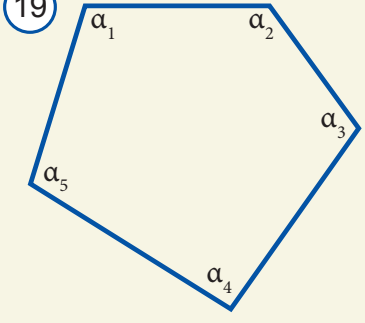
30. 23-rasmdagi noma’lum – x burchakni toping.

31. Ikkita uchburchakning barcha mos tomonlari o‘zaro perpendikulyar bo‘lsa, ularning mos burchaklari teng bo‘ladimi? Javobingizni asoslang.

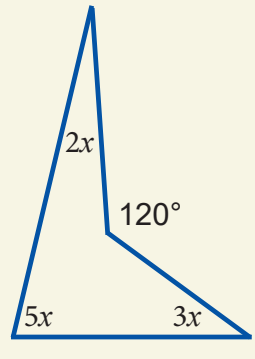
32. Biror uchburchakni faqat bitta to‘g‘ri chiziq bo‘ylab qirqib ikkita o‘tkir burchakli uchburchak hosil qilish mumkinmi? Javobingizni asoslang.

33. ABC uchburchakda $\angle A+\angle B = 110°$ va $\angle B+\angle C = 100°$. Uning ichki burchaklarini toping.

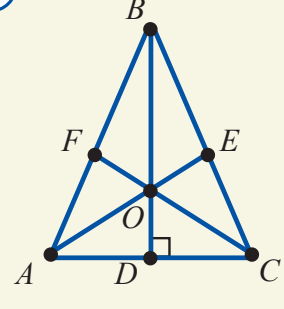
19



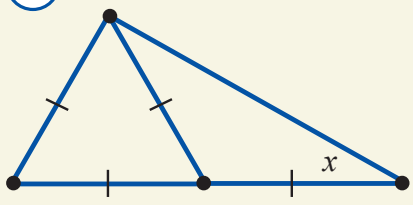
20



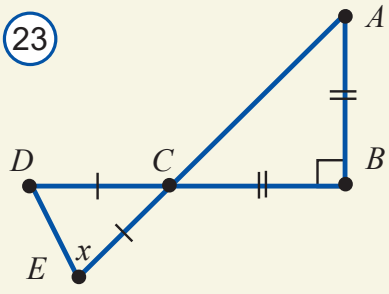
21



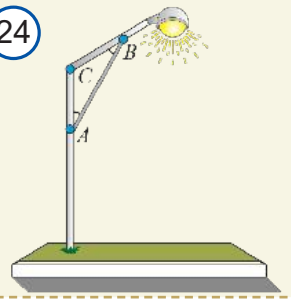
22



23



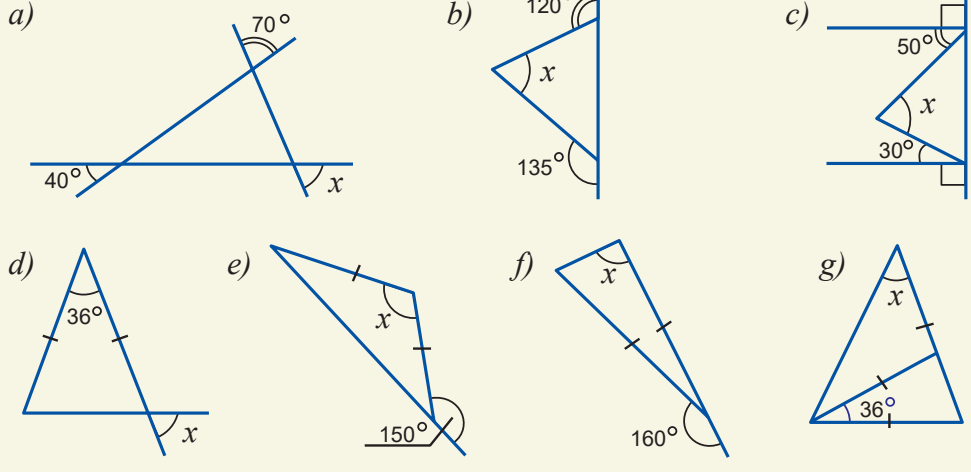
24



34. 24-rasmda tasvirlangan stol chirog‘i shunday yasalganki, $\angle C = 110°$ va $\angle A = \angle B$. A va B burchaklarning gradus o‘lchovini toping. Nima deb o‘ylaysiz, nega u aynan shunday shaklda yasalgan?

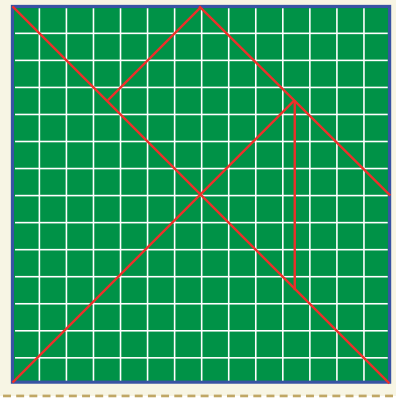
35. 25-rasmdagi noma’lum burchakni toping.

25



Geometrik boshqotirma

26

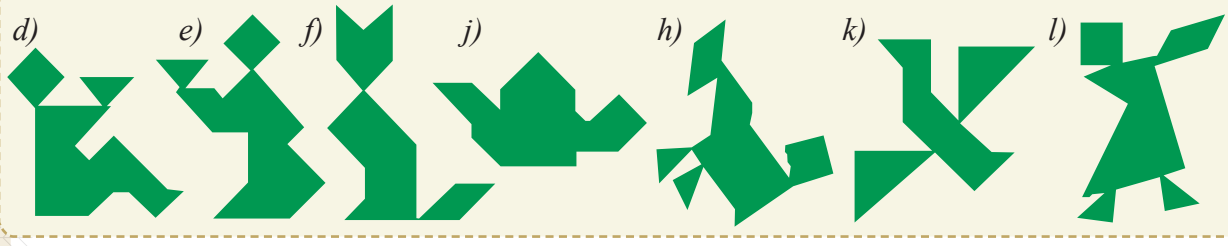


“Tangram” nomli xitoy o‘yinchog‘ini yasang. Buning uchun kvadratni qalin qog‘ozga chizing va 26-rasmda ko‘rsatilgandek, uni yetti bo‘lakka bo‘lib, qirqib oling.

So‘ng “Tangram” bo‘lakchalarining hammasidan foydalanib, 27-rasmda tasvirlangan shakllarni hosil qiling.



27



19 TO‘G‘RI BURCHAKLI UCHBURCHAKLAR

19.1. To‘g‘ri burchakli uchburchakning xossalari

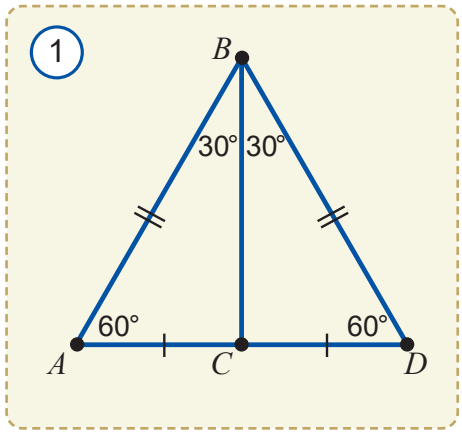
Eslatib o‘tamiz, to‘g‘ri burchakli uchburchakning bitta burchagi to‘g‘ri (90°), qolgan ikki burchagi esa o‘tkir burchaklardan iborat. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagi qarshisidagi tomoni *gipotenuza*, qolgan ikki tomoni esa *katet* deb ataladi. To‘g‘ri burchakli uchburchakning ba’zi bir xossalarini ko‘rib chiqaylik.

1-xossa. **To‘g‘ri burchakli uchburchakning ikkita o‘tkir burchaklari yig‘indisi 90° ga teng.**

Haqiqatan, uchburchak ichki burchaklari yig‘indisi 180° ga teng. To‘g‘ri burchakli uchburchakning bitta burchagi esa 90° ga teng. Shuning uchun uning qolgan ikki burchagi yig‘indisi 180° – 90° = 90° ga teng bo‘ladi.

2-xossa. **To‘g‘ri burchakli uchburchakning 30° li burchagi qarshisidagi kateti gipotenuzasining yarmiga teng.**

Isbot. Aytaylik, 1-rasmda tasvirlangan ABC to‘g‘ri burchakli uchburchak berilgan bo‘lib, unda $\angle ACB = 90°$ va $\angle ABC = 30°$ ga teng bo‘lsin. U holda $\angle BAC = 60°$ bo‘ladi.



$AC = \frac{AB}{2}$ ekanini ko‘rsatamiz.

Berilgan uchburchakka teng DBC uchburchakni 1-rasmda ko‘rsatilgandek qilib yasaymiz. Natijada hamma burchaklari 60° ga teng bo‘lgan ABD uchburchakka ega bo‘lamiz.

Demak, ABD uchburchak teng tomonli. Xususan, $AB = AD$ bo‘ladi. Lekin:

$$AD = AC + CD = 2AC.$$

Shunday qilib, $AB = 2AC$, ya’ni $AC = \frac{AB}{2}$.

Xossa isbotlandi.

3-xossa. **To‘g‘ri burchakli uchburchakning katetlaridan biri gipotenuzaning yarmiga teng bo‘lsa, bu katet 30° li burchak qarshisida yotadi.**

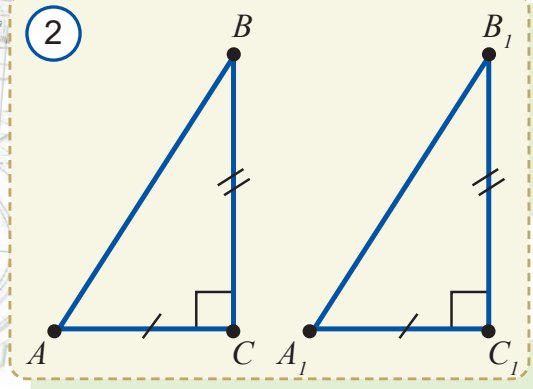
Bu xossa 2-xossaga teskari tasdiq bo‘lib, uni mustaqil isbotlang.

19.2. To‘g‘ri burchakli uchburchaklarning tenglik alomatlari

ABC va $A_1B_1C_1$ to‘g‘ri burchakli uchburchaklar berilgan bo‘lsin. Bu uchburchaklarning bittadan burchagi to‘g‘ri bo‘lgani sababli bu burchaklar doim o‘zaro teng. Shu bois to‘g‘ri burchakli uchburchaklar uchun uchburchaklarning tenglik alomatlari ancha soddalashadi.

To‘g‘ri burchakli uchburchaklar uchun ikki katet bo‘yicha (KK alomat), katet va o‘tkir burchak bo‘yicha (KB alomat), gipotenuza va o‘tkir burchak bo‘yicha (GB alomat) va gipotenuza va katet bo‘yicha (GK alomat) kabi tenglik alomatlarini keltiramiz.

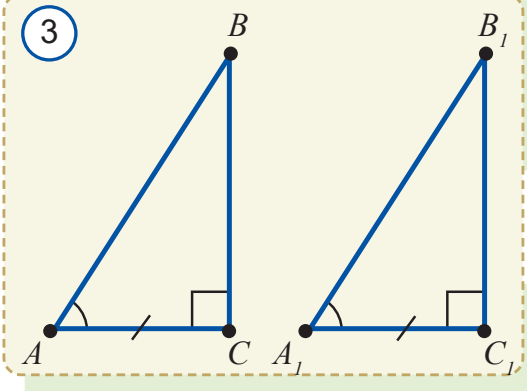
2



Teorema. KK alomat. Bir to‘g‘ri burchakli uchburchakning katetlari ikkinchi to‘g‘ri burchakli uchburchakning katetlariga mos ravishda teng bo‘lsa, bu uchburchaklar o‘zaro teng bo‘ladi *(2-rasm)*.

Bu alomat uchburchaklar tengligining TBT alomatidan bevosita kelib chiqadi.

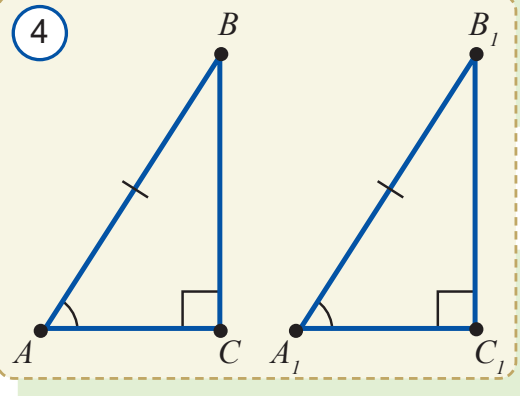
3



Teorema. KB alomat. Bir to‘g‘ri burchakli uchburchakning kateti va unga yopishgan o‘tkir burchagi ikkinchi to‘g‘ri burchakli uchburchakning kateti va unga yopishgan o‘tkir burchagiga teng bo‘lsa, bu uchburchaklar o‘zaro teng bo‘ladi *(3-rasm)*.

Bu alomat uchburchaklar tengligining BTB alomatidan bevosita kelib chiqadi.

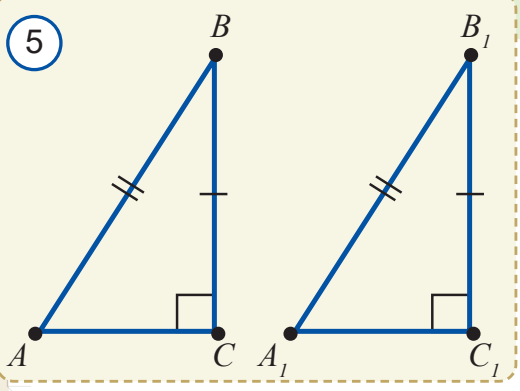
4



Teorema. GB alomat. Bir to‘g‘ri burchakli uchburchakning gipotenuzasi va bitta o‘tkir burchagi ikkinchi to‘g‘ri burchakli uchburchakning gipotenuzasi va bitta o‘tkir burchagiga teng bo‘lsa, bu uchburchaklar o‘zaro teng bo‘ladi *(4-rasm)*.

Bu alomat uchburchaklar tengligining BTB alomatidan bevosita kelib chiqadi.

5



Teorema. GK alomat. Bir to‘g‘ri burchakli uchburchakning gipotenuzasi va bitta kateti ikkinchi to‘g‘ri burchakli uchburchakning gipotenuzasi va bitta katetiga teng bo‘lsa, bu uchburchaklar o‘zaro teng bo‘ladi.

Keling, bu alomatni isbotlaylik.

ABC va $A_1B_1C_1$ uchburchaklar berilgan *(5-rasm)* va ularda $\angle C = 90°$, $\angle C_1 = 90°$, $AB = A_1B_1$, $BC = B_1C_1$ bo‘lsin.

U holda $\Delta ABC = \Delta A_1B_1C_1$ ekanini ko‘rsatamiz.

Isbot. ABC va $A_1B_1C_1$ uchburchaklarning ikkitadan tomonlari o‘zaro teng: $AB = A_1B_1$, $BC = B_1C_1$. Agar ABC va $A_1B_1C_1$ burchaklarining tengligini ko‘rsatsak, TBT alomatiga ko‘ra, bu uchburchaklar o‘zaro teng bo‘ladi.

Buning uchun $A_1B_1C_1$ uchburchakni ABC uchburchak bilan BC va B_1C_1 katetlar ustma-ust tushadigan qilib yonma-yon qo‘yamiz *(6-rasm)*.

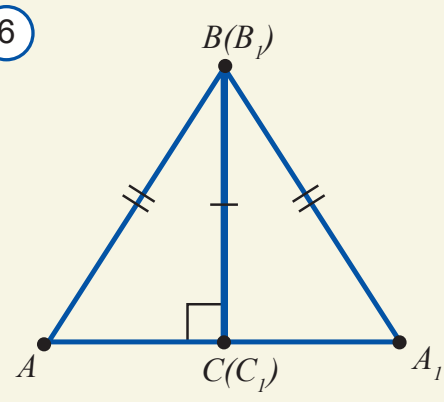
U holda $\angle C$ va $\angle C_1$ to‘g‘ri burchak bo‘lganligi uchun CA va C_1A_1 nurlar yoyiq burchakni tashkil qiladi, ya’ni A, C, C_1 va A_1 nuqtalar bir to‘g‘ri chiziqda yotadi.

Natijada ABA_1 teng yonli uchburchak bo‘ladi. Lekin teng yonli uchburchakda asosga tushirilgan balandlik bissektrisa ham bo‘ladi (83-betdagi teorema xulosasiga ko‘ra).

Demak, $\angle ABC = \angle A_1B_1C_1$.

Teorema isbotlandi.

6



Mavzuga doir savollar

1. *To‘g‘ri burchakli uchburchakning tomonlari qanday nomlanadi?*
2. *To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi nimaga teng?*
3. *To‘g‘ri burchakli uchburchakning burchaklaridan birortasi o‘tmas bo‘lishi mumkinmi?*
4. *To‘g‘ri burchakli uchburchakning nechta balandligi bor?*
5. *To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 120°ga teng bo‘lishi mumkinmi? Nega?*
6. *To‘g‘ri burchakli uchburchakning 30° li burchak qarshisidagi kateti bilan gipotenuzasi orasida qanday munosabat bor?*
7. *To‘g‘ri burchakli uchburchaklar xossalarini ayting va izohlang.*
8. *To‘g‘ri burchakli uchburchaklar tengligining alomatlarini ayting va izohlang.*
9. *To‘g‘ri burchakli uchburchaklarning bir kateti va bir burchagi mos ravishda teng bo‘lsa, bu uchburchaklar teng bo‘ladimi?*
10. *To‘g‘ri burchakli ABC uchburchakda $\angle A + \angle B = \angle C$ bo‘lsa, uning katetlari va gipotenuzasini ayting.*

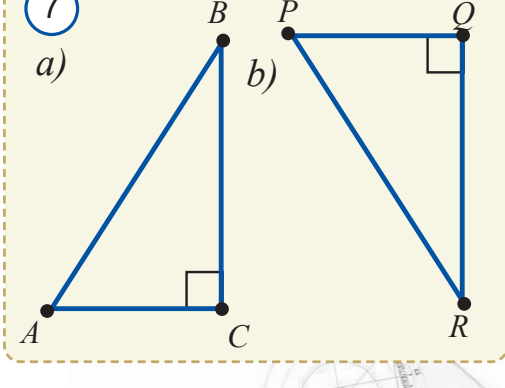
Amaliy mashq va tatbiq

1. 7a-rasmdagi ACB to‘g‘ri burchakli uchburchakning gipotenuzasi va katetlarini yozing. Bu uchburchakning qaysi tomoni uzun: a) AB yoki BC; *b)* AB yoki AC?

2. 7b-rasmdagi PQR to‘g‘ri burchakli uchburchakning gipotenuzasi va katetlarini yozing. Bu uchburchakning qaysi tomoni uzun: a) PR yoki PQ; *b)* PR yoki QR?

3. To‘g‘ri burchakli uchburchakning bitta o‘tkir burchagi 23° ga teng. Bu uchburchakning uchinchi burchagini toping.

7



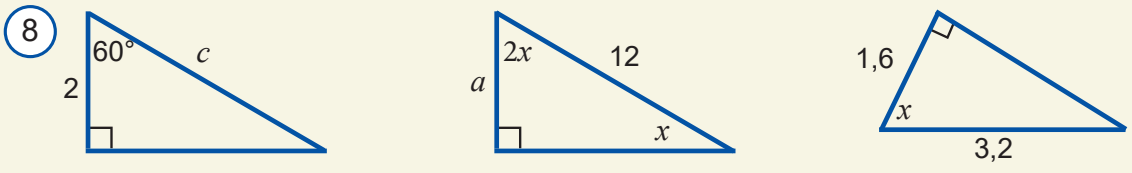
4. To‘g‘ri burchakli uchburchakning bitta burchagi a) 78°; b) 43°. Uchburchakning uchinchi burchagini toping.

5. To‘g‘ri burchakli uchburchakning o‘tkir burchaklaridan biri 30°, gipotenuzasi 34 ga teng. Bu uchburchakning burchaklari va katetlaridan birini toping.

6. To‘g‘ri burchakli uchburchakning gipotenuzasi 14 ga, katetlaridan biri 7 ga teng. Bu uchburchakning burchaklarini toping.

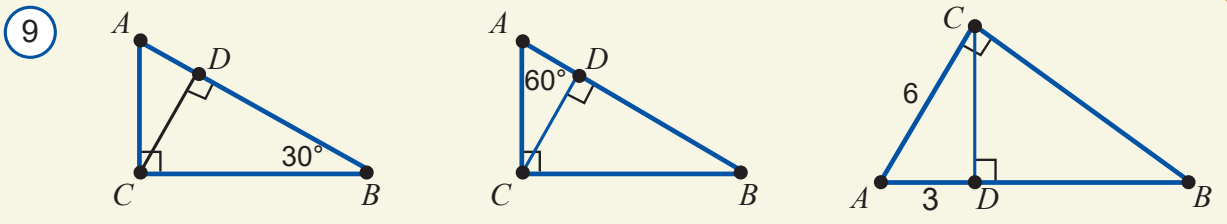
7. 8-rasm: a) $c=?$; b) $a=?$; c) $x=?$.

8



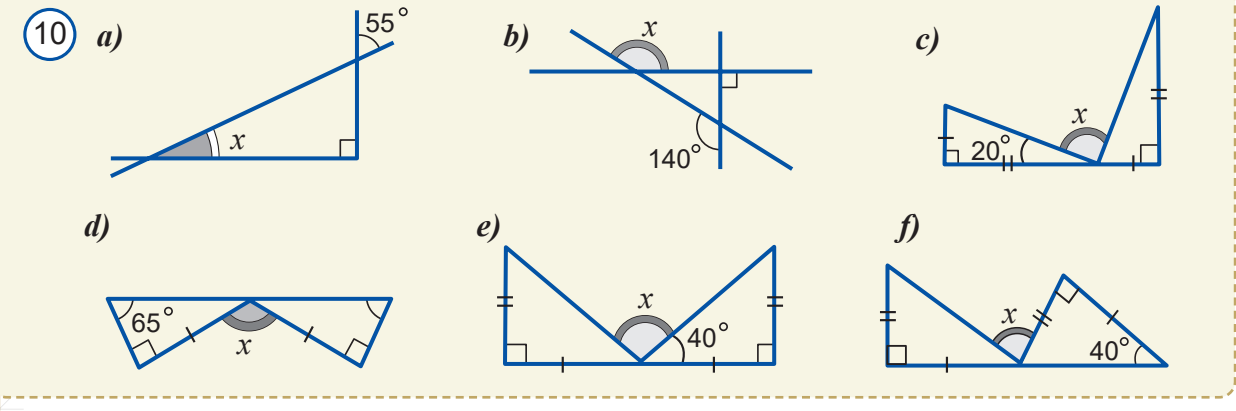
8. 9-rasm: a) $AB=20$, $AD=?$; b) $AB=18$, $BD=?$; c) $BD=?$.

9

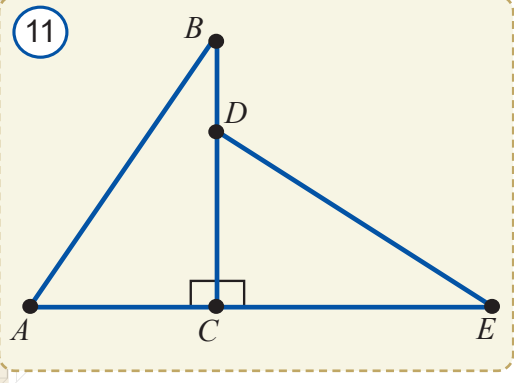


9. 10-rasmdagi noma’lum burchaklarni toping.

10



11



10. Agar 11-rasmda: a) $\angle A=\angle D$, $\angle B=\angle E$; b) $BC=DE$, $AB=CE$; d) $AC=CD$, $BC=CE$; e) $AB=DE$ bo‘lsa, ACB va DCE uchburchaklar teng bo‘ladimi?

11. To‘g‘ri burchakli ABC va $A_1B_1C_1$ uchburchaklarda A va A_1 to‘g‘ri burchaklar, BD va BD_1 bissektrisalar va $\angle B=\angle B_1$, $BD=B_1D_1$ bo‘lsa, $\Delta ABC=\Delta A_1B_1C_1$ ekanini isbotlang.

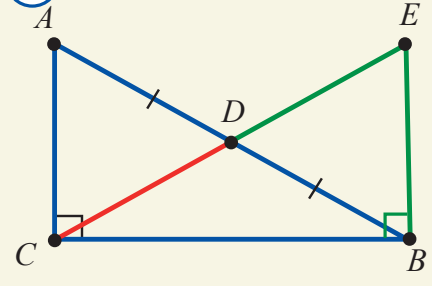
20 BURCHAK BISSEKTRISASINING XOSSASI

Teorema. **To‘g‘ri burchakli uchburchakning gipotenuzaga tushirilgan medianasi gipotenuzaning yarmiga teng.**

ABC to‘g‘ri burchakli uchburchakda AB – gipotenuza va CD – mediana, ya’ni $AD=DB$ bo‘lsin *(1-rasm)*. $CD=\frac{AB}{2}$ ekanini isbotlaymiz.

Isbot. Qo‘shimcha yasashlarni amalga oshiramiz: CB tomonning B nuqtasidan perpendikulyar chiqaramiz. Unda E nuqtani $AC=EB$ qilib belgilaymiz va CE kesmani yasaymiz.

1



ABC va EBC to‘g‘ri burchakli uchburchaklarga qaraymiz. Ularda CB katet umumiy va yasashga ko‘ra $AC=EB$. Unda to‘g‘ri burchakli uchburchaklarning KK alomatiga ko‘ra, bu uchburchaklar o‘zaro teng bo‘ladi. Xususan, $\angle ABC = \angle ECB$ bo‘ladi.

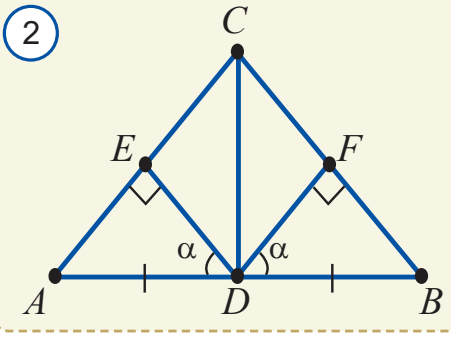
Bu CDB uchburchak teng yonli va $CD=DB$ ekanini bildiradi.

Lekin shartga ko‘ra, $DB=\frac{AB}{2}$.

Bundan $CD=\frac{AB}{2}$ ekanligi kelib chiqadi.

Teorema isbotlandi.

2



Masala. 2-rasmdagi ABC uchburchak teng yonli ekanini isbotlang.

Yechish. $\Delta AED=\Delta BFD$, chunki ularning gipotenuzalari va bittadan o‘tkir burchaklari teng. CED va CFD – to‘g‘ri burchakli uchburchaklar. $ED=FD$ hamda CD gipotenuza umumiy bo‘lgani uchun to‘g‘ri burchakli uchburchaklar tengligining GK alomatiga ko‘ra, $\Delta CED=\Delta CFD$.

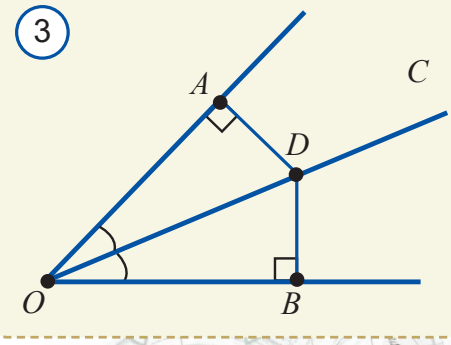
Demak, $\Delta ADC=\Delta BDC$, ya’ni $AC=BC$ va ΔABC teng yonli.

Burchak bissektrisasining xossasi

Yodingizda bo‘lsa, nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa deb nuqtadan to‘g‘ri chiziqqa tushirilgan perpendikulyar uzunligiga aytilgan edi.

Teorema. **Burchak bissektrisasining ixtiyoriy nuqtasidan burchak tomonlarigacha bo‘lgan masofalar o‘zaro teng.**

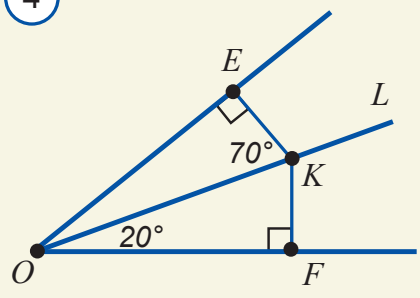
3



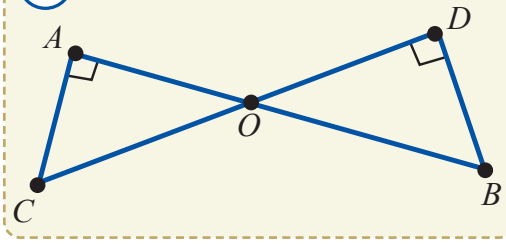
Isbot. Aytaylik, O burchak va uning bissektrisasi OC berilgan bo‘lsin *(3-rasm)*. OC bissektrisada ixtiyoriy D nuqta olamiz va berilgan burchak tomonlariga DA va DB perpendikulyarlar tushiramiz.

OAD va OBD to‘g‘ri burchakli uchburchaklarda:

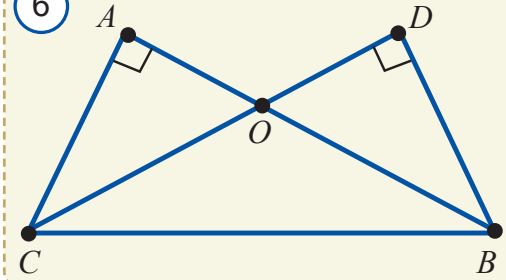
4



5



6



1) $\angle AOD = \angle BOD$ shartga ko‘ra;

2) OD – umumiy gipotenuza.

To‘g‘ri burchakli uchburchaklar tengligining GB alomatiga ko‘ra, $\Delta OAD = \Delta OBD$. Xususan, $DA = DB$.

Teorema isbotlandi.

Masala. EOF burchakning OL bissektrisasida K nuqta olingan (*4-rasm*). Agar $EK \perp OE$, $KF \perp OF$ va $\angle KOF = 20°$ bo‘lsa, a) EOK va OKF burchaklarni; b) EOF va EKF burchaklarni toping.

Yechish. **1.** Yuqorida ko‘rilganidek $\Delta EOK = \Delta FOK$.

Shuning uchun $\angle EOK = \angle FOK = 20°$ va $\angle OKF = \angle OKE = 90° - 20° = 70°$.

2. $\angle EOF = 2 \cdot \angle KOF = 40°$,

$\angle FKE = \angle FKO + \angle OKE = 70° + 70° = 140°$.

Javob: a) 20° va 70°; b) 40° va 140°.

