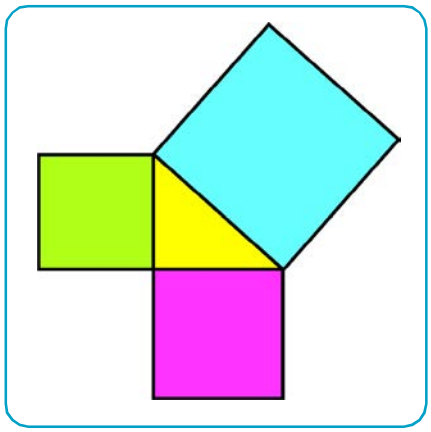
GEOMETRIYA 8

Umumiy o‘rta ta’lim maktablarining 8- sinfi uchun darslik

Qayta ishlangan va to‘ldirilgan 4- nashri

O‘zbekiston Respublikasi Xalq ta’limi vazirligi tasdiqlagan



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«O‘ZBEKISTON»
2019

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Taqrizchilar:

Darslik Respubllika ta’lim markazi tomonidan 2018-yil 25-noyabrda berilgan «Aniq fanlar blok moduli bo‘yicha umumiy o‘rta ta’limning o‘quv dasturi (VIII sinf)» asosida yozilgan. Darslikda belgilangan umumiy o‘rta ta’limda matematika fanini o‘qitishning maqsadi va vazifalari, o‘quvchilarga o‘quv faoliyati natijasida qo‘yiladigan talablar aks etgan. Darslik o‘quvchilarda shakllantiriladigan tayanch kompetensiyalar elementlarini qamrab olgan.

Qayta ishlash jarayonida ekspertlar va taqrizchilarning takliflari inobatga olindi.

Har bir bob oxirida yozma nazorat ishlaridan namunalar va testlar keltirilgan bo‘lib, ular o‘quvchilarning nazorat ishiga puxta tayyorgarlik ko‘rishlarida yordam beradi.

Tarixiy ma’lumotlar ruknida yurtimiz va dunyo olimlarining fanga qo‘shgan ulkan hissalari va tarixiy-ilmiy ishlari bilan tanishasiz.

«Ingiliz tilini o‘rganamiz» ruknida mavzularda uchraydigan muhim geometrik tushunchalarning ingliz tilidagi tarjimasi berib o‘tilgan.

Takrorlashga berilgan masalalardan yil davomida foydalanishingiz mumkin.

Mavzularda yoritilgan bilimlarni o‘rganishingizda Sizlarga muvaffaqiyatlar tilaymiz!

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- ? — faollashtiruvchi savol va topshiriqlar;
- — sinfda ishlanadigan mashqlar;
- — rivojlantiruvchi mashqlar;
- — masala yechish namunasi;
- — uy vazifasi uchun mashqlar.

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7- SINFDA O‘TILGANLARNI TAKRORLASH



1. Uchburchakning perimetri, bissektrisasi va balandligiga doir masalalar

Savol, masala va topshiriqlar

1. Uchburchakning perimetri, medianasi, balandligi va bissektrisasi deb nimaga aytiladi?

2. Perimetri 18 cm ga teng bo‘lgan uchburchakning bissektrisasi uni perimetri 12 cm va 15 cm ga teng bo‘lgan uchburchaklarga ajratadi. Uchburchakning bissektrisasini toping (1- rasm).

3. Uchburchakning asosiga tushirilgan medianasi uni perimetri 18 cm va 24 cm ga teng ikkita uchburchakka ajratadi. Berilgan uchburchakning kichik yon tomoni 6 cm ga teng. Uning katta yon tomonini toping (2- rasm).

4. ABC uchburchakda $AB = BC$ va BD mediana 6 cm ga teng. ABD uchburchakning perimetri 24 cm ga teng. Berilgan uchburchakning perimetrini toping (3- rasm).

Berilgan: $\triangle ABC$ da: $AB = BC$, $BD = 6$ cm $-$ mediana, $P_{ABD} = 24$ cm.

Topish kerak: $P_{ABC} = ?$

Yechish. 1) $P_{ABD} = AB + BD + AD$, bundan:

$$24 = AB + AD + 6, \quad AB + AD = 24 - 6, \quad AB + AD = 18 \text{ (cm)}.$$

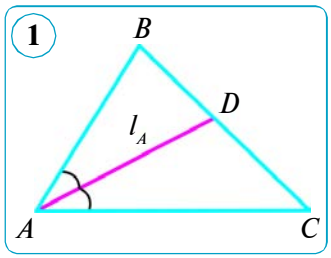
2) $AB = BC$ va $AC = 2AD$, u holda

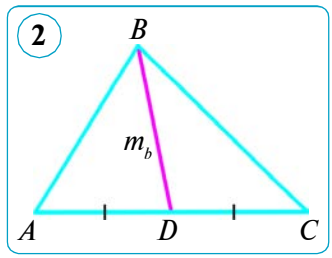
$$P_{ABC} = AB + BC + AC = 2(AB + AD) = 2 \cdot 18 = 36 \text{ (cm)}.$$

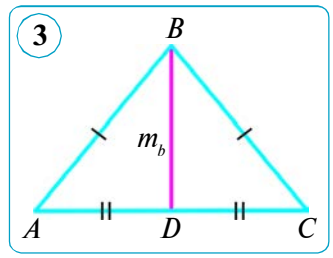
Javob: $P_{ABC} = 36$ cm.

5. Uchburchakning ikki tomoni 0,5 dm va 8,7 dm ga teng. Uchinchi tomoni uzunligi natural son ekanini bilgan holda shu tomonini toping.

6. Perimetri 30 cm ga teng bo‘lgan uchburchakning bissektrisasi uni perimetrlari 16 cm va 24 cm ga teng bo‘lgan uchburchaklarga ajratadi. Uchburchakning bissektrisasini toping.







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7. Perimetri 36 cm ga teng bo‘lgan uchburchakning balandligi uni perimetrlari 18 cm va 24 cm ga teng bo‘lgan uchburchaklarga ajratadi. Uchburchakning balandligini toping.

8. Teng yonli uchburchakning perimetri 22,5 cm, yon tomoni esa 0,6 dm. Shu uchburchakning asosini toping.

2. Uchburchaklar tengligining alomatlari, uchburchak burchaklarining yig‘indisi va tashqi burchagining xossasiga doir masalalar

9. ABC va DEF uchburchaklarda: $AB = DE$, $AC = DF$, $\angle A = \angle D$. Bu uchburchaklar tengmi?

10. Uchburchakning 117° li tashqi burchagiga qo‘shni bo‘lmagan ichki burchaklarining nisbati 5 : 4 kabi. Uchburchakning ichki burchaklarini toping.

11. Teng tomonli ABC uchburchakning AD va BE bissektrisalari O nuqtada kesishadi. Bissektrisalar orasidagi AOE burchakni toping.

12. Teng yonli uchburchakning asosidagi burchagi o‘tmas bo‘la oladimi?

Yechish. Bizga ma’lumki, teng yonli uchburchakning asosidagi burchaklari teng. Ammo ikkita o‘tmas burchakning yig‘indisi 180° dan katta bo‘ladi. Bu uchburchak ichki burchaklarining yig‘indisi haqidagi teoremaga zid. *Javob:* yo‘q, bo‘la olmaydi.

13. Uchburchakning 108° li tashqi burchagiga qo‘shni bo‘lmagan ichki burchaklarining nisbati 2 : 7 kabi. Uchburchak ichki burchaklarini toping.

14. Bir uchburchakning ikki tomoni va burchagi mos ravishda ikkinchi uchburchakning ikki tomoni va burchagiga teng. Bundan shu uchburchaklarning tengligi kelib chiqadimi?

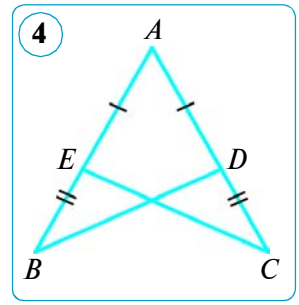
15. ABC va $A_1B_1C_1$ uchburchaklarda AB va A_1B_1, BC va B_1C_1 tomonlar teng hamda mos ravishda AB va A_1B_1 tomonlarga o‘tkazilgan CD va C_1D_1 medianalar ham teng. Uchburchaklarning tengligini isbotlang.

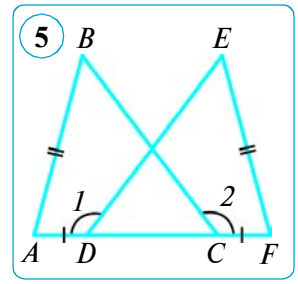
16. 4- rasmda $AB = AC$ va $AE = AD$. $BD = CE$ ekanini isbotlang.

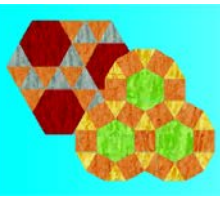
17. 5- rasmda $AD = CF$, $AB = FE$ va $CB = DE$. $\angle 1 = \angle 2$ ekanini isbotlang.

18. ABC uchburchakning B burchagi 42° ga, A uchidagi tashqi burchagi esa 100° ga teng. ACB burchakni toping.

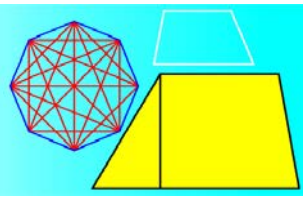
19. To‘g‘ri burchakli ABC uchburchakning C burchagi to‘g‘ri, A uchidagi tashqi burchagi esa 136° ga teng. B burchakni toping.







I BOB
TO‘RTBURCHAKLAR

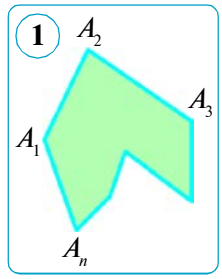


1- §. ASOSIY TO‘RTBURCHAKLAR VA ULARNING XOSSALARI

1. KO‘PBURCHAK ICHKI VA TASHQI BURCHAKLARINING XOSSASI

1. Ko‘pburchaklar. A_1A_2, A_2A_3, ..., $A_{n-1}A_n$, A_nA_1 kesmalardan tuzilgan shaklni ko‘rib chiqamiz. Kesmalar shunday joylashganki, hech qaysi ikki *qo‘shni kesma* (ular umumiy uchga ega) bir to‘g‘ri chiziqda yotmaydi, qo‘shni bo‘lmagan kesmalar esa umumiy nuqtaga ega emas (1- rasm). Bunday shakl ***ko‘pburchak*** deyiladi. A_1, A_2, ..., A_n nuqtalar (uchlar) *ko‘pburchakning* ***uchlari***, A_1A_2, A_2A_3, ..., $A_{n-1}A_n$, A_nA_1 kesmalar esa *ko‘pburchakning* ***tomonlari*** deb ataladi.

1

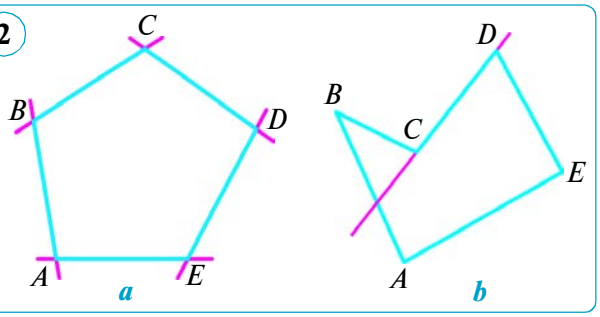


Ko‘pburchak tomonlari soni uning uchlari soniga, ya’ni burchaklari soniga teng. Ko‘pburchaklar uchlari (tomonlari) soniga ko‘ra *uchburchaklar*, *to‘rtburchaklar*, *beshburchaklar* va hokazolarga bo‘linadi.

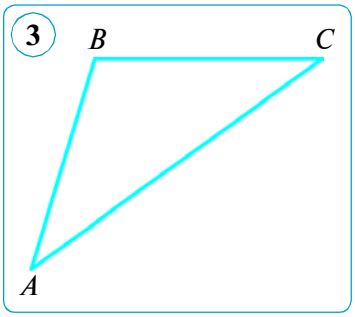
Agar yopiq siniq chiziq o‘z-o‘zi bilan kesishmasa, bunday siniq chiziq ***sodda yopiq siniq chiziq*** deyiladi. U tekislikning shu siniq chiziqqa tegishli bo‘lmagan nuqtalarini *ikki sohaga* – *ichki* va *tashqi sohaga* ajratadi hamda umumiy chegara vazifasini bajaradi. 1- rasmda ichki soha bo‘yab ko‘rsatilgan.

1- ta’rif. *Agar ko‘pburchak uning ixtiyoriy tomonini o‘z ichiga olgan to‘g‘ri chiziq bilan bitta yarim tekislikda yotsa, u* ***qavariq ko‘pburchak*** *deyiladi. Bunda to‘g‘ri chiziqning o‘zi ham shu yarim tekislikka tegishli hisoblanadi.*

2



3



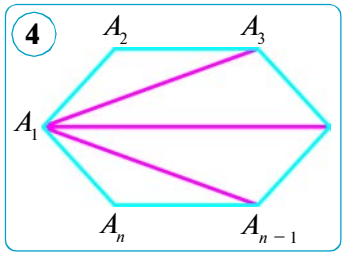
2- *a* va 3- rasmda qavariq ko‘pburchak, 2- *b* rasmda esa noqavariq ko‘pburchak tasvirlangan. Ixtiyoriy uchburchak – qavariq ko‘pburchakdir (3- rasm).

2. Ko‘pburchak ichki va tashqi burchaklarining xossasi.

2- ta’rif. ***Ko‘pburchakning berilgan uchidagi ichki burchagi*** *deb, uning shu uchida uchrashuvchi tomonlari hosil qilgan burchakka aytiladi.*

1- teorema.

Qavariq *n* burchak ichki burchaklarining yig‘indisi $180^\circ(n-2)$ ga teng, bunda *n* – tomonlar soni.

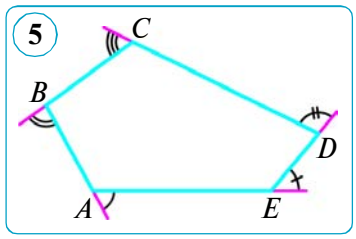


Isbot. $A_1A_2A_3...A_n$ – berilgan qavariq n burchak va $n > 3$ bo‘lsin (4- rasm). Biror uchidan, masalan A_1 dan, ko‘pburchakning barcha diagonallarini o‘tkazamiz. Bu diagonallar uni $(n-2)$ ta uchburchakka ajratadi. Haqiqatan, *ikki chetki uchburchaklar* ($\triangle A_1A_2A_3$ va $\triangle A_1A_{n-1}A_n$) ko‘pburchakning ikki tomoni va bir diagonali, qolgan uchburchaklar esa ko‘pburchakning bir tomoni va ikki diagonalidan tuzilgan. Shuning uchun uchburchaklar soni $(n-2)$ ta, ya’ni ko‘pburchakning tomonlari sonidan ikkitaga kam bo‘ladi. Ko‘pburchakning burchaklari yig‘indisi uni tashkil qiluvchi uchburchak burchaklari yig‘indisiga, ya’ni $S_n = 180^\circ(n-2)$ ga teng bo‘ladi. Teorema isbotlandi.

3- ta’rif. ***Ko‘pburchakning berilgan uchidagi tashqi burchagi*** *deb, uning shu uchidagi ichki burchagiga qo‘shni burchakka aytiladi.*

2- teorema.

Qavariq *n* burchakning har bir uchidan bittadan olingan tashqi burchaklarining yig‘indisi 360° ga teng.



Isbot. Ko‘pburchakning har qaysi uchida bittadan tashqi burchak yasaymiz. Ko‘pburchak ichki burchagi va u bilan qo‘shni bo‘lgan tashqi burchagining yig‘indisi 180° ga teng (5- rasm). Shu sababli barcha ichki va har bir uchidan bittadan olingan tashqi burchaklarining yig‘indisi $180^\circ n$ ga teng. Ammo ko‘pburchakning hamma ichki burchaklari yig‘indisi $180^\circ(n-2)$ ga teng. U holda har qaysi uchidan bittadan olingan tashqi burchaklarning yig‘indisi

$$180^\circ n - 180^\circ(n-2) = 180^\circ n - 180^\circ n + 360^\circ = 360^\circ$$

ga teng bo‘ladi. Teorema isbotlandi.

1- masala. Tomonlari teng bo‘lgan (muntazam) n burchakning har bir ichki burchagi (α_n) nimaga teng?

Yechish. Bizga ma'lumki, ixtiyoriy qavariq n burchakning burchaklari yig'indisi $180^\circ(n-2)$ ga teng. Muntazam ko'pburchakning burchaklari teng bo'lgani uchun ularning har biri quyidagiga teng: $\alpha_n = \frac{180^\circ(n-2)}{n}$.

2- masala. Tomonlari teng bo'lgan (muntazam) n burchakning har bir tashqi burchagi (β_n) nimaga teng?

Yechish. Bizga ma'lumki, ixtiyoriy qavariq n burchakning har bir uchidan bittadan olingan tashqi burchaklarining yig'indisi 360° ga teng.

Shunday qilib, tomonlari teng bo'lgan n burchakning har bir tashqi burchagi quyidagiga teng: $\beta_n = \frac{360^\circ}{n}$.

Savol, masala va topshiriqlar

1. 1) Ko'pburchakning berilgan uchidagi ichki burchagi deb qanday burchakka aytiladi? Tashqi burchagi deb-chi?

2) Qavariq n burchakning ichki burchaklari yig'indisi nimaga teng?

2. Ko'pburchak burchaklarining yig'indisi: 1) 1080° ga; 2) 1620° ga; 3) 3960° ga teng. Ko'pburchakning nechta tomoni bor?

3. 1) To'rtburchak; 2) o'nikkiburchak; 3) o'ttizburchak; 4) ellikburchakning ichki burchaklari yig'indisini toping.

Namuna. 1) $S_{13} = 180^\circ \cdot (13-2) = 180^\circ \cdot 11 = 1980^\circ$.

4. Agar to'rtburchakning uchtadan olingan burchaklari yig'indisi mos ravishda 240°, 260° va 280° bo'lsa, uning eng kichik burchagini toping.

5. Har bir ichki burchagi: 1) 150° ga; 2) 170° ga; 3) 171° ga teng bo'lgan qavariq ko'pburchakning nechta tomoni bor?

6. Ko'pburchak ichki burchaklarining yig'indisi har bir uchidan bittadan olingan tashqi burchaklari yig'indisidan uch marta katta. Shu ko'pburchakning tomonlari soni nechta? Bo'sh joylarga mos sonlarni qo'ying.

Yechish. Masala shartiga ko'ra, $180^\circ(n-2) = \ldots \cdot 360^\circ$. Bundan

$$180^\circ(n-2) = \ldots \cdot 2 \cdot 180^\circ,\ n-2 = 6,\ n = \ldots .$$

Javob: $n = \ldots$.

7. Tashqi burchagining har biri: 1) 18° ga; 2) 24° ga; 3) 60° ga teng bo'lgan qavariq ko'pburchakning nechta tomoni bor?

8. Agar to'rtburchakning uchta burchagi o'tmas bo'lsa, u holda to'rtinchi burchagi o'tkir bo'ladi. Shuni isbotlang.

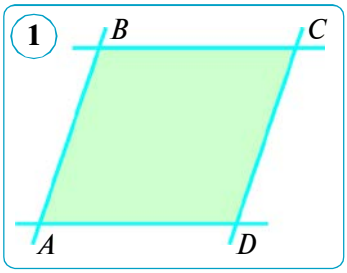
9. Tashqi burchagining har biri: 1) 15° ga; 2) 45° ga; 3) 72° ga teng bo'lgan qavariq ko'pburchakning nechta tomoni bor?

10. Qavariq to'rtburchakning burchaklari 1, 2, 3 va 4 sonlariga proporsional. Shu burchaklarni toping.

2. PARALLELOGRAMM VA UNING XOSSALARI

1. Parallelogramm. Tekislikda ikkita parallel to‘g‘ri chiziqning boshqa ikkita parallel to‘g‘ri chiziq bilan kesishishidan hosil bo‘lgan to‘rtburchakni ko‘rib chiqamiz (1- rasm). Bu to‘rtburchak *maxsus* nomga ega bo‘lib, uni ***parallelogramm*** deb ataymiz.

1



Ta’rif. *Qarama-qarshi tomonlari o‘zaro parallel bo‘lgan to‘rtburchak* ***parallelogramm*** *deb ataladi.*

Agar $ABCD$ parallelogramm bo‘lsa, $AB \parallel DC$ va $AD \parallel BC$ bo‘ladi (1- rasm).

1- masala. 2- rasmda $\triangle ABC = \triangle CDA$. $ABCD$ to‘rtburchak parallelogramm ekanini isbotlang.

Yechish. ABC va CDA uchburchaklarning tengligidan quyidagi kelib chiqadi: $\angle 1 = \angle 3$ va $\angle 2 = \angle 4$. 1 va 3 burchaklar – AB va CD parallel to‘g‘ri chiziqlar va AC kesuvchi hosil qilgan ichki almashinuvchi burchaklar bo‘lgani uchun teng. Xuddi shuningdek, 2 va 4 burchaklar BC va AD parallel to‘g‘ri chiziqlar hamda AC kesuvchi hosil qilgan ichki almashinuvchi burchaklar bo‘lgani uchun teng. Parallel to‘g‘ri chiziqlarning alomatiga ko‘ra quyidagiga ega bo‘lamiz: $AB \parallel DC$ va $BC \parallel AD$. Demak, $ABCD$ to‘rtburchakda qarama-qarshi tomonlar jufti-jufti bilan parallel, ya’ni ta’rifga ko‘ra, $ABCD$ – parallelogramm.

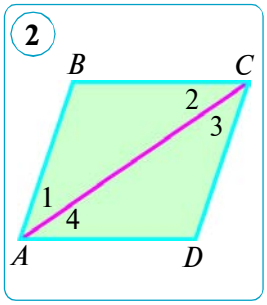
Parallelogrammning bir tomonida yotgan nuqtadan qarama-qarshi tomonni o‘z ichiga olgan to‘g‘ri chiziqqa tushirilgan perpendikular parallelogrammning ***balandligi*** deyiladi. Parallelogrammning bir tomoniga cheksiz ko‘p balandliklar o‘tkazish mumkinligi ravshan (3- rasm), ular parallel to‘g‘ri chiziqlar orasidagi masofalar bo‘lgani uchun o‘zaro teng. Parallelogrammning bir uchidan uning turli tomoniga bir-biridan farq qiladigan ikkita balandlik o‘tkazish mumkin. Masalan, 4- rasmda BP va BF – balandliklardir.

2. Parallelogrammning xossalari.

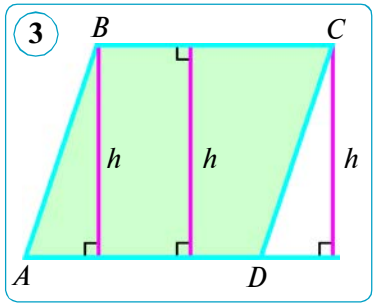
1- teorema.

(1-xossa.) Parallelogrammning bir tomoniga yopishgan burchaklari yig‘indisi 180° ga teng.

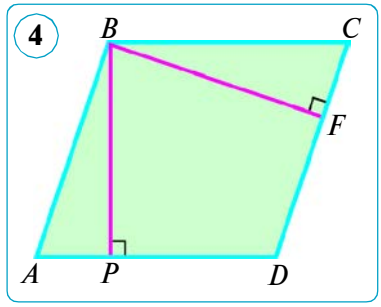
2



3



4



Isbot. Parallelogrammning bir tomoniga yopishgan burchaklar ichki bir tomonli burchaklar bo‘ladi. Shuning uchun ularning yig‘indisi 180° ga teng. Teorema isbotlandi.

2- teorema.

(2-xossa.) Parallelogrammning qarama-qarshi tomonlari va qarama-qarshi burchaklari o‘zaro teng.

Isbot. $ABCD$ – berilgan parallelogramm bo‘lsin, ya’ni $AB \parallel CD$ va $BC \parallel AD$. Parallelogramning AC diagonalini o‘tkazamiz (2- rasmga q.) hamda ABC va CDA uchburchaklarni ko‘rib chiqamiz. Ularda AC tomon – umumiy, 1 va 3 burchaklar – AB va CD parallel to‘g‘ri chiziqlar hamda AC kesuvchi hosil qilgan ichki almashinuvchi burchaklar bo‘lgani uchun teng, 2 va 4 burchaklar esa AD va BC parallel to‘g‘ri chiziqlar hamda AC kesuvchi hosil qilgan ichki almashinuvchi burchaklar bo‘lgani uchun teng. Demak, uchburchaklar tengligining ikkinchi alomatiga ko‘ra, ABC va CDA uchburchaklar teng. Xususan bundan, $AB = CD$, $AD = BC$ va $\angle B = \angle D$ hamda $\angle 1 + \angle 4 = \angle 2 + \angle 3$, ya’ni $\angle A = \angle C$ ekani kelib chiqadi.

2- masala. Parallelogramm burchaklaridan ikkitasining yig‘indisi 172° ga teng. Uning burchaklarini toping.

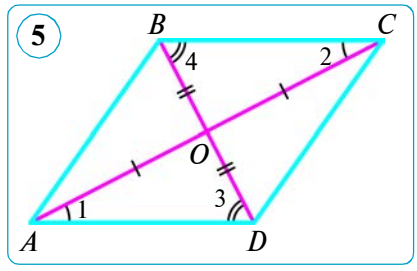
Yechish. $ABCD$ parallelogramm berilgan bo‘lsin. Parallelogrammning qo‘shni burchaklari yig‘indisi 180° ga teng bo‘lgani uchun berilgan burchaklar qo‘shni burchaklar bo‘la olmaydi, demak, ular qarama-qarshi burchaklardir. $\angle A + \angle C = 172°$ bo‘lsin. Parallelogrammning qarama-qarshi burchaklari teng bo‘lgani uchun bu holda burchaklarning har biri $\angle A = \angle C = 172° : 2 = 86°$ bo‘ladi. Parallelogrammning hamma burchaklari yig‘indisi 360° ga teng, shuning uchun qolgan ikki burchagi $\angle B = \angle D = (360° - 172°) : 2 = 94°$ dan bo‘ladi. *Javob:* 86°, 94°, 86°, 94°.

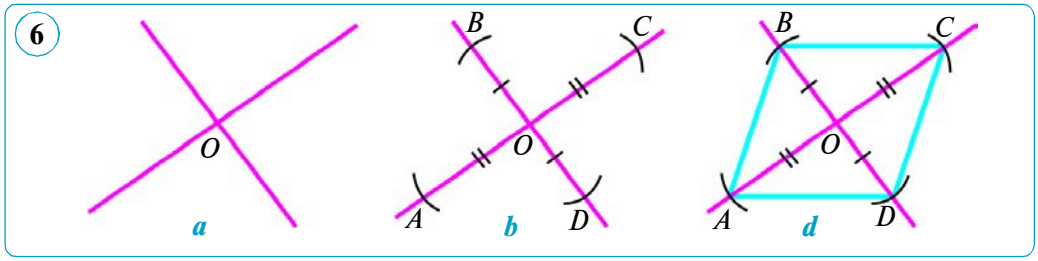
3- teorema.