Mavzuga doir savollar

1. *To‘g‘ri burchakli uchburchakning gipotenu-zaga tushirilgan medianasi gipotenuzaning qanday qismini tashkil qiladi?*
2. *Burchak bissektrisasining xossasini ayting va sharhlang.*

Amaliy mashq va tatbiq

1. Agar 5-rasmda: a) $OC=OB$; b) $AC=BD$; c) $AO=OD$; d) $AC=OD$; e) $\angle OCA=\angle OBD$ bo‘lsa, OAC va ODB uchburchaklar teng bo‘ladimi?

2. Agar 6-rasmda: a) $AC=BD$; b) $OA=OD$; c) $\angle OCB=\angle OBC$; d) $BC=OD$; e) $\angle ACB=\angle DBC$ bo‘lsa, BAC va CDB uchburchaklar teng bo‘ladimi?

3. ABC uchburchakda BD balandlik o‘tkazilgan. Agar $AD=DC$ bo‘lsa, ABC uchburchakning teng yonli ekanini isbotlang.

4. O‘tkir burchakli ABC uchburchakda AA_1 va CC_1 balandliklar teng. $\angle BAC = \angle BCA$ tenglikni isbotlang.

5. Burchak bissektrisasining ixtiyoriy nuqtasi uning tomonlaridan teng uzoqlikda joylashganini isbotlang.

6. Burchak AOB bissektrisasida olingan nuqtadan OA nurgacha bo‘lgan masofa 7 cm bo‘lsa, shu nuqtadan OB nurgacha bo‘lgan masofani toping.

7. O burchak va uning bissektrisasida C nuqta berilgan. Agar $\angle O$=60° va OC = 14 cm bo‘lsa, C nuqtadan burchak tomonlarigacha bo‘lgan masofani toping.

8*. Teng yonli to‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan balandlik gipotenuzaning yarmiga tengligini ko‘rsating.

9. AOB burchak ichida N nuqta olingan. Agar $AN=BN$, $OA\perp AN$ va $OB\perp BN$ bo‘lsa, N nuqta AOB burchak bissektrisasida yotishini isbotlang.

10*. 7-rasmda katakli qog‘ozga chizilgan burchakning bir qismi tasvirlangan. Qog‘ozning burchak uchi joylashgan qismi yirtilib ketgan. A va B nuqtalar burchak tomonlaridan teng uzoqlashgani ma’lum. Burchak bissektrisasini qanday yasash mumkin.

11. 8-rasmda tasvirlangan beshburchak burchaklari yig‘indisini toping.

12. 9-rasmda tasvirlangan kub sirtida joylashgan uchburchakning raqamlangan burchaklari yig‘indisini toping.

13. ABC to‘g‘ri burchakli uchburchakda C to‘g‘ri burchak va AB=12 va $CD=DB$ bo‘lsa, CD ni toping *(10-rasm)*.

14. To‘g‘ri burchakli uchburchakning gipotenuzaga tushirilgan medianasi 8 cm. Agar uchburchakning bir burchagi 60° ga teng bo‘lsa, bu burchakka yopishgan tomonlarni toping.

15*. Uchburchakning ikkita bissektrisasi kesishgan nuqta uchburchak uchala tomonidan teng uzoqlikda bo‘lishini isbotlang.

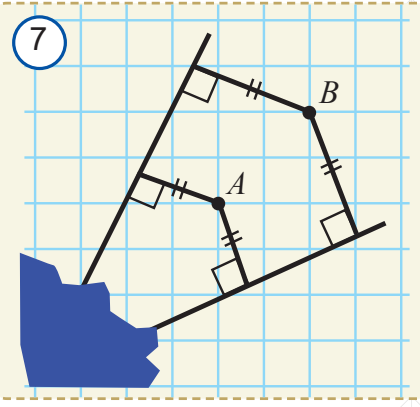
16*. Teng yonli ABC va $A_1B_1C_1$ uchburchaklarning AC va A_1C_1 asoslari va asoslarga tushirilgan BD va B_1D_1 balandliklari teng. $\Delta ABC=\Delta A_1B_1C_1$ tenglikni isbotlang.

17*. Teng yonli ABC uchburchakda AC asos va AD hamda BE balandliklar 50° li burchak ostida kesishadi. Uchburchakning burchaklarini toping.

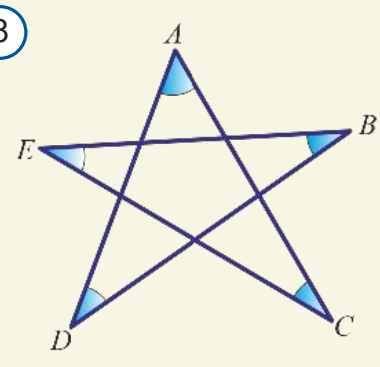
18*. To‘g‘ri burchakli uchburchaklarning bittadan kateti va gipotenuzaga tushirilgan balandligi bo‘yicha tengligini isbotlang.

19*. To‘g‘ri burchakli uchburchaklarning bittadan kateti va gipotenuzaga tushirilgan bissektrisasi bo‘yicha tengligini isbotlang. Ma’lumki, matematik jumla aniq, yetarlicha to‘la va shu bilan birga qisqa, ortiqcha so‘zlarsiz bo‘lishi lozim. Quyidagi jumlalardagi ortiqcha so‘zlarni aniqlab ko‘ring-chi?

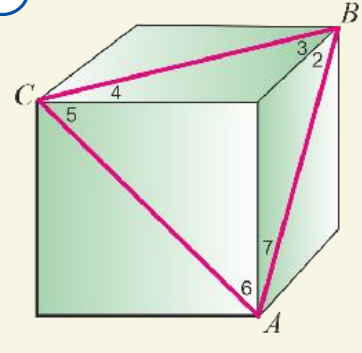
7



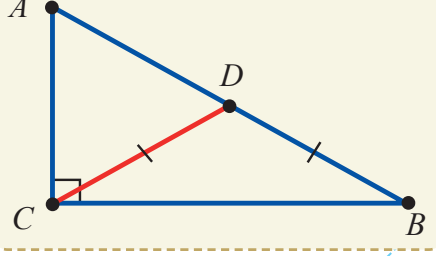
8



9

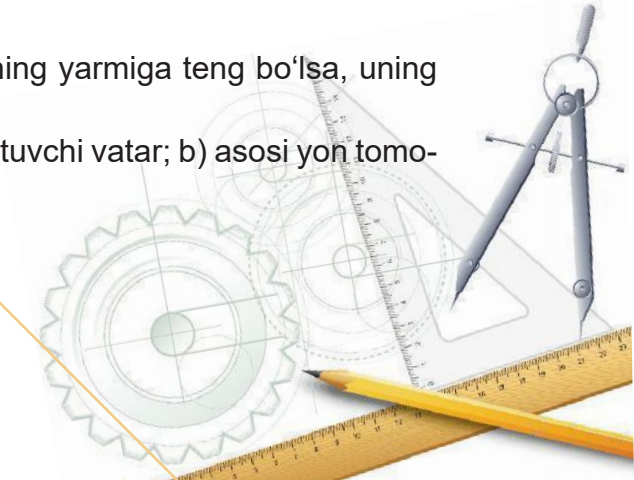


10



Geometriyada aniqlik va qisqalik

1. To‘g‘ri burchakli uchburchakning ikki o‘tkir burchaklari yig‘indisi 90° ga teng.
2. Agar to‘g‘ri burchakli uchburchakda katet gipotenuzaning yarmiga teng bo‘lsa, uning qarshisida yotuvchi o‘tkir burchak 30° ga teng bo‘ladi.
3. Eng kam tomonli ko‘pburchak: a) aylana markazidan o‘tuvchi vatar; b) asosi yon tomoniga teng bo‘lgan teng yonli uchburchak.



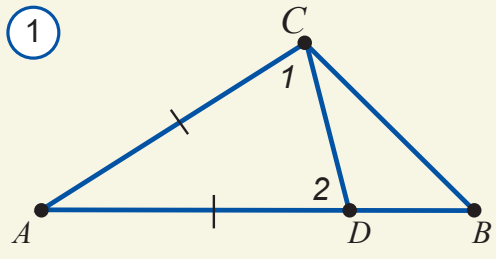
21 UCHBURCHAKNING TOMONLARI VA BURCHAKLARI ORASIDAGI MUNOSABATLAR

21.1. Uchburchakning tomonlari va burchaklari orasidagi munosabatlar

Teorema. **Uchburchakning katta tomoni qarshisida katta burchak yotadi.**

$\Delta ABC,\ AB > AC$ (1-rasm) ⇒ $\angle C > \angle B$

1



Isbot. AB kesmani chizamiz va unga AC tomonga teng AD kesmani qo‘yamiz. $AB>AD$ bo‘lgani uchun D nuqta AB kesmaga tegishli bo‘ladi. Demak, CD kesma C burchakning ichki sohasida yotadi va C burchakni ikki burchakka ajratadi. Shunga ko‘ra, $\angle C > \angle 1$.

ACD uchburchakni teng yonli qilib yasaganimiz uchun $\angle 1 = \angle 2$.

$\angle 2$ CDB uchburchakning tashqi burchagi bo‘lgani uchun $\angle 2 > \angle B$.

Bu ajratib ko‘rsatilgan uchta munosabatdan $\angle C > \angle 1 = \angle 2 > \angle B$, ya’ni $\angle C > \angle B$ ekanini hosil qilamiz.

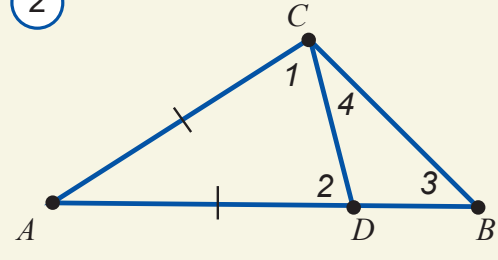
Teorema isbotlandi.

Shuningdek, bu teoremaga teskari teorema ham o‘rinli.

Teskari teorema. **Uchburchakning katta burchagi qarshisida katta tomon yotadi.**

Bu teoremaning isbotini mustaqil bajaring.

2



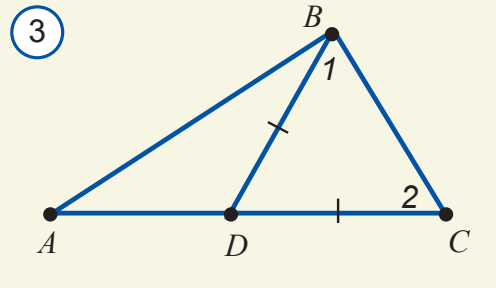
1-natija. Teng yonli uchburchakda teng tomonlar qarshisida teng burchaklar yotadi.

Uning to‘g‘riligini oldin ham isbotlagan edik.

1-masala. 2-rasmda berilgan ma’lumotlardan foydalanib $\angle 1 > \angle 3$ ekanini isbotlang.

Yechish. $\angle 2 > \angle 3$ ekanligi ravshan, chunki $\angle 2$ BDC uchburchakning tashqi burchagi bo‘lib, tashqi burchak xossasiga ko‘ra, $\angle 2 = \angle 3 + \angle 4$ va $\angle 4 > 0$.

3



ACD teng yonli uchburchak bo‘lgani uchun $\angle 1 = \angle 2$. Demak, $\angle 1 > \angle 3$ bo‘ladi.

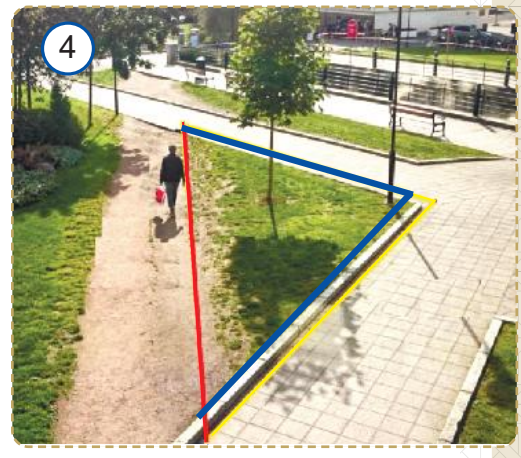
2-masala. 3-rasmda berilganlardan foydalanib $AB < AC$ ekanini ko‘rsating.

Yechish. BDC – teng yonli uchburchak (chunki $BD = DC$). Demak, $\angle 1 = \angle 2$ bo‘ladi. $\angle 1 < \angle ABC$ bo‘lgani uchun $\angle 2 < \angle ABC$. Katta burchak qarshisida katta tomon yotgani uchun $AB < AC$ bo‘ladi.

21.2. Uchburchak tengsizligi

Faollashtiruvchi savol.

Odamlar shahar ko‘chalarida yo‘lak qolib, o‘tloqlar ustidan yurib qoldirgan izlarga ko‘zingiz tushgan bo‘lsa kerak *(4-rasm)*. Odatda ular shoshib turganda yo‘llarini qisqartirish uchun shunday yo‘l tutadi va o‘zlari bilmagan holda “uchburchak tengsizligi” deb nomlangan uchburchakning geometrik xossasidan foydalanadi. Bu qanday xossa? Quyida gap shu haqda boradi.


4

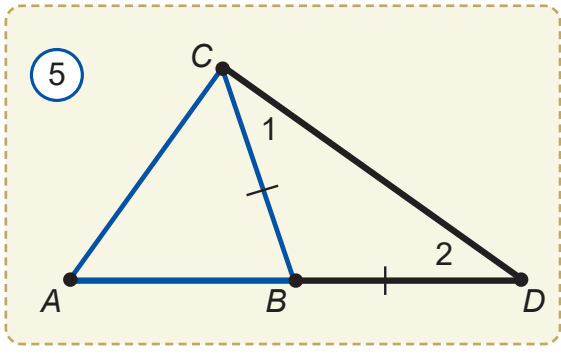
Uchburchakning istalgan bir tomoni qolgan ikki tomoni yig‘indisidan kichik.

ΔABC *(1-rasm)* ⇒ $AC < AB + BC$

Isbot. AB to‘g‘ri chiziqqa BC kesmaga teng BD kesmani qo‘yamiz va C va D nuqtalarni tutashtiramiz *(5-rasm)*. Natijada BCD teng yonli uchburchak hosil bo‘ladi. Unda $\angle 1 = \angle 2$, chunki $BC = BD$. Shakldan ravshanki, $\angle ACD > \angle 1$.

U holda $\angle ACD > \angle 2$, chunki $\angle 1 = \angle 2$.

Bu burchaklar ACD uchburchakka tegishli. Endi katta burchak qarshisida katta tomon yotishini hisobga olsak, $AC < AD$ tengsizlikka ega bo‘lamiz. U holda $AC < AB + BD$, chunki $AD = AB + BD$. Undan $BD = BC$ ekanini hisobga olsak, $AC < AB + BC$ ni hosil qilamiz.



Teorema isbotlandi.

Bu teoremadan quyidagi natija kelib chiqadi.

2-natija. **Bir to‘g‘ri chiziqda yotmagan ixtiyoriy uchta – A, B va C nuqta uchun $AB < AC + BC$, $AC < AB + BC$ va $BC < AB + AC$ tengsizliklar o‘rinli.**

Bu tengsizliklarning har biri *uchburchak tengsizligi* deb ataladi.

Agar A, B va C nuqtalar bir to‘g‘ri chiziqda yotsa, yuqoridagi tengsizliklardan biri tenglikka aylanadi, qolganlari esa to‘g‘riligicha qoladi.

Masalan, bu nuqtalar bir to‘g‘ri chiziqda yotib, A nuqta B va C nuqtalar orasida yotsa, $AC = AB + BC$, $AB < AC + BC$ va $BC < AB + AC$ munosabatlar o‘rinli bo‘ladi.

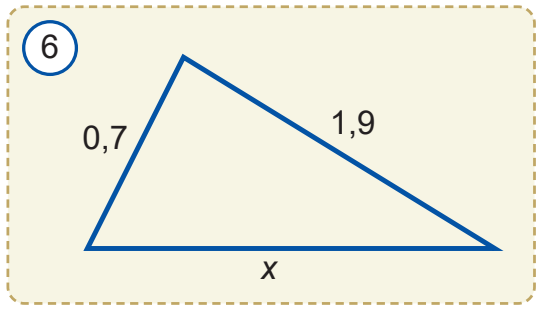
3-natija. **Uchburchakning istalgan bir tomoni qolgan ikki tomoni uzunliklari ayirmasidan katta.**

Haqiqatan ham, $AB < AC + BC$ ko‘rinishdagi uchburchak tengsizliklaridan birini olib, quyidagi shakl almashtirishlarni bajaramiz: $AB - AC < BC$ yoki $BC > AB - AC$

Xuddi shu yo‘l bilan $AC < AB + BC$ tengsizlikdan $BC > AC - AB$ tengsizlikni hosil qilamiz. $BC > AB - AC$ va $BC > AC - AB$ tengsizliklardan $BC > |AB - AC|$ bo‘ladi.

Shunday qilib, **uchburchakning ixtiyoriy tomoni qolgan tomonlarining yig‘indisidan kichik va ayirmasi modulidan katta bo‘ladi.**

Masala. Uchburchakning ikki tomoni 0,7 va 1,9. Agar uchinchi tomoni butun son ekanligi ma'lum bo'lsa, uni toping *(6-rasm).*



Yechilishi. Berilgan uchburchakning ikkita tomoni ma'lum: 0,7 va 1,9.

Uchinchi tomonini uchburchak tengsizligidan foydalanib topamiz: *1)* $x + 0{,}7 > 1{,}9$ yoki $x > 1{,}2$; 2) $1{,}9 + 0{,}7 > x$ yoki $x < 2{,}6$.

Bu ikki tengsizlikdan $1{,}2 < x < 2{,}6$ ni hosil qilamiz.

x – butun son, faqat $x = 2$ qiymat bu qo'sh tengsizlikni qanoatlantiradi.

Demak, uchburchakning noma'lum tomoni 2 ga teng.

Javob: 2.

Mavzuga doir savollar

1. *Uchburchakning katta burchagi 60° dan kichik bo'lishi mumkinmi? Uchburchakning kichik burchagi 60° dan katta bo'lishi mumkinmi?*
2. *ABC uchburchakda: a) $AB < BC < AC$; b) $AB = AC < BC$ bo'lsa, uchburchak burchaklarini taqqoslang. A burchak o'tmas bo'lishi mumkinmi?*
3. *Uchburchakning o'tmas burchagi qarshisida kichik tomon yotishi mumkinmi?*
4. *Uchburchak tengsizligining mazmuni nimadan iborat?*
5. *Uchburchak tengsizligi qanday masalalarni yechishda qo'llanadi?*
6. *To'g'ri burchakli uchburchakning gipotenuzasi uzunmi yoki kateti?*

Amaliy mashq va tatbiq

1. ABC uchburchakda $\angle A > \angle B > \angle C$ bo'lsa, uchburchakning eng uzun tomonini aniqlang.

2. ABC uchburchakda $\angle A = \angle B < \angle C$ bo'lsa, uchburchakning eng kichik tomonini aniqlang.

3. ABC uchburchakda $AB = BC > AC$ bo'lsa, uchburchakning: a) eng katta; b) eng kichik burchagini aniqlang.

4. ABC uchburchakda $AB > BC > AC$ bo'lsa, uchburchakning: a) eng katta; b) eng kichik burchagini aniqlang.

5. Uzunliklari a) 2 m, 2 m va 3 m; b) 1 dm, 2 dm va 3 dm bo'lgan kesmalardan uchburchak yasash mumkinmi?

6. Uzunliklari: a) 12 cm, 23 cm va 35 cm; b) 45 m, 22 m va 33 m bo'lgan kesmalardan uchburchak yasash mumkinmi?

7. Tomonlari: a) 2; 3; 4; b) 2; 2; 4; c) 3,6; 1,8; 5; d) 56; 38; 19 bo'lgan uchburchak mavjudmi?

8. Teng yonli uchburchak tomonlari: a) 7 va 3; b) 10 va 5; c) 8 va 5 bo'lsa, uchinchi tomonini toping.

9. Uchburchakning perimetri 34 dm. Uning bitta tomoni: a) 16 dm; b) 17 dm; c) 18 dm bo'lishi mumkinmi? Nega?

10. Uchburchakning perimetri 12 ga teng. Uning bitta tomoni: a) 5; b) 6; c) 8 ga teng bo'lishi mumkinmi? Nega?

11*. ABC uchburchakda: a) BC= 9, AC = 8, AB = 7; b) BC = 9, AC = 8; AB = 8; c) BC =9, AC = 9, AB = 8 bo‘lsa, uning eng katta va eng kichik burchaklarini aniqlang.

12. Agar a) $\angle A$ = 25°, $\angle B$ = 100°; b) $\angle A$=40°, $\angle B$ = 100°; c) $\angle A$ = 60°, $\angle B$ = 60° bo‘lsa, ABC uchburchak tomonlarini o‘zaro taqqoslang.

13. Uchburchakning istalgan tomoni uning qolgan ikkita tomoni ayirmasidan katta bo‘lishini isbotlang.

14. Teng yonli uchburchakning perimetri 25 cm, bir tomoni ikkinchi tomonidan 4 cm ortiq va tashqi burchaklaridan biri o‘tkir bo‘lsa, uchburchakning tomonlarini toping.

15.* Uzunliklari 2, 3, 4, 5 va 6 ga teng kesmalardan nechta turli uchburchak yasash mumkin?

16. Tekislikdagi uchta A, B, C nuqtalar uchun $AB+BC \geq AC$ tengsizlik bajarilsa, AB, BC va AC kesmalar qanday geometrik shaklni ifodalaydi?

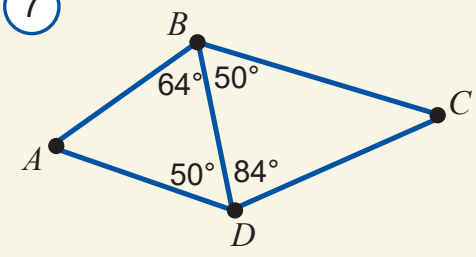
17.* Uchburchak medianasi uchburchakning yarim perimetridan (perimetrining yarmidan) kichik ekanini isbotlang.

18. ABC uchburchakda AB= 12 cm, BC= 10 cm, CA= 7 cm bo‘lsa, uchburchakning eng katta va eng kichik burchaklarini toping.

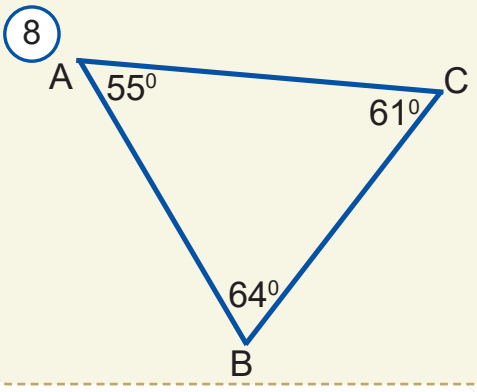
19. Teng yonli uchburchakning uchidagi burchagi 62° bo‘lsa, uning qaysi tomoni katta bo‘ladi? 58° bo‘lsa-chi?

20. ABC uchburchakda: a) $\angle A > \angle B > \angle C$; b) $\angle A = \angle B < \angle C$ bo‘lsa, uchburchak tomonlarini taqqoslang.

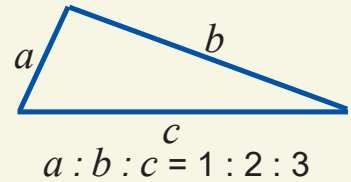
7



8

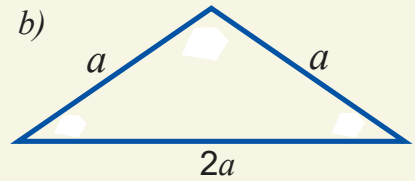


9 a)



$a : b : c = 1 : 2 : 3$

b)



21. Teng tomonli uchburchakning ikkita bissektrisasi kesishganda hosil bo‘ladigan burchaklarni toping.

22*. ABC uchburchakda $AB > BC$ va $\angle A$= 60° bo‘lsa, B burchak qanday qiymatlar qabul qilishi mumkin?

23.* Uchburchakning α, β va γ burchaklari uchun $\alpha<\beta+\gamma$, $\beta<\alpha+\gamma$, $\gamma<\alpha+\beta$ munosabatlar o‘rinli bo‘lsa, bu qanday uchburchak bo‘ladi?

24.* 7-rasmdan eng katta va eng kichik kesmalarni ko‘rsating. Javobingizni izohlang.

25. 8-rasmdagi uchburchakning eng kichik tomonini toping.

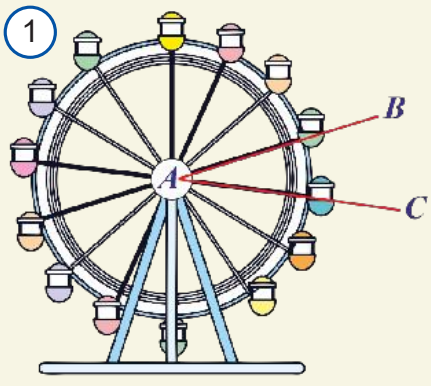
26. Bunday uchburchaklar mavjudmi? Masalaning berilishi to‘g‘rimi (*9-rasm*)?

27. Perimetri 20 cm, bir tomoni ikkinchi tomonidan 2 cm uzun, uchinchi tomonidan esa 4 cm qisqa uchburchak mavjudmi?



22 AMALIY MASHQ VA TATBIQ. BILIMINGIZNI SINAB KO‘RING

1



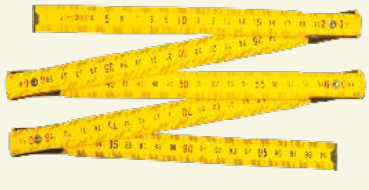
2



3



4



22.1. Amaliy mashq va tatbiq

1. Odam uchburchak shaklidagi maydon bo‘ylab harakatlanib, dastlab turgan joyiga qaytib kelsa, u jami necha gradusga burilgan bo‘ladi? Agar kvadrat shaklidagi maydon bo‘ylab harakatlansa-chi?

2. 1-rasmda suv tegirmoni tasvirlangan. Tegirmonning bir qismi ABC uchburchak shaklida bo‘lib, unda $\angle A=30°$, $AB=AC$. $\angle B$ va $\angle C$ ni toping.

3. Sim bo‘lagi uzunligi butun santimetrlarda o‘lchanadi. Undan uchburchak yasalmoqda. Uchburchakning bir tomoni 1 cm, ikkinchi tomoni 10 cm. Uchburchakning uchinchi tomoni uzunligi qancha bo‘ladi?

4. Qog‘ozga burchak chizildi va uning uchi yirtib tashlandi. Bu burchakning bissektrisasini yasay olasizmi?

5. Varaqni buklash orqali parallel to‘g‘ri chiziqlarni hosil qilish mumkinmi?

6. 2-rasmda tasvirlangan “Stop” yo‘l belgisi shakli tomonlari teng bo‘lgan sakkizburchak ko‘rinishida. Uning ichki va tashqi burchaklari yig‘indisini toping.

7. 3-rasmda tasvirlangan avtomobil g‘ildiragi bitta sektorining gradus o‘lchovini toping.

8. Uzunlik o‘lchov asbobi – yig‘ma metr *(4-rasm)* yordamida qanday uchburchaklarni yasay olasiz?

9. Bu bobda o‘zlashtirgan bilimlaringiz asosida 5-rasmda tasvirlangan daraxt, ko‘l va jarliklarning bevosita o‘lchab bo‘lmaydigan o‘lchamlari uzunligini aniqlash algoritmini tuzing va asoslang.

10. To‘g‘ri burchakli uchburchaklar tengligi alomatidan foydalanib, 6-rasmda tasvirlangan daryoning kengligini aniqlash uchun bajarilgan yasash ishlarini sharhlang va daryoning kengligini topish usulini bayon qiling.

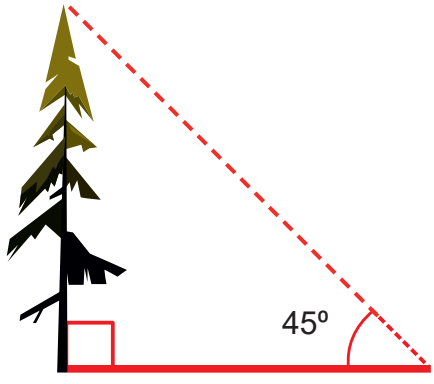
11. Quruvchilar to‘g‘ri chiziqli tonnelni bir vaqtda tog‘ning ikki tomonidan boshlab o‘yib kelmoqda *(7-rasm)*. O‘lchashlar $\angle A_1= 50°10’$, $\angle B_1 = 48°20’$ va $\angle C = 80°5’$ ekanini ko‘rsatayotgan bo‘lsa, quruvchilar to‘g‘ri yo‘nalishni tanlaganmi?

Qiziqarli geometriya

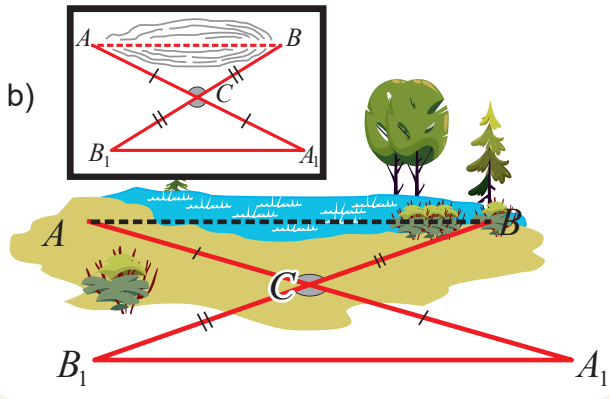
1. 8-rasmda tasvirlangan ikki tomonli, ya’ni har ikki

5

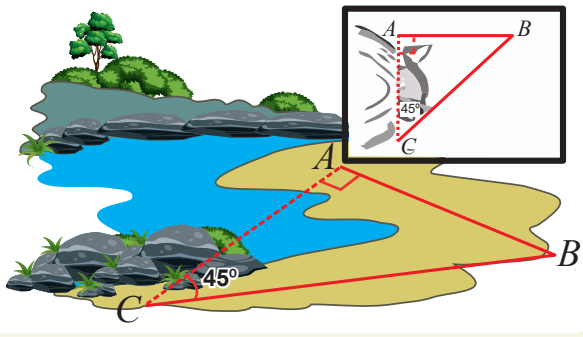
a)



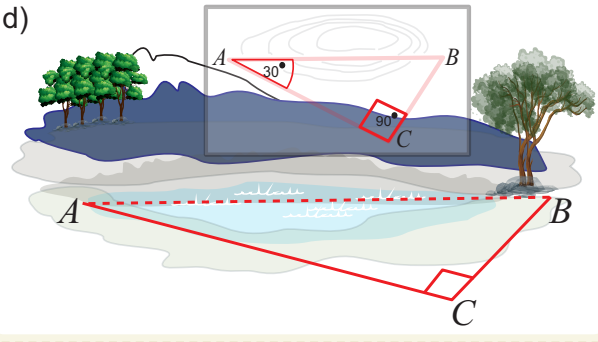
b)



c)



d)



tomonidan foydalanib to‘g‘ri chiziq chizish mumkin bo‘lgan odatdagi o‘quv chizg‘ichidan foydalanib:

a) berilgan burchakni teng ikkiga bo‘ling;

b) berilgan kesmani teng ikkiga bo‘ling;

c) berilgan burchakdan ikki marta katta burchakni yasang;

d) berilgan kesmadan ikki marta uzun kesmani yasang;

e) to‘g‘ri chiziqning nuqtasiga perpendikulyar tushiring;

f) berilgan to‘g‘ri chiziqqa parallel va berilgan nuqtadan o‘tuvchi to‘g‘ri chiziqni chizing;

g) berilgan nuqtadan berilgan to‘g‘ri chiziqqa perpendikulyar tushiring.

2. Kvadratni faqat:

a) bitta tomoni;

b) diagonali;

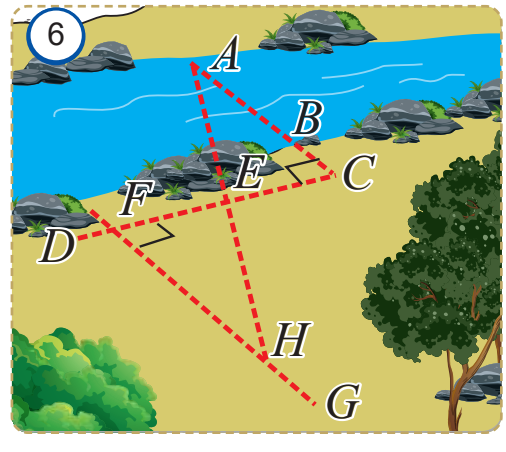
c) ikki qarama-qarshi tomonlari o‘rtalari;

d) ikki qo‘shni tomonlari o‘rtalari;

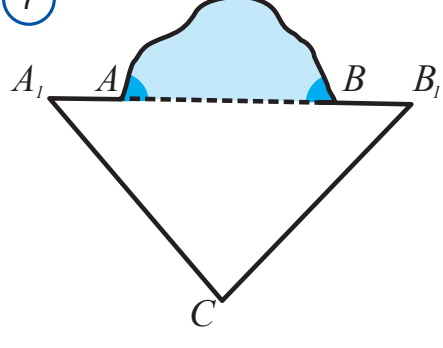
e) bitta tomoni va markazi;

f) markazi va bitta tomonida berilgan ikkita nuqtasiga ko‘ra qanday tiklash mumkin?

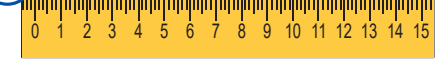
6



7



8



22.2. Bilimingizni sinab ko‘ring

1. Jumlada bo‘sh qoldirilgan joylarni mantiqan to‘g‘ri so‘zlar bilan to‘ldiring.

1. Uchburchakning ichki burchagiga ..... “uchburchakning tashqi burchagi” deb ataladi.
2. Uchburchak ..... 180° ga teng.
3. Ikkita burchagining yig‘indisi 90° ga teng bo‘lgan uchburchak ..... bo‘ladi.
4. Uchburchakning tashqi burchagi unga qo‘shni bo‘lmagan ..... ga teng.
5. Agar uchburchakning bir burchagi o‘tmas bo‘lsa, qolgan ikkita ..... .
6. To‘g‘ri burchakli uchburchakning burchaklari ..... bo‘la olmaydi.
7. Uchburchakning har bir tomoni qolgan tomonlar yig‘indisidan ..... .
8. Ikkita to‘g‘ri burchakli uchburchakning gipotenuzasi va ..... teng bo‘lsa, bu uchburchaklar teng bo‘ladi.
9. To‘g‘ri burchakli uchburchakning katetlari teng bo‘lsa, u ..... bo‘ladi.
10. To‘g‘ri burchakli uchburchakning gipotenuzasiga tushirilgan ..... shu gipotenuzaning yarmiga teng.
11. To‘g‘ri burchakli uchburchakning kateti ..... bo‘lsa, u 30° li burchak qarshisida yotadi.
12. Burchak tomonlaridan teng masofada uzoqlashgan nuqta shu burchakning .....yotadi.

2. Quyida keltirilgan jumlalarda xato bo‘lsa, uni toping va tuzating.

1. To‘g‘ri burchakli uchburchaklarning gipotenuzasi va bittadan burchagi mos ravishda teng bo‘lsa, bu uchburchaklar teng bo‘ladi.
2. Uchburchakning ichki va tashqi burchaklari yig‘indisi 180° ga teng.
3. Uchburchakning tashqi burchagi ikkita ichki burchaklari yig‘indisiga teng.
4. Uchburchakning katta tomoni qarshisida kichik burchak, katta burchagi qarshisida kichik tomon yotadi.
5. Uchburchakning har bir tomoni qolgan tomonlari ayirmasidan kichik.
6. To‘g‘ri burchakli uchburchakning faqat bitta balandligi bor.
7. To‘g‘ri burchakli uchburchakning kateti gipotenuzaning yarmiga teng.
8. To‘g‘ri burchakli uchburchakning balandligi gipotenuzaning yarmiga teng.
9. To‘g‘ri burchakli uchburchaklarning gipotenuzalari teng bo‘lsa, bu uchburchaklar teng bo‘ladi.
10. Uchburchakning ichki burchagi uning qolgan ikkita ichki burchagi yig‘indisidan har doim kichik bo‘ladi.
11. Uchburchakning tashqi burchaklari har doim o‘tmas bo‘ladi.
12. To‘g‘ri burchakli uchburchaklarning balandliklari uning to‘g‘ri burchakli uchida kesishadi.
13. Burchak tomonlaridan teng masofaga uzoqlashgan nuqtalar burchakning bissektrisasida yotadi.
14. Burchak bissektrisasi burchakning tomonlaridan teng masofaga uzoqlashgan nuqtalarning geometrik o‘rnidan iborat.

3. Jadvalda keltirilgan xossalar va talqinlarga mos keluvchi geometrik tushunchalarni yozing.

1	Ichki burchaklari yig‘indisi 180° ga teng	
2	O‘tkir burchaklari yig‘indisi 90° ga teng	
3	Tomonlari kesmalardan iborat	
4	Uchburchak tomonlari orasidagi munosabat	
5	Gipotenuzaning yarmiga teng	
6	Uchta balandligi ham bir uchda kesishadi	
7	Katetdan har doim katta	
8	Nuqtalari burchak tomonlaridan teng uzoqlashgan	

4. Testlar.

1. Agar uchburchak burchaklari 2:3:4 kabi nisbatda bo‘lsa, uning burchaklarini toping.

A) 20°, 30°, 40° B) 40°, 60°, 80° C) 36°, 54°, 90° D) 18°, 27°, 36°

2. Agar uchburchak burchaklari 3:2:1 kabi nisbatda bo‘lsa, uning turini aniqlang.

A) o‘tkir burchakli B) o‘tmas burchakli

C) to‘g‘ri burchakli D) aniqlab bo‘lmaydi

3. Agar uchburchakning bitta tashqi burchagi o‘tkir bo‘lsa, uning turini aniqlang.

A) o‘tkir burchakli B) o‘tmas burchakli

C) to‘g‘ri burchakli D) aniqlab bo‘lmaydi

4. Agar uchburchakning bir burchagi uning qolgan ikki burchagi yig‘indisidan katta bo‘lsa, uning turini aniqlang.

A) o‘tkir burchakli B) o‘tmas burchakli

C) to‘g‘ri burchakli D) aniqlab bo‘lmaydi

5. Qaysi uchburchakning balandliklari uning bir uchida kesishadi?

A) teng yonli uchburchak

B) teng tomonli uchburchak

C) to‘g‘ri burchakli uchburchak

D) bunday uchburchak mavjud emas

6. ABC uchburchakda A uchidagi tashqi burchak 120° ga, C uchidagi ichki burchak esa 80° ga teng. B uchidagi tashqi burchakni toping.

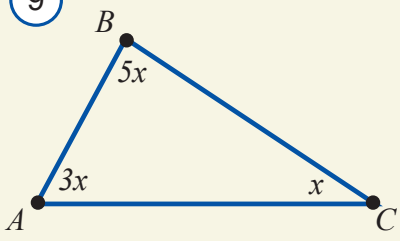
A) 120° B) 140° C) 160° D) 40°

7. Uchburchakning tashqi burchaklaridan biri 120° ga, shu burchakka qo‘shni bo‘lmagan ichki burchaklarining ayirmasi 30° ga teng. Uchburchakning ichki burchaklaridan kattasini toping.

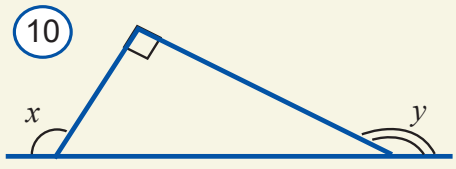
A) 70° B) 75° C) 85° D) 90°



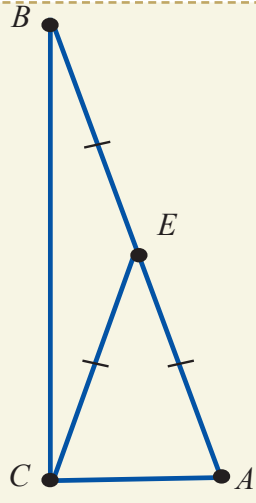
9



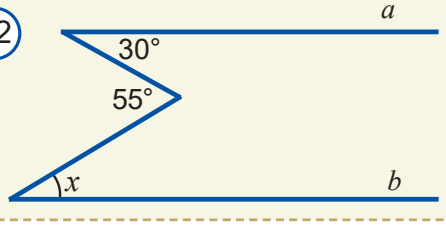
10



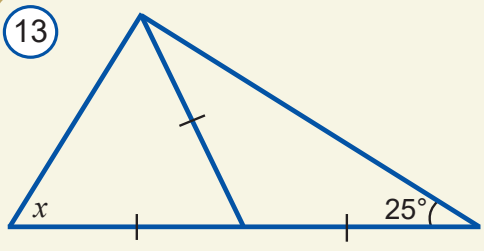
11



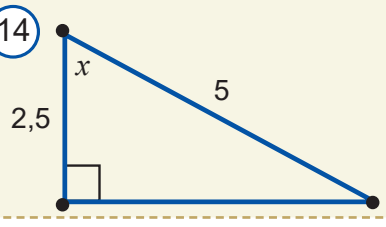
12



13



14



8. Uchburchakning ikkita burchagi qiymatlarining nisbati 1:2 kabi. Uchinchi burchagi shu burchaklarning kichigidan 40° ga katta. Uchburchakning katta burchagini toping.

A) 105° B) 75° C) 80° D) 90°

9. Teng yonli uchburchakning perimetri 48 ga teng. Uning tomonlaridan biri 12 ga teng bo‘lsa, qolgan tomonlarini toping.

A) 18; 12 B) 16; 16 C) 18; 24 D) 18; 18

10. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagidan bissektrisa va balandlik chiqarilgan bo‘lib, ular orasidagi burchak 24° ga teng. Uchburchakning kichik burchagini toping.

A) 21° B) 24° C) 36° D) 16°

11. 9-rasmdagi $\angle A$ ni toping.

A) 10° B) 20° C) 60° D) 100°

12. Uzunliklari 3, 5, 7 va 11 ga teng kesmalardan nechta turli tomonli uchburchak yasash mumkin?

A) 2 B) 3 C) 5 D) 6

13. 10-rasmdagi $x + y$ ni toping.

A) 90° B) 180° C) 270° D) aniqlab bo‘lmaydi.

14. 11-rasmdagi $\angle BCA$ ni toping.

A) 90° B) 96° C) 144° D) 84°

15. 12-rasmda $a||b$ bo‘lsa, x ni toping.

A) 35° B) 45° C) 25° D) 20°

16. 13-rasmdagi x ni toping.

A) 60° B) 55° C) 65° D) 70°

17. 14-rasmdagi x ni toping.

A) 30° B) 45° C) 15° D) 75°

18. Uzunligi 2 cm, 3 cm, 4 cm va 5 cm bo‘lgan kesmalardan nechta turli tomonli uchburchak yasash mumkin?

A) 1 ta B) 2 ta C) 3 ta D) 4 ta

19. Uchburchakning istalgan tomoni uning qolgan ikkita tomoni ayirmasidan

A) katta B) kichik C) teng D) katta yoki teng bo‘ladi.

5. Masalalar.

1. Bo‘g‘inlarining uzunligi 1 m, 2 m, 4 m, 8 m va 16 m bo‘lgan yopiq siniq chiziq yasash mumkinmi?

2. Agar uchburchakning tomonlari butun sonlar bo‘lib, perimetri 15 ga teng bo‘lsa, uning tomonlarini aniqlang.

3. Uchburchakning balandligi uning tomonlaridan har doim ham kichik bo‘ladimi?

4. Katta tomoni 36 ga teng bo‘lgan uchburchakning burchaklari 1:2:3 kabi nisbatda bo‘lsa, shu uchburchakning kichik tomonini toping.

5. Uchburchakning asosiga tushirilgan balandlik uning yon tomonlari bilan 27° va 36° li burchaklar tashkil qiladi. Uchburchakning burchaklarini toping.

6. To‘g‘ri burchakli ABC va $A_1B_1C_1$ uchburchaklarda A va A_1 to‘g‘ri burchaklar, BD va B_1D_1 bissektrisalar va $\angle B=\angle B_1, BD=B_1D_1$ bo‘lsa, $\Delta ABC=\Delta A_1B_1C_1$ ekanini isbotlang.

7. 15-rasmdagi x ni toping.

8. 16-rasmdagi $\angle ABC$ ni toping.

9. 17-rasmda $AB||CD$ ekanini isbotlang.

10. Teng yonli uchburchakning bir burchagi 100° ga teng. Uchburchakning qolgan burchaklarini toping.

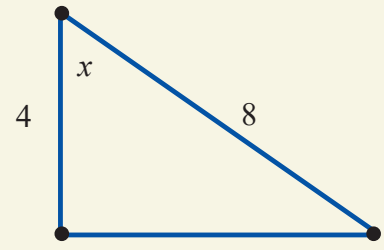
11. Agar teng yonli uchburchakning burchaklaridan biri 60° ga teng bo‘lsa, bu uchburchak teng tomonli bo‘ladimi?

12. Asosi AC va B burchagi 36° ga teng bo‘lgan teng yonli ABC uchburchakning AD bissektrisasi o‘tkazilgan. CDA va ADB uchburchaklarning teng yonli ekanini isbotlang.

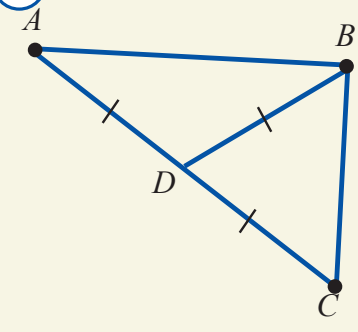
13. Bir uchburchak 60° va 38° li burchaklarga, ikkinchi uchburchak esa 38° va 82° li burchaklarga ega. Bu uchburchaklar teng bo‘lishi mumkinmi?

14. 18-rasmda $BD = CD$ = 10 bo‘lsa, AB ni toping.

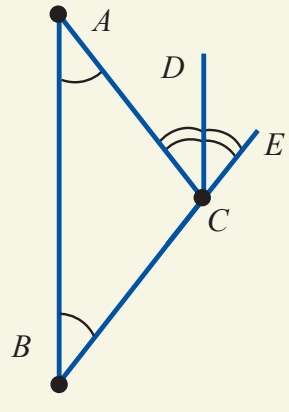
15



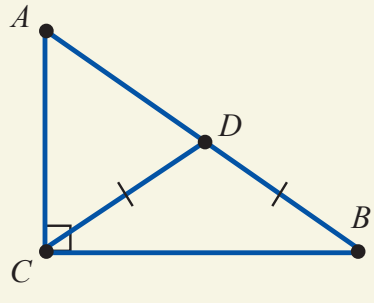
16



17



18



15. Uchburchak perimetri tomonlaridan 14 cm, 16 cm va 24 cm katta bo‘lsa, uchburchakning eng katta tomonini toping.

16. To‘g‘ri burchakli ABC uchburchakning to‘g‘ri burchagi uchidan CD balandlik tushirilgan. Agar a) A = 24°; b) A = 70° bo‘lsa, CDB burchakni toping.

17. Teng yonli uchburchakning bir tashqi burchagi 70° ga teng. Uning ichki burchaklarini toping.

18. ABC uchburchakning A va C uchlaridan tushirilgan balandliklar N nuqtada kesishadi. Agar $\angle A$ = 50° va $\angle C$ = 84° bo‘lsa, ANC burchakni toping.

19. ABC uchburchakda BD mediana AC tomonning yarmiga teng. Uchburchakning B burchagini toping.

20. Perimetri 23 m, bir tomoni ikkinchi tomonidan 2 m qisqa, uchinchi tomonidan esa 1 m uzun uchburchak mavjudmi?

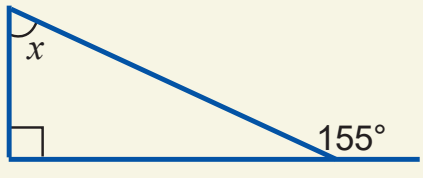
21. ABC uchburchakda A burchak B burchakdan 3 marta kichik, C burchakdan esa 15° ga katta. Uchburchak burchaklarini toping.

22*. Uchburchak ikkita balandligining har ikkisi ham kesishish nuqtasida teng ikkiga bo‘linmasligini isbotlang.

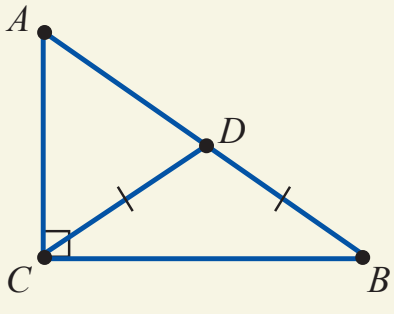
23*. Uchburchakni to‘rtta parallel to‘g‘ri chiziq kesib o‘tyapti. Ularning kamida bittasi uchburchakning uchidan o‘tmasligini isbotlang.

5-nazorat ishi namunasi

1



2



Namunaviy nazorat ishi ikki qismdan iborat:

1. 145-betdagi testlarga o‘xshash 5 ta test;
2. Quyidagi masalalarga o‘xshash 3 ta masala (4-masala “a’lo” baho olmoqchi bo‘lgan o‘quvchilar uchun qo‘shimcha).

1. Noma’lum burchakni toping *(1-rasm)*.

2. Uchburchakning tashqi burchagi 120° bo‘lib, unga qo‘shni bo‘lmagan ichki burchagi 1:2 nisbatda bo‘lsa, uchburchakning burchaklarini toping.

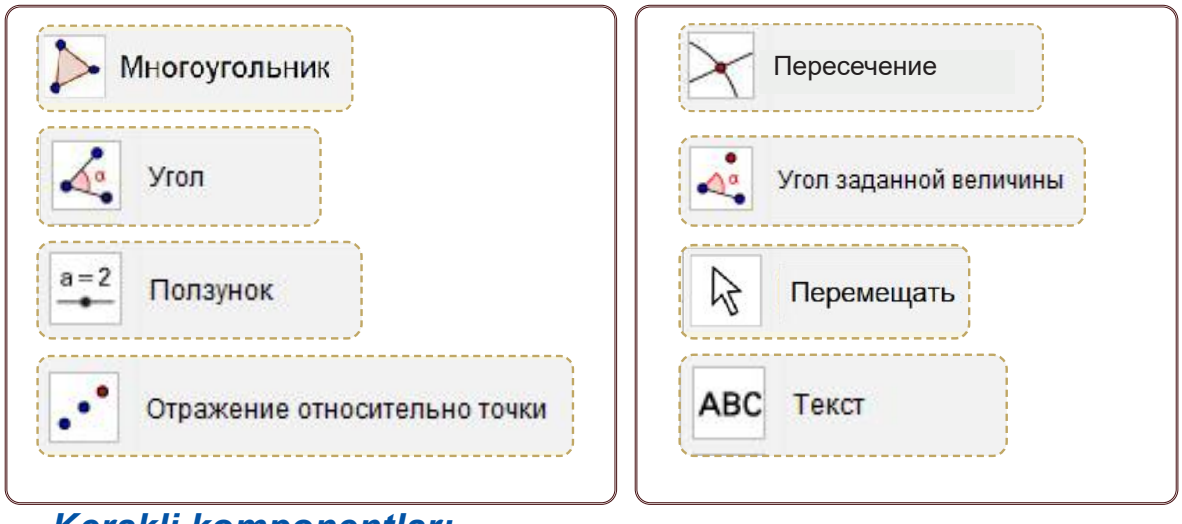
3. Agar 2-rasmda $\angle ACB$=90°, CD=BD va AB= 24 cm bo‘lsa, CD kesmani toping.

4. ABC uchburchak BD bissektrisasi AC tomonni 100° burchak ostida kesadi. Agar BD=DC bo‘lsa, uchburchak burchaklarini toping.

Amaliy topshiriq

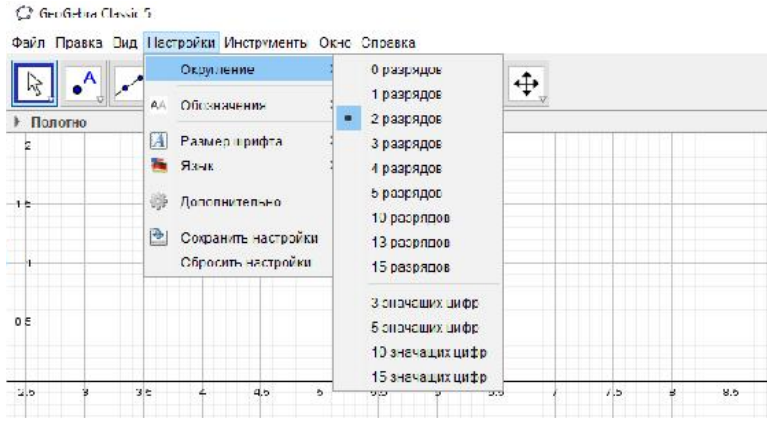
Uchburchakning ichki burchaklari yig‘indisini vizuallashtirish

Uchburchakning ichki burchaklari yig‘indisini vizuallashtirish uchun quyidagi vositalar kerak bo‘ladi.

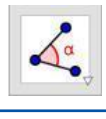


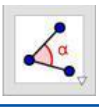
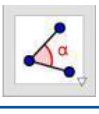
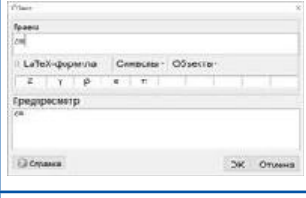
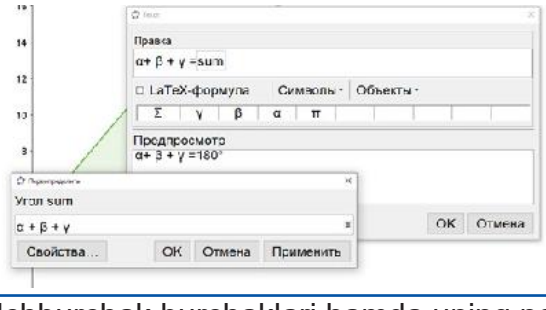
Kerakli komponentlar:

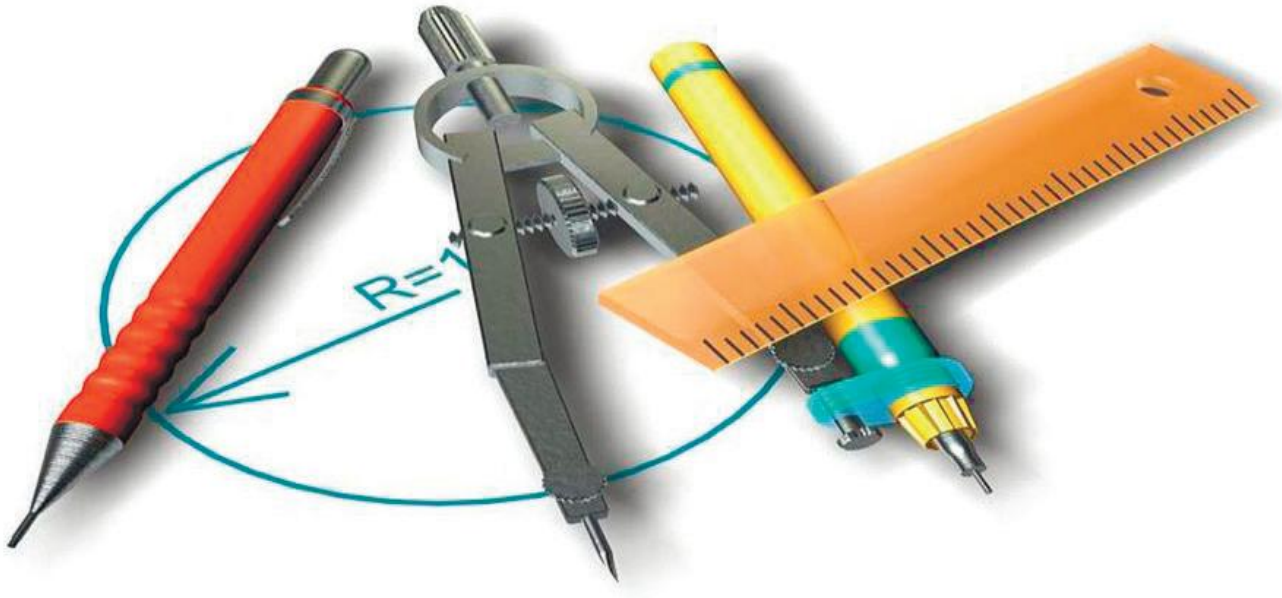
- ***GeoGebra***da yangi oyna oching.
- ***GeoGebra*** interfeysini **“Настройки”** –  **“Геометрия”** ko‘rinishiga o‘tkazing.
- **“Вид”** menyusi orqali **“Строка ввода”** (Kiritish satri) maydonini faollashtiring.
- **“Настройки”** menyusidan **“Округление – 0 разрядов”,** ya’ni sonning o‘nli kasr qismini yaxlitlashni belgilang.



Uchburchakning ichki burchaklari yig‘indisini vizuallashtirish algoritmi

1		Ixtiyoriy *ABC* uchburchakni sichqonchani soat strelkasiga qarshi yo‘nalishda harakatlantirish orqali yasang.
2		*ABC* uchburchakning α, β va γ burchaklarini belgilang.

3		δ burchak uchun 0º dan 180º oraliqda 10º qadam bilan siljitgich (ползунок) hosil qiling.
4		ε burchak uchun 0º dan 180º oraliqda 10º qadam bilan siljitgich (ползунок) hosil qiling.
5		AC kesmaning o‘rtasi – D va AB kesmaning o‘rtasi – E nuqtalarni hosil qiling.
6		Uchburchakni D nuqta atrofida δ burchakka buring (soat strelkasi yo‘nalishida).
7		Uchburchakni E nuqta atrofida ε burchakka buring (soat strelkasi yo‘nalishiga qarshi)
8		Ikkala δ va ε bilan siljitgich (ползунок)larni 180º ga o‘zgartiring.
9		$A'C'B'$ burchakni ζ hosil qiling.
10		$C'_1B'_1A'_1$ burchakni η hosil qiling.
11		Obyekt ustida sichqonchaning o‘ng tugmasini bosing va kontekst menyudan **“Свойства”** buyrug‘ini tanlang. Obyekt rangi, ko‘rinishi, chiziq qalinligi kabilarni o‘zgartiring.
12		Uchburchakning ichki burchaklarini tasvirlovchi dinamik matnni hosil qiling. Masalan, α = **“Объекты”** bo‘limidan α ni tanlang. 
13		**“Ввод”** – kiritish satriga sum = $\alpha+\beta+\gamma$ ni kirirtish orqali uchburchakning ichki burchaklari yig‘indisini hisoblang.
14		Uchburchakning ichki burchaklari yig‘indisini dinamik matn ko‘rinishida tasvirlang: $\alpha+\beta+\gamma$ = matnini kiriting, so‘ng “**Панель объектов”** maydonidagi **sum=180º** ustiga bosing. 
15		Uchburchak burchaklari hamda uning nomlari rangi va ko‘rinishini o‘zgartiring.



V BOB

GEOMETRIK YASASHGA DOIR MASALALAR

Bu bobni o‘rganib chiqqach, quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilim:

– sirkul va oddiy chizg‘ich yordamida geometrik yasashga doir masalalarni yechishda rioya qilinadigan maxsus qoidalar;
– geometrik yasashga doir masalalarni yechish bosqichlari.

Amaliy ko‘nikmalar:

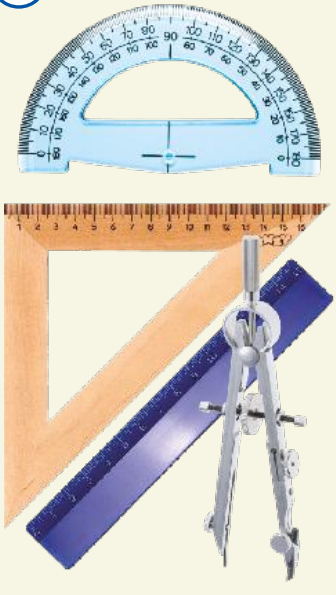
– chizg‘ich va sirkuldan to‘g‘ri foydalanish;
– berilgan burchakka teng burchakni yasay olish;
– burchak bissektrisasini yasay olish;
– perpendikulyar to‘g‘ri chiziqlarni yasay olish;
– kesmani teng ikkiga bo‘lish;
– berilgan elementlariga ko‘ra uchburchaklarni yasay olish.

23 SIRKUL VA CHIZG‘ICH YORDAMIDA GEOMETRIK YASASHGA DOIR MASALALAR

23.1. Geometrik yasashga oid qoidalar

Yasashga doir masalalarni faqat oddiy chizg‘ich va sirkul vositasida yechish mantiqiy mushohada qilish qobiliyatini o‘stiradi. Bu usul Qadimgi Yunonistonda san’at darajasiga yetgan edi. Shu paytgacha turli xil asboblar yordamida har xil geometrik shakllarni yasab keldik. Masalan, chizg‘ich yordamida to‘g‘ri chiziq, nur, kesma, uchburchak va boshqa shakllarni chizdik. Chizg‘ich va transportir yordamida turli xil burchaklarni yasadik. Sirkul yordamida esa aylana va yoylarni tasvirladik *(1-rasm)*.

1



Ma’lum bo‘lishicha, ko‘plab geometrik shakllarni faqat *oddiy chizg‘ich va sirkul (2-rasm)* yordamida yasash mumkin ekan. Oddiy chizg‘ich deganda masshtabli bo‘linmalarga ega bo‘lmagan va bir tomoni tekis chizg‘ichni tushunamiz.