(3-xossa.) Parallelogrammning diagonallari kesishadi va kesishish nuqtasida teng ikkiga bo‘linadi.

Isbot. $ABCD$ berilgan parallelogramm va O – AC va BD diagonallarning kesishish nuqtasi bo‘lsin (5- rasm). $AO = OC$ va $DO = OB$ ekanini isbot qilamiz.

AOD va COB uchburchaklarni ko‘rib chiqamiz. Bu uchburchaklarda $AD = BC$ (parallelogrammning 2-xossasiga ko‘ra uning qarama-qarshi tomonlari teng), $\angle 1 = \angle 2$ va $\angle 3 = \angle 4$ (AD va BC parallel to‘g‘ri chiziqlarning, mos ravishda, AC va BD kesuvchilar bilan kesishishidan hosil bo‘lgan ichki almashinuvchi burchaklar bo‘lgani uchun). Demak, uchburchaklar tengligining ikkinchi alomatiga ko‘ra, $\triangle AOD = \triangle COB$. Bundan $AO = CO$ va $DO = OB$, ya’ni AC va BD diagonal-





larning har biri kesishish nuqtasi O da teng ikkiga bo‘linishi kelib chiqadi. Teorema isbotlandi.

3- masala. 3- xossadan foydalanib parallelogramm yasang.

1- qadam. Ikkita kesishuvchi to‘g‘ri chiziq o‘tkazamiz va ularning kesishish nuqtasini O harfi bilan belgilaymiz (6- *a* rasm).

2- qadam. Sirkul yordamida to‘g‘ri chiziqlarning birida o‘zaro teng OA va OC, ikkinchisida esa o‘zaro teng OB va OD kesmalarni qo‘yamiz (6-*b* rasm).

3- qadam. A, B, C va D nuqtalarni ketma-ket tutashtirib, izlanayotgan $ABCD$ parallelogrammni hosil qilamiz (6- *d* rasm).

Savol, masala va topshiriqlar

1. 1) Qanday to‘rtburchakka parallelogramm deyiladi? Parallelogrammning bir tomoniga yopishgan burchaklari yig‘indisi nimaga teng?
2) Parallelogrammning diagonallari haqida nima deyish mumkin?

2. Parallelogrammning perimetri 152 cm, tomonlaridan biri ikkinchisidan 25 cm ortiq. Parallelogramm tomonlarini toping.

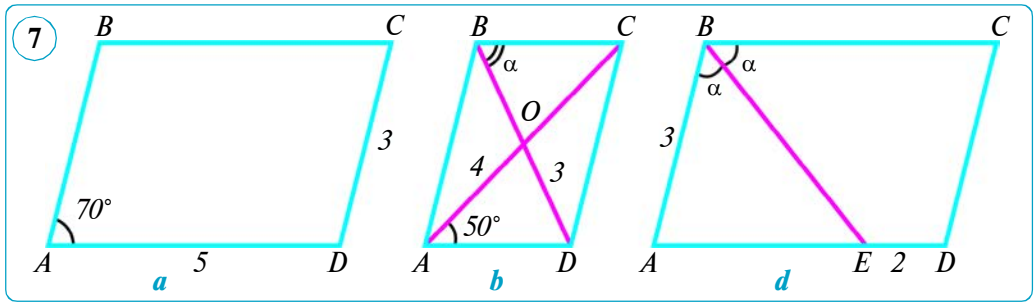
3. Parallelogramm burchaklaridan ikkitasining yig‘indisi: 1) 70° ga; 2) 110° ga; 3) 170° ga teng bo‘lsa, uning hamma burchaklarini toping.

4. $ABCD$ parallelogrammda: $AB = 7$ cm, $BC = 11$ cm, $AC = 14$ cm, $BD = 12$ cm; O – diagonallarning kesishish nuqtasi ekani ma’lum. ABO va BOC uchburchaklarning perimetrlarini toping.

5. Parallelogrammning qo‘shni tomonlari yig‘indisi 20 cm ga, ayirmasi esa 12 cm ga teng. Shu parallelogrammning tomonlarini toping.

6. Parallelogrammning ikki tomoni nisbati 5 : 3 ga, perimetri esa 6,4 dm ga teng. Parallelogramm tomonlarini toping.

7. 7- rasmda parallelogramm ayrim elementlarining kattaligi ko‘rsatilgan. Yana qaysi kattaliklarni topish mumkin?

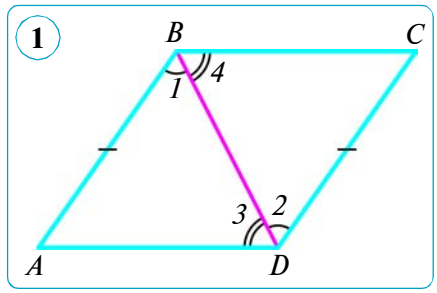


3. PARALLELOGRAMMNING ALOMATLARI

Avvalgi mavzuda ko‘rib chiqqanimizdan ma’lum bo‘ldiki, parallelogrammning xossalarini tatbiq etish uchun ko‘p hollarda berilgan to‘rtburchakning haqiqatan ham parallelogramm ekaniga ishonch hosil qilish kerak. Buni ta’rifga ko‘ra (2- mavzudagi 1- masalaga q.) yoki berilgan to‘rtburchakning parallelogramm ekanini tasdiqlovchi shartlar – alomatlar orqali isbotlash kerak bo‘ladi. Ko‘pincha amaliyotda qo‘llaniladigan parallelogrammning alomatlarini isbotlaymiz. Endi parallelogrammning alomatlari bilan tanishamiz.

1- teorema.

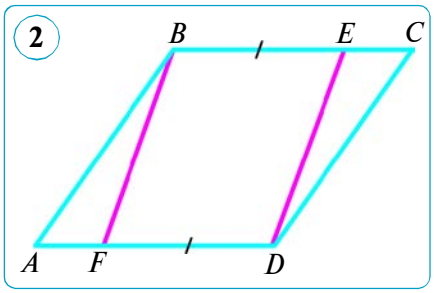
(1- a l o m a t .) Agar to‘rtburchakning ikkita tomoni teng va parallel bo‘lsa, bu to‘rtburchak parallelogrammdir.



Isbot. $ABCD$ to‘rtburchakda $AB \parallel CD$ va $AB = CD$ bo‘lsin (1- rasm). Uning BD diagonalini o‘tkazamiz. Natijada ikkita teng ABD va CDB uchburchaklarga ega bo‘lamiz (ikki tomoni va ular orasidagi burchagiga ko‘ra), chunki ularda $AB = CD$ (shartga ko‘ra), BD tomon – umumiy, $\angle 1 = \angle 2$ (AB va CD parallel to‘g‘ri chiziqlar hamda BD kesuvchi kesishishidan hosil bo‘lgan ichki almashinuvchi burchaklar bo‘lgani uchun). Uchburchaklarning tengligidan, $\angle 3 = \angle 4$ ekani kelib chiqadi. Bu burchaklar AD va BC to‘g‘ri chiziqlar hamda BD kesuvchi kesishishidan hosil bo‘lgan ichki almashinuvchi burchaklar, demak, $AD \parallel BC$. Shunday qilib, $ABCD$ to‘rtburchakning qarama-qarshi tomonlari jufti-jufti bilan parallel. Shuning uchun, parallelogramm ta’rifiga ko‘ra, $ABCD$ to‘rtburchak – parallelogramm.

Teorema isotlandi.

1- masala. $ABCD$ parallelogrammning BC va AD tomonlariga teng kesmalar qo‘yilgan: $BE = DF$ (2- rasm). $BEDF$ to‘rtburchak parallelogramm bo‘ladimi?

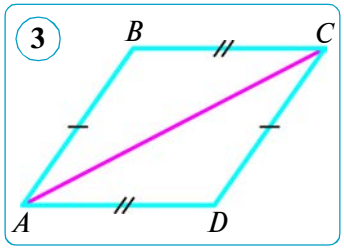


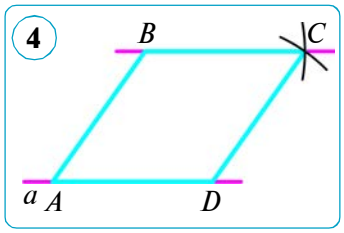
Yechish. $BEDF$ to‘rtburchakning BE va DF qarama-qarshi tomonlari teng hamda parallel. Shuning uchun, parallelogrammning 1- alomatiga ko‘ra, $BEDF$ to‘rtburchak – parallelogramm.

Javob: ha, bo‘ladi.

2- teorema.

(2- a l o m a t .) Agar to‘rtburchakning qarama-qarshi tomonlari jufti-jufti bilan teng bo‘lsa, bu to‘rtburchak parallelogrammdir.





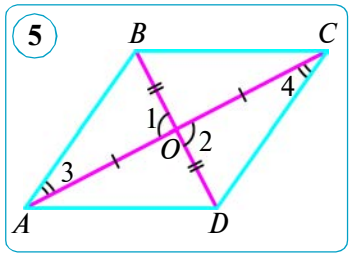
Isbot. $ABCD$ to‘rtburchakda $AB = CD$ va $BC = DA$ bo‘lsin. Uning AC diagonalini o‘tkazamiz (3- rasm). Natijada ABC va CDA uchburchaklar hosil bo‘ladi. Uchburchaklar tengligining 3- alomatiga ko‘ra, bu uchburchaklar teng (AC tomon – umumiy, teorema shartiga ko‘ra esa $AB = CD$ va $BC = DA$). Uchburchaklarning tengligidan CAB va ACD burchaklarning tengligi kelib chiqadi. Bu burchaklar esa AB va DC to‘g‘ri chiziqlar hamda AC kesuvchi hosil qilgan ichki almashinuvchi burchaklardir. To‘g‘ri chiziqlarning parallellik alomatiga ko‘ra, $AB \parallel CD$. Shunday qilib, $ABCD$ to‘rtburchakda AB va CD tomonlar teng hamda parallel, demak, parallelogrammning 1- alomatiga ko‘ra, $ABCD$ to‘rtburchak – parallelogramm. Teorema isbotlandi.

2- masala. Berilgan nuqtadan o‘tuvchi va berilgan to‘g‘ri chiziqqa parallel to‘g‘ri chiziqni yasang.

Yechish. a – to‘g‘ri chiziq, B – unda yotmaydigan nuqta bo‘lsin. a to‘g‘ri chiziqda A va D nuqtalarni belgilaymiz (4- rasm). B, D nuqtalardan radiuslari mos ravishda AD va AB bo‘lgan aylanalar o‘tkazamiz. Ularning kesishish nuqtasini C bilan belgilaymiz. BC to‘g‘ri chiziqni o‘tkazamiz, u izlanayotgan to‘g‘ri chiziq bo‘ladi. Haqiqatan ham, $ABCD$ to‘rtburchakning qarama-qarshi tomonlari teng. Parallelogrammning 2- alomatiga ko‘ra, $ABCD$ to‘rtburchak – parallelogramm. Shuning uchun, $BC \parallel AD$.

3- teorema.

(3- alomat.) Agar to‘rtburchakning diagonallari kesishish nuqtasida teng ikkiga bo‘linsa, bu to‘rtburchak parallelogrammdir.



Isbot. O – $ABCD$ to‘rtburchakning diagonallari kesishgan nuqta bo‘lsin. Shartga ko‘ra, $AO = OC$ va $BO = DO$ (5- rasm). AOB va COD uchburchaklarni ko‘rib chiqamiz. Bu uchburchaklarda: $\angle 1 = \angle 2$ (vertikal burchaklar), $AO = CO$ va $BO = DO$ (shartga ko‘ra). Demak, uchburchaklar tengligining birinchi alomatiga ko‘ra, AOB va COD uchburchaklar teng. Bu uchburchaklar tengligidan ularning mos tomonlari va burchaklarining tengligi kelib chiqadi: $AB = CD$, $\angle 3 = \angle 4$. To‘g‘ri chiziqlarning parallellik alomatiga ko‘ra, $AB \parallel CD$, chunki 3 va 4 burchaklar AB va CD to‘g‘ri chiziqlar hamda AC kesuvchi hosil qilgan ichki almashinuvchi burchaklardir. $ABCD$ to‘rtburchakda $AB = CD$ va $AB \parallel CD$ bo‘lgani uchun parallelogrammning 1- alomatiga ko‘ra, $ABCD$ to‘rtburchak parallelogramm bo‘ladi. Teorema isbotlandi.

Savol, masala va topshiriqlar

1. 1) Agar to‘rtburchakning ikkita tomoni teng va parallel bo‘lsa, bu to‘rtburchak parallelogramm bo‘lishini isbotlay olasizmi?

2) Parallelogrammning 2–3- alomatlarini ifodalang.

2. (*Faollashtiruvchi masala.*) 1) Ikkita teng va parallel kesmalar berilgan. Ularning oxirlari o‘zaro kesishmaydigan kesmalar bilan tutashtirilgan. Hosil bo‘lgan to‘rtburchak parallelogramm bo‘ladimi?

2) Agar to‘rtburchakning ikkita qarama-qarshi burchagi teng bo‘lsa, u parallelogramm bo‘ladimi?

3. $ABCD$ to‘rtburchakda AB va CD tomonlar parallel, $AB = CD = 11$ cm, $AD = 5$ cm. Shu to‘rtburchakning perimetrini toping.

4. Agar: 1) $\angle 1 = 70°$, $\angle 3 = 110°$, $\angle 2 \neq \angle 4$; 2) $\angle 1 = \angle 2 = 60°$, $\angle 3 = \angle 115°$ bo‘lsa (6- rasm), u holda $ABCD$ to‘rtburchak parallelogramm bo‘ladimi?

Yechish. 1) $ABCD$ to‘rtburchakda ikkita AB va CD tomon parallel, chunki $\angle 1 + \angle 3 = 70° + 110° = 180°$. Bu burchaklar – AB va DC to‘g‘ri chiziqlar hamda AD kesuvchi hosil qilgan ichki bir tomonli burchaklar. $AB \parallel DC$ bo‘lgani sababli, $\angle 1 = \angle 4$ bo‘ladi (mos burchaklar). $ABCD$ to‘rtburchakning qolgan ikki AD va BC tomoni parallel emas, chunki ichki almashinuvchi 1 va 2 burchaklar teng emas ($\angle 1 = \angle 4 \neq \angle 2$). Demak, $ABCD$ to‘rtburchak parallelogramm bo‘la olmaydi.

Javob: yo‘q, $ABCD$ to‘rtburchak parallelogramm bo‘la olmaydi.

2) 1- bandga o‘xshash yechiladi.

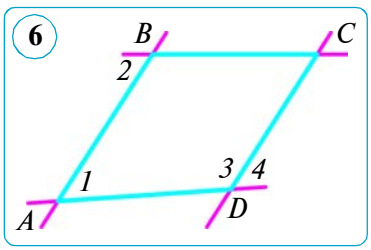
5. $ABCD$ parallelogrammning AB tomoni o‘rtasi E nuqtadan, CD tomoni o‘rtasi F nuqtadan iborat. $EBFD$ to‘rtburchakning parallelogramm ekanini isbotlang (7- rasm).

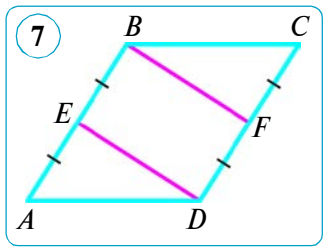
6. $ABCD$ to‘rtburchakda: $AD = BC$, $\angle 1 = \angle 2$ (8- rasm). $ABCD$ to‘rtburchakning parallelogramm ekanini isbotlang.

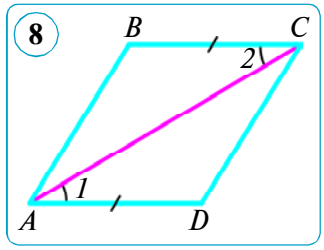
7. $ABCD$ to‘rtburchakda AB va CD tomonlar parallel, $AB = CD = 9$ cm, $AD = 4$ cm. Shu to‘rtburchakning perimetrini toping.

8. $ABCD$ to‘rtburchakda: $AB = CD$, $AD = BC$, A burchak B burchakdan uch marta katta. Shu to‘rtburchakning burchaklarini toping.

9. Parallelogramm burchaklaridan birining bissektrisasi o‘zi kesib o‘tadigan tomonni 4 cm va 5 cm li kesmalarga bo‘ladi. Parallelogrammning perimetrini toping.



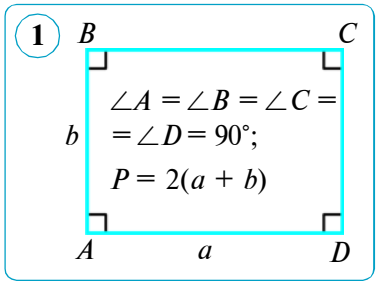




4. TO‘G‘RI TO‘RTBURCHAK VA UNING XOSSALARI

Ta’rif. *Hamma burchaklari to‘g‘ri bo‘lgan parallelogramm* ***to‘g‘ri to‘rtburchak*** *deb ataladi* (1- rasm).

1



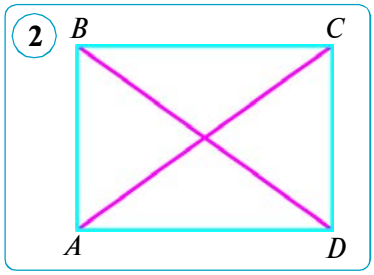
To‘g‘ri to‘rtburchak parallelogrammning xususiy holi bo‘lgani uchun u parallelogrammning barcha xossalariga ega bo‘ladi: to‘g‘ri to‘rtburchakning qarama-qarshi tomonlari teng, diagonallari kesishish nuqtasida teng ikkiga bo‘linadi, to‘g‘ri to‘rtburchakning diagonali uni ikkita teng to‘g‘ri burchakli uchburchakka ajratadi.

To‘g‘ri to‘rtburchakning o‘ziga xos xossasini ko‘rib chiqamiz.

Teorema.

To‘g‘ri to‘rtburchakning diagonallari o‘zaro teng.

2



Isbot. $ABCD$ to‘g‘ri to‘rtburchakda AC va BD diagonallar berilgan bo‘lsin. $AC = BD$ bo‘lishini isbotlaymiz (2- rasm).

To‘g‘ri burchakli ACD va DBA uchburchaklar ikki katetiga (AD – umumiy tomon, $CD = BA$) ko‘ra teng. Bundan ushbu uchburchaklar gipotenuzalarining tengligi, ya’ni $AC = BD$ kelib chiqadi.

Yuqoridagi teoremadan quyidagi teskari teorema kelib chiqadi (**to‘g‘ri to‘rtburchakning alomati**).

Teskari teorema.

Agar parallelogrammning diagonallari teng bo‘lsa, u to‘g‘ri to‘rtburchakdir.

Isbot. $ABCD$ parallelogrammda AC va BD diagonallar teng bo‘lsin (2- rasm). ABD va DCA uchburchaklar uch tomoniga ko‘ra teng ($AB = DC$, $BD = CA$, AD – umumiy tomon). Bundan $\angle A = \angle D$ kelib chiqadi. Parallelogrammning qarama-qarshi burchaklari teng, shuning uchun $\angle A = \angle C$ va $\angle B = \angle D$. Shunday qilib, $\angle A = \angle B = \angle C = \angle D$. Parallelogramm – qavariq to‘rtburchak, shuning uchun: $\angle A + \angle B + \angle C + \angle D = 360°$. Bundan $\angle A = \angle B = \angle C = \angle D = 90°$, ya’ni $ABCD$ parallelogrammning to‘g‘ri to‘rtburchak ekani kelib chiqadi. Teorema isbotlandi.

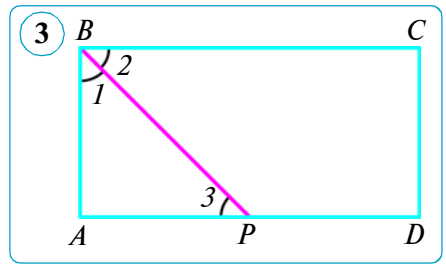
1- masala. $ABCD$ to‘g‘ri to‘rtburchakning perimetri 24 cm ga, BD diagonali esa 9 cm ga teng. ABD uchburchakning perimetrini toping.

Yechish. $AB + AD = P_{ABCD} : 2 = 24 : 2 = 12$ (cm) – qo‘shni tomonlar yig‘indisi (2- rasmga q.). $P_{ABD} = AB + AD + BD = 12 + 9 = 21$ (cm).

Javob: $P_{ABD} = 21$ cm.

2- masala. $ABCD$ to‘g‘ri to‘rtburchak B burchagining bissektrisasi AD tomonni P nuqtada kesadi hamda uni $AP = 17$ cm va $PD = 21$ cm li kesmalarga ajratadi (3- rasm). Shu to‘g‘ri to‘rtburchakning perimetrini toping.

3



Yechish. 1) $ABCD$ – to‘g‘ri to‘rtburchak bo‘lgani uchun $AD \parallel BC$ va shuning uchun $\angle 2 = \angle 3$ (ichki almashinuvchi burchaklar). Biroq, shartga ko‘ra, $\angle 2 = \angle 1$, demak, $\angle 1 = \angle 3$ hamda $\triangle ABP$ – asosi BP bo‘lgan teng yonli uchburchak. Shunday qilib, $AB = AP = 17$ cm.

2) $AD = AP + PD = 17 + 21 = 38$ (cm);

$$P_{ABCD} = 2\,(AB + AD) = 2 \cdot (17 + 38) = 2 \cdot 55 = 110 \text{ (cm)}.$$

Javob: $P_{ABCD} = 110$ cm.

Savol, masala va topshiriqlar

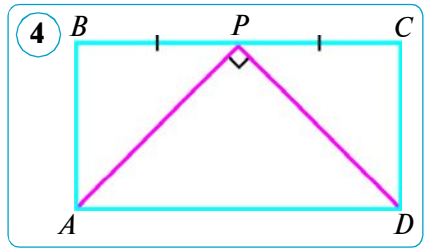
1. 1) Qanday parallelogramm to‘g‘ri to‘rtburchak deb ataladi?
2) To‘g‘ri to‘rtburchakning qanday o‘ziga xos xossasi bor?
3) To‘g‘ri to‘rtburchakning alomatini ifodalang.

2. $ABCD$ to‘g‘ri to‘rtburchakda: $AB = 9$ cm, $BC = 7$ cm.
1) C nuqtadan AD tomongacha bo‘lgan masofani toping.
2) AB va CD to‘g‘ri chiziqlar orasidagi masofani toping.

3. To‘g‘ri to‘rtburchakning perimetri 24 cm. To‘g‘ri to‘rtburchakning ixtiyoriy ichki nuqtasidan uning tomonlarigacha bo‘lgan masofalar yig‘indisini toping.

4. $ABCD$ to‘g‘ri to‘rtburchakning perimetri 24 cm ga teng. P nuqta BC tomonning o‘rtasi, $\angle APD = 90°$ (4- rasm). To‘g‘ri to‘rtburchakning tomonlarini toping.

4



5. Agar to‘rtburchakda diagonallar teng va ular kesishish nuqtasida teng ikkiga bo‘linsa, bu to‘rtburchak to‘g‘ri to‘rtburchak bo‘lishini isbotlang.

6. Parallelogrammning tomonlari 4 cm va 7 cm. Bu parallelogrammning diagonallari: 1) 12 cm va 5 cm; 2) 10 cm va 3 cm bo‘lishi mumkinmi?

7. To‘g‘ri to‘rtburchakning perimetri 42 cm, tomonlaridan biri esa ikkinchisidan ikki marta katta. To‘g‘ri to‘rtburchakning tomonlarini toping.

5–6. ROMB VA KVADRATNING XOSSALARI

1. Romb va uning xossalari.

Ta'rif. *Tomonlari teng bo'lgan parallelogramm* ***romb*** *deyiladi* (1- rasm).

Romb parallelogrammning umumiy xossalariga ega bo'lgan holda yana quyidagi xossaga ega.

Teorema.

Rombning diagonallari o'zaro perpendikulyar hamda rombning burchaklarini teng ikkiga bo'ladi.

Isbot. $ABCD$ – berilgan romb (2- rasm), O – uning diagonallari kesishgan nuqta bo'lsin. $AC \perp BD$ va har bir diagonal rombning mos burchaklarini teng ikkiga bo'lishini (masalan, $\angle BAC = \angle DAC$) isbotlaymiz.

Rombning ta'rifiga ko'ra, $AB = AD$, shuning uchun BAD – BD asosli teng yonli uchburchak. Romb parallelogramm bo'lgani uchun uning diagonallari kesishish nuqtasida teng ikkiga bo'linadi, ya'ni $BO = OD$. Demak, AO – teng yonli BAD uchburchakning medianasi. Teng yonli uchburchakning xossasiga ko'ra, uning asosiga o'tkazilgan mediana ham balandlik, ham bissektrisa bo'ladi. Shuning uchun $AC \perp BD$ va $\angle BAC = \angle DAC$. Shuni isbotlash talab qilingan edi.

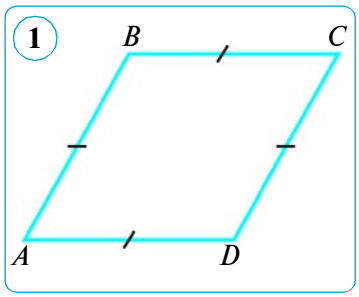
1- masala. $ABCD$ rombning BD diagonali tomoni bilan 35° li burchak hosil qiladi. Uning burchaklarini toping.

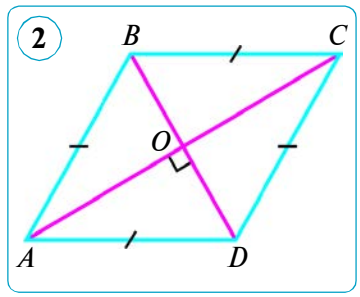
Yechish. $\angle ABD = 35°$, deylik (3- rasm). U holda $\angle CBD = 35°$ (rombning xossasiga ko'ra). $\angle ABC = 2\angle ABD = 2 \cdot 35° = 70°$, $\angle ADC = \angle ABC = 70°$ (parallelogrammning 2- xossasiga ko'ra), $\angle DAB = 180° - \angle ABC$ (parallelogrammning 1- xossasiga ko'ra). Demak, $\angle DAB = 180° - 70° = 110°$, $\angle BCD = \angle DAB = 110°$ (parallelogrammning 2- xossasiga ko'ra).

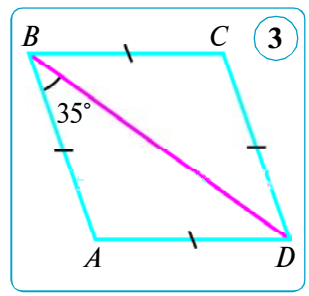
Javob: 70°, 110°, 70°, 110°.