Shu sababdan geometriyada shu ikki asbob yordamida yasashga doir masalalar maxsus ajratib ko‘rsatiladi. Bu ikki asbobdan foydalanishning maxsus qoidalari bor.

Ular vositasida faqat quyidagi ishlarni bajarishga ruxsat beriladi:

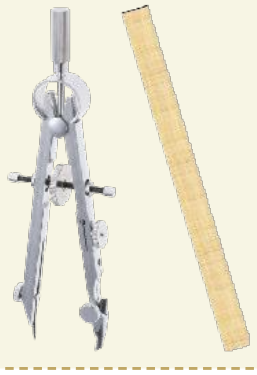
Oddiy chizg‘ich yordamida faqat:

- *ixtiyoriy to‘g‘ri chiziq chizish;*
- *tayin nuqtadan o‘tuvchi to‘g‘ri chiziq chizish;*
- *ikki nuqtadan o‘tuvchi to‘g‘ri chiziq chizish mumkin.*

Sirkul yordamida faqat:

- *ixtiyoriy aylana chizish;*
- *markazi berilgan nuqtada bo‘lgan ixtiyoriy radiusli aylana chizish;*
- *tayin radiusli, markazi esa ixtiyoriy nuqtada bo‘lgan aylana chizish;*
- *markazi berilgan nuqtada, radiusi berilgan kesmadan iborat aylana chizish;*
- *berilgan kesmaga teng kesmani to‘g‘ri chiziqqa uning belgilangan nuqtasidan boshlab har ikki yo‘nalishda qo‘yish mumkin.*

2



Boshqa har qanday geometrik yasashlar mana shu amallarga keltirilishi lozim. Yasashga doir masalalarda nafaqat biror geometrik shaklni yasash yo‘lini, usulini topish talab qilinadi, balki hosil bo‘lgan geometrik shakl haqiqatan berilgan shartlarni qanoatlantirishini asoslash, ya’ni isbotlash ham kerak bo‘ladi.

Shu bois yasashga doir ba’zi murakkabroq masalalarni yechish jarayonining bayoni uzundan uzun bo‘lishi mumkin. Shuning uchun yasashga doir masalalar ichidan asosiy, tayanch masalalarni ajratamiz va ularni yechish jarayonini to‘la izohi bilan keltiramiz.

Murakkabroq masalalarni yechayotib bunday masalalarga duch kelsak, qisqalik uchun ularning tavsifini to‘la keltirmaymiz.

23.2. Geometrik yasashga doir asosiy masalalar

Berilgan burchakka teng burchakni yasash

Yasash. A burchak berilgan. Unga teng burchak yasaymiz, ya’ni O nurga *(3-rasm)* A burchakka teng burchak qo‘yamiz.

1-qadam. Markazi A nuqtada bo‘lgan ixtiyoriy aylana chizamiz *(3–4-rasm)*. Bu aylana berilgan A burchak tomonlarini B va C nuqtalarda kesib o‘tsin.

2-qadam. Radiusi chizilgan aylana radiusiga teng va markazi O nuqtada bo‘lgan aylana chizamiz *(4-rasm)*. Bu aylanani O nur bilan kesishish nuqtasi D bilan belgilaymiz.

3-qadam. Markazi D nuqtada, radiusi esa BC ga teng bo‘lgan uchinchi aylanani chizamiz *(5-rasm)*. Uning ikkinchi aylana bilan kesishish nuqtalaridan birini, aytaylik, yuqori yarimtekislikda yotganini E bilan belgilaymiz *(6-rasm)*.

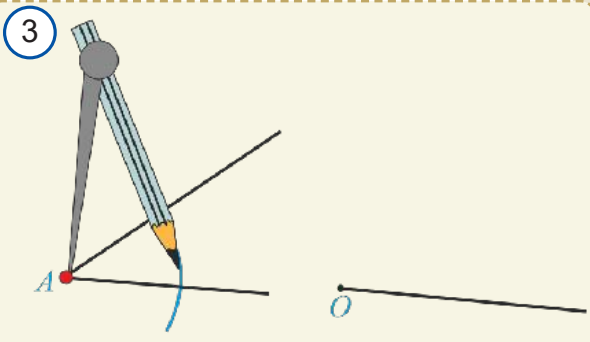
4-qadam. OE nurni o‘tkazamiz *(6-rasm)*. Hosil bo‘lgan EOD burchak O nurga qo‘yilgan, berilgan A burchakka teng burchak bo‘ladi.

Asoslash. 7-rasmda tasvirlangan ABC va ODE uchburchaklarda yasashga ko‘ra: $AB=OD$, $AC=OE$ va $BC=DE$.

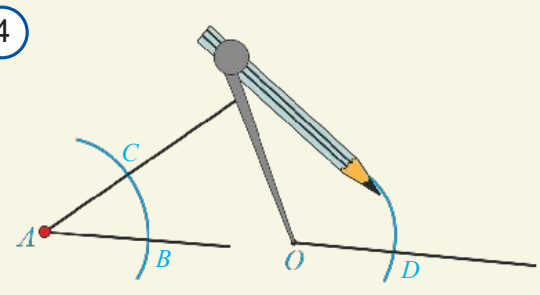
Demak, uchburchaklar tengligining TTT alomatiga ko‘ra: $\Delta ABC = \Delta ODE$. Xususan, $\angle DOE = \angle A$.

Eslatma. *Bu masala ikkita yechimga ega bo‘lib, yechimlar 3-qadamda* O *nur yotgan to‘g‘ri chiziq ajratgan qaysi yarimtekislik olinishiga bog‘liq bo‘ladi.*

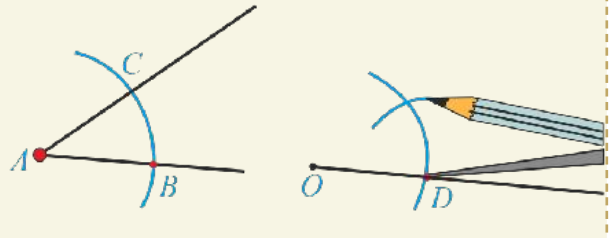
3



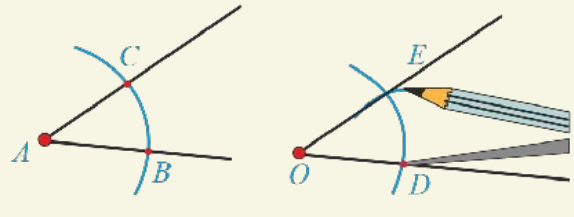
4



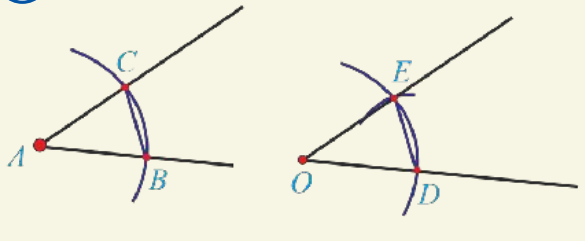
5



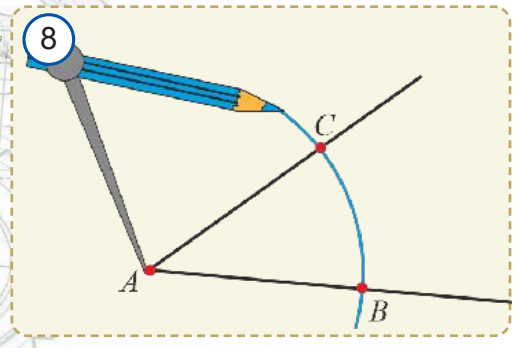
6



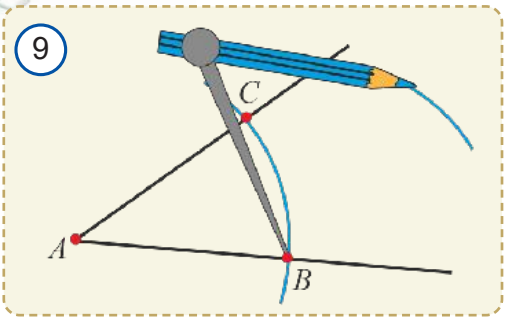
7



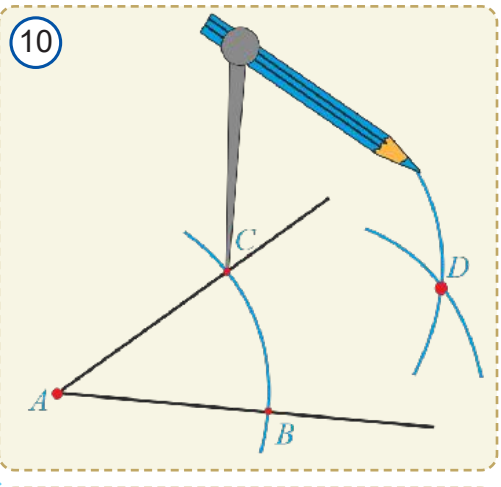
8



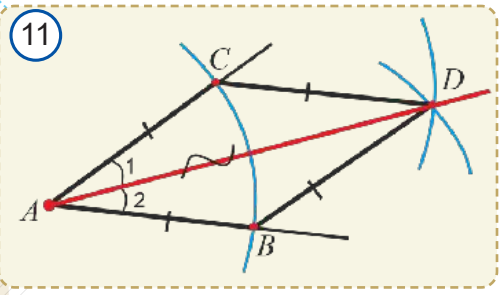
9



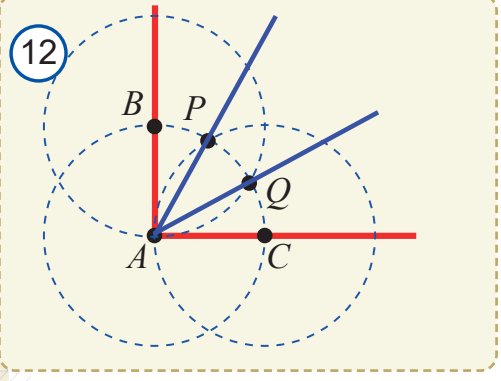
10



11



12



2.Burchak bissektrisasini yasash

Aytaylik, A burchak berilgan bo‘lsin. Bu burchakni teng ikkiga bo‘lish, ya’ni uning bissektrisasini yasash uchun quyidagicha yo‘l tutiladi:

Yasash.

1-qadam. Markazi A nuqtada bo‘lgan ixtiyoriy radiusli aylana chiziladi *(8-rasm)* va uning burchak tomonlari bilan kesishish nuqtalari B va C belgilanadi.

2-qadam. Radiusni o‘zgartirmasdan, markazlari B va C nuqtalarda bo‘lgan ikkita aylana chiziladi (*9-rasm*). Bu ikki aylana kesishuvidan hosil bo‘lgan D nuqta belgilanadi *(10-rasm)*.

3-qadam. A va D nuqtadan o‘tuvchi AD nur o‘tkaziladi *(10–11-rasmlar)*.

AD nur – berilgan burchak bissektrisasi bo‘ladi.

Asoslash. ABD va ACD uchburchaklarda *(11-rasm)*

1) yasashga ko‘ra $AB = AC$;

2) yasashga ko‘ra $BD = CD$;

3) AD – umumiy tomon.

Uchburchaklar tengligining TTT alomatiga ko‘ra, $\Delta ABD = \Delta ACD$. Xususan, $\angle BAD = \angle CAD$.

Masala. Berilgan to‘g‘ri burchakni teng uchga bo‘ling.

Yechish. $\angle A$ to‘g‘ri burchak berilgan bo‘lsin. Uning uchini markaz qilib, ixtiyoriy radiusli aylana chizamiz *(12-rasm)*. Aylana to‘g‘ri burchak tomonlarini B va C nuqtalarda kesib o‘tsin. Radiusni o‘zgartirmasdan markazi B va C nuqtalarda bo‘lgan yana ikkita aylana chizamiz. Bu aylanalar birinchi aylana bilan kesishgan nuqtalardan to‘g‘ri burchak ichida yotganlarini P va Q bilan belgilaymiz. AP va AQ nurlarni chizamiz. Bu nurlar berilgan to‘g‘ri burchakni uchta teng burchakka ajratadi. Bu tasdiqning to‘g‘riligini mustaqil asoslang.

Eslatma. *Berilgan ixtiyoriy burchakni uchga bo‘lish masalasi juda qadimgi va mashhur masala bo‘lib, bu haqda ko‘p olimlar bosh qotirgan. Faqat XIX asrga kelib ayrim burchaklar istisno bo‘lib, odatda burchakni teng uchga bo‘lib bo‘lmasligi isbotlangan. Masalan, 60° li burchakni teng uchga bo‘lib bo‘lmaydi. Gap, albatta, oddiy chizg‘ich va sirkul bilan aniq yasash haqida bormoqda. Bu asboblar bilan juda katta aniqlikda taqribiy yasash yoki boshqa asboblardan foydalanib aniq yasash bajarilishi mumkin.*

3. Berilgan to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziq yasash

Berilgan a to‘g‘ri chiziqqa uning O nuqtasidan o‘tuvchi perpendikulyar to‘g‘ri chiziqni yasaymiz.

Yasash.

1-qadam. O nuqtani markaz qilib ixtiyoriy aylana chizamiz. U berilgan to‘g‘ri chiziqni A va B nuqtalarda kesib o‘tsin *(13-rasm)*.

2-qadam. A va B nuqtalarni markaz qilib, radiusi AB ga teng aylanalar chizamiz *(14–15-rasmlar)*. Bu aylanalarning kesishish nuqtalaridan birini P deb belgilaymiz.

3-qadam. P va O nuqtalardan o‘tuvchi OP to‘g‘ri chiziqni yasaymiz *(15–16-rasm)*.

OP to‘g‘ri chiziq berilgan a to‘g‘ri chiziqqa uning O nuqtasidan o‘tuvchi perpendikulyar bo‘ladi.

Asoslash. AOP va BOP uchburchaklarga qaraymiz. Ularda yasashga ko‘ra:

1) $AO = BO$;

2) $AP = BP$;

3) PO esa umumiy tomon.

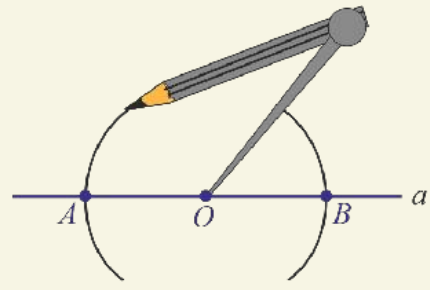
Demak, uchburchaklar tengligining TTT alomatiga ko‘ra: $\Delta AOP = \Delta BOP$. U holda, $\angle AOP = \angle BOP$.

Lekin $\angle AOP + \angle BOP = 180°$.

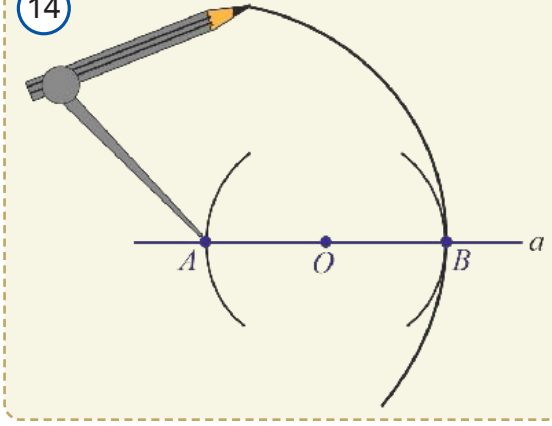
Bundan $\angle AOP = \angle BOP = 90°$ ekanligi kelib chiqadi.

Demak, haqiqatan ham $OP \perp a$.

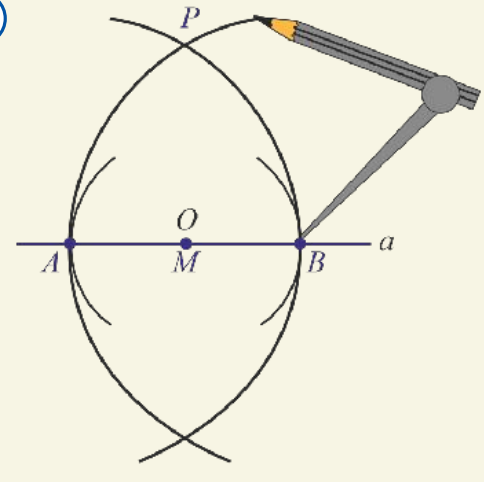
13



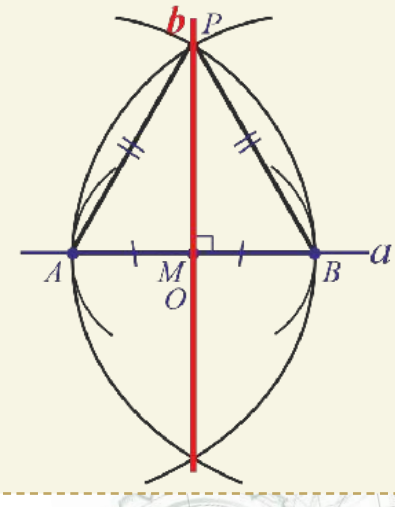
14



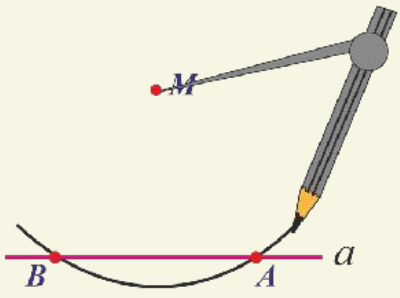
15



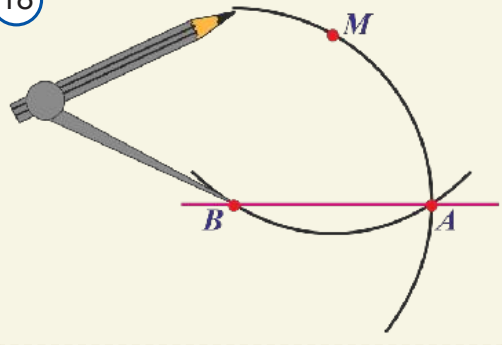
16



17



18



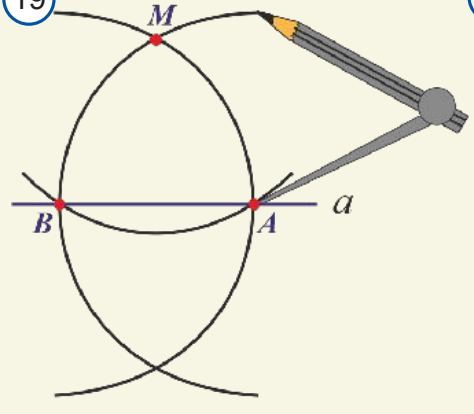
4. Berilgan to‘g‘ri chiziqqa unda yotmagan nuqtadan perpendikulyar tushirish

Berilgan a to‘g‘ri chiziqqa unda yotmagan M nuqtadan o‘tuvchi perpendikulyar to‘g‘ri chiziqni yasaymiz.

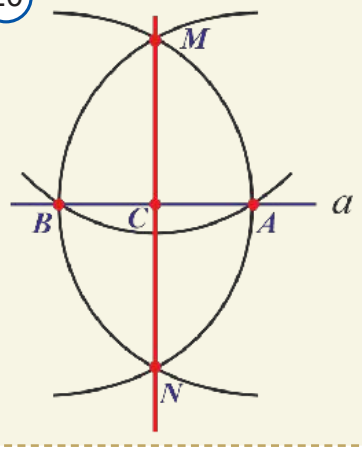
Yasash. 1-qadam. Markazi M nuqtada bo‘lgan, a to‘g‘ri chiziqni kesib o‘tuvchi ixtiyoriy aylana chizamiz. U berilgan to‘g‘ri chiziqni A va B nuqtalarda kesib o‘tsin *(17-rasm)*.

2-qadam. Markazlari A va B nuqtada bo‘lgan, radiusi birinchi chizilgan aylana radiusiga teng aylanalar chizamiz *(18–19-rasmlar)*. Bu aylanalarning kesishish nuqtalaridan biri M nuqta bo‘ladi. Ikkinchisini N bilan belgilaymiz *(20-rasm)*.

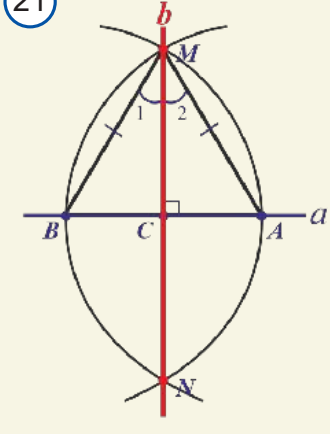
19



20



21



3-qadam. M va N nuqtalardan o‘tuvchi to‘g‘ri chiziq chizamiz. MN berilgan a to‘g‘ri chiziqqa perpendikulyar va unda yotmagan M nuqtadan o‘tuvchi to‘g‘ri chiziq bo‘ladi.

Asoslashni 21-rasm asosida mustaqil bajaring.

Bu masalani yechib, a to‘g‘ri chiziqdan tashqaridagi nuqta orqali a to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziq o‘tkazish mumkin degan xulosaga kelamiz. Bundan va 14-darsda keltirilgan teorema natijasidan quyidagi teoremaning o‘rinli ekanligi kelib chiqadi.

Teorema. **To‘g‘ri chiziqda yotmagan nuqta orqali bu to‘g‘ri chiziqqa perpendikulyar bo‘lgan yagona to‘g‘ri chiziq o‘tkazish mumkin.**

5. Berilgan kesmani teng ikkiga bo‘lish

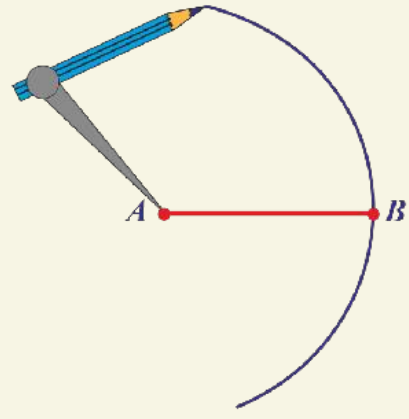
Yasash. Aytaylik, AB kesma berilgan bo‘lsin. Bu kesmani teng ikkiga bo‘luvchi nuqtani topish uchun quyidagicha yo‘l tutiladi:

1-qadam. Radiusi berilgan AB kesmaga teng bo‘lgan, markazlari esa A va B nuqtalarda bo‘lgan ikkita aylana chiziladi *(22-rasm)*.

2-qadam. Aylanalar kesishgan P va D nuqtalari kesma bilan tutashtiriladi *(23-rasm)*. PD to‘g‘ri chiziq va AB kesmaning kesishish nuqtasi O berilgan kesmaning o‘rtasi bo‘ladi.

O nuqta haqiqatan ham AB kesmaning o‘rtasi bo‘lishini mustaqil asoslang.

22



6. Berilgan kesmaning o‘rta perpendikulyarini yasash

Yasash. AB kesma berilgan bo‘lsin. Markazlari A va B nuqtalarda bo‘lgan AB radiusli aylanalar chizamiz *(24-rasm)*. Bu aylanalar P va D nuqtalarda kesishadi va yasashga ko‘ra:

$AP=AD=BP=BD$ bo‘ladi.

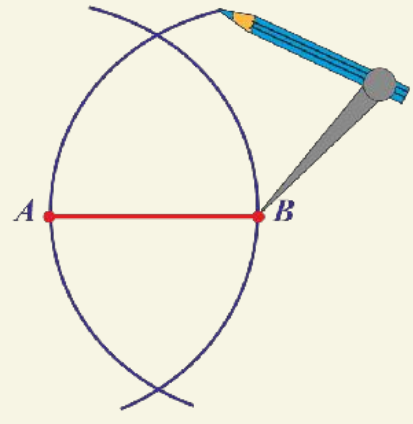
PD to‘g‘ri chiziqni o‘tkazamiz.

Bu to‘g‘ri chiziq AB kesmaning o‘rta perpendikulyaridir.

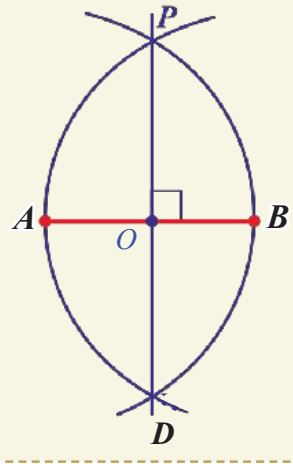
Asoslash. P va D nuqtalar AB kesmaning uchlaridan teng uzoqlikda yotgani uchun shu kesmaning o‘rtasidan o‘tuvchi perpendikulyarda yotadi.

Demak, bu nuqtalardan o‘tuvchi to‘g‘ri chiziq berilgan kesmaning o‘rta perpendikulyari bo‘ladi.

23



24



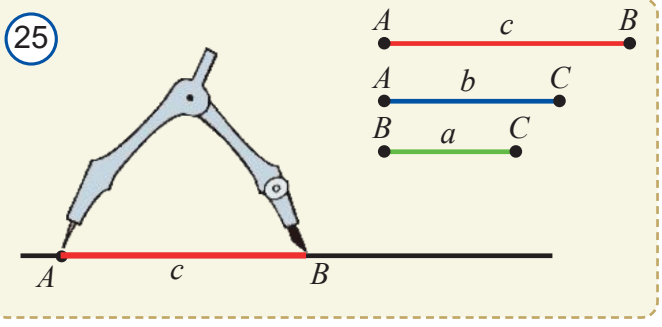
Geometrik boshqotirma

Sardor aylana chizib bo‘lgach, uning markazini qalam bilan belgilashni unutganini payqab qoldi. O‘chakishganday, daftarda sirkulning izi ham qolmabdi. Lekin aylananing radiusi 12 cm ekanligi uning esida edi. Bu ma’lumotdan foydalanib, faqat sirkul yordamida chizilgan aylananing markazini topib bo‘ladimi?

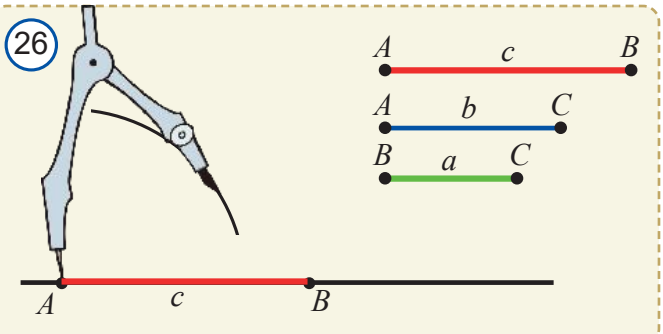


7. Uchburchakni berilgan uch tomoniga ko‘ra yasash

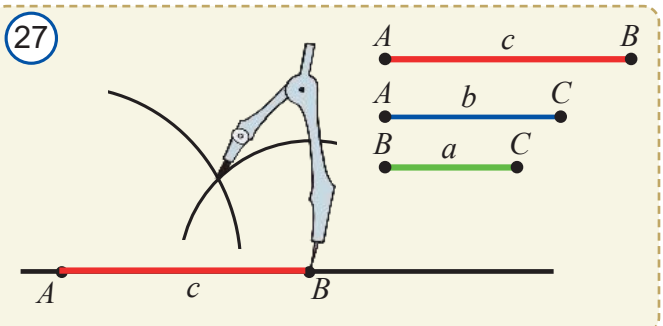
25



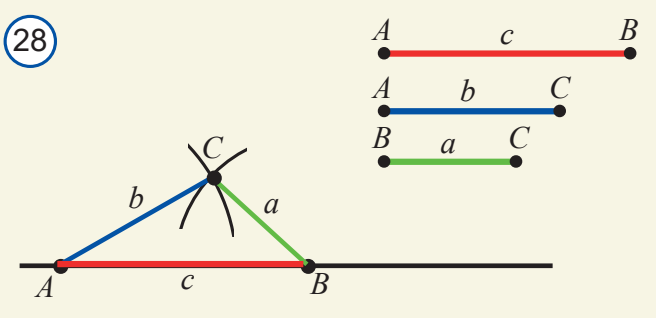
26



27



28



Aytaylik, 25-rasmda tasvirlangandek, uzunliklari mos ravishda a, b va c ga teng kesmalar berilgan bo‘lib, c ulardan eng kattasi bo‘lsin. Tomonlari mos ravishda $AB = c$, $BC = a$ va $AC = b$ bo‘lgan uchburchak yasash uchun quyidagicha yo‘l tutamiz:

1-qadam. Ixtiyoriy to‘g‘ri chiziq chizamiz. To‘g‘ri chiziqda uzunligi c ga teng bo‘lgan AB kesmani sirkul yordamida ajratamiz *(25-rasm)*.

2-qadam. A va C nuqtalar orasidagi masofa b ga teng bo‘lishi kerak. Bundan foydalanib markazi A nuqtada, radiusi b ga teng bo‘lgan aylana chizamiz *(26-rasm)*.

3-qadam. B va C nuqtalar orasidagi masofa esa a ga teng bo‘lishi kerak. Bundan foydalanib markazi B nuqtada, radiusi a ga teng bo‘lgan yana bitta aylana chizamiz *(27-rasm)*.

4-qadam. Bu aylanalar kesishish nuqtasini C bilan belgilaymiz. C nuqtani A va B nuqtalar bilan tutashtiramiz (28-rasm). Yasashga ko‘ra: $AC = b$ va $BC = a$ bo‘ladi. Unda hosil bo‘lgan ABC uchburchakning tomonlari mos ravishda a, b va c ga teng bo‘ladi.

Tahlil. Yasashdan ko‘rinib turibdiki, agar 2- va 3-qadamda yasalgan aylanalar kesishsagina yechim mavjud bo‘ladi. Buning uchun $a + b > c$ bo‘lishi kerak.

Eslatma. *Mazkur usuldan foydalanib, berilgan uchburchakka teng uchburchakni ham yasash mumkin. Bu holda* a, b *va* c *kesmalar uzunligi sirkulning uchini berilgan uchburchakning mos uchlariga qo‘yib aniqlanadi.*

Shuningdek, 1-banddagi berilgan burchakka teng burchakni yasash masalasini ham yuqoridagi usul yordamida hal qilsa bo‘ladi. Buning uchun berilgan A *burchak tomonlarida mos ravishda* B *va* C *nuqtalar tanlanib,* ABC *uchburchak hosil qilinadi. So‘ng yuqorida keltirilgan usul bilan shu uchburchakka teng uchburchak yasaladi. Uchburchaklar teng bo‘lgani uchun ularning mos burchaklari ham teng bo‘ladi.*

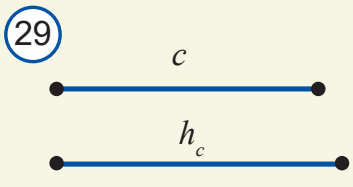
23.3. Geometrik yasashga doir murakkabroq masalalarni yechish

Geometrik yasashga doir masalalarni yechish jarayoni *tahlil, yasash, asoslash va tadqiq* kabi bosqichlardan iborat.

Geometrik yasashlarga doir masalalarni yechish bosqichlari	
1. Tahlil bosqichi	Yasaladigan shaklning taxminiy chizmasi chiziladi. Uning elementlari va masala shartida berilgan elementlar orasidagi munosabatlar aniqlanadi. Geometrik shaklni yasash rejasi tuziladi.
2. Yasash bosqichi	Geometrik shakl tuzilgan rejasiga ko‘ra yasaladi.
3. Asoslash bosqichi	Yasalgan shakl masalaning barcha shartlarini qanoatlantirishi isbotlanadi.
4. Tadqiq bosqichi	Masala yechimining mavjudligi, yechimlar soni aniqlanadi yoki yechimning yo‘qligi, ya’ni bunday shaklni yasab bo‘lmasligi ko‘rsatiladi.

Agar masala soddaroq bo‘lsa, yasash bosqichida boshqa ba’zi bosqichlar og‘zaki bajariladi. Ba’zi hollarda tadqiq bosqichi chuqurroq, 8–9-sinfda o‘tiladigan bilimlarni talab qilsa uni vaqtincha tushirib qoldiramiz.

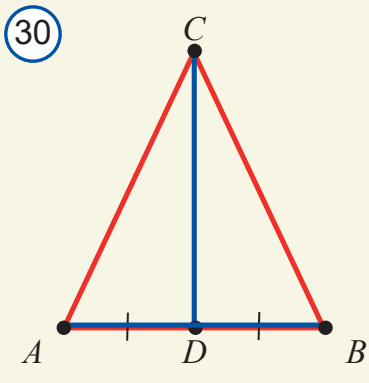
29



Masala. Teng yonli uchburchakni uning asosi va asosga tushirilgan balandlikka ko‘ra yasang *(29-rasm)*.

Yasash. $AB=c$ asos va $CD=h_c$ balandlik yasaladigan ABC teng yonli uchburchakning berilgan elementlari bo‘lsin *(30-rasm)*.

30



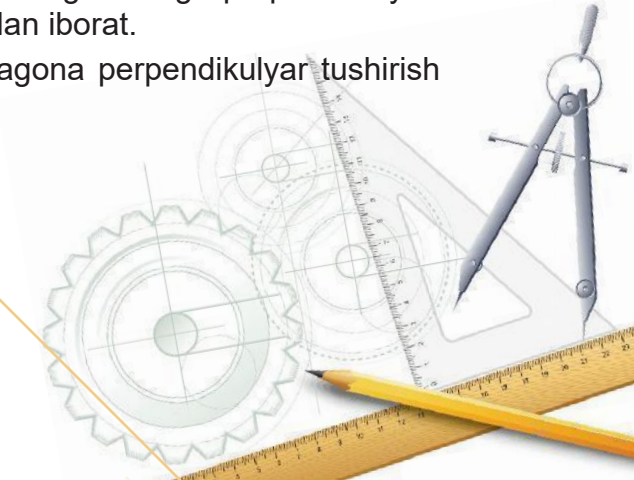
1. Tahlil bosqichi. Aytaylik, ABC uchburchak yasalgan bo‘lsin. Uning AB asosi berilganligi uchun A va B uchlarining o‘rni aniq. C uchning o‘rnini topsak, masala hal bo‘ladi. Ma’lumki, CD balandlik teng yonli uchburchak asosining o‘rtasiga tushadi.

Demak, berilgan AB asosning o‘rtasi D nuqtani topib, shu nuqtadan asosga perpendikulyar tushirsak va perpendikulyarda D nuqtadan boshlab berilgan balandlik kesmasini ajratsak, C nuqtaning o‘rnini topgan bo‘lamiz.

2. Yasash bosqichi: *a)* berilgan AB kesmaning o‘rtasi D nuqtani topamiz; b) D nuqtadan o‘tuvchi va AB kesmaga perpendikulyar bo‘lgan to‘g‘ri chiziqni o‘tkazamiz; c) perpendikulyarga D nuqtadan boshlab h_c balandlik kesmasini qo‘yamiz; d) C nuqtani A va B nuqtalar bilan tutashtiramiz.

3. Asoslash bosqichi. Hosil bo‘lgan ABC uchburchakning asosi va balandligi berilgan kesmalarga teng hamda u teng yonli, chunki uning balandligi asosiga perpendikulyar. Demak, yasalgan uchburchak izlanayotgan uchburchakdan iborat.

4. Tadqiq bosqichi. Ixtiyoriy kesmaning o‘rtasiga yagona perpendikulyar tushirish mumkinligi uchun masala yagona yechimga ega bo‘ladi.



Mavzuga doir savollar

1. Yasashga doir masalalarni yechishda oddiy chizg'ich va sirkul yordamida qanday ishlarni amalga oshirish mumkin?

3. Nima sababdan yasashga doir masalalarni yechishda bajarilgan yasashlarni asoslash zarur?

4. Geometrik yasashlarga doir asosiy masalalarni sanang.

5. Geometrik yasashga doir murakkabroq masalalarni yechishning qanday bosqichlari bor?

Amaliy mashq va tatbiqlar

1. a) 30°; b) 60°; c) 15°; d)120°; e) 45° li burchaklar berilgan. Oddiy chizg'ich va sirkuldan foydalanib ularga teng burchaklarni yasang.

2. a) 75°; b) 90°; c) 135° li burchaklar berilgan. Oddiy chizg'ich va sirkuldan foydalanib, ularga teng burchaklarni yasang.

3. $\angle A=\alpha$ va $\angle B=\beta$ burchaklar berilgan ($\alpha>\beta$). O'lchovi: a) 2α; b) $\alpha-\beta$; c) $2\alpha+\beta$ bo'lgan burchaklarni yasang.

4. 45° va 30° burchaklar berilgan. O'lchovi: a) 15°; b) 75°; c) 105°; d) 120° bo'lgan burchaklarni yasang.

5. Burchak chizing va uni to'rtta teng burchakka ajrating.

6. Oddiy chizg'ich va sirkul yordamida: a) 90°; b) 60°; c) 30° li burchaklarni teng ikkiga bo'ling.

7. 45° li burchakni uchta teng burchaklarga bo'ling.

8*. 36° li burchak berilgan. Sirkul va oddiy chizg'ich yordamida 99° li burchak yasang.

9*. 54° li burchak berilgan. Sirkul va oddiy chizg'ich yordamida bu burchakni teng uchga bo'ling.

10. Kesmani teng ikkiga bo'lishning qanday usulini bilasiz? Kesma chizing va uni teng ikkiga bo'ling.

11. To'g'ri chiziqda A va B nuqtalar berilgan. BA nurda B nuqtadan boshlab shunday BC kesmani qo'yingki, $BC=2AB$ bo'lsin.

12. A va B nuqtalar berilgan. Faqat sirkuldan foydalanib shunday C nuqta yasangki, $AC=3AB$ bo'lsin.

13. a va b uzunlikdagi kesmalar berilgan: $(a>b)$. a) $a+b$; b) $a-b$ uzunlikdagi kesmalarni yasang.

14. Uzunligi 12 cm va 5 cm bo'lgan kesmalar berilgan. Uzunligi a) 17 cm; b) 7 cm; c) 12 cm; d) 22 cm; e) 29 cm bo'lgan kesmalarni yasang.

15*. A va B nuqtalardan bir xil uzoqlashgan hamda a to'g'ri chiziqda yotuvchi nuqtani toping.

16. Faqat chizg'ich yordamida a to'g'ri chiziqda yotmaydigan M nuqta orqali a to'g'ri chiziqqa parallel bo'lgan b to'g'ri chiziqni o'tkazing.

17. Berilgan kesmani to'rtta teng bo'lakka bo'ling.

18. Quyidagilarga ko'ra ABC uchburchak yasang: a) $AB=3$; $BC=5$; $\angle B=45°$; b) $AB=9$; $BC=5$; $AC=12$; c) $AB=22$; $\angle A=30°$; $\angle B=56°$.

19. Quyidagilarga ko‘ra ABC uchburchak yasang: a) $AB = 7$; $BC = 3$; $\angle B = 38°$; b) $AB = 3$; $BC = 8$; $AC = 6$; c) $AB = 9$; $\angle A = 90°$; $\angle B = 50°$.

20. Tomonlari $a = 3\ cm$, $b = 8\ cm$ va $c = 9\ cm$ bo‘lgan uchburchak yasang.

21. a) Tomonlari $a = 3\ cm$, $b = 4\ cm$ va $c = 7\ cm$ bo‘lgan uchburchak yasash mumkinmi?
b) Uchburchak yasash uchun uning a, b va c tomonlari qanday shartni qanoatlantirishi lozim?

22. Berilgan katta tomoni va o‘tkir burchagi bo‘yicha to‘g‘ri burchakli uchburchak yasang.

23. Uchburchak chizing. Uning balandliklarini yasang.

24. Berilgan uchburchakning medianalarini yasang.

25. To‘g‘ri burchakni qanday yasash mumkin?

26. Berilgan to‘g‘ri chiziqqa parallel to‘g‘ri chiziq chizing.

27. Asosi va unga tushirilgan balandligi bo‘yicha teng yonli uchburchak yasang.

28*. Ikki tomoni va ular orasidagi burchak bo‘yicha uchburchak yasang.

29*. Berilgan tomon bo‘yicha kvadrat yasang.

30*. Bir tomoni va unga yopishgan burchaklar bo‘yicha uchburchak yasang.

31*. Gipotenuza va kateti bo‘yicha to‘g‘ri burchakli uchburchak yasang.

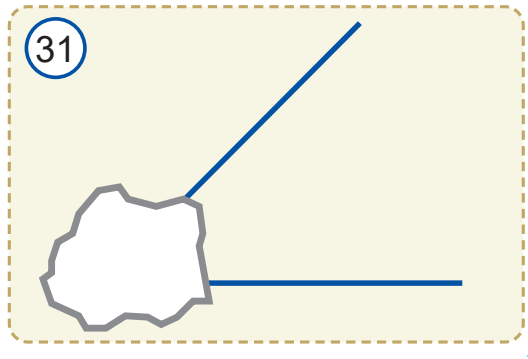
32*. Bir tomoni va unga yopishgan burchagi va shu tomonga tushirilgan balandlikka ko‘ra uchburchak yasang.

33*. Bir tomoni va unga yopishgan burchagi va qolgan ikki tomoni yig‘indisiga ko‘ra uchburchak yasang.

34*. Bir tomoni va unga yopishgan burchagi va qolgan ikki tomoni ayirmasiga ko‘ra uchburchak yasang.

Geometrik boshqotirma

Shohjahon dadasining yozuvlari ichidan 31-rasmda tasvirlangan chizma topib oldi. Afsuski, bu burchakning bir qismiga siyoh to‘kilib, o‘chib ketgan ekan. Shohjahon bu burchakning bissektrisasini yasay oladimi?





24 AMALIY MASHQ VA TATBIQ. BILIMINGIZNI SINAB KO‘RING

1. Testlar

1. Kesmalarning uzunliklari a, b va c larning qaysi qiymatlarida bu kesmalardan uchburchak yasash mumkin emas?

A) $a = 1, b = 2, c = 3$ B) $a = 2, b = 3, c = 4$

C) $a = 3, b = 4, c = 5$ D) $a = 6, b = 4, c = 3$

2. Geometrik yasashlarni bajarish uchun qaysi o‘quv qurollaridan foydalanishga ruxsat beriladi?

A) Transportir B) Transportir, chig‘ich

C) Sirkul, chizg‘ich D) Sirkul, transportir

3. Geometrik yasashlarni bajarishda chizg‘ichdan qanday vazifalarni bajarishga ruxsat beriladi?

A) Kesmani o‘lchashga

B) Kesma, to‘g‘ri chiziq chizishga

C) Nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziqni chamalab chizishga

D) Kesmani o‘lchab, uning o‘rtasini topishga

2. Masalalar.

1. a va b uzunlikdagi kesmalar berilgan: $(a>b)$. a) $2a + 3b$; b) $2a - b$ uzunlikdagi kesmalarni yasang.

2. Faqat bir yarimtekislikda yasash ishlarini bajarib, berilgan kesmani teng ikkiga bo‘ling.

3. Faqat uchburchakli chizg‘ichdan foydalanib berilgan kesmani teng ikkiga bo‘ling.

4. AB kesmaning o‘rtasini to‘g‘ridan to‘g‘ri aniqlashning iloji bo‘lmasa, uning o‘rtasidan o‘tuvchi perpendikulyarni yasash mumkinmi?

5. $\angle A=\alpha$ va $\angle B=\beta$ burchaklar berilgan: $(\alpha>\beta)$. O‘lchovi: a) 3α; b) $\alpha+2\beta$; c) $3\alpha+\beta$ bo‘lgan burchaklarni yasang.

6. Oddiy chizg‘ich va sirkul yordamida: a) 124°; b) 68°; c) 46° li burchaklarni teng ikkiga bo‘ling.

7. Agar aylanadan tashqaridagi nuqtadan aylananing eng yaqin va uzoq nuqtalarigacha bo‘lgan masofalar mos ravishda 2 cm va 10 cm bo‘lsa, aylana radiusini toping.

8. Berilgan gipotenuza bo‘yicha teng yonli to‘g‘ri burchakli uchburchak yasang.

9*. Uzunligi $a+b$, $b+c$ va $a+c$ kesmalar berilgan. Tomonlari a, b, c bo‘lgan uchburchak yasang.

10*. Ikkita kateti bo‘yicha to‘g‘ri burchakli uchburchak yasang.

11. a to‘g‘ri chiziq va uchburchak berilgan. Bir tomoni A da yotadigan va berilgan uchburchakka teng bo‘lgan uchburchak yasang.

12. To‘g‘ri chiziq chizing va unda yotmagan nuqta belgilang. Shu nuqtadan o‘tuvchi va shu to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziq yasang.

13. To‘g‘ri chiziq chizing va unda yotmagan nuqta belgilang. Shu nuqtadan o‘tuvchi va shu to‘g‘ri chiziqqa parallel to‘g‘ri chiziq yasang.

6-nazorat ishi namunasi

Namunaviy nazorat ishi ikki qismdan iborat bo‘ladi:

1. Nazariy – 5 ta test.

2. Quyidagi masalalarga o‘xshash 3 ta masala (4-masala “a’lo” baho olmoqchi bo‘lgan o‘quvchilar uchun qo‘shimcha).

1. 120° li burchak berilgan. Sirkul va chizg‘ich yordamida unga teng burchak yasang.

2. Tomonlari $a = 5\ cm$, $b = 6\ cm$ va $c = 7\ cm$ bo‘lgan uchburchak yasang.

3. 2-masalada qurilgan uchburchakning a tomoniga mediana o‘tkazing.

4. Uchburchakni uning asosi, bir tomoni va asosga tushirilgan balandligiga ko‘ra yasang.

“GeoGebra”da amaliy topshiriq bajarish

1. Muntazam oltiburchak yasash

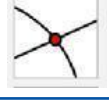
Muntazam oltiburchak yasash uchun quyidagi vositalar kerak bo‘ladi.

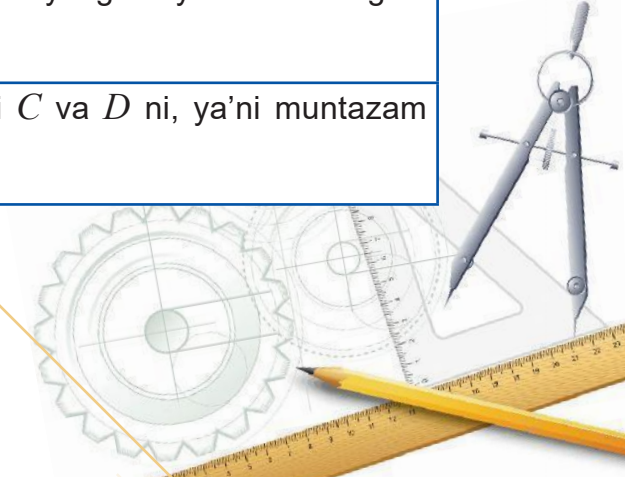


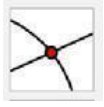
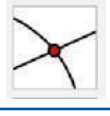
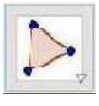
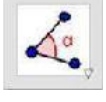
Kerakli komponentlar:

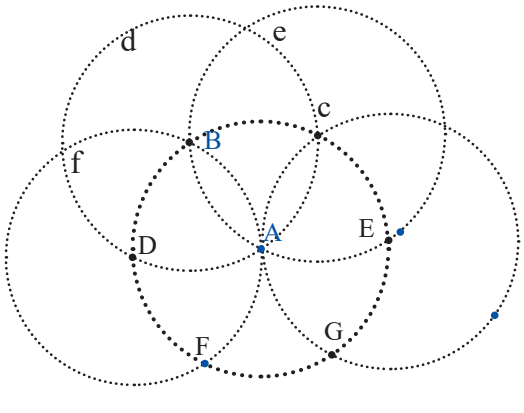
- ***GeoGebra***da yangi oyna oching.
- ***GeoGebra*** interfeysini **“Настройки” –**  **“Геометрия”** ko‘rinishiga o‘tkazing.
- **Yangi nuqta uchun sozlamalarni o‘zgartiring.**

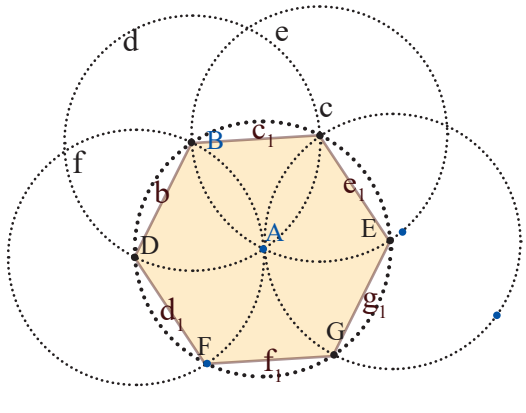
Muntazam oltiburchak yasash algoritmi

1		Markazi A nuqtada va B nuqtadan o‘tuvchi c aylanani chizing.
2		Markazi B nuqta va A nuqtadan o‘tuvchi yangi d aylanani chizing.
3		c va d aylanalar kesishish nuqtalari C va D ni, ya’ni muntazam oltiburchakning uchlarini belgilaymiz.



4		Markazi C nuqta va A nuqtadan o‘tuvchi yangi e aylanani chizing
5		e va c aylanalar kesishish nuqtasida muntazam oltiburchakning E uchini belgilaymiz.
6		Markazi D nuqta va A nuqtadan o‘tuvchi yangi f aylanani chizing.
7		f va c aylanalar kesishish nuqtasida muntazam oltiburchakning F uchini belgilaymiz.
8		Markazi E nuqta va A nuqtadan o‘tuvchi yangi g aylanani chizing.
9		g va c aylanalar kesishish nuqtasida muntazam oltiburchakning G uchini belgilaymiz.
10		Muntazam $FGECBD$ oltiburchakni yasang.
11		Aylanalarni yashirin holatga o‘tkazing.
12		Oltiburchakning ichki burchaklarini ko‘rsating.
13		Oltiburchak to‘g‘ri yasalganini tekshiring.





Topshiriq. Oltiburchakning yasalish jarayonini tushuntirishga harakat qiling.

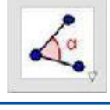
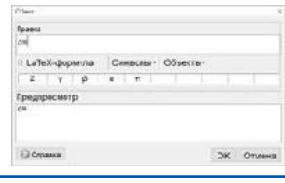
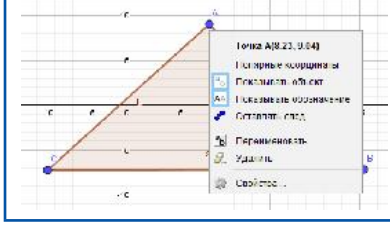
Aylana radiusi qanday bo‘lishi kerak va nima uchun? Javobingizni tushuntiring.

2. Geometrik tushuncha va ma’lumotlarni vizuallashtirish

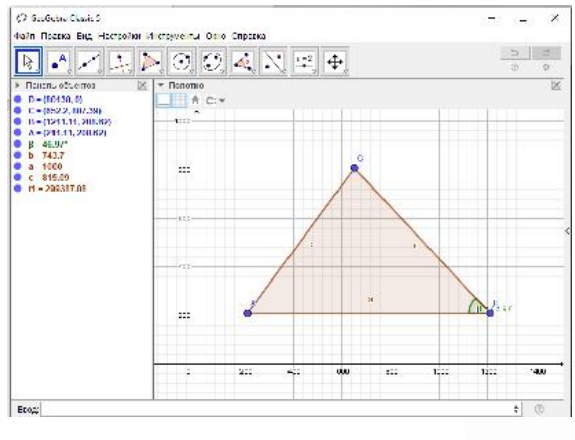
1-topshiriq

Teorema. ***Uchburchakning tashqi burchagi unga qo‘shni bo‘lmagan ikkita ichki burchaklar yig‘indisiga teng.***

Teoremani ***GeoGebra***dan foydalanib vizuallashtiramiz. Quyida keltirilgan amallar ketma-ketligini bajaring.

	ABC uchburchakni yasang.
	AB nurni o‘tkazing.
	Uchburchakning A va C ichki burchaklarini, shuningdek, B tashqi burchakni belgilang.
	$\beta=\alpha+\gamma$ – uchburchakning ikkita ichki burchaklari yig‘indisini tasvirlovchi dinamik matnni hosil qiling. Masalan, **β= “Объекты”** bo‘limidan α va γ ni tanlang. 
	Uchburchak burchaklari va nomlarining rangi va ko‘rinishini o‘zgartiring. Obyekt ustida sichqonchaning o‘ng tugmasini bosib, “**Свойства**”... buyrug‘ini tanlang.

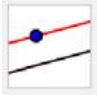
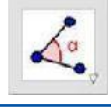
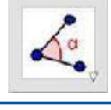
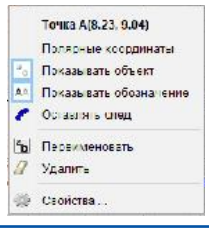
Natijada quyidagi ko‘rinishdagi dinamik chizma hosil bo‘ladi. Bunda uchburchakning shaklini o‘zgartirilishi mumkin, lekin B tashqi burchak har doim A va C ichki burchaklarining yig‘indisiga teng bo‘ladi.



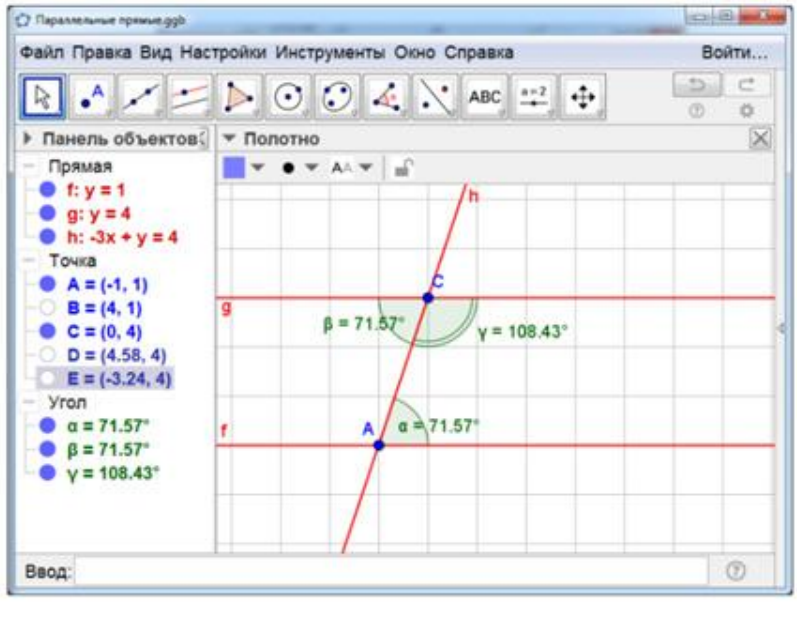
2-topshiriq

Teorema. Ikki parallel to‘g‘ri chiziq va kesishuvchi hosil qilgan bir tomonli burchaklar yig‘indisi 180^o ga teng bo‘ladi.

Teoremani ***GeoGebra*** dan foydalanib vizuallashtiramiz. Quyida keltirilgan amallar ketma-ketligini bajaring.

	AB to‘g‘ri chiziqni o‘tkazing.
	C nuqtani tanlang va u orqali AB to‘g‘ri chiziqqa parallel to‘g‘ri chiziq o‘tkazing.
	A va C nuqtalar orqali parallel chiziqlarni kesuvchi h to‘g‘ri chiziqni o‘tkazing.
	Ichki bir tomonlama va unga qo‘shni burchaklarni belgilang.
	To‘g‘ri chiziqlar va burchaklarning rangi va ko‘rinishini o‘zgartiring. Obyekt ustida sichqonchaning o‘ng tugmasini bosib, “**Свойства”** buyrug‘ini tanlang.

Natijada quyidagi ko‘rinishdagi dinamik chizma hosil bo‘ladi. Bunda kesuvchi va parallel chiziqlar o‘rnini o‘zgartirish mumkin, lekin kesishuvchi chiziq hosil qilgan bir tomonli burchaklar yig‘indisi har doim 180^o ga teng bo‘ladi.





VI BOB

TAKRORLASH

Bu bobni o‘rganib chiqqach, quyidagi bilim va amaliy ko‘nikmalarga ega bo‘lasiz:

Bilim:

– geometrik masalalarni yechish bosqichlari;
– geometrik masalalar turlari;
– masalalar yechishda uchraydigan ba’zi tipik xatoliklar.

Amaliy ko‘nikmalar:

– geometrik masalalarni turlarga ajratish va yechish bosqichlariga ko‘ra faoliyatni tashkil qilish;
– masalalar yechishda uchraydigan xatolarning oldini olish;
– planimetriya bo‘yicha yillik yakuniy nazorat ishiga tayyor bo‘lish.

25 TAKRORLASHGA DOIR MASALALAR

25.1. Geometrik masalalarni yechish bosqichlari

Geometrik masalalarni yechishda quyidagilarga e'tibor berish kerak:

1) geometriyaning asosiy tushunchalari, ularning xossalarini yaxshi bilish va yodda tutish;

2) turli geometrik shakllarning xossalari haqidagi teoremalarni isbotlash usullarini egallash;

3) berilgan geometrik masalaning mohiyatini tushunib yetish.

Odatda geometrik masalalarni yechish jarayoni quyidagi bosqichlardan iborat bo'ladi:

1-bosqich: *masalani tushunish.* Bu bosqichda masalaning sharti va xulosasi alohida ajratib olinadi. Nimalar berilgan, nimani topish, isbotlash yoki yasash kerakligi aniqlanadi. Masalaga oid chizma chiziladi. Chizma katta va aniq bo'lishi maqsadga muvofiq. Berilgan barcha ma'lumotlar chizmada belgilanadi.

2-bosqich: *rejalashtirish.* Bu bosqichda masalani yechish usuli tanlanadi. Uni qo'llash uchun qanday qo'shimcha ma'lumotlar zarurligi aniqlanadi. Yordamchi shakllar chiziladi.

3-bosqich: *yechish.* Bu bosqichda masala bevosita, berilgan reja asosida yechiladi.

4-bosqich: *tekshirish.* Bu bosqichda masalaning topilgan yechimi bevosita tekshiriladi. Yechish jarayoniga tanqidiy nazar tashlanadi. Agar xato aniqlansa, u tuzatiladi. Tuzatishning imkoni bo'lmasa, masalani yechishning boshlang'ich bosqichiga qaytiladi va hamma ish qaytadan bajariladi.

Masala yechishni o'rganish uchun ko'proq masala yechish kerak! Masalaga oid chizmani to'g'ri chizish – masalaning yarmini yechish demak.

Geometrik masalalar qo'yilishi va mohiyatiga qarab uch xil turda bo'lishi mumkin:

1) hisoblashga doir masalalar;

2) isbotlashga doir masalalar;

3) yasashga doir masalalar.

Geometrik masalalar yechish faqat qandaydir geometrik shaklning xossasini o'rganishdan iborat faoliyat emas, albatta. U to'g'ri fikrlash, mantiqiy mulohaza yuritish va ular asosida to'g'ri va oqilona qarorlar qabul qilish, xulosa chiqarish ko'nikma va malakalarini ham shakllantiradi. Bunday ko'nikma va malakalar nafaqat matematikada, balki kundalik turmushda uchraydigan muammolarni hal qilishda ham qo'l keladi.

Albatta, masalani yechish bu faqat to'g'ri javobni topish degani emas. Masalalar yechish davomida ma'lum xossalarni, teoremalarni va ularning natijalarini qo'llay olish, turli usullardan foydalanishni bilish zarur bo'ladi.

Quyidagi masalaning yechilish jarayonini kuzataylik.

Masala. Uchlari teng tomonli uchburchak tomonlarining o‘rtalari bo‘lgan uchburchakning teng tomonli ekanini isbotlang.

ΔABC – teng tomonli, $K-AB$ tomon o‘rtasi, $N-BC$ tomon o‘rtasi, $L-AC$ tomon o‘rtasi ⇒ ΔKNL – teng tomonli

1. Masalani tushunish bosqichi.

Masala shartlari asosida chizma chizib olamiz *(1-rasm)*.

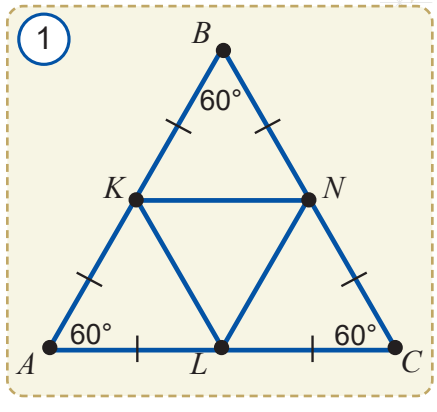
2. Rejalashtirish bosqichi. Teng tomonli uchburchakning xossasidan va uchburchakning TBT alomatidan foydalanamiz.

3. Yechish bosqichi. Shartga ko‘ra,

$LA=AK=KB=BN=NC=CL$ va $\angle A=\angle B=\angle C=60°$. Unda ΔLAK ning AL, AK tomonlari va A burchagi ΔKBN ning BK, BN tomonlari va B burchagiga hamda ΔNCL ning CN, CL tomonlari va C burchagiga mos ravishda teng.

Demak, $\Delta LAK=\Delta KBN=\Delta NCL$. U holda bu uchburchaklarning uchinchi tomonlari ham o‘zaro teng bo‘ladi: $KL=KN=NL$.

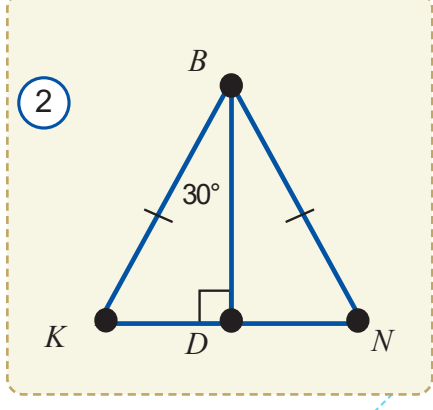
Demak, ΔKNL – teng tomonli.