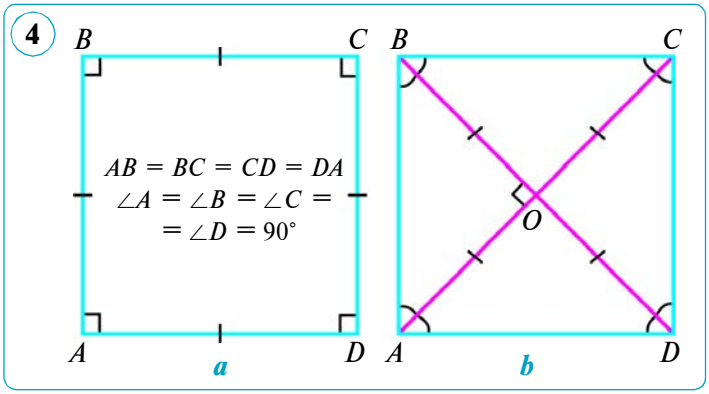
2- masala. Turli romblarning perimetrlari teng bo'lishi mumkinmi?

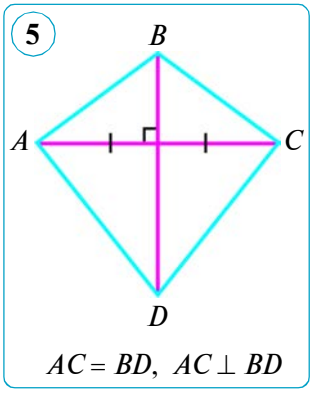
Yechish. Perimetrlari teng bo'lgan romblar bir-biridan burchaklari bilan farq qiladi. Agar rombning o'tkir burchagi: 1) 40° ga teng bo'lsa, u holda qolgan burchaklari mos ravishda 140°, 40°, 140° bo'ladi; 2) 15° ga teng











bo‘lsa, u holda qolgan burchaklari mos ravishda 165°, 15°, 165° bo‘ladi va h.k. Shuningdek, o‘tkir burchak o‘rniga turli o‘tmas burchaklarni olish mumkin. *Javob:* ha, mumkin.

2. Kvadrat va uning xossalari.

Ta’rif. *Tomonlari teng bo‘lgan to‘g‘ri to‘rtburchak* ***kvadrat*** *deb ataladi.*

Kvadrat va rombning ta’riflaridan kvadrat burchaklari to‘g‘ri bo‘lgan romb ekanligi kelib chiqadi (4- *a* rasm). Kvadrat ham parallelogramm, ham to‘g‘ri to‘rtburchak, ham romb bo‘lgani uchun ularning barcha xossalariga ega. Kvadratning asosiy xossalarini keltiramiz.

1. Kvadratning hamma burchaklari to‘g‘ri.

2. Kvadratning diagonallari o‘zaro teng.

3. Kvadratning diagonallari o‘zaro perpendikular va kesishish nuqtasida teng ikkiga bo‘linadi hamda kvadratning burchaklarini teng ikkiga bo‘ladi (4- rasm).

Shu xossalarni mustaqil isbot qiling.

3- masala. Agar rombning diagonallari teng bo‘lsa, u holda bunday romb kvadrat ekanini isbotlang.

Isbot. Romb parallelogramm bo‘lgani uchun to‘g‘ri to‘rtburchakning alomatidan diagonallari teng bo‘lgan rombning to‘g‘ri to‘rtburchak ekanligi kelib chiqadi va demak, u kvadrat bo‘ladi.

4- masala. To‘rtburchakning diagonallari perpendikular va o‘zaro bir-biriga teng. Shu to‘rtburchak kvadrat bo‘ladimi?

Yechish. Masala shartini qanoatlantiruvchi to‘rtburchaklardan biri 5- rasmda tasvirlangan. Bu holda diagonallardan biri teng ikkiga bo‘lingan. Ammo bu kvadratning 2- xossasini hamda 3- xossada keltirilgan shartning bir qismi – o‘zaro perpendikularlik shartini qanoatlantiradi, xolos. Keltirilgan holatda faqat diagonallaridan biri teng ikkiga bo‘lingan, shu sababli bu to‘rtburchak kvadrat bo‘la olmaydi. Ma’lum bir holatda to‘rtburchakning har ikkala diagonali kesishish nuqtasida teng ikkiga bo‘linishi mumkin. Faqat shu holdagina to‘rtburchak kvadrat bo‘la oladi.

Javob: to‘rtburchak kvadrat bo‘lishi shart emas.

Savol, masala va topshiriqlar

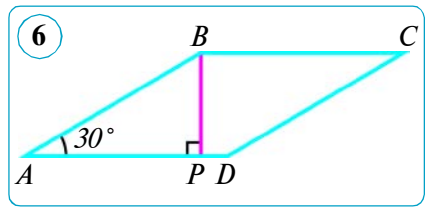
1. 1) Romb deb nimaga aytiladi? Rombning xossasini ayting.

? 2) Kvadrat deb nimaga aytiladi? Uning xossalarini ayting.

3) Kvadratga: a) «parallelogramm»; b) «romb»; d) «to‘g‘ri to‘rtburchak» tushunchalari yordamida ta’rif bering.

2. Kvadratning tomoni 20 cm ga teng. Diagonallari kesishish nuqtasidan tomonlaridan birigacha bo‘lgan masofani toping.

3. *ABCD* rombning tomoni 24 cm ga, *A* burchagi esa 30° ga teng. *B* uchidan unga qarama-qarshi *AD* tomongacha bo‘lgan masofani toping (6- rasm).



Bo‘sh joylarga mos sonlarni qo‘ying.

Yechish. *B* nuqtadan *AD* to‘g‘ri chiziqqacha bo‘lgan masofa *B* nuqtadan shu to‘g‘ri chiziqqa tushirilgan perpendikular, ya’ni *BP* kesma uzunligiga teng. *ABP* uchburchakni ko‘rib chiqamiz. Unda $\angle APB = ...^\circ$, $\angle A = ...^\circ$, $AB = ...$. U holda $BP = 0{,}5 \cdot ... = 0{,}5 \cdot ... = ...$ (cm) (...° li burchak qarshisida yotgan katetning xossasiga ko‘ra). *Javob:* $BP = ...$ cm.

4. 1) (*Amaliy topshiriq.*) 1) Ikkita teng uchburchakdan; 2) to‘rtta teng uchburchakdan qanday qilib romb va kvadrat yasash mumkin? Mumkin bo‘lgan hamma yechimlarni ko‘rsating.

5. Teng yonli to‘g‘ri burchakli uchburchak ichiga kvadrat shunday chizilganki, uning ikkita uchi gipotenuzada, qolgan ikkita uchi esa katetlarda yotadi. Gipotenuza 21 cm ga teng ekani ma’lum bo‘lsa, kvadrat tomonini toping.

6. Rombning diagonallari bilan tomonlari orasida hosil bo‘lgan burchaklarning nisbati 2 : 7 kabi. Rombning burchaklarini toping.

7. Kvadrat tomonlarining o‘rtalari birin-ketin birlashtirilgan. Natijada qanday shakl hosil bo‘ladi?

8. Rombning hamma balandliklari o‘zaro teng ekanini isbot qiling.

9. To‘rtburchakning tomonlari 2 : 4 : 5 : 7 kabi nisbatda, perimetri esa 108 cm ga teng. Shu to‘rtburchakning tomonlarini toping.

10. Burchaklaridan biri 60°, kichik diagonalining uzunligi 16 cm bo‘lgan rombning perimetrini toping.

11. Rombning diagonallari bilan tomonlari orasida hosil bo‘lgan burchaklarning nisbati 5 : 4 kabi. Rombning burchaklarini toping.

12. To‘g‘ri to‘rtburchakning bo‘yi 32 cm, eni esa 28 cm ga teng. Shu to‘g‘ri to‘rtburchakning perimetriga teng bo‘lgan kvadratning tomonini toping.

13. To‘rtburchakning eng kichik tomoni 5 cm ga teng, qolgan tomonlarining har biri oldingisidan mos ravishda 2 cm ga katta. Shu to‘rtburchakning perimetrini toping.

7–8. TRAPETSIYA VA UNING XOSSALARI

1. Trapetsiyaning ta'rifi. Bizga ma'lumki, har qanday parallelogrammda ikki juft parallel tomonlar bo'ladi. Endi biz faqat bir juft parallel tomonlarga ega bo'lgan to'rtburchaklarni ko'rib chiqamiz.

1- ta'rif. *Ikkita tomoni parallel, qolgan ikki tomoni parallel bo'lmagan to'rtburchak* ***trapetsiya*** *deb ataladi.*

Trapetsiyaning parallel tomonlari uning *asoslari*, parallel bo'lmagan tomonlari esa *yon tomonlari* deb ataladi. 1- rasmdagi $ABCD$ trapetsiyada AD va BC tomonlar *asoslar*, AB va CD tomonlar esa *yon tomonlar* bo'ladi.

2- ta'rif. *Tomonlaridan biri asosiga perpendikular bo'lgan trapetsiya* ***to'g'ri burchakli trapetsiya*** *deyiladi* (2- rasm).

3- ta'rif. *Yon tomonlari teng bo'lgan trapetsiya* ***teng yonli trapetsiya*** *deyiladi.*

3- rasmda teng yonli $ABCD$ trapetsiya tasvirlangan: $AB = CD$.

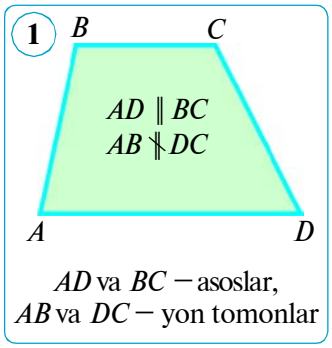
2. Trapetsiyaning alomati. Endi $ABCD$ to'rtburchakning trapetsiya bo'lishi uchun qanday shartni qanoatlantirishinini ko'rib chiqamiz.

Teorema.

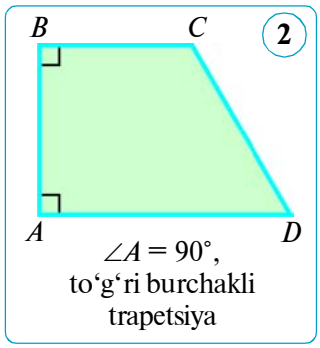
Agar to'rtburchakning bir tomoniga yopishgan ikki burchagining yig'indisi 180° ga teng hamda unga qo'shni tomonlarga yopishgan ikki burchagining yig'indisi 180° dan farqli bo'lsa, bunday to'rtburchak *trapetsiya* bo'ladi.

Isbot. $ABCD$ to'rtburchakda: $\angle A + \angle B = 180^\circ$, $\angle A + \angle D \neq 180^\circ$ bo'lsin. $ABCD$ to'rtburchakning trapetsiya ekanini isbotlaymiz.

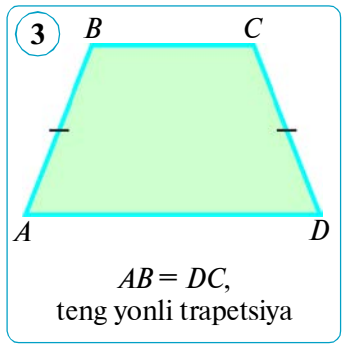
Birinchidan, bir juft qarama-qarshi tomonlar parallel ekanini ko'rsatamiz. AB, BC (l_1) va AD (l_2) to'g'ri chiziqlarni o'tkazamiz (4- rasm). Shartga ko'ra, $\angle A + \angle B = 180^\circ$, u holda AD va BC kesmalar parallellik alomatiga ko'ra parallel bo'ladi. (*Ikki a va b to'g'ri chiziqlarni uchinchi c to'g'ri chiziq kesganda ichki bir tomonli burchaklarning yig'indisi 180° ga teng bo'lsa, u holda a va b to'g'ri chiziqlar parallel bo'ladi.*)



AD va BC – asoslar,
AB va DC – yon tomonlar



$\angle A = 90^\circ$,
to'g'ri burchakli
trapetsiya



$AB = DC$,
teng yonli trapetsiya

Ikkinchidan, $ABCD$ to‘rtburchakning qolgan ikki tomoni parallel emasligini ko‘rsatamiz. Shartga ko‘ra, $\angle A + \angle D \neq 180^\circ$, bu holda AB va DC kesmalar parallel bo‘la olmaydi (*Yevklidning parallel to‘g‘ri chiziqlar to‘g‘risidagi 5- aksiomasiga ko‘ra, ya’ni to‘g‘ri chiziqlar parallel bo‘lishining zaruriy sharti bajarilmadi*). Demak, $ABCD$ to‘rtburchak trapetsiya ekan. Shuni isbotlash talab qilingan edi.

Bu teoremadan quyidagi natija kelib chiqadi.

Natija. Trapetsiyaning bir burchagi 90° bo‘lsa, uning yana bitta 90° li burchagi mavjuddir.

4- ta’rif. *Trapetsiyaning asoslaridan birida yotgan nuqtadan ikkinchi asosni o‘z ichiga olgan to‘g‘ri chiziqqa tushirilgan perpendikulyar* ***trapetsiyaning balandligi*** *deb ataladi.*

Trapetsiya asoslariga perpendikulyar bo‘lgan har qanday kesmani uning balandligi sifatida olish mumkin. Har qanday trapetsiyada istalgancha balandlik o‘tkazsa bo‘ladi (5- rasm).

3. Teng yonli trapetsiyaning xossasi.

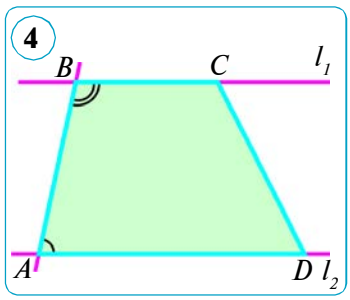
$ABCD$ teng yonli trapetsiyani ko‘rib chiqamiz. Bunda $AD = a$ – katta asos, $BC = b$ – kichik asos bo‘lsin. Kichik asosning B uchidan BP balandlik o‘tkazaylik (6-rasm). Balandlikning P asosi AD asosni AP va PD kesmalarga ajratsin.

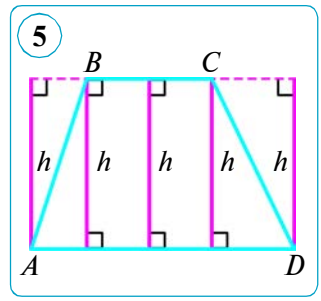
Teorema.

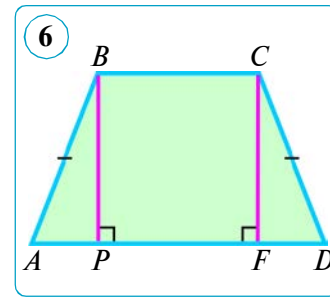
Teng yonli trapetsiyaning o‘tmas burchagi uchidan o‘tkazilgan balandlik katta asosini uzunliklari asoslari ayirmasining yarmiga va asoslari yig‘indisining yarmiga teng bo‘laklarga ajratadi, ya’ni:

$$AP = \frac{a-b}{2}, \quad PD = \frac{a+b}{2}.$$

Isbot. C uchidan $CF \perp AD$ ni o‘tkazamiz. To‘g‘ri burchakli ABP va DCF uchburchaklar teng: $AB = DC$ – shartga ko‘ra, $BP = CF$ esa BC va AD parallel to‘g‘ri chiziqlar orasidagi masofa bo‘lgani uchun. Uchburchaklar tengligidan $AP = FD$ kelib chiqadi. To‘g‘ri chiziqlarning parallellik alomatiga ko‘ra, $BP \parallel CF$, chunki $BP \perp AD$, $CF \perp AD$. Parallel to‘g‘ri chiziqlar orasidagi masofa teng bo‘lganligi uchun $BC = PF = b$. Demak,







$$AP = FD = \frac{AD - PF}{2} = \frac{a-b}{2}, \quad PD = AD - AP = a - \frac{a-b}{2} = \frac{a+b}{2}.$$

Shunday qilib, $AP = \frac{a-b}{2}$ va $PD = \frac{a+b}{2}$ ekan. Teorema isbotlandi.

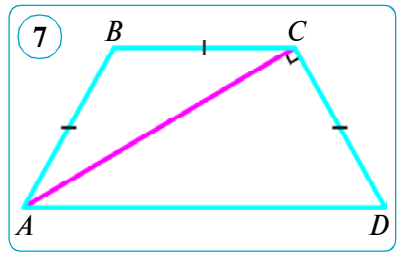
1- masala. Teng yonli trapetsiyaning asosidagi burchaklari teng ekanini isbotlang.

Yechish. $ABCD$ – teng yonli trapetsiya, ya'ni $AB = DC$ va $AD \parallel BC$. Teng yonli trapetsiyaning AD va BC asoslariga yopishgan burchaklari tengligini isbotlaymiz ($\angle A = \angle D$, $\angle B = \angle C$).

Trapetsiyaning o'tmas burchaklari (B va C) uchlaridan AD asosiga perpendikular o'tkazamiz: $BP \perp AD$, $CF \perp AD$ (6- rasmga q.). To'g'ri burchakli ABP va DCF uchburchaklar (gipotenuza va katetiga ko'ra) teng: $AB = DC$ – shartga ko'ra, $BP = CF$ esa BC va AD parallel to'g'ri chiziqlar orasidagi masofa bo'lgani uchun. Uchburchaklar tengligidan $\angle A = \angle D$ kelib chiqadi.

A va B, C va D burchaklar AD va BC parallel to'g'ri chiziqlarning, mos ravishda, AB va CD kesuvchilar bilan kesishishidan hosil bo'lgan ichki bir tomonli burchaklar, shuning uchun $\angle A + \angle B = 180°$ va $\angle C + \angle D = 180°$. Bundan $\angle B = \angle C$ ekani kelib chiqadi. Shunday qilib, teng yonli trapetsiyaning asosidagi burchaklari teng ekan: $\angle A = \angle D$ va $\angle B = \angle C$. Shuni isbotlash talab qilingan edi.

2- masala. Teng yonli trapetsiyaning kichik asosi yon tomoniga teng, diagonali esa yon tomoniga perpendikular. Trapetsiyaning burchaklarini toping.

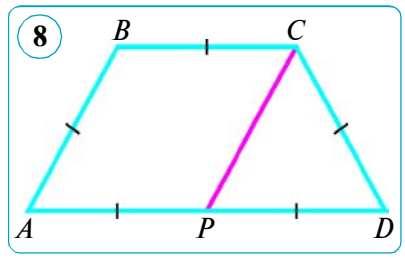


Yechish. Teng yonli $ABCD$ trapetsiya berilgan, unda $AD \parallel BC$, $AB = BC = CD$, $AC \perp CD$ bo'lsin (7- rasm). Masala shartiga ko'ra, AC – teng yonli ABC uchburchakning asosi, demak, $\angle BCA = \angle CAB$. Biroq $\angle A = \angle D$, chunki teng yonli trapetsiyaning asosidagi burchaklari teng, CAD va BCA burchaklar esa $AD \parallel BC$ hamda AC kesuvchi hosil qilgan ichki almashinuvchi burchaklar bo'lgani uchun teng, ya'ni $\angle CAD = \angle BCA$.

Demak, $\angle A = 2\angle CAD$. Shartga ko'ra, ACD – to'g'ri burchakli, shuning uchun $\angle CAD + \angle D = 90°$, lekin $\angle D = \angle A$, u holda $90° = 3\angle CAD$, demak, $\angle CAD = 30°$ va u holda $\angle D = \angle A = 60°$, $\angle C = \angle B = 120°$.

Javob: $\angle A = \angle D = 60°$, $\angle B = \angle C = 120°$.

3- masala. Teng yonli trapetsiyaning tomonlari nisbati $1 : 1 : 1 : 2$ kabi. Shu trapetsiyaning burchaklarini toping.



Yechish. $ABCD$ trapetsiyada $AB = BC = CD = 1$ va $AD = 2$ bo'lsin. AD tomonning o'rtasini P bilan belgilaymiz (8- rasm). $ABCP$ to'rtburchakning AP va BC tomonlari teng va parallel.

Demak, parallelogrammning alomatiga ko‘ra, bu to‘rtburchak parallelogramm bo‘ladi. Shunga ko‘ra, $PC = AB = 1$. PCD uchburchakning hamma tomonlari 1 ga teng, shuning uchun $\angle PDC = 60^\circ$. Shunday qilib, $ABCD$ trapetsiyada $\angle A = \angle D = 60^\circ$ va $\angle B = \angle C = 120^\circ$.

Javob: $\angle A = \angle D = 60^\circ$, $\angle B = \angle C = 120^\circ$.

Savol, masala va topshiriqlar

1. 1) Qanday to‘rtburchak trapetsiya deyiladi?

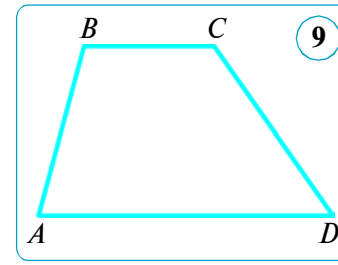
2) Qanday trapetsiya: a) teng yonli trapetsiya; b) to‘g‘ri burchakli trapetsiya deb ataladi?

2. Trapetsiya uchidan o‘tmagan balandligi uni ikkita to‘g‘ri burchakli trapetsiyaga ajratadi. Shaklni chizib ko‘rsating.

3. To‘g‘ri burchakli trapetsiya yon tomonlarining nisbati 1 : 2 kabi. Trapetsiyaning eng katta burchagini toping.

4. Trapetsiyaning asoslari 12 cm va 20 cm, yon tomonlari esa 4 cm va 11 cm. Kichik asosining uchidan kichik tomoniga parallel to‘g‘ri chiziq o‘tkazilgan. Shu parallel to‘g‘ri chiziq ajratgan uchburchakning perimetrini toping.

5. AD va BC asosli $ABCD$ trapetsiyaning B va C burchaklarini toping, bunda $\angle A = 75^\circ$ va $\angle D = 55^\circ$ (9- rasm). Bo‘sh joylarga mos sonlarni qo‘ying.



Yechish. A va B, C va D burchaklar AD va BC parallel to‘g‘ri chiziqlarni ... va ... kesuvchilar bilan kesishishidan hosil bo‘lgan ..., shuning uchun $\angle A + \angle B = ...^\circ$ va $\angle C + \angle D = ...^\circ$. Shartga ko‘ra, $\angle A = 75^\circ$ va $\angle D = 55^\circ$, u holda $\angle B = ...^\circ - \angle A = ...^\circ - ...^\circ = ...^\circ$ va $\angle C = ...^\circ - \angle D = ...^\circ - ...^\circ = ...^\circ$.

Javob: $\angle B = ...^\circ$, $\angle C = ...^\circ$.

6. Teng yonli trapetsiyaning o‘tkir burchaklaridan biri 60° ga, yon tomoni esa 16 cm ga teng. Agar trapetsiyaning asoslari yig‘indisi 38 cm ga teng bo‘lsa, uning asoslarini toping.

7. Teng yonli trapetsiyaning o‘tmas burchagi uchidan o‘tkazilgan balandlik katta asosini 3 cm va 17 cm li kesmalarga bo‘ladi. Uning asoslarini toping.

8. Teng yonli trapetsiyaning diagonallari teng ekanligini isbotlang.

9. Trapetsiyada: 1) uchta to‘g‘ri burchak; 2) uchta o‘tkir burchak; 3) uchta burchak yig‘indisi 180° ga teng bo‘la oladimi? Javobingizni asoslang.

10. To‘g‘ri burchakli trapetsiyaning eng katta va eng kichik burchaklari nisbati 5 : 4 ga teng. Shu trapetsiyaning burchaklarini toping.

11. $ABCD$ trapetsiyaning kichik asosi 6 cm ga, ABE uchburchakning ($BE \parallel CD$) perimetri 36 cm ga teng. Shu tapetsiyaning perimetrini toping.

12. Teng yonli trapetsiyaning diagonali o‘tmas burchagini teng ikkiga bo‘ladi. Trapetsiyaning asoslari 10 cm va 20 cm. Uning perimetrini toping.

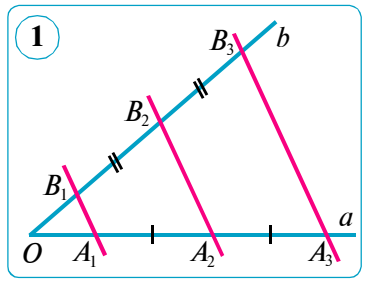
2- §. FALES TEOREMASI VA UNING TATBIQLARI

9. FALES TEOREMASI

Teorema.

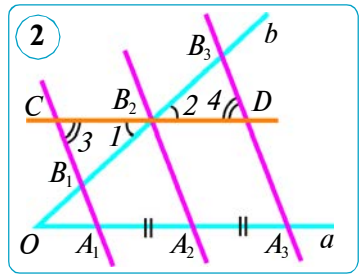
Agar burchak tomonlarini kesuvchi parallel to‘g‘ri chiziqlar uning bir tomonidan teng kesmalar ajratsa, ular ikkinchi tomonidan ham teng kesmalar ajratadi.

Isbot. O burchakning bir tomonida (a nurda) o‘zaro teng A_1A_2 va A_2A_3 kesmalar qo‘yilgan hamda ularning oxirlari (A_1, A_2, A_3) orqali ikkinchi tomonni (b nurni) B_1, B_2, B_3 nuqtalarda kesuvchi o‘zaro parallel A_1B_1, A_2B_2 va A_3B_3 to‘g‘ri chiziqlar o‘tkazilgan bo‘lsin (1- rasm).



Endi hosil bo‘lgan B_1B_2 va B_2B_3 kesmalarning o‘zaro tengligini, ya’ni $A_1A_2 = A_2A_3$ bo‘lsa, $B_1B_2 = B_2B_3$ bo‘lishini isbotlaymiz.

Buning uchun B_2 nuqtadan a nurga parallel CD to‘g‘ri chiziq o‘tkazamiz (2- rasm). Bu to‘g‘ri chiziq A_1B_1 va A_3B_3 to‘g‘ri chiziqlar bilan mos ravishda C va D nuqtalarda kesishsin. $A_1CB_2A_2$ va $A_2B_2DA_3$ to‘rtburchaklar – parallelogramm (ta’rifga ko‘ra), chunki ularning qarama-qarshi tomonlari shartga va yasashga ko‘ra parallel. Shartga ko‘ra, $A_1A_2 = A_2A_3$ hamda parallelogrammning qarama-qarshi tomonlari bo‘lgani uchun $A_1A_2 = CB_2$ va $A_2A_3 = B_2D$ dan $CB_2 = B_2D$ ga ega bo‘lamiz.



B_1B_2C va B_3B_2D uchburchaklarda $CB_2 = B_2D$ (isbotga ko‘ra), shuningdek, $\angle 1 = \angle 2$ (vertikal burchaklar), $\angle 3 = \angle 4$ (A_1B_1 va A_3B_3 parallel to‘g‘ri chiziqlar hamda CD kesuvchi kesishishidan hosil bo‘lgan ichki almashinuvchi burchaklar bo‘lgani uchun).

Uchburchaklar tengligining ikkinchi alomatiga ko‘ra, bu uchburchaklar o‘zaro teng: $\triangle B_1B_2C = \triangle B_3B_2D$. Bundan $B_1B_2 = B_2B_3$ kelib chiqadi.

Shunday qilib, agar $A_1A_2 = A_2A_3$ bo‘lsa, $B_1B_2 = B_2B_3$ bo‘lishi isbotlandi. Shuni isbotlash talab qilingan edi.

Eslatma! *Fales teoremasi shartida burchak o‘rniga har qanday ikki to‘g‘ri chiziqni olish mumkin, bunda teoremaning xulosasi o‘zgarmaydi.*

Natija. **Berilgan ikki to‘g‘ri chiziqni kesuvchi va to‘g‘ri chiziqlarning biridan teng kesmalar ajratuvchi parallel to‘g‘ri chiziqlar ikkinchi to‘g‘ri chiziqdan ham teng kesmalar ajratadi.**

1- masala. (*Kesmani teng bo‘laklarga bo‘lish.*) Berilgan AB kesmani n ta teng bo‘lakka bo‘ling.

Yechish. AB kesma berilgan bo‘lsin. Uni n ta teng bo‘lakka bo‘lishni ko‘rsatamiz. A nuqtadan AB to‘g‘ri chiziqda yotmaydigan AC nurni o‘tkazamiz va unda A nuqtadan boshlab n ta AA_1, A_1A_2, A_2A_3, …, $A_{n-1}A_n$ teng kesmalarni, ya’ni berilgan AB kesmani masala shartidan kelib chiqib nechta bo‘lakka bo‘lish zarur bo‘lsa, shuncha teng kesmani qo‘yamiz (3- rasm, $n=6$). So‘ngra A_nB to‘g‘ri chiziqni (A_n nuqta – oxirgi kesmaning oxiri) va A_1, A_2, A_3, …, A_{n-1} nuqtalar orqali A_nB to‘g‘ri chiziqqa parallel to‘g‘ri chiziqlarni o‘tkazamiz. Bu to‘g‘ri chiziqlar AB kesmani B_1, B_2, B_3, …, B_{n-1} nuqtalarda kesadi va uni Fales teoremasiga ko‘ra n ta teng bo‘lakka bo‘ladi:

$$AB_1 = B_1B_2 = \ldots = B_{n-1}B.$$

Demak, har qanday kesmani istalgancha teng bo‘lakka bo‘lish mumkin.

2- masala. ABC uchburchakning BC tomoni to‘rtta teng kesmaga bo‘linib, bo‘linish nuqtalari orqali uzunligi 18 cm ga teng bo‘lgan AB tomonga parallel ravishda to‘g‘ri chiziqlar o‘tkazilgan. Shu to‘g‘ri chiziqlarning uchburchak ichida qolgan kesmalarining uzunliklarini toping.

Berilgan: $\triangle ABC$ da:

$BB_1 = B_1B_2 = B_2B_3 = B_3C$, $AB = 18$ cm; $B_1C_3 \parallel B_2C_2 \parallel B_3C_1 \parallel AB$.

Topish kerak: B_1C_3, B_2C_2, B_3C_1 (4- rasm).

Yechish. 1) $A_1B_3 \parallel A_2B_2 \parallel A_3B_1 \parallel AC$ o‘tkazamiz.

2) Fales teoremasiga ko‘ra:

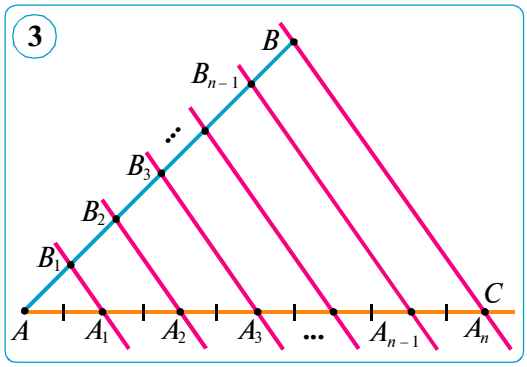
$$AA_1 = A_1A_2 = A_2A_3 = A_3B = AB : 4 = 18 : 4 = 4{,}5 \text{ (cm)}.$$

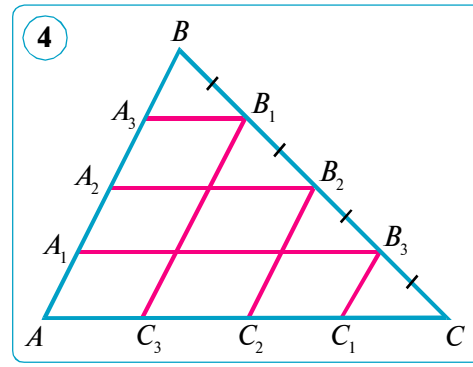
2) Ta’rifga ko‘ra, $AA_1B_3C_1$ to‘rtburchak – parallelogramm, chunki $AA_1 \parallel C_1B_3$ (shartga ko‘ra) va $A_1B_3 \parallel AC_1$ (yasashga ko‘ra).

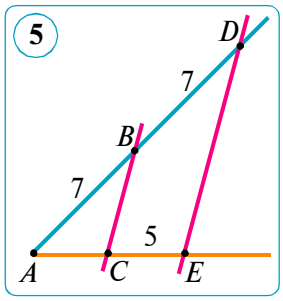
Demak, $AA_1 = C_1B_3 = 4{,}5$ (cm).

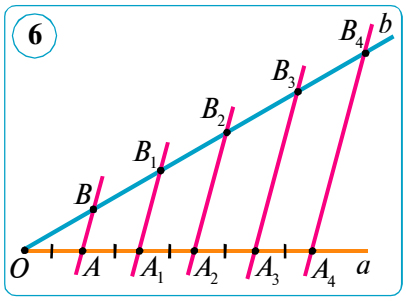
3) Ta’rifga ko‘ra, $AA_2B_2C_2$ to‘rtburchak – parallelogramm, chunki $AA_2 \parallel C_2B_2$ (shartga ko‘ra) va $A_2B_2 \parallel AC_2$ (yasashga ko‘ra). Demak,

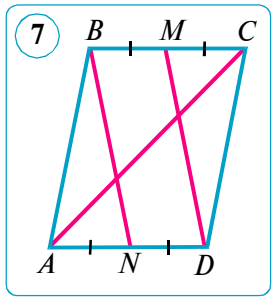
$$AA_2 = C_2B_2 = 2AA_1 = 2 \cdot 4{,}5 = 9 \text{ (cm)}.$$











4) Ta'rifga ko'ra, $AA_3B_1C_3$ to'rtburchak – parallelogramm, chunki $AA_3 \parallel C_3B_1$ (shartga ko'ra) va $A_3B_1 \parallel AC_3$ (yasashga ko'ra). Demak,

$$AA_3 = C_3B_1 = 3AA_1 = 3 \cdot 4{,}5 = 13{,}5 \text{ (cm)}.$$

Javob: $C_1B_3 = 4{,}5$ cm, $C_2B_2 = 9$ cm, $C_3B_1 = 13{,}5$ cm.

Savol, masala va topshiriqlar

1. 1) Fales teoremasini ayting.

2) Fales teoremasi faqat burchak uchun o'rinlimi?

3) Berilgan kesma qanday qilib n ta teng bo'lakka bo'linadi?

2. (*Amaliy topshiriq.*) Sirkul va chizg'ich yordamida berilgan AB kesmani: 1) ikkita; 2) uchta; 3) oltita; 4) yettita teng bo'lakka bo'ling.

3. *Berilgan*: $\angle A$, $AB = BD = 7$ cm, $BC \parallel DE$, $CE = 5$ cm (5- rasm).
Topish kerak: AC.

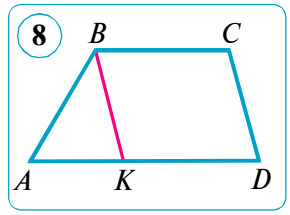
4. *Berilgan:* $\angle aOb$, $OA = AA_1 = A_1A_2 = A_2A_3 = A_3A_4$,
$AB \parallel A_1B_1 \parallel A_2B_2 \parallel A_3B_3 \parallel A_4B_4$, $OB_4 = 8$ cm (6- rasm).
Topish kerak: OB_1, OB_2, OB_3.

5. $ABCD$ parallelogrammda M nuqta BC tomonning, N nuqta AD tomonning o'rtasi. BN va MD to'g'ri chiziqlar parallelogrammning AC diagonalini teng uchta bo'lakka bo'lishini isbotlang (7- rasm).

6. $ABCD$ trapetsiyada B uchi orqali CD tomonga parallel BK to'g'ri chiziq o'tkazilgan (8- rasm).

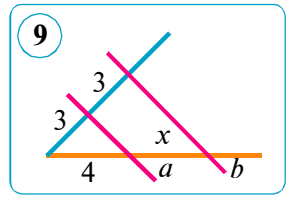
1) $KBCD$ – parallelogramm ekanini isbotlang.

2) Agar $BC = 4$ cm, $P_{ABK} = 11$ cm bo'lsa, trapetsiyaning perimetrini toping.



7. Sirkul va chizg'ich yordamida berilgan AB kesmani: 1) to'rtta; 2) beshta teng bo'lakka bo'ling.

8. $a \parallel b$ ekani ma'lum. 9- rasmda berilgan ma'lumotlardan foydalanib, x ni toping.



9. *Berilgan:* $\angle aOb$, $OA = AA_1 = A_1A_2 = A_2A_3 = A_3A_4$,
$AB \parallel A_1B_1 \parallel A_2B_2 \parallel A_3B_3 \parallel A_4B_4$, $OB_4 - B_3B_4 = 18$ cm (6- rasmga q.).
Topish kerak: OB_1, OB_2, OB_3.

10–11. UCHBURCHAK O‘RTA CHIZIG‘INING XOSSASI. TRAPETSIYA O‘RTA CHIZIG‘INING XOSSASI

1. Uchburchak o‘rta chizig‘ining xossasi.

Ta’rif. *Uchburchakning* ***o‘rta chizig‘i*** *deb, uning ikki tomoni o‘rtalarini tutashtiruvchi kesmaga aytiladi.*

ABC uchburchakda $AD = DB$ va $CE = EB$ bo‘lsin, u holda DE o‘rta chiziq bo‘ladi (ta’rifga ko‘ra). DE o‘rta chiziqqa nisbatan AC tomon *asos* deb ataladi (1- rasm). Har qanday uchburchakning uchta o‘rta chizig‘i bo‘ladi (2- rasm).

1- teorema.

Uchburchakning o‘rta chizig‘i uning uchinchi tomoniga parallel bo‘lib, uzunligi bu tomon uzunligining yarmiga teng.

Berilgan: $\triangle ABC$ da: $AD = DB$, $CE = EB$, DE – o‘rta chiziq (3- rasm).

Isbot qilish kerak: 1) $DE \parallel AC$; 2) $DE = \frac{1}{2} AC$.

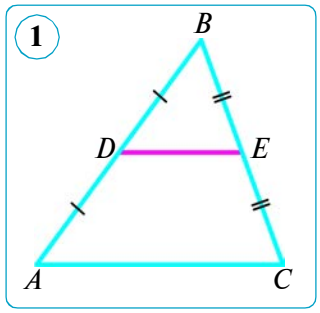
Isbot. 1) DE kesma ABC uchburchakning o‘rta chizig‘i bo‘lsin. D nuqta orqali AC tomonga parallel to‘g‘ri chiziq o‘tkazamiz. Bu to‘g‘ri chiziq Fales teoremasiga ko‘ra BC kesmani o‘rtasidan kesib o‘tadi, ya’ni DE o‘rta chiziqni o‘z ichiga oladi. Yasalishiga ko‘ra, $DE \parallel AC$.

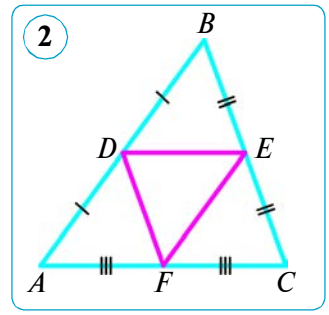
2) Endi EF o‘rta chiziqni o‘tkazamiz. 1- bandda isbotlanganga ko‘ra, u AB tomonga parallel bo‘ladi: $EF \parallel AB$, bundan $EF \parallel AD$. $ADEF$ to‘rtburchakning qarama-qarshi tomonlari o‘zaro parallel bo‘lgani uchun u ta’rifga ko‘ra parallelogramm bo‘ladi. Parallelogrammning xossasiga ko‘ra $DE = AF$, Fales teoremasiga ko‘ra $AF = FC$ bo‘lgani uchun $DE = \frac{1}{2} AC$.

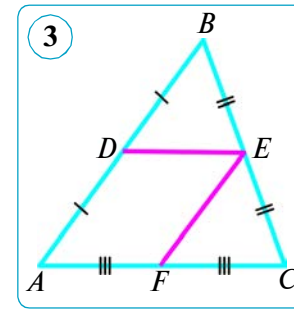
Teorema isbotlandi.

1- masala. Uchburchakning perimetri p ga teng. Uchlari berilgan uchburchak tomonlarining o‘rtalarida bo‘lgan uchburchakning perimetrini toping.

Yechish. Hosil bo‘lgan uchburchakning tomonlari berilgan uchburchakning o‘rta chiziqlari bo‘ladi (2- rasm). Demak, ular mos tomonlarining yar-







miga teng. Shu sababli izlanayotgan perimetr berilgan uchburchak perimetrining yarmiga teng bo‘ladi:

$$P_{DEF} = DE + EF + FD = 0{,}5(AC + AB + BC) = 0{,}5p.$$

Javob: $0{,}5p$.

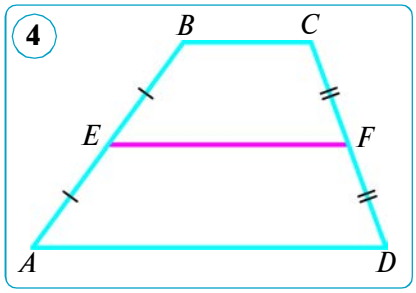
2. Trapetsiya o‘rta chizig‘ining xossasi.

Ta’rif. *Trapetsiya yon tomonlari o‘rtasini tutashtiruvchi kesma* ***trapetsiyaning o‘rta chizig‘i*** *deyiladi.*

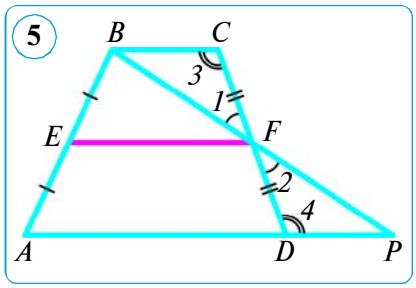
Bizga $ABCD$ trapetsiya berilgan bo‘lib, unda AD va BC – trapetsiya asoslari, AB va DC – yon tomonlari, E va F nuqtalar yon tomonlarining o‘rtalari bo‘lsin (4- rasm). Bunda EF trapetsiyaning o‘rta chizig‘i bo‘ladi.

2- teorema.

Trapetsiyaning o‘rta chizig‘i uning asoslariga parallel va uning uzunligi trapetsiya asoslari uzunliklari yig‘indisining yarmiga teng.



Isbot. EF – asoslari AD va BC bo‘lgan $ABCD$ trapetsiyaning o‘rta chizig‘i bo‘lsin ($AD \parallel BC$). BF to‘g‘ri chiziq o‘tkazamiz va uning AD to‘g‘ri chiziq bilan kesishish nuqtasini P deb belgilaymiz (5- rasm). Uchburchaklar tengligining ikkinchi alomatiga ko‘ra, BCF va PDF uchburchaklar teng ($CF = DF$ shartga ko‘ra, $\angle 1 = \angle 2$ – vertikal burchaklar va $\angle 3 = \angle 4$ – BC va AD parallel to‘g‘ri chiziqlar hamda CD kesuvchi hosil qilgan ichki almashinuvchi burchaklar bo‘lgani uchun). Bu uchburchaklarning tengligidan tomonlar teng degan xulosa chiqadi: $BF = PF$ va $BC = DP$. Demak, trapetsiyaning EF o‘rta chizig‘i ABP uchburchakning o‘rta chizig‘i ekan. Uchburchak o‘rta chizig‘ining xossasiga ko‘ra:



$$EF \parallel AP \text{ va } EF = \frac{1}{2}AP.$$

$AD \parallel BC$ bo‘lgani sababli, EF har ikkala asosga parallel bo‘ladi va quyiqagicha ifodalanishi mumkin:

$$EF = \frac{1}{2}AP = \frac{1}{2}(AD + DP) = \frac{1}{2}(AD + BC).$$

Demak, $EF \parallel AD \parallel BC$ va $EF = \frac{1}{2}(AD + BC)$. Teorema isbotlandi.

Natija. **Trapetsiyaning yon tomoni o‘rtasidan o‘tuvchi va asoslariga parallel to‘g‘ri chiziq ikkinchi yon tomonini teng ikkiga bo‘ladi.**

Buni mustaqil isbot qiling.

Savol, masala va topshiriqlar

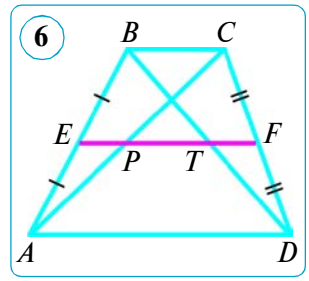
1. 1) Uchburchakning o‘rta chizig‘i deb nimaga aytiladi?

? 2) Uchburchakda nechta o‘rta chiziq yasash mumkin?

2. Uchburchakning tomonlari 5 cm, 7 cm va 11 cm ga teng. Uchlari shu uchburchak tomonlarining o‘rtalarida yotgan uchburchak tomonlarini toping.

3. Uchburchakning o‘rta chiziqlari 6 cm, 7 cm va 9 cm ga teng bo‘lgan uchburchakning tomonlarini toping.

4. Trapetsiyaning diagonallari uning o‘rta chizig‘i EF ni E uchidan boshlab 5 cm, 7 cm va 4 cm li kesmalarga bo‘ladi (6- rasm). Trapetsiya asoslarini toping.



5. ABC uchburchak tomonlarining har biri uchta teng kesmaga bo‘lingan va bo‘linish nuqtalari kesmalar bilan tutashtirilgan. ABC uchburchakning perimetri p ga teng bo‘lsa, 7- rasmda hosil bo‘lgan shaklning perimetrini toping.

6. Trapetsiyaning asoslari: 1) 4,5 dm va 8,2 dm; 2) 9 cm va 21 cm ga teng. Uning o‘rta chizig‘i uzunligi qancha?

7. $ABCD$ trapetsiyada (8- rasm) EF kesma CD tomonga parallel, E nuqta esa AB ning o‘rtasi. $EF = 0{,}5CD$ ekanligini isbotlang.

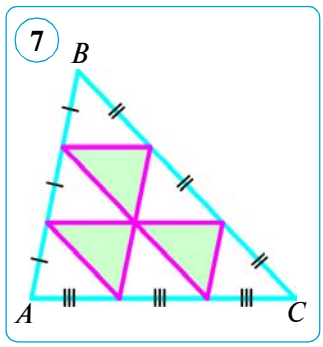
8. 9- rasmdagi noma’lum uzunliklarni hisoblang.

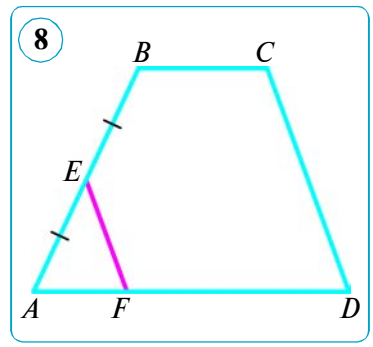
9. Trapetsiyaning diagonallari uning o‘rta chizig‘ini har biri 6 cm li kesmalarga bo‘ladi. Shu trapetsiya asoslarini toping.

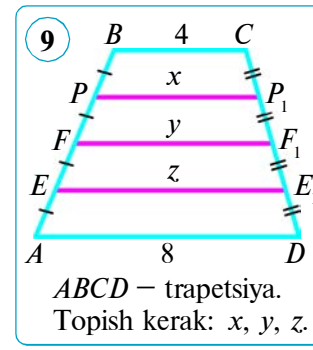
10. Teng yonli trapetsiyada uzunligi 6 cm ga teng diagonali asosi bilan 60° li burchak tashkil qiladi. Trapetsiyaning o‘rta chizig‘ini toping.

11. Trapetsiyaning katta asosi kichik asosidan 3 marta katta va uning o‘rta chizig‘i 20 cm ga teng. Trapetsiyaning asoslarini toping.

12. Trapetsiyaning perimetri 40 cm ga, parallel bo‘lmagan tomonlarining yig‘indisi esa 16 cm ga teng. Shu trapetsiyaning o‘rta chizig‘ini toping.







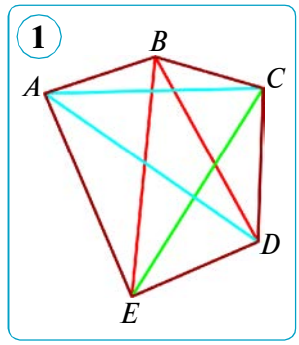
$ABCD$ – trapetsiya.
Topish kerak: x, y, z.

12. AMALIY MASHQ VA TATBIQ

Tadqiqot uchun masalalar.

1- masala. Tomonlar soni n bo‘lgan ko‘pburchakni yasang va uning diagonallarini o‘tkazing, bunda: 1) $n = 5$; 2) $n = 7$; 3) $n = 8$. Ko‘pburchakning turli diagonallari sonini (d_n) hisoblash formulasini mulohaza yuritib toping.

Yechish. 1) $n = 5$. A uchidan **2** ta AC va AD, B uchidan **2** ta BD va BE diagonallar chiqadi va h.k. Beshta uchning har biridan **2** tadan diagonal chiqadi (1- rasm).



Bundan qavariq beshburchakning har bir uchidan chiqqan diagonallari soni tomonlari (uchlari) sonidan 3 taga kamligi, ya’ni $5 - 3 = 2$ ga teng ekani kelib chiqadi. Hamma uchlaridan chiqqan diagonallari sonini topish uchun tomonlari sonini **2** ga ko‘paytiramiz:

$$\mathbf{5 \cdot (5 - 3) = 5 \cdot 2 = 10}.$$

Bu ko‘paytmada har bir diagonal ikki martadan hisobga olingan. Ammo AC va CA, BD va DB va h.k. bir diagonalning ikki xil belgilanishidir, ya’ni ular yangi diagonallar emas. Shu sababli hosil qilingan ko‘paytmani **2** ga bo‘lib, jami turli diagonallar sonini topamiz:

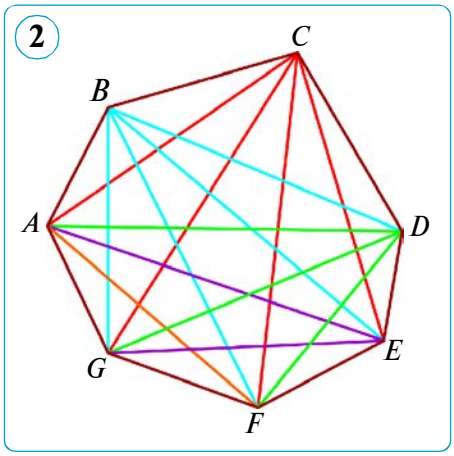
$$5 \cdot 2 : 2 = \mathbf{5}.$$

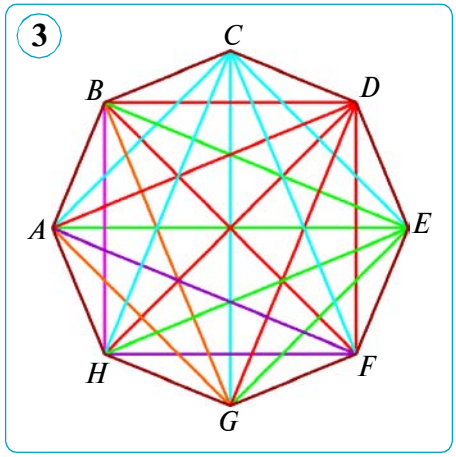
Demak, qavariq beshburchakning jami turli diagonallari soni quyidagiga teng:

$$d_5 = \frac{5 \cdot (5-3)}{2} = \frac{5 \cdot \cancel{2}^{1}}{1\cancel{2}} = \mathbf{5}.$$

Javob: 5 ta.

2) $n = 7$. Qavariq yettiburchakning ja’mi turli diagonallari soni yuqorida ko‘rsatib o‘tilgan masala yechimiga o‘xshab topiladi. Qilingan muhokamalarda aniqlangan qonuniyatga asoslanib, qavariq yettiburchakning diagonallari sonini quyidagicha topamiz (2- rasm):





$$d_7 = \frac{7\cdot(7-3)}{2} = \frac{7\cdot \cancel{4}^{2}}{\cancel{2}_{1}} = \mathbf{14}.$$

Javob: 14 ta.

3) $n = 8$. Qavariq sakkizburchakning ja'mi turli diagonallari soni yuqorida ko'rsatib o'tilgan masala yechimiga o'xshab topiladi. Qilingan muhokamalarda aniqlangan qonuniyatga asoslanib, qavariq sakkizburchakning diagonallari sonini topamiz (3- rasm):

$$d_8 = \frac{8\cdot(8-3)}{2} = \frac{\cancel{8}^{4}\cdot 5}{\cancel{2}_{1}} = \mathbf{20}.$$

Javob: 20 ta.

Demak, istalgan qavariq ko'pburchakning turli diagonallari soni quyidagi formula bo'yicha topiladi:

$$\boldsymbol{d_n = \frac{n(n-3)}{2}}.$$

Eslatma! *Qavariq n burchakning bir uchidan chiqqan diagonallari uni (n – 2) ta uchburchakka ajratadi.*

2- masala. Ko'pburchakning 25 ta diagonali bo'lishi mumkinmi?

Yechish. n burchakning jami turli diagonallari soni $d_n = \frac{n(n-3)}{2}$ ga teng. Demak, $\frac{n(n-3)}{2} = 25$. U holda $n(n-3) = 50$ yoki $n(n-3) = 2\cdot 5\cdot 5$. Bundan ko'rinadiki, 50 ni bir-biridan 3 ga farq qiladigan ikkita natural sonning ko'paytmasi ko'rinishida ifodalab bo'lmaydi. Shuning uchun jami turli diagonallari soni 25 ta bo'ladigan ko'pburchak mavjud emas.

Javob: yo'q, mavjud emas.

3- masala. Matematika xonasidagi rasmlarda tasvirlangan uchburchak va to'rtburchaklarning soni 15 ta. Bu shakllarning jami tomonlari soni 53 ta. Rasmlarda nechta uchburchak va nechta to'rtburchak tasvirlangan?