4. Tekshirish bosqichi.

Masalaning yechilish jarayonini yana bir bor ko‘zdan kechirib, unda har bir mulohaza mantiqan to‘g‘ri olib borilganini tekshiramiz.

Bu masalani boshqa usulda ham yechish mumkin. Bunda uchidagi burchagi 60° bo‘lgan teng yonli uchburchakning xossasidan foydalanamiz. ΔKBN teng yonli uchburchakning BD balandligini tushiramiz *(2-rasm)*. BD bissektrisa ham bo‘lgani uchun $\angle KBD=60°:2=30°$ va $\angle BKD = \angle BND= 90° - 30° = 60°$ bo‘ladi.



Demak, ΔKBN teng tomonli uchburchak ekan. Shu tariqa ΔKAL va ΔNCL lar ham teng tomonli uchburchaklar ekanligi aniqlanadi va $BK=KN=NL=LK$ ekanligi ma’lum bo‘ladi. Bundan esa ΔKNL ning nafaqat teng tomonli uchburchak, balki $\Delta KNL=\Delta KBN=\Delta NCL=\Delta KAL$ ekanligi ham ma’lum bo‘ladi.

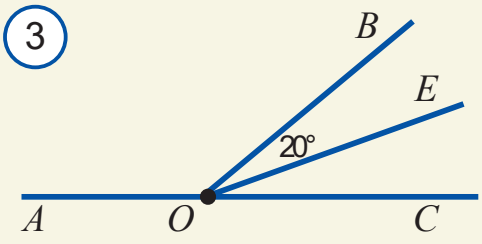
25.2. Hisoblashga doir masalalar

Hisoblashga doir masalalar arifmetik va algebraik masalalarga o‘xshab ketadi. Turli geometrik formulalar yordamida, berilgan sonli kattaliklar asosida ketma-ket hisob-kitob ishlari bajariladi va izlanayotgan kattalik topiladi.

Bu masalalarda ko‘pincha chizmani to‘g‘ri chizib olish va kerakli belgilashlarni kiritish ishni ancha osonlashtiradi.

1-masala. Qo‘shni burchaklardan birining bissektrisasi ikkinchi burchakning tomonlaridan biri bilan 20° li burchak hosil qiladi. Shu burchaklarni toping.

3



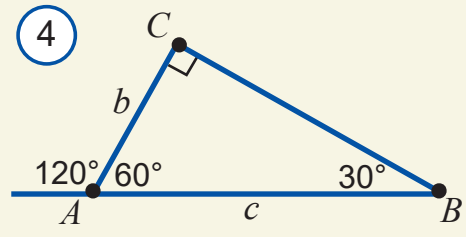
Yechilishi. Masala shartini chizmada tasvirlaymiz *(3-rasm)*. Bundan OE bissektrisa o‘tkir burchakning bissektrisasi ekanligi ma’lum bo‘ladi. Demak, $\angle BOC = 2\cdot 20° = 40°$,

$\angle AOB = 180° - 40° = 140°$ bo‘ladi.

2-masala. ABC to‘g‘ri burchakli uchburchakda $\angle C$ – to‘g‘ri burchak, A uchidagi tashqi burchak 120° ga teng. Agar $AC+AB = 18\ cm$ bo‘lsa, uchburchakning gipotenuzasini toping.

Yechilishi. Masala shartiga binoan chizmani tasvirlaymiz *(4-rasm)*. Uchburchak tashqi burchagining ta’rifidan $\angle A = 180° - 120° = 60°$, $\angle B = 90° - \angle A = 30°$ ekanini aniqlaymiz. $AC = b$, $AB = c$ bo‘lsin. U holda $b + c = 18$. O‘tkir burchagi 30° ga teng bo‘lgan to‘g‘ri burchakli uchburchakning xossasiga ko‘ra, $c = 2b$ bo‘ladi. Bundan $b + c = b + 2b = 18$, ya’ni $b = 6$. Unda $c = 12$ ekanligi ma’lum bo‘ladi.

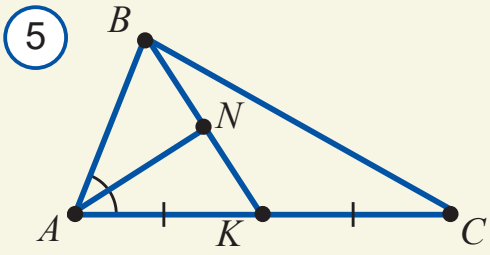
4



Javob: 12.

3-masala. ABC uchburchakda $AB = 1$, A burchakning bissektrisasi B uchdan tushirilgan medianaga perpendikulyar. Agar BC tomonning uzunligi butun son bilan ifodalansa, uchburchakning perimetrini toping.

5



Yechilishi. Masala shartini chizmada tasvirlaymiz *(5-rasm)*:

$AK = KC$. $AN \perp BK$. $\Delta ANB = \Delta ANK$ ekanini aniqlaymiz, chunki AN katet umumiy va bittadan burchaklari teng (katet va unga yopishgan o‘tkir burchak bo‘yicha). Bundan esa $AB = AK = KC = 1$, ya’ni $AC = 1 + 1 = 2$ ekanligi ma’lum bo‘ladi.

$BC = x$ – butun son, uchburchak tengsizligiga ko‘ra, $2 + 1 > x$ va $x + 1 > 2$ yoki $x < 3$ va $x > 1$, ya’ni $1 < x < 3$ bo‘lishi kerak. 1 bilan 3 ning orasida bitta butun son bor: 2.

Demak. $BC = 2$ va $P_{ABC} = 1 + 2 + 2 = 5$.

Javob: 5

25.3. Isbotlashga doir masalalar

Isbotlashga doir masalalar o‘ziga xos kichkina teoremalardir. Ularni yechish masalada keltirilgan tasdiqni isbotlashdan iborat bo‘ladi. Misol tariqasida quyidagi masalalarga qaraylik.

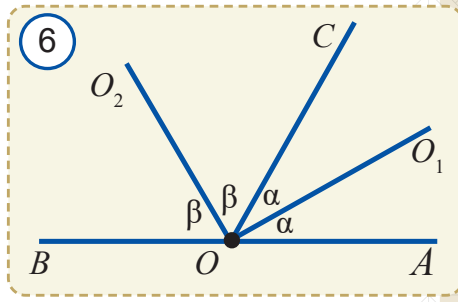
1-masala. Qo‘shni burchaklarning bissektrisalari o‘zaro perpendikulyar ekanini isbotlang.

$\angle AOC$ va $\angle BOC$ – qo‘shni burchaklar, OO_1 va OO_2 – bissektrisalar *(6-rasm)* ⇒ $OO_1 \perp OO_2$.

Isboti. OO_1 va OO_2 bissektrisalar ajratgan burchaklarni mos ravishda (1-rasmda tasvirlangandek) α va β deb belgilaymiz. U holda $2\alpha+2\beta=180°$ yoki $\alpha+\beta=90°$, ya’ni $\angle O_1OO_2=\alpha+\beta=90°$.

Demak, $OO_1 \perp OO_2$. Shuni isbotlash talab qilingan edi.

6



2-masala. 7a-rasmda tasvirlangan $ABCD$ to‘rtburchakda $\angle D = \angle A+\angle B+\angle C$ ekanini isbotlang.

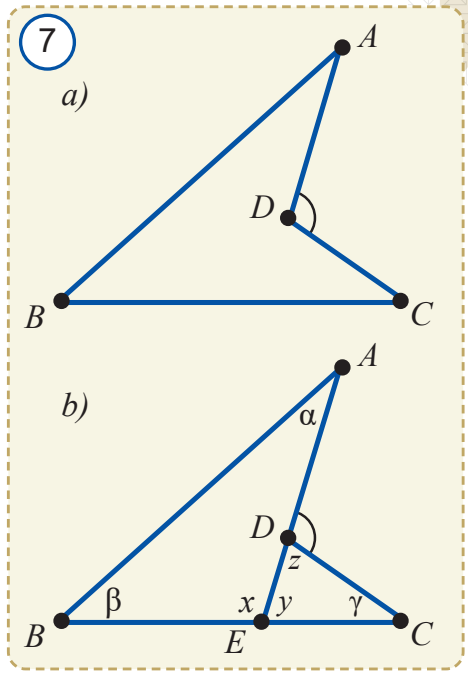
Isboti. AD to‘g‘ri chiziqning BC tomon bilan kesishgan nuqtasini E bilan belgilaymiz *(AD* tomonni davom ettiramiz) va burchaklar uchun zarur belgilashlarni kiritamiz *(7b-rasm)*. Ma’lumki, $\alpha+\beta+x=180°$ va $y+z+\gamma=180°$. Bu tengliklarni qo‘shib, $\alpha+\beta+\gamma+x+y+z=360°$ tenglikka ega bo‘lamiz. Qo‘shni burchakning xossasiga ko‘ra, $x+y=180°$ bo‘lgani uchun $\alpha+\beta+\gamma+180°+z=360°$ yoki $\alpha+\beta+\gamma=180°-z=\angle D$, ya’ni:

$\angle D=\alpha+\beta+\gamma=\angle A+\angle B+\angle C$ bo‘ladi.

Tenglik isbotlandi.

Yuqoridagi ikki masalani tayyor chizmaga tayanib ishladik, 2-masalada qo‘shimcha yasash va zarur belgilashlarni amalga oshirdik, bu esa masalani oson yechishimizga yordam berdi.

7



Amaliy mashq va tatbiq

1. AB kesma uzunliklari $1:2:3:4$ kabi nisbatdagi kesmalarga (shu ketma-ketlikda) ajratilgan. Agar chetki kesmalarning o‘rtalari orasidagi masofa 15 cm ga teng bo‘lsa, AB kesmaning uzunligini toping.

2. $\angle ABC=160°$ bo‘lgan burchakning uchidan shu burchak tomonlari orasida yotuvchi BO va BE nurlar chiqarilgan. Agar BO nur berilgan burchakni teng ikkiga, BE nur esa $3:5$ kabi nisbatda bo‘lsa, OBE burchakni toping.

3. AOB burchak OC nur orqali biri ikkinchisidan $30°$ ga katta bo‘lgan ikkita burchakka ajratilgan. Berilgan burchak bissektrisasi bilan OC nur orasidagi burchakni toping.

4. Teng yonli uchburchakning asosidagi burchagi $30°$ ga teng. Shu uchburchakning yon tomoni va ikkinchi yon tomoniga tushirilgan balandligi orasidagi burchakni toping.

5. Uchburchakning bir tashqi burchagi 100°, unga qo‘shni bo‘lmagan burchaklar nisbati 2:3 kabi. Uchburchakning burchaklarini toping.

6. A, B, C, D nuqtalar ko‘rsatilgan tartibda bir to‘g‘ri chiziqda yotadi va $AB = BC = 1$, $CD = 2$. K nuqta BC kesmada shunday joylashganki, u BC va AD kesmalarni bir xil nisbatdagi bo‘laklarga bo‘ladi: $BK : KC = AK : KD$. Bu nisbatlarni toping.

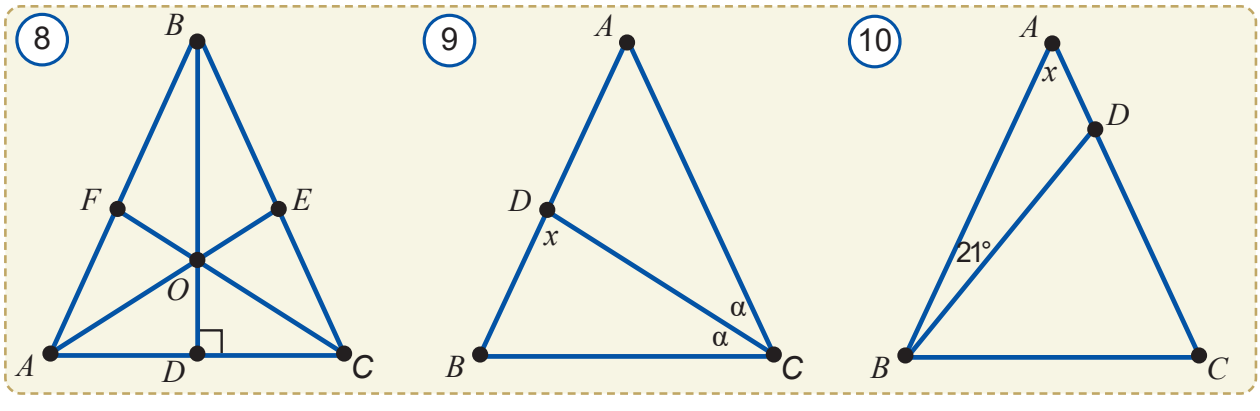
7. Uchburchak ikkita burchagining bissektrisalari kesishgandan hosil bo‘lgan burchak 128° ga teng. Uchburchakning uchinchi burchagini toping.

8. Teng yonli uchburchakning uchidagi burchagi 96° ga teng. Asosidagi burchaklarning bissektrisalari kesishishidan hosil bo‘lgan o‘tkir burchakni toping.

9. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagidan bissektrisa va balandlik chiqarilgan bo‘lib, ular orasidagi burchak 24° ga teng. Uchburchakning qolgan burchaklarini toping.

10. Agar 8-rasmda $AB = BC$, $\angle ABC = 50°$, AE va FC bissektrisalar bo‘lsa, AOB, EOC burchaklarni toping.

11. Agar 9-rasmda $AB = AC$, $AD = DC$ bo‘lsa, x ni toping.



12. Agar 10-rasmda $AB = AC$, $BD = BC$ bo‘lsa, x ni toping.

13. Uchburchakning bir burchagi o‘ziga qo‘shni bo‘lmagan tashqi burchaklarning ayirmasiga teng. Bu uchburchakning to‘g‘ri burchakli uchburchak ekanini isbotlang.

14. Bir burchagi 150° bo‘lgan teng yonli uchburchakning asosidagi uchlaridan tushirilgan balandliklari teng bo‘lishini isbotlang.

15. Teng tomonli uchburchakning medianalari kesishish nuqtasida 2 : 1 nisbatda bo‘linishini isbotlang.

16. Teng yonli uchburchakning uchidagi tashqi burchagi bissektrisasi uchburchak asosiga parallel bo‘lishini isbotlang.

17. 16-masalaga teskari teoremani ifodalang va uni isbotlang.

18. Teng tomonli uchburchakning ixtiyoriy ikkita medianasi 60° li burchak ostida kesishishini isbotlang.

19*. Uchburchaklarning tengligini ularning ikki tomoni va uchinchi tomonga tushirilgan medianasi bo‘yicha isbotlang.

20*. ABC va $A_1B_1C_1$ uchburchaklarda BM va B_1M_1 medianalar o‘tkazilgan. Agar $AB = A_1B_1$, $AC = A_1C_1$ va $BM = B_1M_1$ bo‘lsa, $\Delta ABC = \Delta A_1B_1C_1$ ekanini isbotlang.

21*. ABC va $A_1B_1C_1$ uchburchaklarda AD, A_1D_1 – bissektrisalar. Agar $AB = A_1B_1$, $BD = B_1D_1$ va $AD = A_1D_1$ bo‘lsa, $\Delta ABC = \Delta A_1B_1C_1$ ekanini ko‘rsating.

22. ABC va $A_1B_1C_1$ uchburchaklarda BH va B_1H_1 balandliklar o‘tkazilgan. Agar $\angle A = \angle A_1$, $\angle B = \angle B_1$ va $BH = B_1H_1$ bo‘lsa, $\Delta ABC = \Delta A_1B_1C_1$ bo‘lishini isbotlang.

23*. Uchburchakning ikkita balandligi teng bo‘lsa, uning teng yonli uchburchak ekanini isbotlang.

24*. 11-rasmda $\alpha+\gamma=\beta+\delta=90°$ ekanini isbotlang.

25*. 12-rasmda $\alpha<\beta<\gamma$ ekanini isbotlang.

26. Ikki parallel to‘g‘ri chiziq va kesuvchi hosil qilgan almashinuvchi burchaklarning bissektrisalari parallel bo‘lishini isbotlang.

27. Uchburchakning istalgan bir tomoni uning qolgan ikki tomoni ayirmasidan katta bo‘lishini isbotlang.

28. Uchburchakning α, β va γ burchaklari uchun $\alpha<\beta+\gamma$, $\beta<\alpha+\gamma$, $\gamma<\alpha+\beta$ munosabatlar o‘rinli bo‘lsa, bu qanday uchburchak bo‘ladi?

29. Berilgan ikki nuqtadan o‘tuvchi aylana yasang. Masala nechta yechimga ega?

30. ABC uchburchakning AA_1 va BB_1 bissektrisalari O nuqtada kesishadi. Agar: a) $\angle AOB=136°$; b) $\angle AOB=111°$ bo‘lsa, ACB burchakni toping.

31. 13-rasmdagi kubda $BD=6$ bo‘lsa, BE, DE, AC kesmalar va $\angle BED$ ni toping.

32. Perimetri 42 cm bo‘lgan ABC uchburchakning medianasi uni perimetri 33 cm va 35 cm bo‘lgan ikkita uchburchakka ajratadi. Mediananing uzunligini toping.

33. To‘g‘ri burchakli uchburchak o‘tkir burchaklarining bissektrisalari qanday burchak ostida kesishadi?

34. 14-rasmda $\angle 1=\angle 2$ ekanini isbotlang.

35. MN va NM nurlarining umumiy qismi qanday shakl bo‘ladi?

36. Aylananing o‘zaro perpendikulyar diametrlarini yasang.

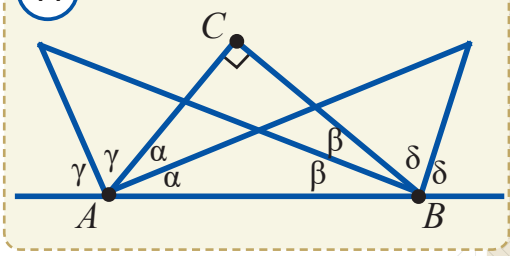
37. Qo‘shni burchaklardan biri ikkinchisidan 4 marta kichik bo‘lsa, shu burchaklardan kattasini toping.

38. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklarning nisbati 7 : 3 ga teng. Shu burchaklardan kichigini toping.

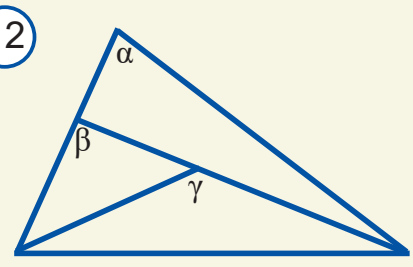
39. A, B va C nuqtalar bir to‘g‘ri chiziqda yotadi. BC kesmaning uzunligi AC kesmaning uzunligidan 3 marta katta, AB kesmaning uzunligi esa BC uzunligidan 3,6 cm ga qisqa. AC kesmaning uzunligini toping.

40. Ikki to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesganda tashqi bir tomonli burchaklarning yig‘indisi 180° ga teng bo‘lsa, bu to‘g‘ri chiziqlarning o‘zaro parallel ekanini isbotlang.

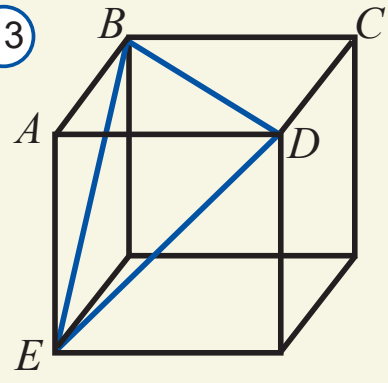
11



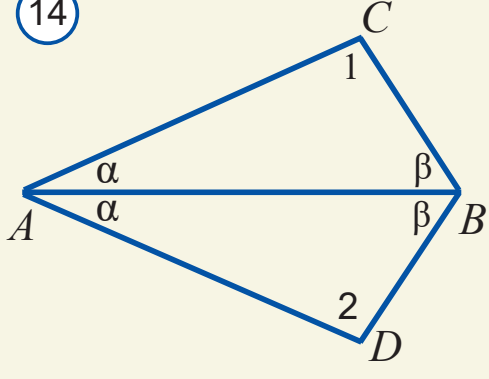
12



13



14



26 AMALIY MASHQ VA TATBIQ. BILIMINGIZNI SINAB KO‘RING.

1. Geometrik diktant. Jumlalarni mohiyatidan kelib chiqib to‘ldiring.

1. Tekislikda ... orqali bitta to‘g‘ri chiziq o‘tkazish mumkin.
2. Burchakning ... burchakni ikkita o‘zaro teng burchakka ajratadi.
3. Kesmaning o‘rtasi uni ikkita ... ajratadi.
4. Tekislikda to‘g‘ri chiziqqa tegishli bo‘lgan ... ham, tegishli bo‘lmagan ... ham mavjud.
5. Agar uchburchak teng yonli bo‘lsa, ... burchaklari teng bo‘ladi.
6. Ikkita teng uchburchaklarning mos ... va mos ... teng bo‘ladi.
7. Teng tomonli uchburchakning har bir ... gradusga teng.
8. To‘g‘ri burchakli uchburchakning o‘tkir ... 90° ga teng.
9. Yoyiq burchak bissektrisasi uni ikkita ... burchakka ajratadi.
10. Uchinchi to‘g‘ri chiziqqa parallel bo‘lgan ikkita to‘g‘ri chiziq ... bo‘ladi.
11. Bir to‘g‘ri chiziqqa perpendikulyar bo‘lgan ikkita to‘g‘ri chiziq ... bo‘ladi.
12. Parallel to‘g‘ri chiziqlarni kesuvchi bilan kesganda, hosil bo‘lgan ichki bir tomonli burchaklar ... bo‘ladi.
13. Kesma uchlaridan teng ... kesmaning o‘rta perpendikulyarida yotadi.
14. Aylanadagi nuqtalar aylana markazidan teng ... .

2. Quyida keltirilgan jumlalarda xato bo‘lsa, uni toping va tuzating.

1. Tekislikda ikkita nuqta orqali ikkita to‘g‘ri chiziq o‘tkazish mumkin.
2. To‘g‘ri burchak 180° ga teng bo‘ladi.
3. Qo‘shni burchaklar teng bo‘ladi.
4. Vertikal burchaklar yig‘indisi 180° ga teng.
5. Uchburchakning uchi bilan shu uchi qarshisidagi tomonining o‘rtasini tutashtiruvchi kesma “uchburchakning bissektrisasi” deyiladi.
6. Uchburchakning perimetri deb uning burchaklari yig‘indisiga aytiladi.
7. Uchburchak tomonlarining yig‘indisi 180° ga teng.
8. 90° ga teng burchak ostida kesishgan to‘g‘ri chiziqlar “parallel” deyiladi.
9. Parallel to‘g‘ri chiziqlar bitta nuqtada kesishadi.
10. Aylananing diametri radiusiga teng.
11. To‘g‘ri burchakli uchburchakning katetlari teng bo‘lsa, uning kichik burchagi 30° ga teng bo‘ladi.
12. Teng yonli uchburchakning har bir burchagi 60° ga teng.
13. Burchak bissektrisasida yotgan nuqtalar burchak uchlaridan teng uzoqlikda yotadi.

3. Berilgan xossaga ega bo‘lgan geometrik shaklni o‘ng ustundagi mos qatorga yozing.

1.	Uzunligi 5 cm	
2	Nuqta va uchlari shu nuqtalarda bo‘lgan ikkita nurdan iborat	
3	Kesishmaydigan to‘g‘ri chiziqlar	
4	Uchidan chiqqan balandligi ham, medianasi ham bissektrisasi bo‘ladi	
5	Hamma tomonlari teng uchburchak	
6	Ikkita tomoni teng uchburchak	
7	Burchakni ikkita teng burchakka ajratadi	
8	Ikkita kateti bor	
9	Ikki burchagining yig‘indisi 90° dan katta bo‘lgan uchburchak	

4. Birinchi ustunda berilgan geometrik tushunchalarga ikkinchi ustundan tegishli xossa yoki talqinlarni qo‘ying.

Geometrik tushunchalar	Xossa yoki talqinlari
1. Perpendikulyar to‘g‘ri chiziqlar	(A) tayin uzunlikka ega
2. Teng tomonli uchburchak	(B) ikkita burchagi teng
3. Aylana	(C) gipotenuzaning yarmiga teng
4. Burchak bissektrisasidagi nuqta	(D) uchi bilan qarshisidagi tomon o‘rtasini tutashtiradi
5. Uchburchak balandligi	(E) bitta ichki burchagiga qo‘shni va qolgan ikkita burchagi yig‘indisiga teng
6. 30° li burchak qarshisidagi katet	(F) kesishmaydi
7. Mediana	(G) 90° li burchak ostida kesishadi
8. Uchburchakning tashqi burchagi	(H) tomonlari teng
9. Teng yonli uchburchak	(I) nuqtalari markazidan teng uzoqlashgan
10. Kesma	(J) uning tomonlaridan teng uzoqlikda yotadi
11. Parallel to‘g‘ri chiziqlar	(K) bir uchidan o‘tadi va bir tomoniga perpendikulyar

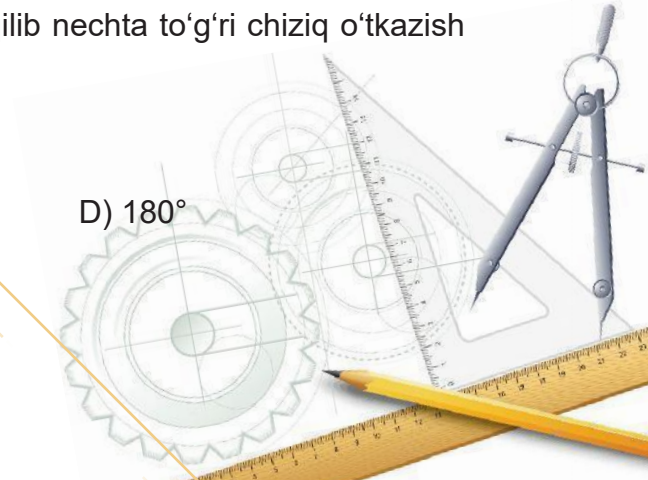
5. Testlar.

1. Berilgan nuqtadan berilgan to‘g‘ri chiziqqa parallel qilib nechta to‘g‘ri chiziq o‘tkazish mumkin?

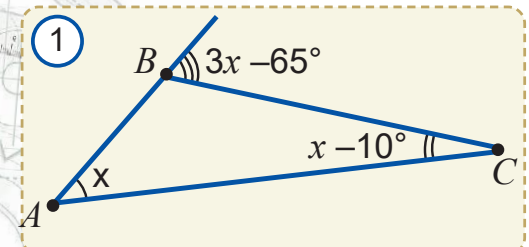
A) 1 B) 2 C) 3 D) 4

2. Yoyiq burchak necha gradusga teng?

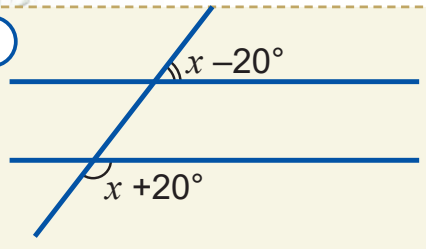
A) 90° B) 90° dan katta C) 90° dan kichik D) 180°



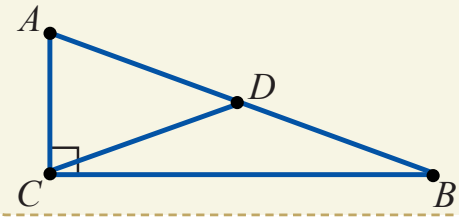
1



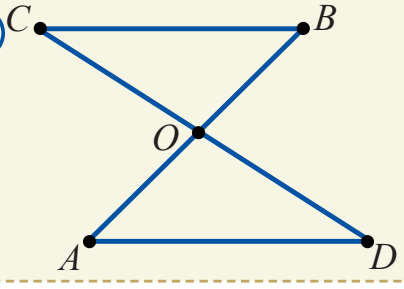
2



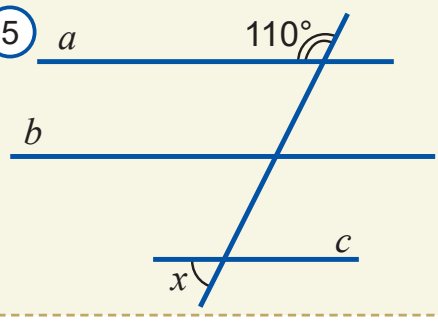
3



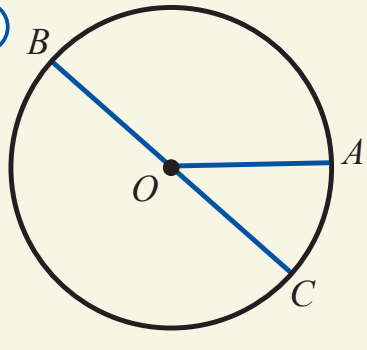
4



5



6



3. 1-rasmdagi $\angle BCA$ burchakni toping.

A) 25° B) 35° C) 45° D) 55°

4. 2-rasmdagi x ni toping.

A) 80° B) 90° C) 100° D) 70°

5. Agar ABC uchburchakda $\angle B = 30°$, $\angle C = 90°$ va $AC = 10\ cm$ bo‘lsa, AB gipotenuzasini toping.

A) 10 cm B) 12 cm C) 15 cm D) 20 cm

6. ABC uchburchakda $AB = BC$, $AB = AC + 7(cm)$. Agar ABC uchburchak perimetri 23 cm bo‘lsa, uchburchakning kichik tomonini toping.

A) 3 cm B) 5 cm C) 7 cm D) 9 cm

7. Qo‘shni burchaklardan biri ikkinchisidan uch marta katta. Bu burchaklar ayirmasini toping.

A) 45° B) 60° C) 75° D) 90°

8. Aylananing radiusi 3,2 cm. Uning diametrini toping.

A) 3,2 B) 5,2 C) 6,4 D) 1,6

9. ABC – to‘g‘ri burchakli uchburchak (*3-rasm*), $\angle C = 90°$, CD – mediana. $\angle BDC = 130°$ bo‘lsa, $\angle A$ ni toping.

A) 45° B) 65° C) 75° D) 85°

10. ABC teng yonli uchburchakning uchidagi B burchagi 80° ga teng. Uning A uchidagi tashqi burchagini toping.

A) 130° B) 120° C) 110° D) 100°

11. Agar $a \perp b$, $b \perp c$, $c \perp d$ bo‘lsa, quyidagi javoblardan qaysi biri to‘g‘ri.

A) $a \| c$ B) $b \perp d$ C) $a \| d$ D) $b \| c$

12. Agar 4-rasmda $AO = OB$, $OC = OD$, $BC = 5\ cm$ va $AO + OC = 7\ cm$ bo‘lsa, AOD uchburchak perimetrini toping.

A) 5 cm B) 7 cm C) 12 cm D) 17 cm

13. Agar 5-rasmda $a \| b$ va $b \| c$ bo‘lsa, $x = ?$

A) 60° B) 70° C) 80° D) 90°

14. ABC uchburchakda $\angle A = 50°$ va $\angle B = 70°$ bo‘lsa, uning katta tomonini aniqlang.

A) AB B) BC C) AC D) aniqlab bo‘lmaydi.