Yechish. To'rtburchakning tomonlari soni natural sonning ixtiyoriy qiymatida to'rtga karrali, ya'ni juft son bo'ladi. Uchburchaklar soni toq son bo'lgandagina yig'indi toq bo'ladi.

Masala shartiga ko'ra tenglama tuzamiz: $3x + 4y = 53$.

Quyida mumkin bo'lgan hollarni ko'rib chiqamiz. Tenglamadagi noma'lumlar o'rniga tegishli qiymatlarni qo'yib, uni qanoatlantiruvchi yechimni topamiz.

1- hol. $x = 1$ va $y = 14$ bo'lsin. U holda $3\cdot 1 + 4\cdot 14 = 53$, ya'ni $59 \neq 53$.

2- hol. $x = 3$, $y = 13$; $3\cdot 3 + 4\cdot 12 = 53$, ya'ni $57 \neq 53$.

3- hol. $x = 5$, $y = 10$; $3\cdot 5 + 4\cdot 10 = 53$, ya'ni $55 \neq 53$.

4- hol. $x = \mathbf{7}$, $y = \mathbf{8}$; $3\cdot 7 + 4\cdot 8 = 53$, ya'ni $53 = 53$.

4- hol masala shartini qanoatlantirdi, shu sababli boshqa hollar qaralmaydi.

Javob: **7** ta uchburchak, **8** ta to'rtburchak.

Mustahkamlash uchun qo'shimcha mashqlar.

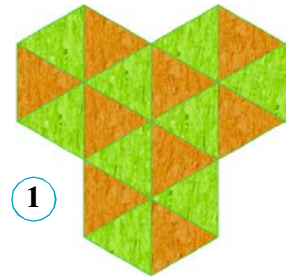
1. Qavariq ko'pburchakning bir uchidan chiqqan diagonallari soni 13 ta. Shu ko'pburchakning tomonlari soni nechta? Jami diagonallari soni-chi?
2. Diagonallarining soni: 1) tomonlari soniga teng; 2) tomonlarining sonidan kam bo'lgan; 2) tomonlarining sonidan ortiq bo'lgan ko'pburchak bormi?

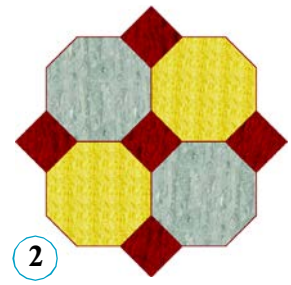
AMALIY KOMPETENSIYANI RIVOJLANTIRUVCHI QO‘SHIMCHA MATERIALLAR

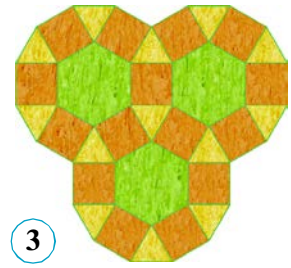
MUNTAZAM KO‘PBURCHAKLI PARKETLAR

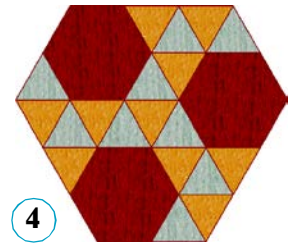
Siz, albatta, parket to‘g‘risida ma’lum bir tasavvurga egasiz. Ko‘pincha uylar, turli inshootlar pollari to‘g‘ri to‘rtburchak, kvadrat va muntazam oltiburchakli parketlar bilan bezatiladi.

Matematik nuqtayi nazardan qaraganda, parket – bu tekislikni geometrik shakllar bilan bir-biriga zich va ularni kesishmaydigan qilib joylashtirishdir. Dastlab muntazam ko‘pburchaklar – kvadrat, to‘rtburchak va oltiburchakli parketlarni ko‘rib chiqamiz. Bir xil kvadratlardan tuzilgan katakli daftaringiz eng sodda parketlarga misol bo‘ladi. 1- rasmda muntazam uchburchaklardan; 2- rasmda kvadrat bilan muntazam oltiburchakdan; 3- rasmda esa muntazam oltiburchaklar, kvadratlar va teng tomonli uchburchaklardan; 4- rasmda muntazam oltiburchaklar va uchburchaklardan tuzilgan chiroyli parketlar tasvirlangan.


1


2


3


4

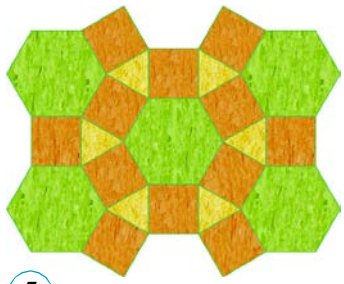
Parket deb, tekislikni ko‘pburchaklar bilan shunday qoplashga aytiladiki, bunda ixtiyoriy ikkita ko‘pburchak umumiy tomonga yoki umumiy uchga ega bo‘ladi, yoxud umumiy uchlarga ega bo‘lmaydi.

Agar parket muntazam ko‘pburchaklardan tashkil topsa va har bir uch atrofida ko‘pburchaklar bir xil usulda joylashgan bo‘lsa, parket *muntazam* deyiladi.

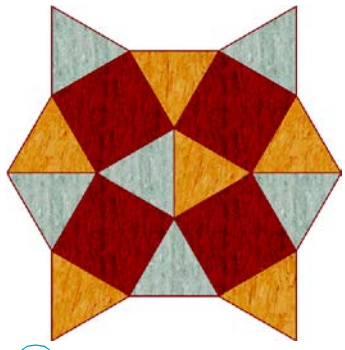
Teng tomonli uchburchaklar, kvadratlar va muntazam oltiburchaklar tekislikni qoplovchi parketlarga misol bo‘ladi. Bulardan boshqa muntazam ko‘pburchaklar bilan tekislikni qoplash mumkin emasligini isbotlaymiz. Buning uchun parketning bir uchidan chiquvchi ko‘pburchaklarning burchaklari yig‘indisi 360° ga teng bo‘lishidan foydalanamiz.







5



6

Buning uchun muntazam beshburchakni ko‘rib chiqamiz. Bizga ma’lumki, muntazam beshburchakning ichki burchaklari 108° ga teng. Parketning bir uchiga uchta muntazam beshburchakni joylashtirish mumkin emas, chunki bu holda burchaklar yig‘indisi $324° < 360°$ bo‘ladi. Agar muntazam beshburchaklar soni 4 ga teng yoki undan katta bo‘lsa, u holda burchaklar yig‘indisi $432° > 360°$ bo‘ladi. Shuning uchun muntazam beshburchaklardan tuzilgan parketlar mavjud emas. Xuddi shunga o‘xshash parketning bir uchiga uchta yoki undan ko‘p bo‘lgan muntazam yettiburchakli, muntazam sakkizburchakli va hokazo parket bo‘lagini joylashtirib bo‘lmaydi, chunki ularning har bir burchagi 120° dan katta va ularning yig‘indisi 360° dan katta bo‘ladi. Shu sababli muntazam yettiburchak, muntazam sakkizburchak va hokazodan tuzilgan parketlar mavjud emas.

5- rasmdagi muntazam oltiburchaklar, kvadratlar va teng tomonli uchburchaklardan tuzilgan parketlar 3- rasmdagi parketlardan joylashishi bilan farq qiladi. 6- rasmda esa teng tomonli uchburchaklar va kvadratlardan tuzilgan parket tasvirlangan. Keltirilgan har ikkala parketda ham umumiy qonuniyat saqlanganini ko‘rish mumkin, ya’ni har uchi atrofida joylashgan shakllarning ichki burchaklari yig‘indisi 360° ga tengligi o‘z-o‘zidan ayon. Masalan, 5- rasmda $60° + 90° + 90° + 120° = 360°$, ya’ni bir uchi atrofida bitta teng tomonli uchburchak, 2 ta kvadrat va bitta muntazam oltiburchak joylashgan; 6- rasmda esa bitta uch atrofida 3 ta teng tomonli uchburchak (har bir ichki burchagi 60° dan) va 2 ta kvadrat (har bir ichki burchagi 90° dan) joylashgan.

Tekislikni qoplovchi muntazam parketlarning boshqa turlarini quyidagi jadvalda keltiramiz. 5–6- rasmlardagi parketlarni yasab ko‘ring.

α_1	α_2	α_3	α_4	α_5	α_6	$\alpha_1 + \alpha_2 + \alpha_3 + \ldots = 360°$
60°	60°	60°	60°	60°	60°	Uchburchaklardan tuzilgan parket
60°	60°	120°	120°			Uchburchaklar va oltiburchaklardan tuzilgan parket
60°	90°	90°	120°			Uchburchak, kvadratlar va oltiburchakdan tuzilgan parket
60°	150°	150°				Uchburchak va o‘nikkiburchaklardan tuzilgan parket
90°	90°	90°	90°			Kvadratlardan tuzilgan parket
120°	120°	120°				Oltiburchaklardan tuzilgan parket

13–14. 1- NAZORAT ISHI. XATOLAR USTIDA ISHLASH

1. To‘g‘ri to‘rtburchakning perimetri 40 cm ga, tomonlarining nisbati 3 : 5 ga teng. Shu to‘g‘ri to‘rtburchakning tomonlarini toping.

2. Parallelogrammning tomonlaridan biri ikkinchisidan 4 marta katta, perimetri esa 30 cm ga teng. Parallelogrammning tomonlarini toping.

3. To‘g‘ri burchakli trapetsiyaning o‘tkir burchagi 45° ga, kichik yon tomoni hamda kichik asosi 16 cm ga teng. Trapetsiyaning katta asosini toping.

4. $ABCD$ trapetsiyada AD – katta asosi. B uchi orqali CD tomonga parallel va AD tomonni E nuqtada kesuvchi to‘g‘ri chiziq o‘tkazilgan, $BC = 7$ cm, $AE = 4$ cm. 1) Trapetsiyaning o‘rta chizig‘ini; 2) ABE uchburchakning perimetri 17 cm bo‘lsa, shu trapetsiyaning perimetrini toping.

1- TEST — O‘zingizni sinab ko‘ring!

1. Qavariq to‘rtburchakning burchaklaridan biri to‘g‘ri burchak, qolganlari esa o‘zaro 3 : 4 : 8 nisbatda. To‘rtburchakning kichik burchagini toping.

A) 72°; B) 54°; D) 144°; E) 90°.

2. Har bir ichki burchagi 156° bo‘lgan qavariq ko‘pburchakning nechta tomoni bor?

A) 10 ta; B) 15 ta; D) 12 ta; E) 8 ta.

3. $ABCD$ parallelogrammning perimetri 32 cm ga, BD diagonali 9 cm ga teng. ABD uchburchakning perimetrini toping.

A) 16 cm; B) 25 cm; D) 23 cm; E) 41 cm.

4. Ikkita burchagining yig‘indisi 100° ga teng bo‘lgan parallelogrammning katta burchagini toping.

A) 120°; B) 110°; D) 150°; E) 130°.

5. Rombning burchaklaridan biri 150° ga teng, kichik diagonali esa 4,5 cm. Rombning perimetrini toping.

A) 27 cm; B) 18 cm; D) 13 cm; E) 21,5 cm.

6. $ABCD$ trapetsiyaning o‘rta chizig‘i uni o‘rta chiziqlari 13 cm va 17 cm ga teng bo‘lgan ikkita trapetsiyaga ajratadi. Trapetsiyaning katta asosini toping.

A) 19 cm; B) 21 cm; D) 18 cm; E) 30 cm.

7. Uchburchakning o‘rta chizig‘i uning asosidan 5,4 cm qisqa. Uchburchakning o‘rta chizig‘i bilan asosining yig‘indisini toping.

A) 13,5 cm; B) 16,2 cm; D) 10,8 cm; E) 21,6 cm.

8. Teng yonli trapetsiyaning perimetri 36 cm, o‘rta chizig‘i 10 cm. Yon tomonining uzunligini toping.

A) 10 cm; B) 8 cm; D) 12 cm; E) 13 cm.

9. Trapetsiyaning o‘rta chizig‘i 9 cm, asoslaridan biri ikkinchisidan 6 cm qisqa. Trapetsiyaning katta asosini toping.

A) 15 cm; B) 18 cm; D) 12 cm; E) 10 cm.

Ingliz tilini o‘rganamiz!

Ko‘pburchak – polygon
To‘g‘ri to‘rtburchak – rectangle
Romb – rhombus
Kvadrat – square
Balandlik – height

Perimetr – perimeter
Diagonal – diagonal
Parallelogramm – parallelogramm
Trapetsiya – trapezoid
Burchak – angle

Tarixiy ma’lumotlar



Abu Rayhon Beruniy
(973–1048)

Qadimda Misr va Bobil matematikasida to‘rtburchaklarning quyidagi turlari uchraydi: kvadratlar, to‘g‘ri to‘rtburchaklar, to‘g‘ri burchakli va teng yonli trapetsiyalar. O‘rta osiyolik olimlardan **Abu Rayhon Beruniy** ham to‘rtburchaklarning turlari bo‘yicha ko‘plab izlanishlar olib borgan. U o‘zining «Astronomiya san’atidan boshlang‘ich ma’lumot beruvchi kitob» nomli asarida «To‘rtburchaklarning turi qanday?» deb savol qo‘yadi va unga quyidagicha javob beradi:

*«Ulardan **birinchisi** – **kvadrat**, uning barcha tomonlari teng, barcha burchaklari to‘g‘ri, diagonallari, ya’ni qarama-qarshi burchaklarini (uchlarini) tutashtiruvchi chiziqlari o‘zaro teng.*

***Ikkinchisi** – **to‘g‘ri to‘rtburchak**, u kvadratga nisbatan uzunroq, barcha burchaklari to‘g‘ri, turli tomonlari turlicha, uning faqat qarama-qarshi tomonlari va diagonallari teng.*

***Uchinchisi** – **romb**, uning to‘rtta tomoni teng, ammo diagonallari turlicha, burchaklari esa to‘g‘ri burchak emas.*

***To‘rtinchisi** – **romboid**, uning diagonallari turlicha, faqat ikkitadan qarama-qarshi tomonlari teng.*

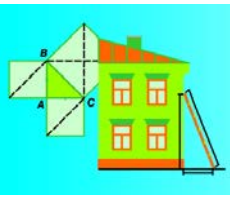
*Bu shakllardan farqli to‘rtburchaklar **trapetsiyalar** deyiladi».*

Kvadrat lotincha so‘z bo‘lib, «to‘rt burchakli» degan ma’noni bildiradi. Beruniy arabcha «*murabba*» atamasini ishlatgan, lotinchaga mana shu atama tarjima qilingan. To‘g‘ri to‘rtburchak arab tilida «*mustatil*» – «cho‘zinchoq» degan ma’noni anglatadi.

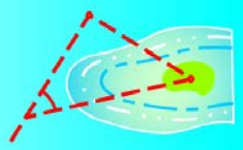
Romb atamasining vujudga kelishi turlicha tushuntiriladi. U grekcha so‘z bo‘lib, romb «*aylanuvchi jism*», «*pildiroq*» ma’nosini beradi. Geometriyaga bu atama pildiroq kesimining rombga o‘xshashligi tufayli kiritilgan. Arabchada «*romb*» uchun «*muayyan*» atamasi olingan.

Trapetsiya grekcha so‘z bo‘lib, tarjimasi «*stolcha*» (ovqat yeyiladigan stol)ga to‘g‘ri keladi, lug‘aviy ma’nosi – to‘rt oyoqlik. Haqiqatan, grekcha «*trapedzion*» so‘zi «stolcha», «xo‘rak stoli» degan ma’noni anglatadi.

Beruniy asarlarida «*trapetsiya*» – «*muxarrif*» deb nomlanib, bu atama yunoncha «*trapedzion*» so‘zining arab tilidagi tarjimasidir.



II BOB
TO‘G‘RI BURCHAKLI UCHBURCHAKNING TOMONLARI VA BURCHAKLARI ORASIDAGI MUNOSABATLAR

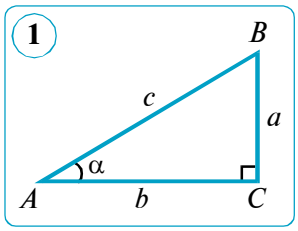


3- §. O‘TKIR BURCHAKNING TRIGONOMETRIK FUNKSIYALARI

15. O‘TKIR BURCHAKNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI

Trigonometriya matematikaning bo‘limi bo‘lib, uchburchakning tomonlari bilan burchaklari orasidagi bog‘lanishlar, trigonometrik funksiyalarning xossalari va ular orasidagi munosabatlarni o‘rganadi. «**Trigonometriya**» so‘zi grekcha «**trigon**» – uchburchak va «**metrezis**» – o‘lchash degan so‘zlaridan olingan bo‘lib, o‘zbek tilida «**uchburchaklarni o‘lchash**» degan ma’noni bildiradi.

Trigonometriyaning asosiy vazifasi *uchburchaklarni yechishdan* iboratdir. Uchburchak geometriyaning eng muhim shakllaridan biri hisoblanadi. Shuning uchun uchburchaklarni o‘rganishni davom ettiramiz. Bobning asosiy maqsadi uchburchaklarning biror elementi (tomonlari va burchaklari)ni boshqa elementlari orqali ifodalashdan iborat.



Katetlari $BC = a$ va $AC = b$, gipotenuzasi $AB = c$ va o‘tkir burchagi $\angle A = \alpha$ bo‘lgan to‘g‘ri burchakli ($\angle C = 90°$) ABC uchburchak berilgan bo‘lsin (1- rasm).

Shu uchburchakning jufti-jufti bilan tomonlari nisbatini olaylik:

$\frac{a}{c}$ – α burchak qarshisidagi katetning gipotenuzaga nisbati;

$\frac{b}{c}$ – α burchakka yopishgan katetning gipotenuzaga nisbati;

$\frac{a}{b}$ – α burchak qarshisidagi katetning shu burchakka yopishgan katetga nisbati;

$\frac{b}{a}$ – α burchakka yopishgan katetning shu burchak qarshisidagi katetga nisbati;

$\frac{c}{b}$ – gipotenuzaning α burchakka yopishgan katetga nisbati;

$\frac{c}{a}$ – gipotenuzaning α burchak qarshisidagi katetga nisbati.

Shunday qilib, jami 6 ta nisbatni hosil qildik.

Xuddi shunga o‘xshash, ikkinchi o‘tkir burchak (B) uchun ham shu tartibda nisbatlarni tuzishimiz mumkin.

Bu nisbatlardan dastlabki to‘rttasi *maxsus nomlar* bilan ataladi.

1- ta’rif. To‘g‘ri burchakli uchburchak o‘tkir burchagining **sinusi** deb, shu burchak qarshisidagi katetning gipotenuzaga nisbatiga aytiladi.

α burchakning sinusi **sinα** kabi belgilanadi va «*sinus alfa*» deb o‘qiladi. Ta’rifga ko‘ra:

$$\sin\alpha = \frac{a}{c}.$$

2- ta’rif. To‘g‘ri burchakli uchburchak o‘tkir burchagining **kosinusi** deb, shu burchakka yopishgan katetning gipotenuzaga nisbatiga aytiladi.

α burchakning kosinusi **cosα** kabi belgilanadi va «*kosinus alfa*» deb o‘qiladi. Ta’rifga ko‘ra:

$$\cos\alpha = \frac{b}{c}.$$

3- ta’rif. To‘g‘ri burchakli uchburchak o‘tkir burchagining **tangensi** deb, shu burchak qarshisidagi katetning burchakka yopishgan katetga nisbatiga aytiladi.

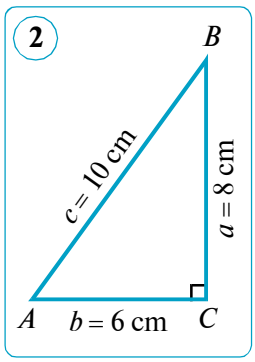
α burchakning tangensi **tgα** kabi belgilanadi va «*tangens alfa*» deb o‘qiladi. Ta’rifga ko‘ra:

$$\text{tg}\alpha = \frac{a}{b}.$$

4- ta’rif. To‘g‘ri burchakli uchburchak o‘tkir burchagining **kotangensi** deb, shu burchakka yopishgan katetning qarshisidagi katetga nisbatiga aytiladi.

α burchakning kotangensi **ctgα** kabi belgilanadi va «*kotangens alfa*» deb o‘qiladi. Ta’rifga ko‘ra:

$$\text{ctg}\alpha = \frac{b}{a}.$$



To‘g‘ri burchakli uchburchakda katet gipotenuzadan kichik bo‘lgani uchun ***o‘tkir burchakning sinusi va kosinusi birdan kichik*** bo‘ladi.

To‘g‘ri burchakli uchburchakda katetlar o‘zaro teng, biri ikkinchisidan katta yoki kichik bo‘lishi mumkin. Shuning uchun tangens va kotangensning qiymatlari ***1 dan kichik***, ***1 ga teng*** hamda ***1 dan katta*** bo‘lishi mumkin.

Masala. ABC uchburchakda $\angle C = 90^\circ$, $AB = 10$ cm, $BC = 8$ cm, $AC = 6$ cm (2- rasm). A burchakning trigonometrik funksiyalari qiymatlarini toping.

Yechish. Ta'rifga ko'ra:

$$\sin A = \frac{BC}{AB} = \frac{8}{10} = 0{,}8; \qquad \operatorname{tg}A = \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{6}}} = \frac{4}{3} = 1\frac{1}{3};$$

$$\cos A = \frac{AC}{AB} = \frac{6}{10} = 0{,}6; \qquad \operatorname{ctg}A = \frac{\overset{3}{\cancel{6}}}{\underset{4}{\cancel{8}}} = \frac{3}{4} = 0{,}75;$$

Javob: $\sin A = 0{,}8$; $\cos A = 0{,}6$; $\operatorname{tg}A = 1\frac{1}{3}$; $\operatorname{ctg}A = 0{,}75$.

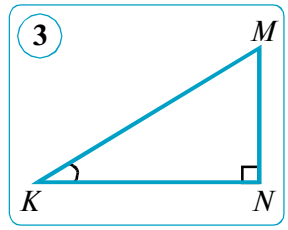
Savol, masala va topshiriqlar

1. 1) To'g'ri burchakli uchburchakning tomonlaridan qanday nisbatlar tuzish mumkin va ular qanday o'qiladi?

2) To'g'ri burchakli uchburchakda o'tkir burchakning sinusi, kosinusi, tangensi va kotangensi deb nimaga aytiladi va ular qanday belgilanadi?

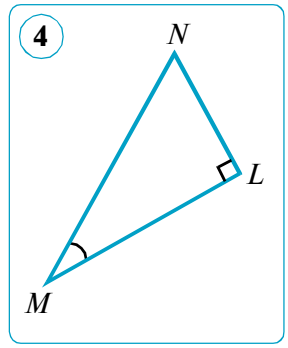
2. Har bir kasr ta'rifga ko'ra K burchakning qaysi trigonometrik funksiyasini ifodalaydi (3- rasm): a) $\frac{KN}{KM}$; b) $\frac{MN}{KN}$; d) $\frac{MN}{KM}$; e) $\frac{KN}{MN}$?

3. $\triangle ABC$ da $\angle C = 90°$, $AB = 6$ cm, $BC = 5$ cm, $AC = \sqrt{11}$ cm (1- rasmga q.). A va B burchaklar sinusi, kosinusi, tangensi va kotangenslari qiymatlarini toping.



4. To'g'ri burchakli uchburchakda o'tkir burchakning sinusi: a) 0,98; b) $\sqrt{2}$; d) $\sqrt{5}-2$ ga teng bo'lishi mumkinmi?

5. To'g'ri burchakli MNL uchburchakda $\sin N = \frac{24}{25}$ ga teng. Bu tenglikdan uchburchakning qaysi tomonlarini topish mumkin (4- rasm)?



6. MNL uchburchakda $\angle L = 90°$, $MN = 13$ cm, $ML = 12$ cm, $NL = 5$ cm (4- rasm). M burchakning sinusi, kosinusi, tangensi va kotangensi qiymatlarini toping.

7. ABC uchburchakda $\angle C = 90°$, $AB = 17$ cm, $BC = 8$ cm, $AC = 15$ cm. A va B burchaklar sinusi, kosinusi, tangensi va kotangensi qiymatlarini toping.

Bilib qo'ygan foydali!



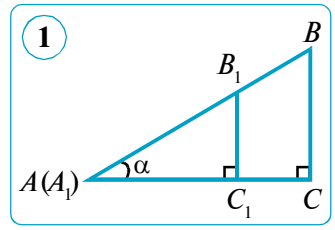
- ***«Sinus»*** *atamasi lotin tilidan olingan bo'lib, «egilish» degan ma'noni anglatadi.*
- ***«Tangens»*** *atamasi lotin tilidan tarjima qilinganda «urinma» degan ma'noni anglatadi.*
- ***«Kosinus»*** *va* ***«kotangens»*** *atamalari «complementi sinus» va «complementi tangens» – «to'ldiruvchi sinus» va «to'ldiruvchi tangens» atamalarining qisqartmalaridan iboratdir.*

16. O‘TKIR BURCHAKNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSI (DAVOMI)

1. O‘tkir burchakning trigonometrik funksiyalari.

To‘g‘ri burchakli uchburchakda o‘tkir burchakning sinusi, kosinusi, tangensi va kotangensining qiymatlari faqat o‘tkir burchak kattaligiga bog‘liqligi va to‘g‘ri burchakli uchburchakning tanlanishiga bog‘liq emasligini ko‘rsatamiz.

To‘g‘ri burchakli ABC va $A_1B_1C_1$ uchburchaklarda ($\angle C = \angle C_1 = 90°$) $\angle A = \angle A_1$ bo‘lsin (1-rasm).



Proporsiyaning asosiy xossasiga ko‘ra:

$$\frac{BC}{AB} = \frac{B_1C_1}{A_1B_1};\quad \frac{AC}{AB} = \frac{A_1C_1}{A_1B_1};\quad \frac{BC}{AC} = \frac{B_1C_1}{A_1C_1};\quad \frac{AC}{BC} = \frac{A_1C_1}{B_1C_1}.$$

Bu tengliklarning chap va o‘ng qismlari mos ravishda A va A_1 o‘tkir burchaklarning sinuslari, kosinuslari, tangenslari va kotangenslariga teng. Demak,

$$\sin A = \frac{BC}{AB} = \frac{B_1C_1}{A_1B_1} = \sin A_1,\quad \operatorname{tg}A = \frac{BC}{AC} = \frac{B_1C_1}{A_1C_1} = \operatorname{tg}A_1,$$

$$\cos A = \frac{AC}{AB} = \frac{A_1C_1}{A_1B_1} = \cos A_1,\quad \operatorname{ctg}A = \frac{AC}{BC} = \frac{A_1C_1}{B_1C_1} = \operatorname{ctg}A_1.$$

Bulardan ko‘rinadiki, A o‘tkir burchakning sinusi, kosinusi, tangensi va kotangensi uchburchakning tanlanishiga bog‘liq emas. Agar o‘tkir burchakning qiymati o‘zgarsa, bu nisbatlar, albatta, o‘zgaradi.