15. Agar 6-rasmda O aylana markazi, $AO = 4\ cm$ bo‘lsa, BC kesma uzunligini toping.

A) 4 cm B) 5 cm C) 2 cm D) 8 cm

16. 7-rasmda tasvirlangan uchburchakning kichik burchagini toping.

A) 30° B) 45° C) 60° D) 90°

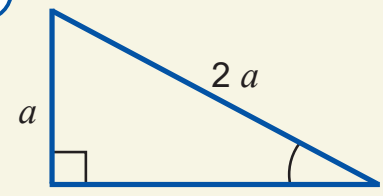
17. Uchburchakning bir balandligi uni perimetrlari 25 cm va 29 cm bo‘lgan uchburchaklarga ajratadi. Agar berilgan uchburchak perimetri 40 cm bo‘lsa, uning balandligini toping.

A) 10 cm B) 7 cm C) 5 cm D) 9 cm

18. 120° ga teng burchakka qo‘shni burchaklar yig‘indisini toping.

A) 30° B) 45° C) 180° D) 120°

7



19. ABC uchburchakning C burchagi 70° ga teng bo‘lsa, A va B burchaklari bissektrisalari orasidagi burchakni toping.

A) 55° B) 60° C) 65° D) 75°.

20. $ABCD$ to‘g‘ri to‘rtburchakning A va D uchlaridan chiqarilgan bissektrisalar BC tomonni 3 ta teng qismga ajratadi. Agar to‘g‘ri to‘rtburchakning tomonlari butun sonlardan iborat bo‘lib, AB = 5 bo‘lsa, uning perimetrini toping.

A) 20 B) 30; C) 40 D) 80.

6. Masalalar

1. Teng yonli ABC uchburchakning uchidan AB asosiga o‘tkazilgan bissektrisasi uni ikkita uchburchakka ajratadi. Bu uchburchaklarning tengligini isbotlang.

2. Perimetri 30 cm bo‘lgan uchburchakning bir tomoni ikkinchi tomonidan 2 cm katta, uchinchi tomonidan esa 2 cm kichik. Uchburchakning katta tomonini toping.

3. Uchburchakning asosiga tushirilgan medianasi uni perimetri 18 cm va 24 cm ga teng ikki uchburchakka ajratadi. Berilgan uchburchakning kichik yon tomoni 6 cm ga teng. Uchburchakning katta yon tomonini toping.

4. Uchburchakning 5 ga teng bo‘lgan balandligi uni perimetri 18 va 26 bo‘lgan ikkita uchburchakka ajratadi. Berilgan uchburchakning perimetrini toping.

5. Teng yonli uchburchakning perimetri 7,6 cm ga, asosi esa 2 cm ga teng. Yon tomonini toping.

6. AB va CD to‘g‘ri chiziqlar O nuqtada kesishadi. BOC va AOD burchaklarning yig‘indisi 194° ga teng. AOC burchakni toping.

7. ABC uchburchakda A burchak C burchakka teng, AD balandlik esa BC tomonni teng ikkiga bo‘ladi. Agar BD = 7,8 cm bo‘lsa, AC ni toping.

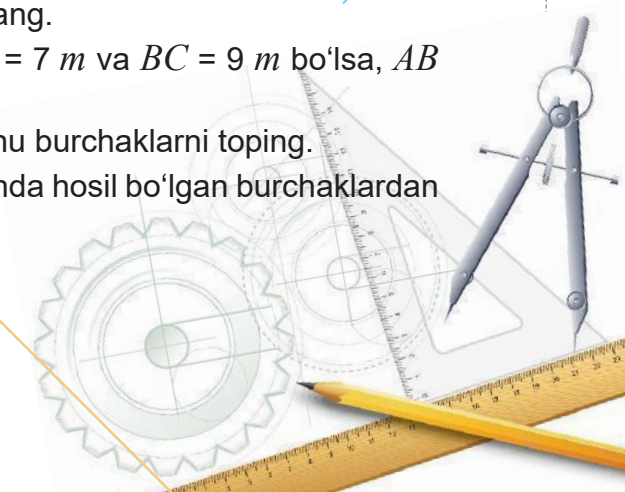
8. Teng yonli uchburchakning yon tomoniga tushirilgan balandligi bilan ikkinchi yon tomoni orasidagi burchak 20° ga teng. Uchburchakning asosidagi burchagini toping.

9. B burchakning bissektrisasida yotgan D nuqtadan burchakning tomonlariga DA va DC perpendikulyarlar o‘tkazilgan. $DA = DC$ ekanini isbotlang.

10. Agar A, B va C nuqtalar bir to‘g‘ri chiziqda yotib, AC = 7 m va BC = 9 m bo‘lsa, AB kesmaning uzunligini toping.

11. Qo‘shni burchaklardan biri ikkinchisidan 18° kichik. Shu burchaklarni toping.

12. Ikki parallel to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesganda hosil bo‘lgan burchaklardan biri 55° ga teng. Qolgan burchaklarini toping.



13. A, B va C nuqtalar bir to‘g‘ri chiziqda yotadi. Agar AB = 2 cm, BC = 3 cm va AC=5 cm bo‘lsa, B nuqta AC kesmaga tegishli bo‘ladimi? Javobingizni asoslang.

14. A nuqta BC to‘g‘ri chiziqning B va C nuqtalari orasida yotadi. Agar BC = 15 cm, AC kesma esa AB kesmadan 3 cm ga qisqa bo‘lsa, AB kesmaning uzunligini toping.

15. Qo‘shni burchaklardan biri ikkinchisidan 5 marta katta bo‘lsa, shu burchaklardan kattasini toping.

16. Ikki to‘g‘ri chiziqning kesishishidan hosil bo‘lgan burchaklarning nisbati 5:4 ga teng. Shu burchaklardan kichigini toping.

17. A, B va C nuqtalar bir to‘g‘ri chiziqda yotadi. BC kesmaning uzunligi AC kesmaning uzunligidan 2 marta kichik, AB kesmaning uzunligi esa BC uzunligidan 5,3 cm ga uzun. AC kesmaning uzunligini toping.

18. Teng yonli uchburchakning uchidagi burchagi 42° ga teng. Asosidagi burchaklarning bissektrisalari kesishishidan hosil bo‘lgan o‘tkir burchakni toping.

19. Ikki parallel to‘g‘ri chiziqni uchinchi to‘g‘ri chiziq kesganda hosil bo‘lgan burchaklardan biri 109° ga teng. Qolgan burchaklarini toping.

7. Murakkab masalalar

1. Yerdan Quyoshgacha bo‘lgan masofa taxminan 149 500 ming km, Yerdan Oygacha bo‘lgan masofa esa 400 ming km. Quyoshdan Oygacha bo‘lgan masofa: a) Oyning; b) Quyoshning to‘liq tutilgan paytida qanchani tashkil etadi?

2. C nuqta AB kesmaning o‘rtasidir. AB kesmada shunday D nuqtani topingki, $DA = 1{,}5\ (DB + DC)$ bo‘lsin.

3. Qo‘shni burchaklardan biri ularning ayirmasidan 4 marta katta. Bu burchaklarni toping.

4. D nuqta ABC uchburchakning ichki sohasida yotibdi. ABC burchak ADC burchakdan kichik ekanini ko‘rsating.

5. Berilgan o‘tkir burchakdan 25° ga katta bo‘lgan burchakni yasang.

6. Uchburchak perimetri uning bir tomonidan a ga, ikkinchi tomonidan b ga va uchinchi tomonidan c ga katta. Uchburchak perimetrini toping.

7. Uchburchak medianalari yig‘indisi uning perimetridan kichik, lekin uning yarim perimetridan katta ekanini isbotlang.

8. Ixtiyoriy uchburchakni bir nechta teng yonli uchburchaklarga bo‘lish mumkinligini isbotlang.

9. Agar BB_1 kesma ABC uchburchakning bissektrisasi bo‘lsa, $AB > AB_1$ va $BC > B_1C$ ekanini isbotlang.

10. AB va CD kesmalar O nuqtada kesishadi. Agar $AC=AO=BO=BD$ bo‘lsa, $OC=OD$ ekanini isbotlang.

11. Uchburchakning balandliklari kesishish nuqtasida teng ikkiga bo‘linishi mumkinmi?

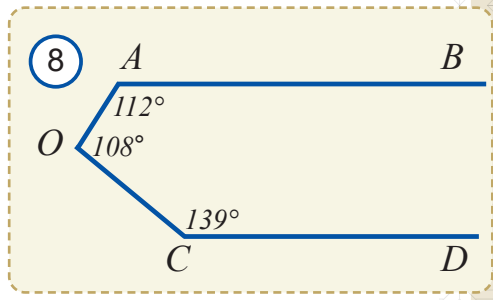
12. A, B, C va D nuqtalar tekislikda shunday joylashganki, $AB=BC=CA$ va $DA=DB=DC$. ADB burchak gradus o‘lchovini toping.

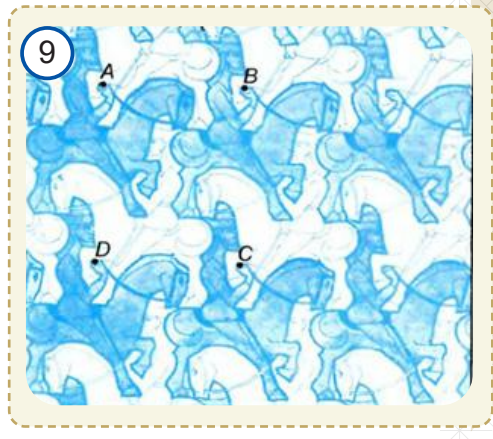
13. 8-rasmdagi AB va CD kesmalar o‘zaro parallel bo‘ladimi?

14. O‘tmas burchakning gradus o‘lchovi α ga teng. Uning tomonlariga perpendikulyar ikkita nur o‘tkazilgan. Bu nurlar o‘zaro gradus o‘lchovi β ga teng burchak hosil qiladi. $\alpha+\beta=180°$ ekanini isbotlang.

15. 9-rasmda keltirilgan otliq-chavandozlar tasviri teng shakllardan iborat. Agar $ABCD$ yuzi S ga teng bo‘lgan to‘rtburchak bo‘lsa, bitta otliq shaklining yuzini aniqlang.

16. ABC teng yonli uchburchakda $AB=AC$, M nuqta – AB tomonning o‘rtasi, N nuqta esa AC tomonning o‘rtasi. N nuqtadan chiquvchi perpendikulyar AB tomonni L nuqtada, M nuqtadan chiquvchi perpendikulyar AC tomonni K nuqtada kesib o‘tadi: $MK\perp AB$ va $NL\perp AC$. $\angle NLK = \angle MKL$ ekanini isbotlang.





Yakuniy nazorat ishi

Yakuniy nazorat ishi ikki qismdan iborat bo‘ladi. Birinchi qismda oldingi darslarda ko‘rilgan geometrik diktant va test savollariga o‘xshash 5 ta diktant savollari va 10 ta testni yechish taklif qilinadi. Nazorat ishining ikkinchi qismida quyidagi variantda berilgan masalalarga o‘xshash 6 ta masala berilishi mumkin.

Yakuniy yozma nazorat ishi namunasi

1. Agar 1-rasmda $a||b$ va $\angle 1 = 45°$ bo‘sa, $\angle 2$, $\angle 3$ va $\angle 4$ ni toping.

2. ABC uchburchakda $\angle A = 32°$, B burchak A burchakdan $12°$ kichik bo‘lsa, C burchakka tashqi bo‘lgan burchakni toping.

3. 2-rasmda berilgan ma’lumotlar asosida

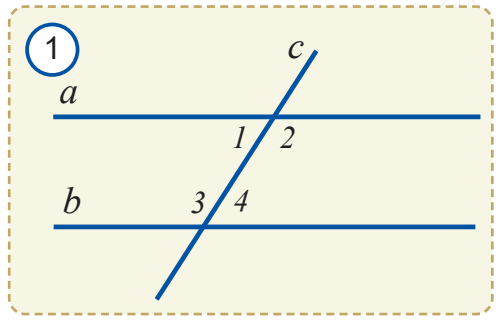
a) $\Delta ABC = \Delta ADC$ ekanini isbotlang;

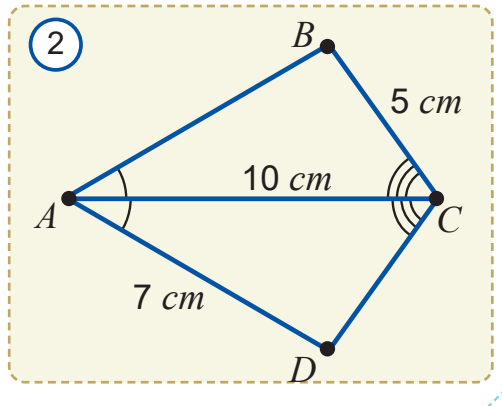
b) ACD uchburchak perimetrini toping.

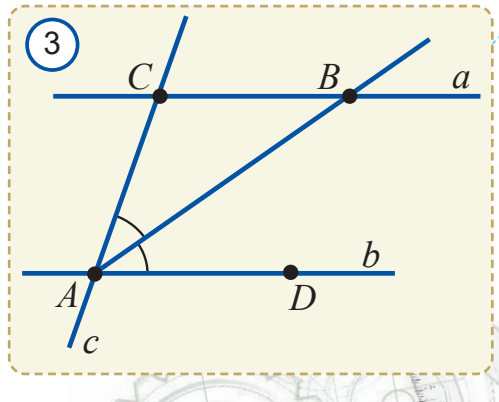
4. 3-rasmda $a||b$ va AB – CAD burchak bissektrisasi, $AC = 7$ cm. BC kesma uzunligini toping.

5. To‘g‘ri burchakli uchburchakning to‘g‘ri burchagidan tushirilgan balandligi uning bissektrisasi ham bo‘ladi. Bu uchburchak burchaklarini toping.

6. Berilgan burchakka teng burchak va uning bissektrisasini yasang.







Qo‘shni va vertikal burchaklar

Qo‘shni burchaklar yig‘indisi 180° ga teng *(1-rasm)*.

Qo‘shni burchaklarning bissektrisalari o‘zaro perpendikulyar.

Vertikal burchaklar teng *(2-rasm)*.

Uchburchaklar

Uchburchaklar tengligi alomatlari *(3-rasm)*:

1) (TBT) ikki tomoni va ular orasidagi burchakka ko‘ra;

2) (BTB) tomon va unga yopishgan ikki burchakka ko‘ra;

3) (TTT) uch tomonga ko‘ra.

Teng yonli uchburchaklar

Teng yonli uchburchakning asosidagi burchaklari teng bo‘ladi *(4-rasm)*.

Agar uchburchakning ikkita teng burchagi bo‘lsa, u teng yon uchburchak bo‘ladi *(4-rasm)*.

Teng yonli uchburchak uchidan asosiga tushirilgan bissektrisa uning balandligi va medianasi ham bo‘ladi.

Agar uchburchakning balandligi uning bissektrisasi yoki medianasi ham bo‘lsa, bu uchburchak teng yon bo‘ladi.

Kesma o‘rta perpendikulyarining xossasi

Kesma o‘rta perpendikulyarining istalgan nuqtasi shu kesmaning uchlaridan teng masofada joylashgan *(5-rasm)*.

Agar nuqta kesma uchlaridan teng masofada joylashgan bo‘lsa, u kesma o‘rta perpendikulyarida yotadi.

O‘rta perpendikulyar – kesma uchlaridan teng masofada joylashgan nuqtalarning geometrik o‘rni.

Parallel to‘g‘ri chiziqlar

Uchinchi to‘g‘ri chiziqqa parallel ikkita to‘g‘ri chiziq o‘zaro parallel bo‘ladi.

Agar almashinuvchi burchaklar teng bo‘lsa, u holda to‘g‘ri chiziqlar parallel va aksincha *(6-rasm).*

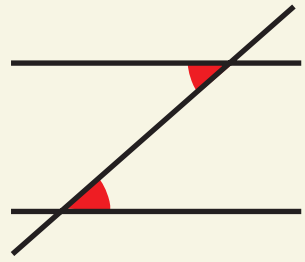
Agar mos burchaklar teng bo‘lsa, u holda to‘g‘ri chiziqlar parallel va aksincha *(7-rasm)*.

Agar bir tomonli burchaklar yig‘indisi 180° bo‘lsa, u holda to‘g‘ri chiziqlar parallel va aksincha *(8-rasm)*.

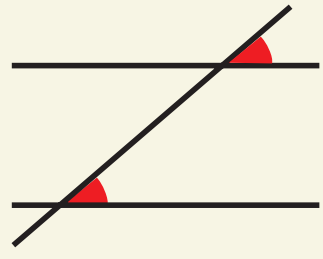
Uchinchi to‘g‘ri chiziqqa perpendikulyar ikkita to‘g‘ri chiziq bir-biriga parallel bo‘ladi *(9-rasm)*.

Parallel chiziqlardan biriga perpendikulyar bo‘lgan to‘g‘ri chiziq ularning ikkinchisiga ham perpendikulyar bo‘ladi.

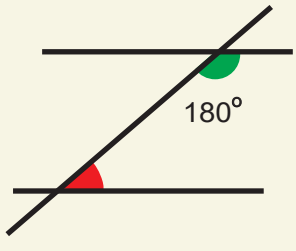
6



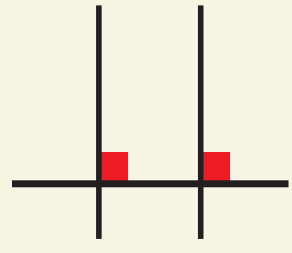
7



8



9



Uchburchak tomonlari va burchaklari orasidagi munosabatlar

Uchburchak ichki burchaklarining yig‘indisi 180° ga teng *(10-rasm)*.

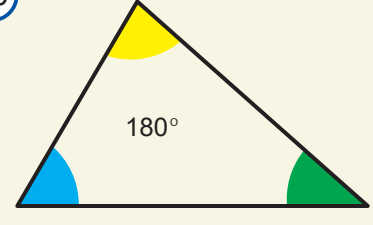
Uchburchakning tashqi burchagi unga qo‘shni bo‘lmagan ikkita ichki burchaklar yig‘indisiga teng *(11-rasm)*.

Uchburchakning tashqi burchagi unga qo‘shni bo‘lmagan har qanday ichki burchakdan katta.

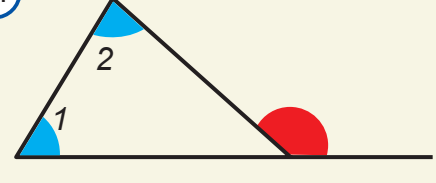
Uchburchakning istalgan tomoni uning qolgan ikki tomoni yig‘indisidan kichik.

Uchburchakda katta burchak qarshisida katta tomon, katta tomon qarshisida esa katta burchak yotadi.

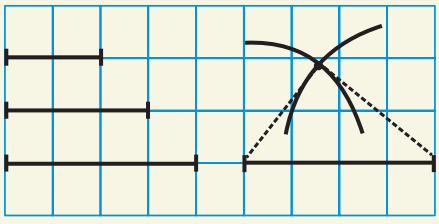
10



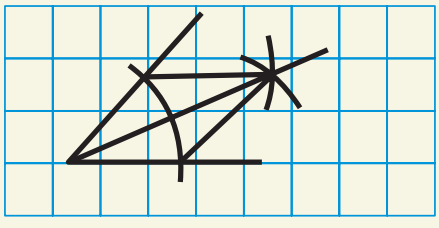
11



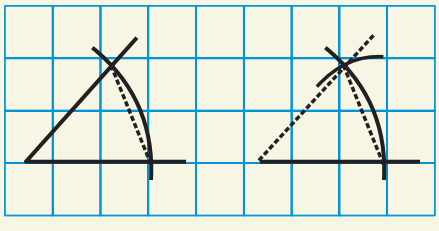
12



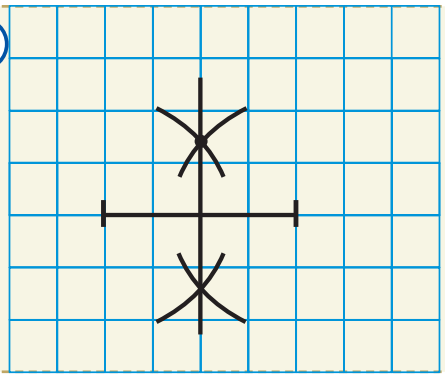
13



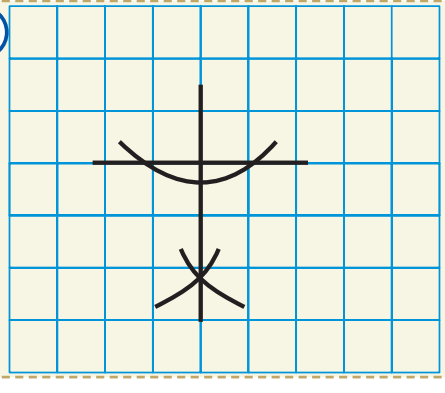
14



15



16



To‘g‘ri burchakli uchburchaklar

To‘g‘ri burchakli uchburchakning ikkita o‘tkir burchagi yig‘indisi 90° ga teng.

To‘g‘ri burchakli uchburchakda 30° burchak qarshisida yotgan katet gipotenuzaning yarmiga teng.

To‘g‘ri burchakli uchburchakning kateti gipotenuzasidan kichik.

To‘g‘ri burchakli uchburchaklar tengligi alomatlari:

1) (KK) ikki katetga ko‘ra;

2) (KB) katet va unga yopishgan o‘tkir burchakka ko‘ra;

3) (GB) gipotenuza va o‘tkir burchakka ko‘ra;

4) (GK) katet va gipotenuzaga ko‘ra.

Burchak bissektrisasining xossasi

Burchak bissektrisasining istalgan nuqtasi burchak tomonlaridan teng masofada joylashgan.

Agar burchak ichki sohasidagi nuqta burchak tomonlaridan teng masofada joylashgan bo‘lsa, u burchakning bissektrisasida yotadi.

Bissektrisa – burchakning tomonlaridan teng masofada joylashgan burchak ichidagi nuqtalarning geometrik o‘rni.

Yasashga doir asosiy masalalar

Uch tomonga ko‘ra uchburchak yasash *(12-rasm)*;

Burchak bissektrisasini yasash *(13-rasm)*;

Berilgan burchakka teng burchakni yasash *(14-rasm)*;

Kesma o‘rtasini yasash *(15-rasm)*;

Berilgan to‘g‘ri chiziqqa perpendikulyar to‘g‘ri chiziqni yasash. *(16-rasm)*

10 dan 99 gacha bo‘lgan natural sonlar kvadratlarining jadvali									
O‘nlik / Birlik	1	2	3	4	5	6	7	8	9
0	100	400	900	1600	2500	3600	4900	6400	8100
1	121	441	961	1681	2601	3721	5041	6561	8281
2	144	484	1024	1764	2704	3844	5184	6724	8464
3	169	529	1089	1849	2809	3969	5329	6889	8649
4	196	576	1156	1936	2916	4036	5476	7056	8836
5	225	625	1225	2025	3025	4225	5625	7225	9025
6	256	676	1296	2116	3136	4356	5776	7396	9216
7	289	729	1369	2209	3249	4489	5929	7569	9409
8	324	784	1444	2304	3364	4624	6084	7744	9604
9	361	841	1521	2401	3481	4761	6241	7921	9801



www.idum.uz saytidan yuklab olindi

Grek alifbosi

Yozilishi	O‘qilishi
$A\ \alpha$	alfa
$B\ \beta$	betta
$\Gamma\ \gamma$	gamma
$\Delta\ \delta$	delta
$E\ \varepsilon$	epsilon
$Z\ \zeta$	dzeta
$H\ \eta$	eta
$\Theta\ \theta$	teta
$I\ \iota$	yota
$K\ \kappa$	kappa
$\Lambda\ \lambda$	lyambda
$M\ \mu$	myu
$N\ \nu$	nyu

Yozilishi	O‘qilishi
$\Xi\ \xi$	ksi
$O\ o$	omikron
$\Pi\ \pi$	pi
$P\ \rho$	ro
$\Sigma\ \sigma$	sigma
$T\ \tau$	tau
$Y\ \upsilon$	yupsilon (ipsilon)
$\Phi\ \varphi$	fi
$X\ \chi$	xi
$\Psi\ \psi$	psi
$\Omega\ \omega$	omega

Javoblar va ko‘rsatmalar

1

2. a) $\begin{cases} O \in a \\ O \in b \end{cases}$ Kesishuvchi to‘g‘ri chiziqlarning har biriga tegishli nuqta ularning kesishish nuqtasidir va u bittadir; b) nuqtalar faqat to‘g‘ri chiziqqa tegishli. B va E nuqtalar faqat to‘g‘ri chiziqqa tegishli; c) F, H, G, K nuqtalar to‘g‘ri chiziqqa ham to‘g‘ri chiziqqa tegishli emas. **3.** a) $\begin{cases} O \in a \\ O \in b \end{cases}$ nuqta to‘g‘ri chiziqqa ham to‘g‘ri chiziqqa ham tegishli, ya’ni ularning kesishish nuqtasidir; b) nuqta faqat to‘g‘ri chiziqqa tegishli.

c) $\begin{cases} A \notin a \\ A \notin b \end{cases}$; $\begin{cases} C \notin a \\ C \notin b \end{cases}$; $\begin{cases} D \notin a \\ D \notin b \end{cases}$; $\begin{cases} E \notin a \\ E \notin b \end{cases}$ A; C; D; E nuqtalar a to‘g‘ri chiziqqa ham to‘g‘ri chiziqqa tegishli emas. **4.** a) A, O, E nuqtalar a to‘g‘ri chiziqqa tegishli; b) D va O nuqtalar b to‘g‘ri chiziqqa tegishli; d) O nuqta a to‘g‘ri chiziqqa ham b to‘g‘ri chiziqqa ham tegishli; e) A; E nuqtalar a to‘g‘ri chiziqqa tegishli, b to‘g‘ri chiziqqa tegishli emas; f) D nuqtalar b to‘g‘ri chiziqqa tegishli, a to‘g‘ri chiziqqa tegishli emas; g) B va C nuqtalar a to‘g‘ri chiziqqa ham b to‘g‘ri chiziqqa tegishli emas. **5.** A nuqta ham AB ga ham AC ga tegishli. 6. 1 ta yoki cheksiz ko‘p. **7.** Yotmaydi. **8.** a) istalgancha; b) 1 ta; c) 1 ta yoki 3 ta. **9.** 2 ta, 3 ta, 4 ta. **10.** 2 ta, 7 ta. **11.** 6 ta to‘g‘ri chiziq. Bu to‘g‘ri chiziqlar tekislikni 12 ta bo‘lakka ajratadi. **12.** a) 3 ta; b) 6 ta. **13.** 6 ta; 10 ta. **14***. Mumkin emas.

2

1. 6 ta. **2.** 6 ta: AB, BC, CD; AC; AD; BD. **3.** To‘ldiruvchi nurlar: 1) AB va AE ; 2) AC va AF ; 3) AD va AH . **4.** a) C nuqtadan chiquvchi nurlar: CA, CD, CB, CM, CN; to‘ldiruvchi nurlar: CA va CD yoki CA va CB; b) D nuqtadan chiquvchi nurlar: DC, DA, DB, DQ, DP; to‘ldiruvchi nurlar: DC va DB yoki DA va DB. **5.** 4 ta, 6 ta. **7.** a) b va d, p va q, n va u ; b) 2 va 5; 6 va 9. **8.** 3 va 14; 4 va 10; 6 va 9; 5 va 12. **9***. a) 2 ta; b) 3 ta; c) 4 ta; d) 11 ta. **10.** 2 ta; 4 ta; 7 ta; 11 ta. **11.** Ha. **12.** 5 ta; **13.** 29 ta qismga ajratadi. **14.** 26,8 cm. **15.** 1 birlik; 2 birlik ; $\frac{1}{2}$ *birlik* ; $\frac{2}{3}$ *birlik* ; **16.** 4 cm; 5 cm; 6,5 cm; 1 cm; 2,5 cm; 1,5 cm. **17.** a) 6,6; b) 1; c) 9. **18.** 12,8 cm. **19.** 0,8. 21. 2 ta hol bo‘lishi mumkin. B nuqta AC kesmada bo‘lsa, AC=800 m. C nuqta AB kesmada bo‘lsa, AC=400 m. **22.** 5. **23.** 46 cm. **24.** a) 24 cm; b) 14 cm. **25.** 5. **26**. a) 2 ta; b) 1 ta. **27.** a) 4 cm; b) 1,6 cm; c) 0,4 cm; d) 2,6 cm. **28.** 26. **29.** $CD = \frac{AB}{2} + 46\ cm$ **31.** a) 5 cm; b) 7,8 m; c) 2,8 km. **32.** a) 70 cm; b) 186 dm; c) 82 m. **33.** a) AC=5 cm ; BC=4 cm; b) AC=11 cm ; BC=2 cm; c) AC=10 cm ;BC=1 cm. **35.** B nuqta A va C nuqtalar orasida yotadi. **36.** a) 5,9 cm; b) 9,5 cm. **37.** a) 3AB; b) $\frac{3AC}{2}$ c) $\frac{3AE}{4}$ **39.** Bir to‘g‘ri chiziqda olingan istalgan uchta nuqtaning faqat bittasi qolgan ikkitasining orasida yotadi. **40.** Har qanday ikki nuqtadan faqat bitta to‘g‘ri chiziq o‘tadi.

3

1. a) burchakning uchi: N; burchakning tomonlari NM va NL; b) burchakning uchi: B; burchakning tomonlari: BA va BO. **2.** burchaklari: ∠BAC,∠BAD,∠CAD ; burchakning uchi: A; burchakning tomonlari: AB, AC, AD. **3.** a) ∠AOC, ∠COD, ∠BOD, ∠AOD, ∠AOB, ∠COD. b) ∠AOB, ∠BOC, ∠COD,∠DOE, ∠AOE, ∠AOC, ∠AOD, ∠BOD, ∠COE,

∠BOE. **4.** a) ha; b) yo‘q. **5.** 1) ∠AOC=30°; ∠BOC=30°. 2) ∠AOC=88°; ∠BOC=88°. **6.** 1) 24,5° ; 2) 39,5°; 3) 71°. **7.** 1) a va h; 2) c, d, e, g; 3) b va f. **8.** a) ha; b) ha; c) yo‘q. **9.** a) 87°56'; b) 128°29' 47"; c) 1°47' 29". **11.** a) 72° ; b) 60°19' ; c) 55°45'. **12.** 1) ∠CBF; 2) ∠DBF. **14.** ∠POQ>∠ABC. **15.** a) ha; b) yo‘q; c) yo‘q. **16.** 176°. **17*.** 61°. **18*.** a) 90°; b) 180°. **19.** ∠AOB=60°, ∠AOC=90°, ∠AOD=130°, ∠BOC=30°, ∠BOD=70°; ∠COD=40°. Bissektrisalari: 1) 30°; 2) 45°; 3) 65°; 4) 15°; 5) 35°; 6) 20° burchaklardan o‘tadi. **20.** 3) 15°; 2) 22,5°; 5) 30°; 6) 45°; 1) 90°; 4) 135°.