Shunday qilib, **o‘tkir burchakning sinusi, kosinusi, tangensi va kotangensi faqat o‘tkir burchak kattaligiga bog‘liq.**

Sinus, kosinus, tangens va kotangens ***o‘tkir burchakning trigonometrik funksiyalari*** deyiladi.

Yuqorida keltirilgan tengliklardan quyidagi muhim xulosaga kelish mumkin:

agar A va A_1 o‘tkir burchaklar uchun trigonometrik funksiyalardan birortasi teng bo‘lsa, u holda A va A_1 o‘tkir burchaklar teng ($\angle A = \angle A_1$) bo‘ladi.

Boshqacha aytganda, ***trigonometrik funksiyaning har bir qiymatiga yagona o‘tkir burchak mos keladi***.

2. Tangens va kotangensning sinus va kosinuslar orqali ifodalanishi.

Sinus va kosinus ta’riflaridan quyidagi tengliklar kelib chiqadi (15- mavzuga q.):

$$\frac{\sin\alpha}{\cos\alpha} = \frac{a}{c} : \frac{b}{c} = \frac{a}{c}\cdot\frac{c}{b} = \frac{a}{b} = \operatorname{tg}\alpha,\quad \text{ya’ni}\quad \operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}; \qquad (1)$$

$$\frac{\cos\alpha}{\sin\alpha} = \frac{b}{c} : \frac{a}{c} = \frac{b}{c}\cdot\frac{c}{a} = \frac{b}{a} = \operatorname{ctg}\alpha,\quad \text{ya’ni}\quad \operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}. \qquad (2)$$

Shunday qilib, o‘tkir burchak tangensi va kotangensi sinus va kosinus orqali quyidagicha ta’riflanadi.

O‘tkir burchak sinusining kosinusiga nisbati shu burchakning ***tangensi*** *deyiladi.*

O‘tkir burchak kosinusining sinusiga nisbati shu burchakning ***kotangensi*** *deyiladi.*

(1) va (2) tengliklarni hadma-had ko‘paytirib, quyidagi tenglikni hosil qilamiz:

$$\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = \frac{\sin\alpha}{\cos\alpha} \cdot \frac{\cos\alpha}{\sin\alpha} = 1 \Leftrightarrow \operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1. \qquad (3)$$

Demak, **α o‘tkir burchak tangensi va kotangensining ko‘paytmasi 1 ga teng**.

Bundan, α o‘tkir burchak tangensi va kotangensi o‘zaro teskari funksiyalar ekani kelib chiqadi.

Shunday qilib, biz α o‘tkir burchak uchun uchta yangi **tenglik** (**ayniyat**)ni keltirib chiqardik.

3. To‘g‘ri burchakli uchburchakning tomonlari bilan burchaklari orasidagi munosabatlar.

Trigonometrik funksiyalarning ta’riflaridan quyidagi qoidalar kelib chiqadi.

1- qoida. α burchak qarshisidagi katet gipotenuza bilan α burchak sinusining ko‘paytmasiga teng:

$$a = c\sin\alpha.$$

2- qoida. α burchak qarshisidagi katet ikkinchi katet bilan α burchak tangensining ko‘paytmasiga teng:

$$a = b\operatorname{tg}\alpha.$$

3- qoida. α burchakka yopishgan katet gipotenuza bilan α burchak kosinusining ko‘paytmasiga teng:

$$b = c\cos\alpha.$$

4- qoida. α burchakka yopishgan katet qarshisidagi katetning α burchak tangensiga nisbatiga teng:

$$b = \frac{a}{\operatorname{tg}\alpha}.$$

5- qoida. Gipotenuza α o‘tkir burchak qarshisidagi katetning α burchak sinusiga nisbatiga teng:

$$c = \frac{a}{\sin\alpha}.$$

6- qoida. Gipotenuza α o‘tkir burchakka yopishgan katetning α burchak kosinusiga nisbatiga teng:

$$c = \frac{b}{\cos\alpha}.$$

Masala. ABC uchburchakda C burchak 90° ga teng. Agar:

1) $AB = 18$ cm va $\sin A = \frac{1}{3}$ bo‘lsa, BC katetni; 2) $AC = 15$ cm va $\cos A = \frac{5}{6}$ bo‘lsa, AB gipotenuzani; 3) $BC = 26$ cm va $\operatorname{tg} A = \frac{13}{15}$ bo‘lsa, AC katetni hisoblang.

Yechish. 1) 1- qoidadan foydalanib, BC katetni topamiz:

$$BC = AB \sin A = {}^{6}\cancel{18} \cdot \frac{1}{\cancel{3}_{1}} = 6 \text{ (cm)}.$$

Javob: 6 cm.

2) 6- qoidadan foydalanib, AB gipotenuzani topamiz:

$$AB = \frac{AC}{\cos A} = 15 : \frac{5}{6} = {}^{3}\cancel{15} \cdot \frac{6}{\cancel{5}_{1}} = 3 \cdot 6 = 18 \text{ (cm)}.$$

Javob: 18 cm.

3) 4- qoidadan foydalanib, AC katetni topamiz:

$$AC = \frac{BC}{\operatorname{tg} A} = 26 : \frac{13}{15} = {}^{2}\cancel{26} \cdot \frac{15}{\cancel{13}_{1}} = 2 \cdot 15 = 30 \text{ (cm)}.$$

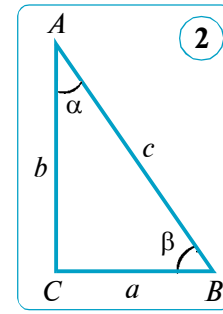
Javob: 30 cm.

Savol, masala va topshiriqlar

1. 1) O‘tkir burchakning trigonometrik funksiyalari deb nimaga aytiladi?

2) O‘tkir burchak sinusi, kosinusi, tangensi va kotangensining kattaliklari nimaga bog‘liq?

2. Quyida berilgan tengliklardan qaysi birlari to‘g‘ri ekanini aniqlang (2- rasm). Javobingizni asoslang.



a) $c = \frac{a}{\sin\alpha}$; b) $b = c\sin\alpha$; d) $c = a\operatorname{tg}\alpha$; e) $a = \frac{b}{\operatorname{ctg}\alpha}$.

3. To‘g‘ri burchakli uchburchak o‘tkir burchagining tangensi $\sqrt{2}$; 0,001 va 100 ga teng bo‘lishi mumkinmi? Javobingizni asoslang.

4. ABC uchburchakda C burchak 90° ga teng. Agar:

1) $BC = 10$ cm va $\operatorname{tg} A = \frac{5}{8}$ bo‘lsa, AC katetni;

2) $BC = 8$ cm va $\sin A = 0{,}16$ bo‘lsa, AB gipotenuzani hisoblang.

5. To‘g‘ri burchakli uchburchakning tomonlari bilan burchaklari orasidagi 6 ta munosabatni β burchak uchun keltirib chiqaring (2- rasm).

6. ABC uchburchakda C burchak 90° ga teng. Agar $BC = 4$ cm va $\sin A = 0{,}25$ bo‘lsa, AB gipotenuzani hisoblang.

7. ABC uchburchakda C burchak 90° ga teng. Agar $AC = 2$ cm va $\cos A = 0{,}4$ bo‘lsa, AB gipotenuzani hisoblang.

8. ABC uchburchakda C burchak 90° ga teng. Agar $BC = 14$ cm va $\cos B = \frac{7}{25}$ bo‘lsa, AB gipotenuzani hisoblang.

4- §. PIFAGOR TEOREMASI VA UNING TATBIQLARI

17. PIFAGOR TEOREMASI VA UNING TURLI ISBOTLARI

1. Pifagor teoremasi – geometriyaning muhim teoremalaridan biridir.

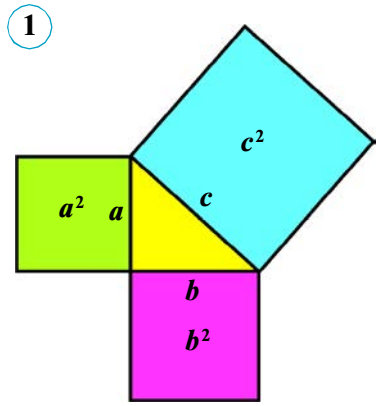
Buyuk yunon matematigi **Pifagor** hayoti haqida ma'lumotlar juda kam. Pifagor maktabi shakllarni ajratish va to'g'ri chiziqli shakllarni tengdosh shakllarga almashtirishning geometrik usulidan teoremalarni isbot qilish va masalalar yechishda foydalangani yunon matematiklarining asarlaridangina ma'lum. Xususan, geometriyaning fan sifatida tarkib topishiga Pifagor va uning maktabi katta hissa qo'shgan. Quyida keltiriladigan teorema Pifagor nomi bilan yuritiladi.

Teorema.

(Pifagor teoremasi.) **To'g'ri burchakli uchburchak gipotenuzasining kvadrati uning katetlari kvadratlarining yig'indisiga teng.**

Bu teorema to'g'ri burchakli uchburchakka oid bo'lib, uchburchak tomonlariga teng kvadratlarning yuzlari orasidagi munosabatni ko'rsatadi. Pifagor bu teoremaning nazariy isbotini keltirgan. Pifagor teoremasi bilan aniqlangan geometrik munosabatning xususiy hollari Pifagordan oldin ham turli xalqlarga ma'lum bo'lgan, ammo teoremaning umumiy shakli Pifagor maktabi tomonidan yaratilgan.

1



Katetlari a va b, gipotenuzasi c bo'lgan to'g'ri burchakli ABC uchburchak berilgan bo'lsin, u holda Pifagor teoremasi

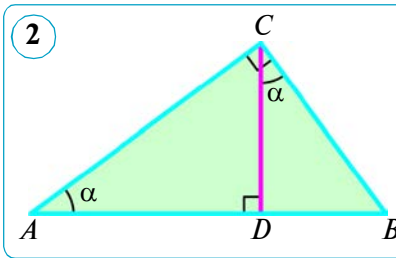
$$c^2 = a^2 + b^2, \qquad (1)$$

formula bilan ifodalanadi, bunda a^2, b^2, c^2 – tomonlari a, b, c bo'lgan kvadratlarning yuzlariga teng. Shuning uchun bu tenglik **tomoni gipotenuzaning uzunligiga teng kvadratning yuzi tomonlari katetlarga teng kvadratlarning yuzlari yig'indisiga teng** ekanini ko'rsatadi (1- rasm).

2. Pifagor teoremasining o'tkir burchak kosinusi orqali isbotlanishi.

Isbot. ABC – berilgan to'g'ri burchakli uchburchak bo'lib, uning C burchagi to'g'ri burchak bo'lsin. To'g'ri burchakli uchburchakning C uchidan CD balandlikni o'tkazamiz (2- rasm).

2



To‘g‘ri burchakli ACD va ABC uchburchaklardan burchak kosinusining ta’rifiga ko‘ra:

$$\cos A = \frac{AD}{AC} = \frac{AC}{AB}.$$

Bundan $AD \cdot AB = AC^2$ (2).

To‘g‘ri burchakli BCD va ABC uchburchaklardan burchak kosinusining ta’rifiga ko‘ra:

$$\cos B = \frac{BD}{BC} = \frac{BC}{AB}.$$

Bundan $BD \cdot AB = BC^2$ (3).

Hosil bo‘lgan (2) va (3) tengliklarni hadma-had qo‘shib va $AD + DB = AB$ ekanini inobatga olib,

$$AC^2 + BC^2 = AB \cdot D + BD \cdot AB = AB \cdot (AD + BD) \cdot AB = AB^2$$

tenglikni hosil qilamiz. Teorema isbotlandi.

To‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakning tomonlarini mos ravishda $a = BC$, $b = AC$, $c = AB$ deb belgilab, Pifagor formulasini hosil qilamiz:

$$\boldsymbol{c^2 = a^2 + b^2.}$$

3. Pifagor teoremasining yuzalar orqali isbotlanishi.

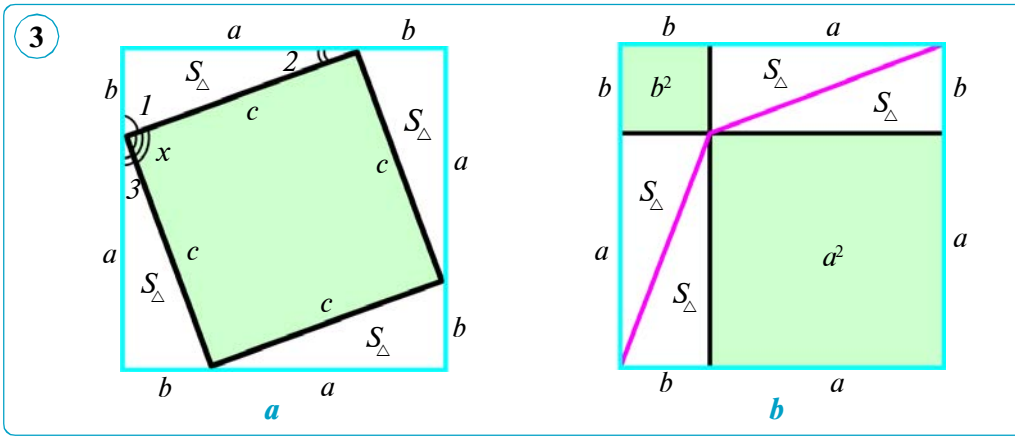
Katetlari a, b va gipotenuzasi c ga teng bo‘lgan to‘g‘ri burchakli uchburchak berilgan. Bu uchburchak uchun Pifagor teoremasi o‘rinli ekanini isbotlaymiz, ya’ni

$$\boldsymbol{a^2 + b^2 = c^2}$$

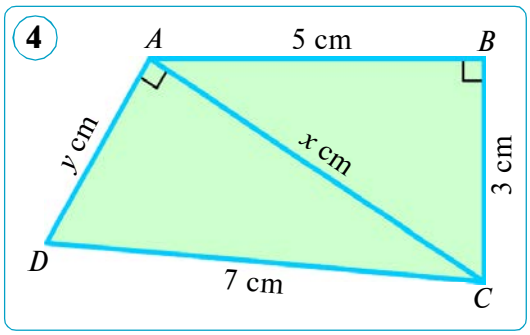
ekanini ko‘rsatamiz.

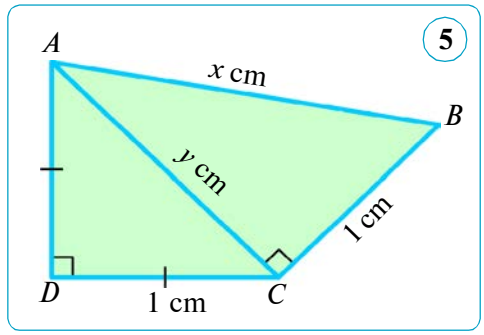
Isbot. Tomoni $(a + b)$ ga teng bo‘lgan ikkita kvadrat yasaymiz. Ularni 3- rasmda ko‘rsatilgan usul bilan to‘g‘ri burchakli uchburchaklar, kvadratlar va to‘g‘ri to‘rtburchaklarga ajratib chiqamiz. 3- *a* rasmdagi to‘rtburchak tomoni c bo‘lgan kvadrat ekanini ko‘rsatamiz. Haqiqatan ham, avvalo bu to‘rtburchak romb, chunki uning tomoni katetlari a va b bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasi c ga teng. Endi chizmadagi x burchak

3



a *b*





to‘g‘ri ekanini ko‘rsatamiz. Haqiqatan, $\angle x + \angle 1 + \angle 3 = 180°$, $\angle 3 = \angle 2$ (chunki uchburchaklar teng) va $\angle 1 = 90° - \angle 2$ ekanini e’tiborga olib topamiz: $\angle x = 90°$. Shuning uchun bu to‘rtburchak burchaklaridan biri 90° ga teng bo‘lgan romb, ya’ni kvadrat bo‘ladi. Qaralayotgan ikkala katta kvadrat tengdosh, ya’ni ularning yuzlari teng. Shuningdek, birinchi kvadrat yuzi $4S_{\triangle} + c^2$ ga, ikkinchi kvadratning yuzi esa $4S_{\triangle} + a^2 + b^2$ ga teng (3- *b* rasm). Shuning uchun

$$4S_{\triangle} + c^2 = 4S_{\triangle} + a^2 + b^2.$$

Demak, $c^2 = a^2 + b^2$. Teorema isbotlandi.

Masala. 4- rasmdagi noma’lum kesmalar uzunligini toping.

Yechish. 1) $\triangle ABC$ – to‘g‘ri burchakli, $\angle B = 90°$ (4- rasm). Pifagor teoremasiga ko‘ra: $x^2 = 5^2 + 3^2$, bundan $x^2 = 34 \Rightarrow x = \sqrt{34}$ $(x > 0)$.

2) $\triangle ACD$ – to‘g‘ri burchakli, $\angle CAD = 90°$ (4- rasm). Pifagor teoremasiga ko‘ra, $y^2 + \left(\sqrt{34}\right)^2 = 7^2$, bundan $y^2 + 34 = 49$, $y^2 = 15$, $y = \sqrt{15}$ $(y > 0)$.

Javob: $x = \sqrt{34}$ cm; $y = \sqrt{15}$ cm.

Savol, masala va topshiriqlar

1. 1) Pifagor teoremasining qanday isbotlarini bilasiz?

2) «Gipotenuzaning kvadrati», «katetning kvadrati» degan iboralarni qanday tushunasiz?

2. To‘g‘ri burchakli uchburchakning *a* va *b* katetlari berilgan. Agar: 1) $a = 5$, $b = 12$; 2) $a = 4\sqrt{2}$, $b = 7$; 3) $a = 0{,}7$, $b = 2{,}4$; 4) $a = 5$, $b = 6$; 5) $a = \frac{5}{13}$, $b = \frac{12}{13}$ bo‘lsa, *c* gipotenuzani toping.

3. Rombning diagonallari: 1) 12 cm va 16 cm; 2) 14 cm va 48 cm. Rombning perimetrini toping.

4. Noma’lum kesmalar uzunligini toping (5- rasm).

5. To‘g‘ri burchakli uchburchakda *a* va *b* – katetlar, *c* – gipotenuza. Agar: 1) $a = 1{,}2$, $c = 1{,}3$; 2) $a = 7$, $c = 9$; 3) $a = 1{,}5$, $c = 1{,}7$; 4) $a = 2$, $c = 2{,}5$ bo‘lsa, *b* katetni toping.

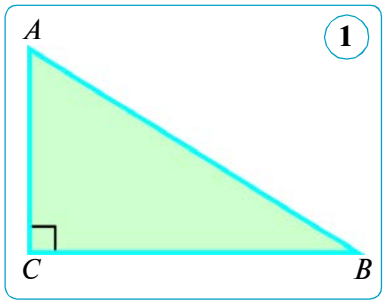
6. To‘g‘ri to‘rtburchakning tomonlari: 1) 2,4 dm va 7 cm; 2) 50 cm va 12 dm; 3) 8 dm va 1,5 m. Uning diagonalini toping.

18. PIFAGOR TEOREMASIGA TESKARI TEOREMA

1. Pifagor teoremasining ba'zi natijalari.

Pifagor teoremasi natijalari ichidan bittasining isbotini keltirib o'tamiz.

Natija. To'g'ri burchakli uchburchakning istalgan kateti gipotenuzadan kichikdir.



Isbot. $\triangle ABC$ – to'g'ri burchakli, $\angle C = 90^\circ$ (1- rasm). Uchburchakning istalgan kateti gipotenuzadan kichik ekanini isbotlaymiz.

Haqiqatan, Pifagor teoremasiga ko'ra katetlar uchun:

$$AC^2 = AB^2 - BC^2 \text{ va } BC^2 = AB^2 - AC^2$$

munosabatlar o'rinli. Bundan

$$AC^2 < AB^2 \text{ va } BC^2 < AB^2$$

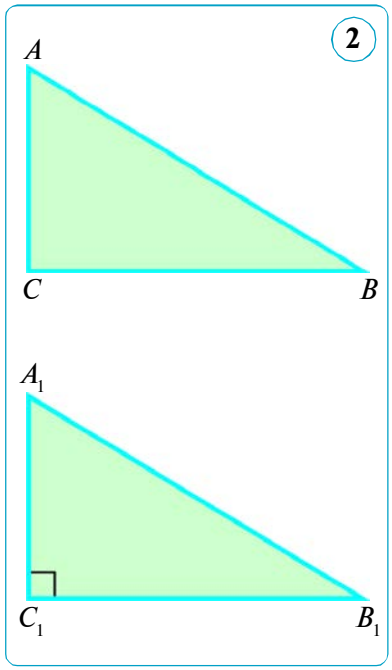
ekani kelib chiqadi.

Demak, $AC < AB$ va $BC < AB$. Natija isbotlandi.

2. Pifagor teoremasiga teskari teorema.

Teorema.

Agar uchburchak tomonlaridan birining kvadrati uning qolgan ikki tomoni kvadratlarining yig'indisiga teng bo'lsa, u holda uchburchak to'g'ri burchakli bo'ladi.



Isbot. $\triangle ABC$ da $AB^2 = AC^2 + BC^2$ bo'lsin. $\angle C = 90^\circ$ ekanini isbotlaymiz (2- rasm).

C_1 burchagi to'g'ri bo'lgan to'g'ri burchakli $A_1B_1C_1$ uchburchakni ko'rib chiqamiz, unda $A_1C_1 = AC$ va $B_1C_1 = BC$. Pifagor teoremasiga ko'ra, $A_1B_1^2 = A_1C_1^2 + B_1C_1^2$ va demak,

$$A_1B_1^2 = AC^2 + BC^2.$$

Teorema shartiga ko'ra,

$$AB^2 = AC^2 + BC^2, \text{ demak, } A_1B_1^2 = AB^2.$$

Bundan $A_1B_1 = AB$ ekanligini topamiz. Shunday qilib, ABC va $A_1B_1C_1$ uchburchaklar uch tomoniga ko'ra teng. Shuning uchun $\angle C = \angle C_1$, ya'ni ABC uchburchakning C uchidagi burchagi to'g'ri burchak ekani kelib chiqadi.

Teorema isbotlandi.

1- masala. Agar uchburchakning tomonlari: 1) $a=5$, $b=11$, $c=12$; 2) $a=\sqrt{85}$, $b=7$, $c=6$ bo‘lsa, u to‘g‘ri burchakli uchburchak bo‘ladimi?

Yechish. 1) Ikkita kichik tomoni kvadratlari yig‘indisini hisoblaymiz:

$$5^2+11^2=25+121=146.$$

Endi katta tomoni kvadratini hisoblaymiz: $12^2=144$.

Olingan natijalarni taqqoslasak, $a^2+b^2\neq c^2$ munosabat kelib chiqadi. Demak, uchburchak to‘g‘ri burchakli emas.

Javob: $a=5$, $b=11$ va $c=12$ bo‘lganda uchburchak to‘g‘ri burchakli bo‘lmaydi.

2) Ikkita kichik tomoni kvadratlari yig‘indisini hisoblaymiz:

$$7^2+6^2=49+36=85.$$

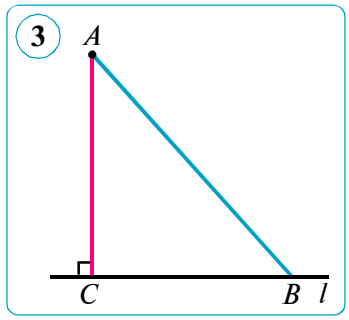
Endi katta tomoni kvadratini hisoblaymiz: $\left(\sqrt{85}\right)^2=85$.

Demak, $85=85$ – o‘rinli. Natijada $b^2+c^2=a^2$ ga ega bo‘lamiz. Bundan uchburchakning to‘g‘ri burchakli ekani kelib chiqadi.

Javob: $a=\sqrt{85}$, $b=7$ va $c=6$ bo‘lganda uchburchak to‘g‘ri burchakli bo‘ladi.

3. Perpendikular va og‘ma.

l – to‘g‘ri chiziq va unda yotmagan A nuqta berilgan bo‘lsin. Ta’rifga ko‘ra, A dan l to‘g‘ri chiziqqacha eng qisqa masofa A dan l ga tushirilgan AC ***perpendikular****ning* uzunligiga teng bo‘ladi (3- rasm).



Haqiqatan, har bir $B\in l$ uchun ACB uchburchak – to‘g‘ri burchakli, bunda AC va CB – katetlar, AB esa gipotenuza bo‘ladi. CB kesma AB og‘maning l to‘g‘ri chiziqdagi ***proyeksiyasi*** deyiladi.

Pifagor teoremasi AB – og‘ma, AC – perpendikular va CB – proyeksiyasi uzunliklarini quyidagi tenglik bilan bo‘g‘laydi:

$$AB^2=AC^2+CB^2.$$

Shuning uchun, *har doim* $AB>AC$ yoki $AB>BC$, boshqacha aytganda, *bir nuqtadan o‘tkazilgan perpendikular va og‘maning proyeksiyasi og‘madan kichik bo‘ladi.*

Shuningdek, *teng og‘malar teng proyeksiyalarga ega*; *ikkita og‘madan qaysi birining proyeksiyasi katta bo‘lsa, o‘sha og‘ma katta bo‘ladi.*

2- masala. Diagonallari 10 cm va 24 cm ga teng bo‘lgan rombning tomonini toping.

Yechish. Rombning diagonallari perpendikular va kesishish nuqtasida teng ikkiga bo‘linishidan foydalanamiz. U holda rombning tomoni katetlari 5 cm va 12 cm ga teng bo‘lgan to‘g‘ri burchakli uchburchakning gipotenuzasi bo‘ladi.

$5^2 + 12^2 = 25 + 144 = 169$, ya'ni $169 = 13^2$.

Demak, rombning tomoni 13 cm ga teng ekan.

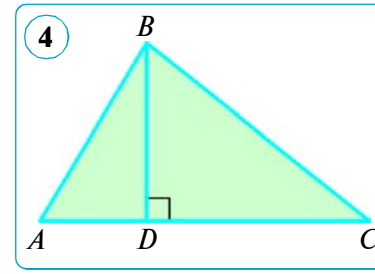
Javob: 13 cm.

Savol, masala va topshiriqlar

1. 1) Pifagor teoremasiga teskari teoremani ifodalang.
2) Og'maning to'g'ri chiziqdagi proyeksiyasi deganda nimani tushunasiz?
3) Katet gipotenuzadan kichik ekani to'g'rimi?

2. To'g'ri burchakli uchburchakning tomonlari quyidagi sonlarga teng bo'lishi mumkinmi: 1) 11 cm, 7 cm, 17 cm; 2) 3 cm, 1,6 cm, 3,4 cm; 3) 3 cm, 4 cm, 6 cm; 4) 2 cm, $\sqrt{7}$ cm, $\sqrt{11}$ cm? Javobingizni asoslang.

3. $\triangle ABC$ da $AB = 13$ cm, $BC = 20$ cm, BD – uchburchakning balandligi va u 12 cm ga teng. AB, BC tomonlarning AC tomonga tushirilgan proyeksiyalari uzunliklari va AC tomon uzunligini toping (4- rasm). Bo'sh joylarga mos javoblarni yozing.



Yechish. $\triangle ABD$ va $\triangle BCD$ – to'g'ri burchakli, chunki $\angle ADB = \angle BDC = 90°$. AB va BC tomonlarning AC tomondagi proyeksiyalari mos ravishda AD va CD kesmalardan iborat.

$\triangle ABD$ dan Pifagor teoremasiga ko'ra:

$AD^2 = AB^2 - BD^2 = 13^2 - ...^2 = ... - ... = ...$ (cm).

Bundan $AD = ...$ cm.

$\triangle BCD$ dan Pifagor teoremasiga ko'ra:

$CD^2 = BC^2 - BD^2 = ...^2 - 12^2 = ... - ... = ...$ (cm). Bundan $CD = ...$ cm.

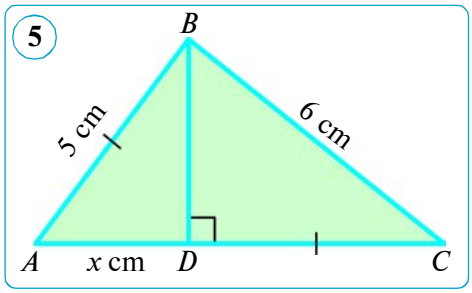
$AC = ... + DC = ... + ... = ...$ (cm).

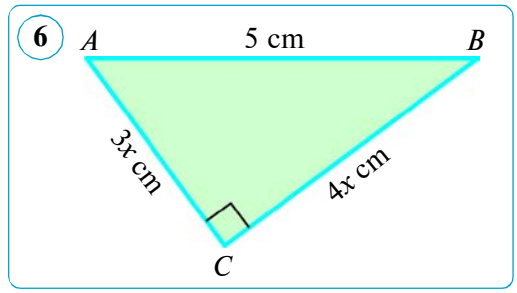
Javob: $AD = ...$ cm, $CD = ...$ cm, $AC = ...$ cm.

4. Noma'lum uzunliklarni toping (5–6- rasmlar).

5. To'g'ri burchakli uchburchakning ikki tomoni 6 cm va 8 cm ga teng. Uchinchi tomonning uzunligini toping. Masala nechta yechimga ega?

6. To'g'ri burchakli uchburchakning tomonlari quyidagi sonlarga teng bo'lishi mumkinmi: 1) $a = 12$, $b = 35$, $c = 37$; 2) $a = 11$, $b = 20$, $c = 25$; 3) $a = 18$, $b = 24$, $c = 30$; 4) $a = 9$, $b = 12$, $c = 15$?





19. PIFAGOR TEOREMASINING BA'ZI TATBIQLARI

Uchta tomoniga ko'ra uchburchakning balandligini topish.

Tomonlari a, b va c bo'lgan ABC uchburchakni ko'rib chiqamiz. Uning C uchidan AB tomonga tushirilgan $CD = h_c$ balandligini topamiz (1- a rasm).

Balandlik asosi D nuqtaning AB kesmaga nisbatan qanday joylashishiga ko'ra uch hol bo'lishi mumkin. Shu hollarni ko'rib chiqamiz.

1-hol. D nuqta AB kesmaning ichki nuqtasi bo'lsin. Agar $AD = x$ belgilashni kiritsak, u holda $DB = c - x$ bo'ladi (1- a rasm). $\triangle ADC$ va $\triangle BDC$ lar to'g'ri burchakli, Pifagor teoremasiga ko'ra:

$$h_c^2 = b^2 - x^2 \quad (1) \quad \text{va} \quad h_c^2 = a^2 - (c-x)^2 \quad (2).$$

Bulardan quyidagi tenglikni hosil qilamiz: $b^2 - x^2 = a^2 - (c-x)^2$.

Bundan

$$b^2 - x^2 = a^2 - c^2 + 2cx - x^2, \quad \text{ya'ni} \quad b^2 = a^2 - c^2 + 2cx\,.$$

Oxirigi tenglamadan x ni topamiz:

$$x = \frac{b^2 + c^2 - a^2}{2c} \quad \text{yoki} \quad x^2 = \frac{\left(b^2 + c^2 - a^2\right)^2}{4c^2}.$$

x^2 ning bu qiymatini (1) tenglikka qo'yib, quyidagiga ega bo'lamiz:

$$h_c^2 = b^2 - \frac{\left(b^2 + c^2 - a^2\right)^2}{4c^2} = \frac{4b^2c^2 - \left(b^2 + c^2 - a^2\right)^2}{4c^2}.$$

Bu kasrning suratini ko'paytuvchilarga ajratib, quyidagiga ega bo'lamiz:

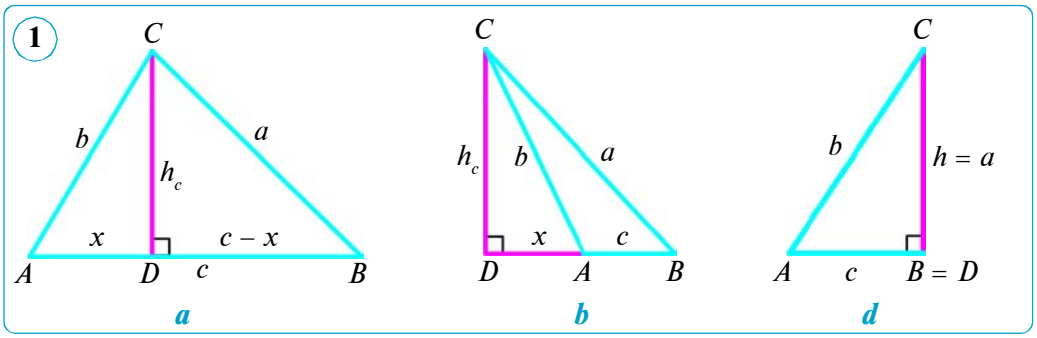
$$h_c^2 = \frac{\left(2bc - \left(b^2 + c^2 - a^2\right)\right)\left(2bc + \left(b^2 + c^2 - a^2\right)\right)}{4c^2} = \frac{\left(2bc - b^2 - c^2 + a^2\right)\left(2bc + b^2 + c^2 - a^2\right)}{4c^2}.$$

Hosil bo'lgan ifodaning suratidagi ikkala ko'paytuvchini quyidagicha shakl almashtiramiz:

$$2bc - b^2 - c^2 + a^2 = a^2 - (b-c)^2 = (a - b + c)(a + b - c) \quad \text{va}$$
$$2bc + b^2 + c^2 - a^2 = (b+c)^2 - a^2 = (b + c - a)(b + c + a).$$

U holda

$$h_c^2 = \frac{(a - b + c)(a + b - c)(b + c - a)(b + c + a)}{4c^2},$$



bundan

$$h_c = \frac{1}{2c}\sqrt{(a+b+c)(a-b+c)(a+b-c)(b+c-a)}\,.$$

Uchburchakning yarim perimetrini p bilan belgilaymiz, u holda:

$$a+b+c = 2p,$$
$$a-b+c = a+b+c-2b = 2p-2b = 2(p-b),$$
$$a+b-c = a+b+c-2c = 2p-2c = 2(p-c),$$
$$b+c-a = a+b+c-2a = 2p-2a = 2(p-a).$$

Hosil qilingan ifodalarni ildiz ostidagi ifodalarning o‘rniga qo‘yib, quyidagi natijani hosil qilamiz:

$$h_c = \frac{1}{2c}\sqrt{16p(p-a)(p-b)(p-c)} = \frac{1}{2c}\cdot 4\sqrt{p(p-a)(p-b)(p-c)} =$$
$$= \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}\,.$$

Xuddi shuningdek,

$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)} \quad \text{va} \quad h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}\,.$$

2- hol. D nuqta AB kesmaning davomida yotadi, ya’ni $DB = c + x$. Bunda ham qayd qilingan natija hosil bo‘ladi (1- *b* rasm).

3- hol. D nuqta B nuqta bilan, ya’ni $h = a$ — balandlik katet bilan ustma-ust tushadi. Bu holda uchburchak to‘g‘ri burchakli bo‘ladi (1- *d* rasm).

Savol, masala va topshiriqlar

1. Tomonlari: 1) 10 cm, 10 cm, 12 cm; 2) 17 dm, 17 dm, 16 dm; 3) 4 dm, 13 dm, 15 dm bo‘lgan uchburchaklarning balandliklarini toping.

2. Balandligi h ga teng bo‘lgan teng tomonli uchburchakning tomonini toping. Agar: 1) $h = 6$ cm; 2) $h = 1{,}5$ dm bo‘lsa, tomonni toping.

3. Uchburchakning tomonlari: 1) $a = 5$ cm, $b = 7$ cm, $c = 6$ cm; 2) $a = 13$ dm, $b = 14$ dm, $c = 15$ dm; 3) $a = 24$ cm, $b = 25$ cm, $c = 7$ cm ga teng. Uchburchakning katta tomoniga tushirilgan balandligini toping.

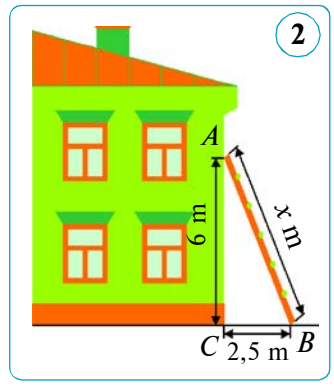
4. Agar teng tomonli uchburchakning tomoni 12 cm ga teng bo‘lsa, uning balandligini toping.

5. Uchburchakning tomonlari $a = 8$ cm, $b = 10$ cm va $c = 12$ cm. Uning eng katta va eng kichik balandliklarini toping.

6. Uchburchakning tomonlari: 1) 17, 65, 80; 2) 8, 6, 4; 3) 24, 25, 7; 4) 30, 34, 16; 5) 15, 17, 8 ga teng bo‘lsa, uning eng kichik tomoniga tushirilgan balandligini toping.

7. Uchburchakning tomonlari $a = 16$ cm, $b = 12$ cm va $c = 8$ cm. Uchburchakning kichik balandligini toping.

8. Narvonning uzunligini toping (2- rasm).



5- §. TRIGONOMETRIK AYNIYATLAR

20–21. ASOSIY TRIGONOMETRIK AYNIYAT VA UNING NATIJALARI

1. Asosiy trigonometrik ayniyatlar.

Bir burchakning trigonometrik funksiyalari orasidagi bog‘lanishni ifodalovchi ayniyatlarni keltirib chiqaramiz.

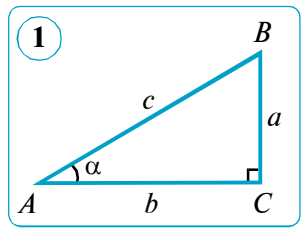
Teorema.

Har qanday o‘tkir α burchak uchun

$$\sin^2\alpha + \cos^2\alpha = 1$$

tenglik o‘rinli.

Isbot. A uchidagi burchagi α ga teng bo‘lgan to‘g‘ri burchakli ixtiyoriy ABC uchburchakni olamiz (1- rasm).



Pifagor teoremasiga ko‘ra: $BC^2 + AC^2 = AB^2$.

Tenglikning ikkala qismini AB^2 ga bo‘lib, quyidagi tenglikka ega bo‘lamiz: $\left(\frac{BC}{AB}\right)^2 + \left(\frac{AC}{AB}\right)^2 = 1$.

Ammo $\frac{BC}{AB} = \sin\alpha$, $\frac{AC}{AB} = \cos\alpha$. Shunday qilib,

$$\sin^2\alpha + \cos^2\alpha = 1. \qquad (1)$$

(1) tenglik **trigonometriyaning asosiy ayniyati** deyiladi.

Bizga bir burchakning trigonometrik funksiyalari orasidagi bog‘lanishni ifodalovchi uchta tenglik ma’lum:

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} \quad (2), \qquad \operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha} \quad (3), \qquad \operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1 \quad (4).$$

(1) tenglikning har ikkala qismini $\cos^2\alpha$ ga bo‘lib, (5) ayniyatni hosil qilamiz:

$$\frac{\sin^2\alpha}{\cos^2\alpha} + 1 = \frac{1}{\cos^2\alpha} \quad \text{yoki} \quad 1 + \operatorname{tg}^2\alpha = \frac{1}{\cos^2\alpha}. \qquad (5)$$

(1) tenglikning har ikkala qismini $\sin^2\alpha$ ga bo‘lib, (6) ayniyatni hosil qilamiz:

$$1 + \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha} \quad \text{yoki} \quad 1 + \operatorname{ctg}^2\alpha = \frac{1}{\sin^2\alpha}. \qquad (6)$$

2. Asosiy trigonometrik ayniyatdan kelib chiqadigan natijalar.

Har qanday α o‘tkir burchak uchun quyidagi tengliklar o‘rinli:

$$\sin^2\alpha = 1 - \cos^2\alpha \Rightarrow \sin\alpha = \sqrt{1 - \cos^2\alpha}. \qquad (7)$$

$$\cos^2\alpha = 1 - \sin^2\alpha \Rightarrow \cos\alpha = \sqrt{1 - \sin^2\alpha}. \qquad (8)$$

1- masala. Agar $\cos\alpha = \frac{2}{3}$ bo‘lsa, $\sin\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ning qiymatlarini hisoblang.

Yechish. 1) $\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(\frac{2}{3}\right)^2} = \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$.

2) $\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\sqrt{5}}{3} : \frac{2}{3} = \frac{\sqrt{5}}{2}$; 3) $\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{2}{3} : \frac{\sqrt{5}}{3} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$.

Javob: $\sin\alpha = \frac{\sqrt{5}}{3}$, $\operatorname{tg}\alpha = \frac{\sqrt{5}}{2}$, $\operatorname{ctg}\alpha = \frac{2\sqrt{5}}{5}$.

2- masala. Ifodani soddalashtiring: 1) $1 + \frac{1-\sin^2\alpha}{\sin^2\alpha}$; 2) $1 - \frac{\cos^2\alpha - 1}{\cos^2\alpha}$.

Yechish. 1) Qo‘shiluvchilarni umumiy maxrajga keltiramiz, so‘ngra suratdagi o‘xshash hadlarni ixchamlab va (6) ayniyatdan foydalanib topamiz:

$$1 + \frac{1-\sin^2\alpha}{\sin^2\alpha} = \frac{\sin^2\alpha + 1 - \sin^2\alpha}{\sin^2\alpha} = \frac{1}{\sin^2\alpha} = 1 + \operatorname{ctg}^2\alpha .$$

Javob: $1 + \operatorname{ctg}^2\alpha$.

2) Ayirmani umumiy maxrajga keltiramiz, so‘ngra suratdagi o‘xshash hadlarni ixchamlab va (5) ayniyatdan foydalanib topamiz:

$$1 - \frac{\cos^2\alpha - 1}{\cos^2\alpha} = \frac{\cos^2\alpha - (\cos^2\alpha - 1)}{\cos^2\alpha} = \frac{\cos^2\alpha - \cos^2\alpha + 1}{\cos^2\alpha} = \frac{1}{\cos^2\alpha} = 1 + \operatorname{tg}^2\alpha .$$

Javob: $1 + \operatorname{tg}^2\alpha$.

3- masala. Ifodani soddalashtiring: $\sin^4\alpha + \cos^4\alpha + 2\sin^2\alpha\cos^2\alpha$.

Yechish. Ikki son yig‘indisining kvadrati formulasi va asosiy trigonometrik ayniyatdan foydalanib, ifodani soddalashtiramiz:

$$\sin^4\alpha + \cos^4\alpha + 2\sin^2\alpha\cos^2\alpha = (\sin^2\alpha + \cos^2\alpha)^2 = 1.$$

Javob: 1.

Savol, masala va topshiriqlar

1. 1) Qaysi tenglik trigonometriyaning asosiy ayniyati deyiladi?
2) Trigonometrik ayniyatlarni ifodalovchi tengliklardan qaysilarini bilasiz?
3) Asosiy trigonometrik ayniyatdan qanday natijalar kelib chiqadi?

2. Agar: 1) $\sin\alpha = \frac{12}{13}$ bo‘lsa, $\cos\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ni; 2) $\cos\alpha = 0{,}8$ bo‘lsa, $\sin\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ni; 3) $\cos\alpha = 0{,}28$ bo‘lsa, $\sin\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ni toping.

3. Agar $\operatorname{tg}\alpha = \frac{5}{12}$ bo‘lsa, $\sin\alpha$ va $\cos\alpha$ ni toping.

Namuna. Agar $\operatorname{tg}\alpha = \frac{4}{3}$ bo‘lsa, $\sin\alpha$ va $\cos\alpha$ ni toping.

Yechish. $\frac{1}{\cos^2\alpha} = 1 + \operatorname{tg}^2\alpha = 1 + \left(\frac{4}{3}\right)^2 = 1 + \frac{16}{9} = \frac{25}{9}$. Demak, $\cos^2\alpha = \frac{9}{25}$.

Bundan $\cos\alpha = \sqrt{\frac{9}{25}} = \frac{3}{5}$.

Endi $\sin\alpha$ ni hisoblaymiz: $\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} \Rightarrow \sin\alpha = \operatorname{tg}\alpha \cdot \cos\alpha = \frac{4}{3}\cdot\frac{3}{5} = \frac{4}{5}$.

Javob: $\sin\alpha = \frac{4}{5}$; $\cos\alpha = \frac{3}{5}$.

4. Ifodani soddalashtiring: 1) $1 + \sin^2\alpha + \cos^2\alpha$; 2) $\frac{\sin^2\alpha}{1-\cos^2\alpha}$; 3) $\frac{1-\sin^2\alpha}{\cos^2\alpha}$.

Namuna. Ifodani soddalashtiring: $1 + \sin^2\alpha - \cos^2\alpha$.

Yechish. Soddalashtirish uchun qo‘shiluvchilarni guruhlab, hosil qilamiz:

$$1+\sin^2\alpha - \cos^2\alpha = \underbrace{1-\cos^2\alpha}_{\sin^2\alpha} + \sin^2\alpha = \sin^2\alpha + \sin^2\alpha = 2\sin^2\alpha.$$

Javob: $2\sin^2\alpha$.

5. Ifodani soddalashtiring: 1) $\frac{(1-\sin\alpha)(1+\sin\alpha)}{\sin^2\alpha}$; 2) $\frac{\sin\alpha\cos\alpha}{\cos^2\alpha}$; 3) $\frac{\operatorname{tg}\alpha}{\sin\alpha}$.

6. To‘g‘ri burchakli uchburchakning katetlari 7 cm va 24 cm ga teng. Uchburchakning eng kichik burchagi trigonometrik funksiyalari qiymatlarini toping.

7. Agar: 1) $\operatorname{tg}A = 2$; 2) $\sin\alpha = \frac{\sqrt{3}}{2}$; 3) $\cos\alpha = \frac{15}{17}$ bo‘lsa, A o‘tkir burchak trigonometrik funksiyalarining qiymatlarini toping.

8. Ifodani soddalashtiring: $\frac{1-\operatorname{tg}^2\alpha+\operatorname{tg}^4\alpha}{\cos^2\alpha}$.

Yechish.

$$\frac{1-\operatorname{tg}^2\alpha+\operatorname{tg}^4\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}\left(1-\operatorname{tg}^2\alpha+\operatorname{tg}^4\alpha\right) = \left(1+\operatorname{tg}^2\alpha\right)\left(1-\operatorname{tg}^2\alpha+\operatorname{tg}^4\alpha\right) = 1+\operatorname{tg}^6\alpha.$$

Soddalashtirishda (5) ayniyatdan va $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ formuladan foydalanildi.

Javob: $1 + \operatorname{tg}^6\alpha$.

9. Agar: 1) $\sin\alpha = \frac{8}{17}$ bo‘lsa, $\cos\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ni; 2) $\cos\alpha = 0{,}6$ bo‘lsa, $\sin\alpha$, $\operatorname{tg}\alpha$ va $\operatorname{ctg}\alpha$ ni toping.

10. Bir burchakning sinusi va kosinusi mos ravishda quyidagi sonlarga teng bo‘lishi yoki bo‘lmasligini aniqlang: 1) $\frac{1}{2}$ va $\frac{\sqrt{3}}{2}$; 2) $\frac{1}{3}$ va $\frac{3}{4}$.

11. Bir burchakning tangens va kotangensi mos ravishda quyidagi sonlarga teng bo‘lishi yoki bo‘lmasligini aniqlang:

1) 0,4 va 2,5; 2) 1,1 va 0,9; 3) $\sqrt{5}+2$ va $\sqrt{5}-2$.

12. Ifodani soddalashtiring: 1) $\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha - \cos^2\alpha$; 2) $\cos\alpha - \cos\alpha \cdot \sin^2\alpha$.

13. To‘g‘ri burchakli uchburchakning katetlari 8 cm va 15 cm ga teng. Uchburchakning eng kichik burchagi trigonometrik funksiyalari qiymatlarini toping.

14. Ifodani soddalashtiring: 1) $(1-\cos\alpha)(1+\cos\alpha)$; 2) $\sin\alpha - \sin\alpha\cos^2\alpha$.

22. TO‘LDIRUVCHI BURCHAKNING TRIGONOMETRIK FUNKSIYALARI UCHUN FORMULALAR

To‘ldiruvchi burchakning trigonometrik funksiyalari uchun formulalar.

To‘ldiruvchi burchaklar *deb, yig‘indisi 90° ga teng bo‘lgan ikki burchakka aytiladi.* To‘g‘ri burchakli uchburchakning o‘tkir burchaklari to‘ldiruvchi burchaklarga misol bo‘ladi, chunki ularning yig‘indisi 90° ga teng.

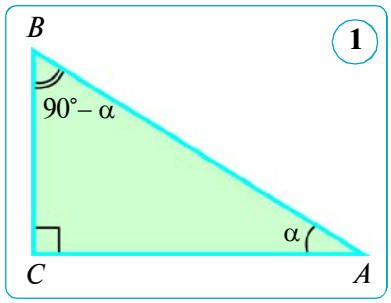
Biz ko‘rib chiqqan trigonometrik ayniyatlar bir burchakning turli trigonometrik funksiyalari orasidagi o‘zaro munosabatlarni o‘rnatishga imkon beradi. Endi to‘g‘ri burchakli uchburchakning ikki o‘tkir burchagi orasidagi munosabatlarni ko‘rib chiqamiz.

Teorema.

Har qanday to‘g‘ri burchakli uchburchakning o‘tkir burchagi α uchun

$$\sin(90^\circ - \alpha) = \cos\alpha; \quad \cos(90^\circ - \alpha) = \sin\alpha$$

tengliklar o‘rinli.



Isbot. Gipotenuzasi AB bo‘lgan to‘g‘ri burchakli ABC uchburchakni qaraymiz (1- rasm). Agar $\angle A = \alpha$ bo‘lsa, u holda $\angle B = 90^\circ - \alpha$ ga teng bo‘ladi. Uchburchakning o‘tkir burchaklarini sinus va kosinuslar orqali ifodalaymiz. Ta’rifga ko‘ra:

$$\sin B = \frac{AC}{AB} \quad \text{va} \quad \cos A = \frac{AC}{AB},$$

ya’ni $\sin(90^\circ - \alpha) = \cos\alpha$;

$$\sin A = \frac{BC}{AB} \quad \text{va} \quad \cos B = \frac{BC}{AB}, \quad \text{ya’ni} \quad \cos(90^\circ - \alpha) = \sin\alpha.$$

Teorema isbotlandi.

Isbot qilingan teoremadan ushbu natija kelib chiqadi.

Natija. Har qanday o‘tkir α burchak uchun

$$\mathbf{tg(90^\circ - \alpha) = ctg\alpha; \quad ctg(90^\circ - \alpha) = tg\alpha}$$

tengliklar o‘rinli.

Bu tengliklarning to‘g‘riligini yuqorida keltirib chiqarilgan formulalardan foydalanib isbotlashni o‘zingizga havola qilamiz.

A va B o‘tkir burchaklar – bir-birini 90° to‘ldiruvchi burchaklardir. Shuni e’tiborga olib, yuqorida keltirib chiqarilgan formulalar quyidagicha o‘qiladi:

- **berilgan burchakning sinusi to‘ldiruvchi burchakning kosinusiga teng;**
- **berilgan burchakning kosinusi to‘ldiruvchi burchakning sinusiga teng;**
- **berilgan burchakning tangensi to‘ldiruvchi burchakning kotangensiga teng;**
- **berilgan burchakning kotangensi to‘ldiruvchi burchakning tangensiga teng.**

1- masala. A va B burchaklar – to‘g‘ri burchakli uchburchakning o‘tkir burchaklari bo‘lsin. Agar $\sin B = \frac{1}{\sqrt{5}}$ bo‘lsa, $\mathrm{tg}A$ ni toping.

Yechish. $\sin B = \cos A$, demak, $\cos A = \sin B = \frac{1}{\sqrt{5}}$. Endi A burchakning sinusini asosiy trigonometrik ayniyatning natijasidan foydalanib topamiz:

$$\sin A = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1 - \frac{1}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}.$$

Burchakning tangensini sinus va kosinus orqali topamiz:

$$\mathrm{tg}A = \frac{\sin A}{\cos A} = \frac{2}{\sqrt{5}} : \frac{1}{\sqrt{5}} = 2.$$

Javob: 2.

2- masala. Agar $\mathrm{ctg}x = \mathrm{tg}20^\circ$ bo‘lsa, o‘tkir x burchakni toping.

Yechish. $\mathrm{tg}20^\circ = \mathrm{ctg}(90^\circ - 20^\circ) = \mathrm{ctg}70^\circ$, demak, $\mathrm{ctg}x = \mathrm{ctg}70^\circ$.

Bundan $x = 70^\circ$. *Javob:* $x = 70^\circ$.

Savol, masala va topshiriqlar

1. 1) To‘ldiruvchi burchaklar deb nimaga aytiladi?

? 2) To‘g‘ri burchakli uchburchakning ikki o‘tkir burchagi orasidagi qanday munosabatlarni bilasiz? Mos formulalarni yozing.

2. Agar: 1) $\sin x = \cos 40^\circ$; 2) $\cos x = \sin 76^\circ$; 3) $\mathrm{tg}x = \mathrm{ctg}56^\circ$; 4) $\mathrm{ctg}x = \mathrm{tg}16^\circ$ bo‘lsa, o‘tkir x burchakni toping.

3. A va B burchaklar – to‘g‘ri burchakli uchburchakning o‘tkir burchaklari. Agar $\cos A = 0{,}6$ bo‘lsa, $\sin B$ va $\cos B$ ni toping.