4

7. a) Toshkent va Farg‘ona shaharlari o‘rtasidagi masofa 248 km. b) Toshkent va Termiz shaharlari o‘rtasidagi masofa 444 km. c) Toshkent va Jizzax shaharlari o‘rtasidagi masofa 126 km. d) Toshkent va Guliston shaharlari o‘rtasidagi masofa 90 km. e) Toshkent va Samarqand shaharlari o‘rtasidagi masofa 183 km. f) Toshkent va Qarshi shaharlari o‘rtasidagi masofa 280 km. g) Toshkent va Navoiy shaharlari o‘rtasidagi masofa 319 km. h) Toshkent va Urganch shaharlari o‘rtasidagi masofa 704 km. i) Toshkent va Nukus shaharlari o‘rtasidagi masofa 777 km. j) Toshkent va Qoraqalpog‘iston o‘rtasidagi masofa 1019 km. **8.** 246 chaqirim. **9.** 1) 120°; 2) 6 soat. **10.** 1) 38,1 cm; 2) 43,18 cm; 3) 48,26 cm. **11.** 1) Yerdan Quyoshgacha 149 637 000 km. 2) Veneradan Quyoshgacha 107 803 000 km. 3) Merkuriydan Quyoshgacha 57 924 000 km. 4) Marsdan Quyoshgacha 226 869 000 km. 5) Yupiterdan Quyoshgacha 777 147 000 km. 6) Saturndan Quyoshgacha 1 427 183 000 km. 7) Urandan Quyoshgacha 2 868 847 000 km. 8) Neptundan Quyoshgacha 4 498 764 000 km. **12.** a) 2235 m; b) 7822,5 m; c) 10 877 m.

1-nazorat ishi: 1. BC=3 cm. 2. BC=12 cm. 3. ∠BOC=35°. 4. ∠O=150°.

5

3. 45°. **4.** a) 8 ta; b) 8 ta; c) 8 ta; d) 8 ta. **5.** 5 ta o‘tkir; 1 ta o‘tmas. **6.** Ikki marta bukish orqali hosil qilish mumkin. **7.** 3^00, 9^00. **8.** a) 105°; b) 75°; c) 105°. **9.** a) 30°; b) 180°; c) 1°. **10.** a) 160°; b) 90°; c) 35°; d) 171°. **11.** A) 146°; b) 71°; c) 175°; d) 13°. **12.** ∠AOB=135°, ∠BOC=45°. **13.** ∠AOB=36°, ∠BOC=144°. **14.** a) Yo‘q; b) ha; c) yo‘q. **15.** a) 140°; b) 137°. **16.** a) 45°; b) 45°; c) 53°. **17.** Ha, agar ikki burchak ham 90° dan iborat bo‘lsa. **18.** a) 40°; 140°; b) 55°; 125°; c) 18°; 162°. **19.** 140°, 40°, 140°.

20.

∠1	34°	62°	48°	19°	175°
∠2	146°	118°	132°	161°	5°

21.

∠1	12°	162°	120°	15°	45°
∠2	168°	18°	60°	165°	135°

22. 1) 110°; 2) 50°; 3) 50°. **23.** 149°; 2) 59°; 3) 59°. **24.** a) 45°; 135°; b) 60°; 120°; c) 30°; 60°; 90°. **25.** a) 75°; 105°; b) 100°; 80°; d) 108°; 72°. **26.** a) 120°; б) 80°; в) 60°; 60°; 60°. **27.** г) 90°; 90°; 90°; д) 75°; 105°; 75°; e) 46°; 46°; 134°. **28.** 135°. **29.** 135°. **30.** a) ∠3=∠4; ∠9=∠10; b) ∠1 va ∠2. **31.** OC nur ∠AOD ning; OD nur ∠COE ning; OE nur ∠DOB ning; OD nur ∠AOB ning bissektrisasi bo‘ladi. **32.** A nuqta B va C nuqtalar orasida yotadi.

6

3. 30°. 5. AO=1 cm ,AB=2 cm, AC=1 cm. **6.** 13700 km. **7.** a) B nuqta qolgan ikkitasi orasida yotadi; b) A nuqta qolgan ikkitasi orasida yotadi. **8.** 90°. 10. Nurlardan OC qolgan ikkitasining orasida yotadi. **11.** 60°; 60°. **12.** 80°, 100°, 80°, 100°. **13.** OE va OF bir to‘g‘ri chiziqda yotadi. **6 – testlar: 1.** E; **2.** D; **3.** D; **4.** A; **5.** E; **6.** B; **7.** E; **8.** E; **9.** B; **10.** A; **11.** A; **12.** D; **13.** E; **14.** B; **15.** A; **16.** A; **17.** B; **18.** E

7

2. 90°. **3.** 60°. **4.** Yo‘q. **5.** Masala 2 ta yechimga ega: 1) 65°; 2) 15°. **6.** 15°. **7.** a) ∠AOC=45°; ∠BOC=45°; b) ∠AOC=30° ; ∠BOC=30°; c) ∠AOC=25° ; ∠BOC=25°; d) ∠AOC=10°; ∠BOC=10°. **8.** ∠DOE=60°. **9.** Yo‘q. **10.** Masala ikkita yechimga ega: 1) 0,5 m; 2) 5,9 m. **11.** a) AC=9m, BC=6 m; b) AC=7,5 m, BC=7,5 m; c) AC=6 m, BC=9m. **13.** a) 15 ta; b)

21ta; c) 45 ta. **15.** 1,3. **16.** 6 ta. **17.** 4.30 yoki 7.30. **18.** 6. **19.** ∠AOB=110°, ∠BOC=70°; b) ∠AOB=36°, ∠BOC=144°; c) ∠AOB=112°, ∠BOC=68°; d)∠AOB= 150°, ∠BOC=30°. **20.** 50°, 130°, 50°, 130°. **21.** a) C∈AB; b) A∈BC. **22.** a) x=180°-α ; y=α . b) x=90° ; y=90°;

c) x=180°-α ; y=α. **23.** a) 6 ta; b) 10 ta; c) $\frac{(n+1)(n+2)}{2}$ *ta* 2 ta. **24.** $b_{50}=\frac{2^{49}}{10}$ *mm.*

8 **6.** . 1,2,3,4,5,7 raqam belgilari siniq chiziq bo‘ladi. **7.** a) a, b, d, e, g; b) c, f, h; c) c, f. **9.** a) to‘g‘ri; b) o‘tkir; c) teng yonli; d) teng tomonli; e) o‘tmas burchakli. **10.** a) to‘g‘ri burchakli uchburchak, b) o‘tkir burchakli uchburchak, c) teng tomonli uchburchak, d) teng yonli uchburchak, e) o‘tmas burchakli uchburchak. **11.** a) 12,2 dm; b) 102,4 m. **12.** a) 139,6 mm; b) 102,4 m. **14.** Har bir burchagi 60°. **15.** a) 3 ta; b) 3 ta; c) 3 ta. **16.** 1) ABE – o‘tmas; 2) ACE, BCE, DCE – to‘g‘ri; 3) BED – teng yonli; 4) AED – turli tomonli. **20.** Balandlik, bissektrisa, mediana va ichki burchaklar. **21.** To‘g‘ri burchakli uchburchak. **22.** Ha. **23.** 3. **24.** 9. **25.** 16. **27.** ΔKLM – to‘g‘ri burchakli uchburchak, ML=5 cm. **28.** 18 cm; 24 cm; 30 cm. Bu to‘g‘ri burchakli uchburchak. **29.** a=36 mm; b=48 mm; c=60 mm.

9 **3.** AB=BA; BC=AC; AC=BC. **4.** ∠MNL= ∠LMN; ∠MLN=∠LNM; ∠NML=∠MLN. **5.** ∠B=80°; ∠D=52°; ∠F=48°. **6.** AB=LM=5; BC=MN=8; AC=LN=9. **7.** x=5. **12.** e) ∠D=35°, ∠C=62°. **13.** 85°. **15.** Yo‘q.

10 **1.** 10 cm. **2.** a=12 ; b=8. **8.** AB=8 cm; BC=8 cm; AC=11 cm.

11 **2.** 4. **3.** ∠E=∠N=35°; ∠D=∠L=135°; ED=NL=7. **10.** AC=BD=7.

12 **5.** 6 ta. ΔABN=ΔCAK; ΔABC=ΔCNK; ΔBNK=ΔBCK; ΔBNC=ΔCKN; ΔBAC= ΔKAN; ΔBAN= ΔKAC. **8.** 3 ta. ΔABD=ΔACD; ΔABC=ΔBCD; ΔAOB=ΔCOD. **10.** 10,4 cm. **12.** 8 cm. **15.** $\angle C_1=90°$, $\angle A_1=30°$, $\angle B_1=60°$. **16.** 10 sm, 10 cm.

13 **5 – testlar: 1.** B; **2.** A; **3.** B; **4.** E; **5.** D. **6.** A. **7.** D; **8.** A; **9.** B; **10.** D; **11.** A; **12.** B; **13.** A; **14.** B; **15.** D; **16.** A. **6 – masalalar:** 1. 1) teng tomonli uchburchak; 2) to‘g‘ri burchakli uchburchak. 3) o‘tkir burchakli uchburchak; 4) teng yonli uchburchak; 5) o‘tmas burchakli uchburchak. 2. 1) uchburchaklar teng emas, TBT alomatiga mos emas; 2) uchburchaklar teng, TBT alomatiga ko‘ra teng; 3) uchburchaklar teng emas, TBT alomatiga mos emas. 4) uchburchaklar teng, BTB alomatiga ko‘ra teng; 5) uchburchaklar teng, TTT alomatiga ko‘ra teng; 6) uchburchaklar teng emas, TBT alomatiga mos emas. **7.** Ha. **8.** ΔABD=ΔCAD; ΔAED=ΔAFD; ΔBED=ΔCFD. **11.** 58°. **12.** 48°. **13.** 120°.

3-nazorat ishi: 1. 10. **3.** $7\frac{1}{3}$, $7\frac{1}{3}$, $3\frac{11}{15}$.

14 **2.** a∥b; c∥d. **4.** Ha. **6.** 1) a∥b, 3 ta bo‘lak hosil bo‘ladi. 2) a∦b, 4 ta bo‘lak hosil bo‘ladi. **8.** a) vertikal burchaklar: ∠1 va ∠4; ∠2 va ∠3; ∠5 va ∠7; ∠6 va ∠8; b) qo‘shni burchaklar: ∠1 va ∠2; ∠2 va ∠4; ∠4 va ∠3; ∠3 va ∠1; ∠5 va ∠6; ∠6 va ∠7; ∠7 va ∠8; ∠8 va ∠5. **9.** ∠1=∠6=63°; ∠2=∠3=∠5==∠7=117°. b) ∠2=∠3=∠5=∠7=117°; ∠4=∠8=63°. **10.** ∠1=∠3==∠5=∠7=117°; ∠4=∠8=63°. **11.** ∠2=∠4=∠6=∠8=58°; ∠3=∠5=122°. **12.** 98°, 82°, 98°; 70°, 110°, 70°. **13.** ∠2=148°; ∠3=148°; ∠4=32°; ∠6=46°; ∠7=134°; ∠8=46°. **14.** ∠3=∠5 ichki almashinuvchi; ∠2=∠8 tashqi almashinuvchi; ∠4=∠6 ichki almashinuvchi burchaklar teng. **15.** Tengliklar bajariladi. **16.** Mumkin. **17.** ∠1+∠8=180°; ∠2+∠7=180°; ∠4+∠5=180°; ∠3+∠6=180°. **18.** almashinuvchi burchaklar o‘zaro teng bo‘ladi.

15 **1.** x=150°. **3.** x=60°; y=120°. **8.** a) ha; b) ha. **9.** a) yo‘q; b) ha. **11.** 1 tasi kesmasligi mumkin yoki hammasi kesib o‘tadi. **12.** AD∥BC . **13.** a) a∦b; b) a∥b. **14.** x=116°. **15.** ∠2=75°; ∠3=105°; ∠4=75°; ∠6=75; ∠7=105°; ∠8=75°. **16.** ∠1=60°; ∠2=120°; ∠4=120°; ∠5=60°;

∠6=120°; ∠7=60°. **17.** ∠2 = ∠4 = ∠6 = ∠8 = 75°. **18.** a) BC ∥ AD; AB ∥ CD. б) AD ∥ BC; в) AD ∥ BD

16 **1.** a) noto‘g‘ri; b) noto‘g‘ri; c) noto‘g‘ri; d) noto‘g‘ri; e) noto‘g‘ri; f) noto‘g‘ri; g) noto‘g‘ri; h) noto‘g‘ri. **2.** a) to‘g‘ri; b) to‘g‘ri; c) to‘g‘ri. **3.** a) to‘g‘ri; b) to‘g‘ri; c) to‘g‘ri. **4.** ∠2=45°; ∠3=135°; ∠4=45°; ∠5=135°; ∠6=45°; ∠7=135°; ∠8=45°. **5.** ∠1=131°; ∠3=131°; ∠4=49°; ∠5=131°; ∠6=49°; ∠7=131°; ∠8=49°. **9.** 45°. **10.** x=11°. **14.** ∠2 = ∠3 = 53°. **15.** ∠1=73°, ∠2=73°. **16.** 70°, 110°. **19.** 70°, 110°. **20.** a) mumkin b) mumkin. **21.** a) mumkin b) mumkin. **23.** x=105°. **24.** x=35°. **25.** ∠1=78°; ∠2=102°; ∠3=78°; ∠4=102°; ∠5=78°; ∠6=102°; ∠7=78°; ∠8=102°. **26.** ∠1=64°; ∠2=116°; ∠3=64°; ∠4=116°; ∠5=64°; ∠6=116°; ∠7=64°; ∠8=116°. **27.** Yo‘q. **28.** x=42°. **29.** 8 ta burchakdan 4 tasi o‘tmas bo‘ladi. Agar bunday bo‘lmasa, burchaklarning hammasi 90° ga teng bo‘ladi. a⊥c; b⊥c; a∥b bo‘ladi. **30.** ∠1=∠3=∠5=∠7=50°; ∠2=∠4=∠6=∠8=130°. **32.** ∠D=98°

17 **5 – testlar: 1.** A; **2.** B; **3.** A; **4.** D; **5.** D; **6.** D; **7.** D; **8.** E; **9.** B; **10.** B; **11.** D; **12.** E; **13.** A; **14.** B; **15.** E; **16.** A. **6 – masalalar: 1.** Ha. **2.** Ha. **3.** ∠4=∠7=118°; ∠2=∠3=∠6=∠8=62°. **4.** a∦b. **5.** x=55°. **6.** 128°. **8.** ∠1=∠3=∠5=∠7=42°; ∠2=∠4=∠6=∠8=133°. **9.** ∠1=∠3=∠5=∠7=20°; ∠2=∠4=∠6=∠8=160°. **10.** ∠1=∠3=∠5=∠7=75°; ∠2=∠4=∠6=∠8=105°. **11.** 59°. **12.** ∠1=∠3=∠5=∠7=72°; ∠2=∠4=∠6=∠8=108°. **4-nazorat ishi: 1.** ∠3=∠5=∠7=34°; ∠2=∠4=∠6=∠8=146°. **2.** 52°.

18 **1.** a) 80°; b) 25°; c) 45°; d) 45°. **2.** a) 63°; b) 90°; c) 113°, d) 15°. **3.** 102°. **4.** a) 80°, 50°; b) 30°; 60°; 90°; c) 50°, 60°, 70°. **5.** a) 65°; b) 45°; 90°; 45°. **6.** a) 79°; b) 105°. **7.** x=20°,y=50°. **9.** x+y=270°. **10.** 60°. **11.** 60°, 60°, 60°. **12.** 45°, 45°, 90°. **13.** a) 65°, 65; b) 60°; 60°; c) 37,5°; 37,5°. **14.** 60°, 45°, 75°. **15.** 30°, 120°. **16.** a) 75°; b) 34°; 68°; c) 50°. **17.** a) 128° ; b) 51°.
18. x+y+z=540°-(α+β+γ). **19.** 50°; 90°; 40°. **20.** a) 90°; b) 50°; c) 110°, d) 60°. **21.** 60°; 48°. **22.** 540°. **23.** 24°, 36°, 60°. **25.** a) 30°, 30°; b) 55°, 55°. **26.** a) 15°, 150°; b) 75°, 30°. **28.** ∠AOB=122,5°; ∠EOC=65°. **29.** 30°. **30.** 67,5°. **31.** Ha. **32.** Mumkin emas. **33.** ∠A=80°, ∠B=30°, ∠C= 70°. **34.** ∠A=35°; ∠B=35°. **35.** a) 70°; b) 75°; d) 80°; e) 72°; f) 120°; g) 80°; h) 36°

19 **1.** a) AB; b) AB. **2.** a) PR; b) PR. **3.** ∠A=67°; ∠B=23°; ∠C=90°. **4.** a) 78°; 12°; 90°; b) 47°; 43°; 90°. **5.** ∠A=60°; ∠B=30°; ∠C=90°; AC=17; BC=17√3. **6.** 60°, 30°, 90°. **7.** 1) 4; 2) 6; 3) 60°. **8.** a) 5; b) 13,5; c) 9. **9.** 1) 35°; 2) 130°; 3) 90°; 4) 130°; 5) 100°; 6) 90°. **10.** a) Yo‘q; b) yo‘q; d) ha; e) yo‘q.

20 **1.** a) bo‘ladi; b) bo‘ladi; d) bo‘ladi; e) yo‘q; f) yo‘q. **2.** a) bo‘ladi; b) bo‘ladi; d) bo‘ladi; e) yo‘q; f) bo‘ladi. **6.** 7 cm. **7.** 7 sm, 7 cm. **11.** 180°. **12.** 270°. **13.** 6. **14.** 8 cm. **17.** ∠B=80°.

21 **1.** BC; **2.** AB va BC (AB=BC). **3.** a) ∠A yoki ∠C; b) ∠B. **4.** a) ∠C; b) ∠B. **5.** a) Mumkin; b) mumkin emas. **6.** a) mumkin emas; b) mumkin. **7.** a) mavjud; b) mavjud emas; c) mavjud; d) mavjud. **8.** a) 7; b) 10; c) 8 yoki 5. **9.** a) mumkin. Uchburchakning ikki tomoni yig‘indisi uchinchi tomonidan katta bo‘lishi zarur. Teng ham, kichik ham bo‘lmasligi kerak. Aks holda uchburchak hosil bo‘lmaydi; b) mumkin emas. Uchburchak hosil bo‘lmaydi; c) mumkin emas. Uchburchak hosil bo‘lmaydi. **10.** a) mumkin. Uchburchakning ikki tomoni yig‘indisi uchinchi tomonidan katta bo‘lishi zarur. Teng ham, kichik ham bo‘lmasligi kerak. Aks holda uchburchak hosil bo‘lmaydi; b) mumkin emas. Uchburchak hosil bo‘lmaydi; c) mumkin emas. Uchburchak hosil bo‘lmaydi. **11.** a) eng katta burchak A, eng kichik burchak C; b) eng katta burchak A, eng kichik burchak C yoki B; c) eng katta burchak A yoki B. eng kichik burchak C. **12.** a) BC<AB<AC; b) BC=AB<AC; c) BC=AC=AB. **14.** 7; 7; 11. **15.** 7

ta. **16.** Uchburchak yoki kesma. **18.** Eng kattasi ∠ACB, eng kichigi ∠ABC. **19.** Asosi, yon tomoni. **20.** a) BC>AC>AB; b) BC=AC<<AB. **21.** 60°; 120°; 60°; 120°. **22.** 0°<∠B<60°. **23.** O‘tkir burchakli uchburchak. **24.** Eng kattasi BC, eng kichigi AB. **25.** Uchburchakning eng kichik tomoni BC. **26.** a) uchburchak mavjud emas. b) uchburchak mavjud emas. **27.** Uchburchak mavjud emas.

22 **1.** 1) 180°; 2) 360°. **2.** ∠B=75°; ∠C=75°. **3.** c=10 cm . **6.** Ichki burchaklar yig‘indisi α=1080° ga teng. Tashqi burchaklar yig‘indisi β=360° ga teng . **7.** 72°. **8.** a=20 cm, b=40 cm, c=40 cm. **9.** a) ∠A=∠B=45° li bo‘lgani uchun daraxt uzunligi uning soyasi uzunligiga teng; b) AB=A_1 B_1 bo‘lgani uchun AB ko‘lning kengligi shuncha bo‘ladi; c) AB qancha uzunlikda bo‘lsa, AC ko‘l uzunligi unga teng bo‘ladi; d) jarning uzunligi AB=2BC bo‘ladi. **10.** Ko‘lning uzunligi AB=AC-CB ga teng. **11.** Yo‘q.

4 – testlar: 1. B; **2.** C; **3.** B; **4.** B; **5.** D; **6.** B; **7.** B; **8.** B; **9.** E; **10.** A; **11.** D; **12.** A; **13.** D; **14.** A; **15.** D; **16.** D; **17.** D; **18.** D. **19.** A.

5 – masala. 1. Mumkin emas. **2.** 1) a=1; b=7 ; c=7. 2) a=2; b=6; c=7. 3) a=3; b=5; c=7. 4) a=4; b=4; c=7. 3. Yo‘q, h_b>b; h_b=a; h_b=b. 4. 18. 5. 63°, 63°, 54°. 7. 60°. 8. 90°. 10. 40°, 40°, 100°. 11. Teng bo‘ladi. **13.** Ha. **14.** 20. **15.** 13. **16.** 1) 24°; 2) 70°. 17. 35°, 110°, 35°. **18.** 134°. **19.** 90°. **20.** Ha. **21.** 39°,117°,24°.

5-nazorat ishi: 1. 65°. **2.** 40°,60°,80°. **3.** 12 cm. **4.** 40°,60°,80°.

23 **Testlar: 1.** A; **2.** D; **3.** B.

24 **1.** 20 cm. **2.** 20°. **3.** 15°. **4.** 30°. **5.** 40°; 60°; 80°. **6.** 1:2. **7.** 76°. **8.** 42°. **9.** 21°, 69°. **10.** ∠AOB=122,5°. **11.** 72°. **12.** 46°. 28. O‘tkir burchakli. **30.** a) 92°; b) 42°. **31.** 6; 6; 6; 60°. **33.** 45°. **35.** Kesma. **37.** 144°. **38.** 54°. **39.** 3,6 cm.

25 **5 – Testlar: 1.** A; **2.** E; **3.** D; **4.** B; **5.** E; **6.** A; **7.** E; **8.** D. **9.** B. **10.** A. **11.** A; **12.** D; **13.** B; **14.** D; **15.** E; **16.** A; **17.** B; **18.** E; **19.** A; **20.** D. **6 – Masalalar: 2.** 12 cm. **3.** 12 sm. **4.** 34. **5.** 2,8 cm. **6.** 83°. **7.** 15,6 cm. **8.** 55°. **10.** 2 m yoki 16 m. **11.** 81°, 99°. **12.** 125°; 55°; 55°; 125°; 125°; 55°; 55°; 125°. **13.** Ha. **14.** 9 cm. **15.** 30°, 150°. **16.** 80°, 100°. **17.** 5,3 cm. **18.** 69°. **19.** 109°; 71°; 71°; 71°; 109°; 109°; 71°; 109°;

7. Murakkab masalalar: 1. a) 149900 ming km. b) 149100 ming km. **2.** D nuqta.

3. $\alpha = \frac{540^\circ}{7}$; $\beta = \frac{720^\circ}{7}$. **6.** $P = \frac{a+b+c}{2}$. **11.** Yo‘q. **12.** 120°;. **13.** Yo‘q.

Yakuniy nazorat ishi: 1. 135°, 135°, 45°. **2.** 52° **3.** b) 22 cm. **4.** 7 cm. **5.** ∠B=45°; ∠C=45°.



Darslikni tuzishda foydalanilgan va qo‘shimcha o‘rganishga tavsiya etilayotgan o‘quv-uslubiy adabiyotlar va elektron resurslar

1. *A’zamov A., Haydarov B.* Matematika sayyorasi. Toshkent: O‘qituvchi, 1993.
2. *Saitov Y.* Matematika va matematiklar haqida. Toshkent: O‘qituvchi, 1992.
4. *Afonina S. I.* Matematika va go‘zallik. Toshkent: O‘qituvchi, 1986.
6. *Ismailov A., Xaydarov B., Karimov N., Sh.Ismailov.* Xalqaro tadqiqotlarda o‘quvchilarning matematik savodxonligini baholash, metodik qo‘llanma. – Toshkent, 2019.
8. *Погорелов А. В.* Геометрия 7–9, учебник. Москва: Просвещение, 2004.
9. *Атанасян С.* Геометрия 7–9 классы, учебник. Москва: Просвещение, 2002.
10. *Шарыгин Ф.* Геометрия 7–9 классы, учебник. Москва: Дрофа, 2000.
11. *Штейнгауз Г.* Математический калейдоскоп. Москва: Наука, 1993.
12. *Perelman Y. I.* Qiziqarli geometriya, Toshkent: O‘qituvchi, 1981.
13. *Кордемский Б. А.* Математическая смекалка. Москва: Наука, 1991.
16. *Бевз Г. П.* и др. Геометрия 7 учебник, Киев: Вежа, 2007.
17. *Александров А. Д.* Геометрия -7 учебник, Москва. Просвещение, 2013.
18. *Johannes Paasonen* Ahaa mathematiikkaa 7, Porvoo-Helsinki-Juva, 1993.
19. *Daniel C. Alexander*, Elementary geometry for college students. Canada, Brooks/Cole, Cengage Learning, 2011.
20. http://www.uzedu.uz – Xalq ta’limi vazirligining rasmiy sayti, axborot ta’lim portali
21. http://centeroko.ru. – Ta’lim sifatini baholash markazi (Rossiya) sayti.
22. http://www.markaz.tdi.uz – Ta’lim sifatini baholash bo‘yicha xalqaro tadqiqotlarni amalga oshirish milliy markazi sayti.
23. http://www. avloniy.uz – A. Avloniy nomidagi pedagoglarni kasbiy rivojlantirish va yangi metodikalarga o‘rgatish milliy-tadqiqot instituti sayti.
24. http://www.masofa.uz – A. Avloniy nomidagi pedagoglarni kasbiy rivojlantirish va yangi metodikalarga o‘rgatish milliy-tadqiqot instituti masofadan o‘qitish portali.
25. http://www.onlinedu.uz – A. Avloniy nomidagi pedagoglarni kasbiy rivojlantirish va yangi metodikalarga o‘rgatish milliy-tadqiqot instituti “Uzluksiz kasbiy ta’lim” elektron platformasi.
26. http://www.ziyonet.uz – “Ziyonet” ijtimoiy ta’lim portali.
27. http://www.rtm.uz – Respublika ta’lim markazi sayti.
28. http://dr.rtm.uz – yangi darsliklar, o‘qituvchilar uchun uslubiy qo‘llanmalarning elektron shakllari, taqdimotlar, multimedia ilovalari, videodarslar va raqamli resurslar platformasi.
29. http://www.maktab.uz – 1–11-sinflar uchun onlayn maktab hamda maktab o‘quv dasturi bo‘yicha videodarslar va boshqa materiallar platformasi.
30. http://www.stesting.uz – O‘quvchilar uchun “Xalqaro baholash tadqiqotlariga tayyorlanish” elektron platformasi (o‘zbek tilida).
31. http://www.skillsgrover.uz – Finlandiyaning matematikani o‘rganish va o‘quvchilar bilimlarini baholash platformasi (o‘zbek tilida).
32. http://www.khanakademy.org – “Xon akademiyasi” masofaviy ta’lim sayti (ingliz tilida).
33. http://www.xanakademiyasi.uz – matematika, informatika, kimyo, fizika, iqtisodiyot, biologiya va astronomiya kabi fanlar bo‘yicha videodarslar platformasi (o‘zbek tilida).
34. http://www.school.edu.ru – umumta’lim portali (rus tilida).
35. http://www.problems.ru/ – matematikadan masalalar izlash tizimi (rus tilida).
36. http://geometry.net/ – algebra va geometriyadan o‘quv materiallari (ingliz tilida).
37. http://mathproblem.narod.ru/ – matematik to‘garaklar va olimpiadalar (rus tilida).
38. http://www.ixl.com – masofadan turib o‘qitish matematika ta’limi portali (ingliz tilida)
39. http://www.mathkang.ru – “Kenguru” xalqaro matematik tanlovi sayti (rus tilida).
40. http://www.olimpia.uz – “Kenguru” xalqaro matematik tanlovi sayti (o‘zbek tilida).
41. http://www.brilliant.org – matematikadan masofaviy ta’lim sayti (ingliz tilida).
42. http://www.geogebra.com – geometriya va algebra fanlari bo‘yicha dinamik (“jonli”) chizmalar yaratish imkoniyatini beradigan bepul dastur.
43. http://www. yaklass.ru – maktab o‘quvchilari va o‘qituvchilar uchun online ta’lim platformasi.
44. www.schulen-ans-netz.de – Germaniya “Internet maktab”i sayti (nemis tilida).
45. www.studienkreis.de – Germaniya o‘quv to‘garaklari sayti (nemis tilida).

www.idum.uz saytidan yuklab olindi

O‘quv nashri

GEOMETRIYA

Umumiy o‘rta ta’lim maktablarining
7-sinfi uchun darslik

Muharrir – Xurshidbek Ibrohimov
Texnik muharrir – Akmal Sulaymonov
Musahhih – Orifjon Madvaliyev
Sahifalovchi va dizayner – Ixvoldin Salohitdinov
Rassom – Umid Sulaymonov

Original-maketdan bosishga ruxsat etildi __.__.2022. Bichimi 70x90 1/16. “Arial” garniturasi. Ofset bosma usulda bosildi. Shartli bosma tabog‘i 14,0. Nashr bosma tabog‘i 13,0. Nusxasi _____. Buyurtma N_____.

Shartnoma N___.

www.idum.uz saytidan yuklab olindi

Ijaraga berilgan darslik holatini ko‘rsatuvchi jadval

T/r	O‘quvchining ismi va familiyasi	O‘quv yili	Darslikning olingandagi holati	Sinf rahbari-ning imzosi	Darslikning topshiril-gandagi holati	Sinf rahbari-ning imzosi
1						
2						
3						
4						
5						
6						

Darslik ijaraga berilib, o‘quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to‘ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko‘chmagan, betlarida yozuv va chiziqlar yo‘q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta’mirlangan. Ko‘chgan varaqlari qayta ta’mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqovaga chizilgan, yirtilgan, asosiy qismdan ajralgan yoki butunlay yo‘q, qoniqarsiz ta’mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo‘yab tashlangan. Darslikni tiklab bo‘lmaydi.