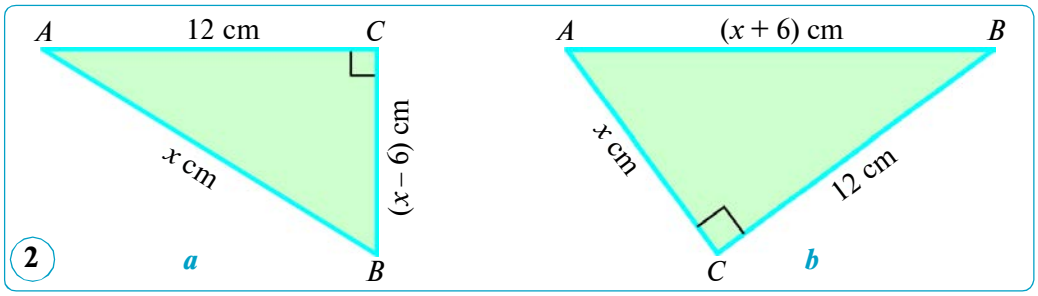
4. Bir burchakning sinusi va kosinusi mos ravishda quyidagi sonlarga teng bo‘lishi yoki bo‘lmasligini aniqlang: 1) $\frac{1}{\sqrt{5}}$ va $\frac{2}{\sqrt{5}}$; 2) 0,3 va 0,4.

5. A va B burchaklar – to‘g‘ri burchakli uchburchakning o‘tkir burchaklari. Agar $\sin B = 0{,}5$ bo‘lsa, $\cos A$ va $\mathrm{tg}A$ ni toping.

6. Noma’lum uzunliklarni toping (2- rasm) hamda o‘tkir burchaklar sinusi, kosinusi, tangensi va kotangensini hisoblang.

7. Agar $\sin(90^\circ - \alpha) = 0{,}8$ bo‘lsa, $\cos\alpha$ va $\sin\alpha$ ni toping.

8. Ifodani soddalashtiring: 1) $\frac{\cos\alpha}{\mathrm{ctg}\alpha}$; 2) $\mathrm{ctg}^2\alpha(2\sin^2\alpha + \cos^2\alpha - 1)$.

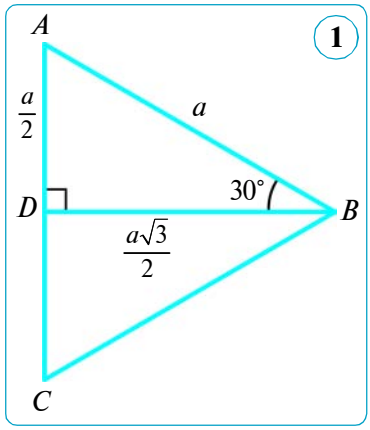


2

23. 30°, 45°, 60° LI BURCHAKLARNING SINUSI, KOSINUSI, TANGENSI VA KOTANGENSINI HISOBLASH

1. 30° li burchakning sinusi, kosinusi, tangensi va kotangensini hisoblash.

Teng tomonli ABC uchburchakni olamiz (1- rasm). Unga BD balandlik o‘tkazsak, u bissektrisa va mediana vazifasini bajaradi. Shu sababli ABD uchburchak B uchidagi o‘tkir burchagi 30° ga teng bo‘lgan to‘g‘ri burchakli ($\angle D = 90°$) uchburchakdir. Teng tomonli uchburchakning tomoni a ga teng bo‘lsin. U holda $AD = \frac{a}{2}$. Pifagor teoremasiga ko‘ra:



$$BD = \sqrt{AB^2 - AD^2} = \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3a^2}{4}} = \frac{a\sqrt{3}}{2}.$$

Ta’riflarga ko‘ra:

$$\sin 30° = \frac{AD}{AB} = \frac{a}{2} : a = \frac{1}{2};$$

$$\cos 30° = \frac{BD}{AB} = \frac{a\sqrt{3}}{2} : a = \frac{\sqrt{3}}{2};$$

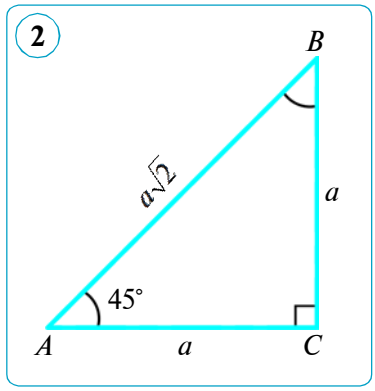
$$\operatorname{tg} 30° = \frac{AD}{BD} = \frac{a}{2} : \frac{a\sqrt{3}}{2} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3};$$

$$\operatorname{ctg} 30° = \frac{BD}{AD} = \frac{a\sqrt{3}}{2} : \frac{a}{2} = \sqrt{3}.$$

To‘ldiruvchi burchakning trigonometrik funksiyalari uchun chiqarilgan formulalar yordamida **60° li burchakning trigonometrik funksiyalari qiymatlarini** topamiz:

$$\sin 60° = \sin(90° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}; \quad \operatorname{tg} 60° = \operatorname{tg}(90° - 30°) = \operatorname{ctg} 30° = \sqrt{3};$$

$$\cos 60° = \cos(90° - 30°) = \sin 30° = \frac{1}{2}; \quad \operatorname{ctg} 60° = \operatorname{ctg}(90° - 30°) = \operatorname{tg} 30° = \frac{\sqrt{3}}{3}.$$



2. 45° li burchakning sinusi, kosinusi, tangensi va kotangensini hisoblash.

45° li burchakning trigonometrik funksiyalarini hisoblash uchun teng yonli to‘g‘ri burchakli ABC uchburchakni qaraymiz (2- rasm). Bu uchburchakda $AC = BC = a$, $\angle A = \angle B = 45°$ bo‘lsin. Pifagor teoremasiga ko‘ra, gipotenuza $AB = a\sqrt{2}$ ga teng bo‘ladi. O‘tkir burchakning trigonometrik funksiyalari ta’rifiga ko‘ra:

$$\sin 45° = \sin A = \frac{BC}{AB} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2};$$

$$\cos 45^\circ = \cos A = \frac{AC}{AB} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2};$$

$$\operatorname{tg} 45^\circ = \operatorname{tg} A = \frac{BC}{AC} = \frac{a}{a} = 1;$$

$$\operatorname{ctg} 45^\circ = \operatorname{ctg} A = \frac{AC}{BC} = \frac{a}{a} = 1.$$

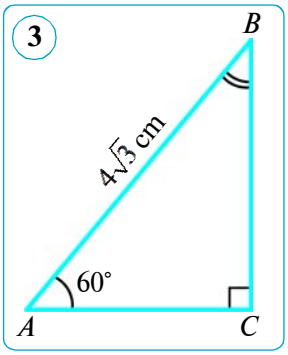
30°, 45° va 60° li burchaklarning sinusi, kosinusi, tangensi va kotangensi qiymatlari jadvalini tuzamiz.

α	30°	45°	60°
$\sin\alpha$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\alpha$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\operatorname{tg}\alpha$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$
$\operatorname{ctg}\alpha$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$

O‘tkir burchakli trigonometrik funksiyalarning qiymatlari, sonlarning kvadratlari va ulardan chiqarilgan arifmetik kvadrat ildizni maxsus jadvallardan bilish yoki kalkulyatordan foydalanib hisoblash mumkin.

Masala. To‘g‘ri burchakli ABC uchburchakning AB gipotenuzasi $4\sqrt{3}$ cm va $\angle A = 60^\circ$ (3- rasm). Shu uchburchakning katetlarini toping.

3



Yechish. Bizga ma’lumki, α burchak qarshisidagi katet gipotenuza bilan α burchak sinusining ko‘paytmasiga teng. Shunga ko‘ra:

$$BC = AB \sin A = 4\sqrt{3} \cdot \sin 60^\circ = 4\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 6 \text{ (cm)}.$$

Bizga ma’lumki, α burchakka yopishgan katet gipotenuza bilan α burchak kosinusining ko‘paytmasiga teng. Shunga ko‘ra:

$$AC = AB \cos A = 4\sqrt{3} \cdot \cos 60^\circ = 4\sqrt{3} \cdot \frac{1}{2} = 2\sqrt{3} \text{ (cm)}.$$

Javob: $BC = 6$ cm, $AC = 2\sqrt{3}$ cm.

Savol, masala va topshiriqlar

1. Hisoblang: 1) $\sin 30^\circ + \operatorname{tg} 45^\circ$; 2) $\cos 30^\circ \cdot \operatorname{tg} 60^\circ$; 3) $\sqrt{2}\sin 45^\circ - \cos 60^\circ$.

2. Teng tomonli uchburchak chizing va uning balandligini o‘tkazing. Kerakli o‘lchashlarni bajarib, 30° va 60° li burchaklarning trigonometrik funksiyalarini hisoblang, natijalarni jadvaldagilari bilan taqqoslang.

3. $ABCD$ parallelogrammning BD diagonali AB tomonga perpendikular va 16 cm ga teng. Agar BDA burchak 30° ga teng bo‘lsa, parallelogrammning tomonlarini toping.

4. To‘g‘ri burchakli uchburchakning bitta kateti $6\sqrt{3}$ ga, bu katet qarshisidagi burchak 60° ga teng. Gipotenuza va ikkinchi katetni toping.

5. Ifodani soddalashtiring: 1) $\operatorname{tg}^2\alpha(2\cos^2\alpha + \sin^2\alpha - 1)$; 2) $\operatorname{tg}^2\alpha - \sin^2\alpha \operatorname{tg}^2\alpha$.

6. To‘g‘ri burchakli uchburchakning bitta kateti 2 ga, bu katet qarshisidagi burchak 60° ga teng. Gipotenuza va ikkinchi katetni toping.

7. Ifodani soddalashtiring: $(\sin\alpha + \cos\alpha)^2 + (\sin\alpha - \cos\alpha)^2$.

8. Hisoblang: 1) $\cos 45^\circ \cdot \sin 45^\circ$; 2) $\sin 60^\circ \cdot \operatorname{tg} 30^\circ$; 3) $\sqrt{3}\cos 30^\circ - \cos 60^\circ$.

6- §. TO‘G‘RI BURCHAKLI UCHBURCHAKLARNI YECHISH

24. TRIGONOMETRIK FUNKSIYALARNING QIYMATLARI JADVALI

Darslik oxirida butun sonli graduslar bilan 1° dan 89° gacha barcha burchaklar uchun mos kelgan trigonometrik funksiyalar (o‘n mingdan birgacha aniqlikda) ko‘rsatilgan jadval keltirilgan. Bu jadval quyidagicha tuzilgan: chap tomondan birinchi ustunga (yuqorisida «graduslar» deb yozilganiga) graduslarning sonlari 1°, 2°, 3°, ..., 45° gacha joylashtirilgan; ikkinchi ustunga (yuqorisida «sinuslar» yozilganiga) sinuslarning birinchi ustunda ko‘rsatilgan burchaklarga mos keluvchi qiymatlari qo‘yilgan; 3- ustunga tangenslar, so‘ngra kotangenslar va undan keyin kosinuslar qiymatlari joylashtirilgan. So‘nggi 6- ustunga yana graduslar sonlari: ya’ni 45°, 46°, 47°, ... va hokazo, 89° gacha joylashtirilgan. Bu (joyni tejash uchun) quyidagiga asosan qilingan: to‘ldiruvchi burchakning trigonometrik funksiyalari uchun formulalarga binoan $\sin\alpha = \cos(90^\circ - \alpha)$, $\cos\alpha = \sin(90^\circ - \alpha)$ va hokazo, demak, $\sin1^\circ = \cos89^\circ$, $\sin2^\circ = \cos88^\circ$ va hokazo. Shuning uchun yuqoridagi «sinuslar» yozilgan ustunning ostiga «kosinuslar»; yuqoridagi «tangenslar» yozilgan (chapdan 3-) ustunning ostiga «kotangenslar» yozilgan va shunga o‘xshash. Shunday qilib, 1° dan 45° gacha burchaklar uchun graduslar sonlarini chap tomondagi birinchi ustundan va trigonometrik funksiyalarning nomlarini yuqoridan o‘qish, 45° dan 89° gacha bo‘lgan burchaklar uchun esa graduslarning sonlarini o‘ng tomondagi so‘nggi ustundan va funksiyalarning nomlarini ustunlarning tagidan o‘qish kerak. Masalan, jadvaldan tangensning qiymatini topamiz: $\text{tg}35^\circ = 0{,}7002$.

1. Berilgan burchakka ko‘ra trigonometrik funksiyalarini topish.

1- masala. $\sin20^\circ$ ni toping.

Yechish. $1^\circ \le 20^\circ \le 45^\circ$ bo‘lgani uchun *chapdagi* «graduslar» so‘zi yozilgan ustundan 20 ni olamiz va unga mos satrning ikkinchi («$\sin\alpha$») ustunidan 0,3420 qiymatni topamiz. Ana shu son $\sin20^\circ$ ning qiymatidir.

Demak, $\sin20^\circ \approx 0{,}3420$.

2- masala. $\sin75^\circ$ ni toping.

Yechish. $45^\circ \le 75^\circ \le 89^\circ$ bo‘lgani uchun *o‘ngdagi* «graduslar» so‘zi yozilgan ustundan 75 ni olamiz va unga mos satrning to‘rtinchi (*pastdagi* «$\sin\alpha$») ustunidan 0,9659 qiymatni topamiz. Ana shu son $\sin75^\circ$ ning qiymatidir.

Demak, $\sin75^\circ \approx 0{,}9659$.

3- masala. $\cos33^\circ$ ni toping.

Yechish. $1^\circ \le 33^\circ \le 45^\circ$ bo‘lgani uchun *chapdagi* «graduslar» so‘zi yozilgan ustundan 33 ni olamiz va unga mos satrning to‘rtinchi («$\cos\alpha$») ustunidan 0,8387 qiymatni topamiz. Ana shu son $\cos33^\circ$ ning qiymatidir.

Demak, $\cos33^\circ \approx 0{,}8387$.

Tangens va kotangenslarning qiymatlari mos ravishda sinus va kosinuslarning qiymatlari jadvaldan qanday topilgan bo‘lsa, shunday topiladi.

2. Burchakni trigonometrik funksiyasiga ko‘ra topish.

4- masala. Agar $\sin x = 0{,}9848$ bo‘lsa, x o‘tkir burchakni toping.

Yechish. Sinusi 0,9848 ga teng bo‘lgan burchakni topish uchun trigonometrik funksiyalarning qiymatlari joylashgan birinchi yoki to‘rtinchi ustundan bu qiymatni izlaymiz. Bu qiymat to‘rtinchi ($\sin\alpha$) ustunda bor, ya’ni izlanayotgan burchak 45° dan katta va 89° dan kichik. Bu satrga mos o‘ngdagi «graduslar» ustunidan 80 sonini topamiz. Demak, izlanayotgan burchak taxminan 80° ga teng. *Javob:* $x \approx 80°$.

5- masala. Agar $\operatorname{tg} x = 0{,}7002$ bo‘lsa, x o‘tkir burchakni toping.

Yechish. Tangensi 0,7002 ga teng bo‘lgan burchakni topish uchun trigonometrik funksiyalarning qiymatlari joylashgan ikkinchi yoki uchinchi ustundan bu qiymatni izlaymiz. Ushbu qiymat ikkinchi ($\operatorname{tg}\alpha$) ustunda bor, ya’ni izlanayotgan burchak 45° dan kichik. Bu satrga mos chapdagi «graduslar» ustunidan 35 sonini topamiz. Demak, izlanayotgan burchak taxminan 35° ga teng. *Javob:* $x \approx 35°$.

Savol, masala va topshiriqlar

1. Jadvaldan foydalanib toping:

a) 1) $\sin 3°$; 2) $\sin 21°$; 3) $\sin 50°$; 4) $\sin 82°$; 5) $\sin 40°$;

b) 1) $\cos 9°$; 2) $\cos 12°$; 3) $\cos 41°$; 4) $\cos 67°$; 5) $\cos 4°$;

d) 1) $\operatorname{tg} 5°$; 2) $\operatorname{tg} 89°$; 3) $\operatorname{tg} 15°$; 4) $\operatorname{tg} 60°$; 5) $\operatorname{tg} 50°$;

e) 1) $\operatorname{ctg} 10°$; 2) $\operatorname{ctg} 30°$; 3) $\operatorname{ctg} 75°$; 4) $\operatorname{ctg} 52°$; 5) $\operatorname{ctg} 5°$.

2. Jadvaldan foydalanib, x o‘tkir burchakni toping:

a) 1) $\sin x \approx 0{,}1392$; 2) $\sin x \approx 0{,}8590$; 3) $\sin x \approx 0{,}5150$;

b) 1) $\cos x \approx 0{,}7431$; 2) $\cos x \approx 0{,}6428$; 3) $\cos x \approx 0{,}0523$;

d) 1) $\operatorname{tg} x \approx 0{,}4663$; 2) $\operatorname{tg} x \approx 11{,}430$; 3) $\operatorname{tg} x \approx 0{,}1763$;

e) 1) $\operatorname{ctg} x \approx 0{,}9004$; 2) $\operatorname{ctg} x \approx 1{,}192$; 3) $\operatorname{ctg} x \approx 0{,}3640$.

3. (*Amaliy ish.*) Transportir yordamida o‘tkir burchagi 40° bo‘lgan to‘g‘ri burchakli uchburchak yasang. Uning tomonlarini o‘lchang hamda shu burchakning sinusi, kosinusi, tangensi va kotangensini hisoblang.

4. Ifodaning qiymatini toping: $\operatorname{tg}15° \cdot \operatorname{tg}30° \cdot \operatorname{tg}60° \cdot \operatorname{tg}75°$.

Yechish. $\operatorname{tg}(90° - \alpha) = \operatorname{ctg}\alpha$ va $\operatorname{tg}\alpha \cdot \operatorname{ctg}\alpha = 1$ formulalardan foydalanib ifodaning qiymatini hisoblaymiz (bo‘sh joylarga mos javobni yozing):

$$\operatorname{tg}15° \cdot \operatorname{tg}30° \cdot \operatorname{tg}60° \cdot \operatorname{tg}75° = \operatorname{tg}15° \cdot \operatorname{tg}30° \cdot \operatorname{tg}(90° - 30°) \cdot \operatorname{tg}(90° - 15°) =$$
$$= (\operatorname{tg}15° \cdot \operatorname{ctg}...°) \cdot (\operatorname{tg}30° \cdot \operatorname{ctg}...°) = ... \cdot ... = ...$$

5. Isbotlang: $\operatorname{tg}1° \cdot \operatorname{tg}2° \cdot ... \cdot \operatorname{tg}88° \cdot \operatorname{tg}89° = 1$.

6. Ifodani soddalashtiring: 1) $\cos^2\alpha + \cos^2(90° - \alpha)$; 2) $\sin^2\alpha - \cos^2(90° - \alpha)$.

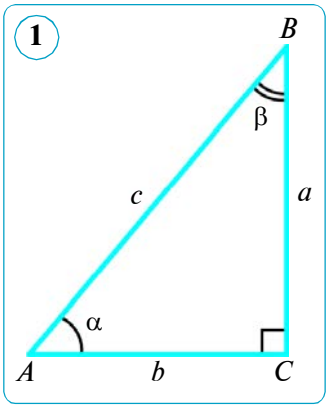
7. Jadvaldan foydalanib toping: 1) $\sin 70°$; 2) $\cos 55°$; 3) $\operatorname{tg} 10°$; 4) $\operatorname{ctg} 18°$.

8. Jadvaldan foydalanib, x o‘tkir burchakni toping: $\sin x \approx 0{,}1392$.

25. TO‘G‘RI BURCHAKLI UCHBURCHAKLARNI YECHISH

Uchburchaklarni yechish uchburchakning ma’lum burchaklari va tomonlari bo‘yicha uning noma’lum tomonlari va burchaklarini topishdan iborat. To‘g‘ri burchakli uchburchakni tomoni va o‘tkir burchagi yoki ikki tomoni bo‘yicha yechish mumkin. To‘g‘ri burchakli uchburchaklarni yechishda 1- rasmdagi belgilashlardan foydalanamiz. Buning uchun masalaning mohiyatidan kelib chiqqan holda, trigonometrik funksiyalarning qiymatlarini o‘n mingdan birlar xonasigacha (darslik oxiridagi ilovaga q.) yoki zarur bo‘lsa, mingdan birlar xonasigacha, tomonlar uzunliklarini yuzdan birgacha, burchakning gradus o‘lchovini birgacha yaxlitlab olishga kelishib olamiz.

1



To‘g‘ri burchakli uchburchakning elementlarini uning ikkita ma’lum elementiga ko‘ra hisoblashning 4 ta holini ko‘rib chiqamiz.

1- hol. Uchburchakni gipotenuzasi va o‘tkir burchagi bo‘yicha yechish.

1- masala. To‘g‘ri burchakli uchburchakning gipotenuzasi $c = 10$ cm va o‘tkir burchagi $\alpha = 50^\circ$ berilgan. a, b katetlar va β o‘tkir burchakni toping.

Yechish. 1) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda $\beta = 90^\circ - \alpha = 90^\circ - 50^\circ = 40^\circ$.

1- usul. 2) α burchak qarshisidagi katet gipotenuza bilan α burchak sinusining ko‘paytmasiga teng, ya’ni $a = c\sin\alpha$.

Demak, $a = 10\sin 50^\circ = 10 \cdot 0,7660 \approx 7,66$ (cm).

3) α burchakka yopishgan katet gipotenuza bilan α burchak kosinusining ko‘paytmasiga teng, ya’ni $b = c\cos\alpha$.

Demak, $b = 10\cos 50^\circ = 10 \cdot 0,6428 \approx 6,43$ (cm).

2- usul. 2) $a = c\cos\beta$; $a = 10\cos 40^\circ = 10 \cdot 0,7660 \approx 7,66$ (cm).

3) $b = c\sin\beta$; $b = 10\sin 40^\circ = 10 \cdot 0,6428 \approx 6,43$ (cm).

Javob: $a \approx 7,66$ cm; $b \approx 6,43$ cm; $\beta = 40^\circ$.

2- hol. Uchburchakni kateti va o‘tkir burchagi bo‘yicha yechish.

2- masala. To‘g‘ri burchakli uchburchakning kateti $a = 6$ cm va o‘tkir burchagi $\beta = 22^\circ$ berilgan. b katet, c gipotenuza va α o‘tkir burchakni toping.

Yechish. 1) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda $\alpha = 90^\circ - \beta = 90^\circ - 22^\circ = 68^\circ$.

1-usul. 2) Gipotenuza β o‘tkir burchakka yopishgan katetning β burchak kosinusiga nisbatiga teng, ya’ni $c = \dfrac{a}{\cos\beta}$.

Demak, $c = \dfrac{a}{\cos\beta} = \dfrac{6}{\cos 22^\circ} = \dfrac{6}{0,9272} \approx 6,47$ (cm).

3) Ta'rifga ko'ra: $\operatorname{tg}\beta = \frac{b}{a}$. Bundan $b = a\operatorname{tg}\beta$, ya'ni

$$b = 6\operatorname{tg}22^\circ = 6 \cdot 0{,}4040 \approx 2{,}42 \text{ (cm)}.$$

2- usul. 2) Gipotenuza α o'tkir burchak qarshisidagi katetning α burchak sinusiga nisbatiga teng, ya'ni $c = \frac{a}{\sin\alpha}$.

Demak, $c = \frac{a}{\sin\alpha} = \frac{6}{\sin 68^\circ} = \frac{6}{0{,}9272} \approx 6{,}47$ (cm).

3) Ta'rifga ko'ra: $\operatorname{tg}\beta = \frac{b}{a}$. Bundan $b = a\operatorname{tg}\beta$, ya'ni

$$b = 6\operatorname{tg}22^\circ = 6 \cdot 0{,}4040 \approx 2{,}42 \text{ (cm)}.$$

Javob: $c \approx 6{,}47$ cm, $b \approx 2{,}42$ cm, $\alpha = 68^\circ$.

Savol, masala va topshiriqlar

1. To'g'ri burchakli uchburchakda uzunligi 7 cm ga teng bo'lgan katet 60° li burchakka yopishgan. Shu uchburchakning gipotenuzasini toping.

2. To'g'ri burchakli uchburchakning gipotenuzasi 12 cm ga, katetlaridan biri esa $6\sqrt{2}$ cm ga teng. Uchburchakning o'tkir burchaklarini toping.

3. To'g'ri burchakli uchburchakning gipotenuzasi $c = 10$ cm va o'tkir burchagi $\alpha = 42^\circ$ berilgan. a, b katetlar va β o'tkir burchakni toping. Masalani ikki usul (matndagi 1- masalaga q.) bilan yeching.

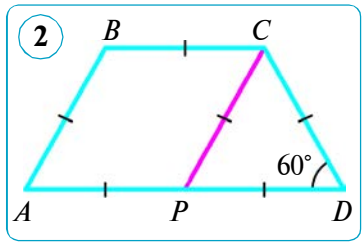
4. To'g'ri burchakli uchburchakning kateti $b = 4$ cm va o'tkir burchagi $\beta = 18^\circ$ berilgan. a katet, c gipotenuza va α o'tkir burchakni toping. Masalani ikki usul (matndagi 2- masalaga q.) bilan yeching.

5. Ifodani soddalashtiring: $\frac{\cos^2\alpha}{(1+\sin\alpha)(1-\sin\alpha)} - \sin\alpha\cos(90^\circ - \alpha)$.

6. Teng yonli trapetsiya asosidagi burchak 60° ga, yon tomoni esa kichik asosiga teng bo'lib, $2\sqrt{2}$ cm ga teng. Shu trapetsiyaning katta asosini toping. Bo'sh joylarga mos javobni yozing.

Yechish. $ABCD$ trapetsiya – teng yonli, $\angle A = \angle D = 60^\circ$, $AB = DC = BC = 2\sqrt{2}$ cm.

$CP \parallel BA$ o'tkazamiz (2- rasm). U holda $\angle A = \angle CPD = 60^\circ$ ($CP \parallel BA$ hamda AD kesuvchi kesishishidan hosil bo'lgan ... burchaklar). CPD uchburchakning burchaklari ...° dan, demak, u ... tomonli. Shuning uchun, $CP = PD = \ldots = 2\sqrt{2}$ cm. U holda $AD = 2 \cdot 2\sqrt{2} = \ldots$ (cm). *Javob:* $4\sqrt{2}$ cm.

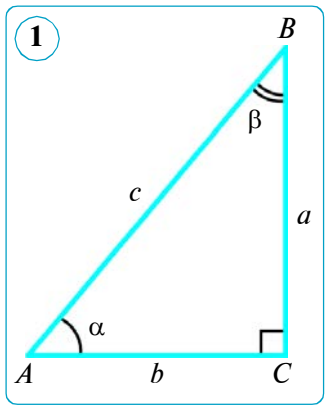


7. To'g'ri burchakli uchburchakning gipotenuzasi $c = 8$ cm va o'tkir burchagi $\alpha = 30^\circ$ berilgan. Uning a, b katetlari va β o'tkir burchagini toping. Masalani ikki usul (matndagi 1- masalaga q.) bilan yeching.

26. TO‘G‘RI BURCHAKLI UCHBURCHAKLARNI YECHISH (DAVOMI)

3- hol. Uchburchakni gipotenuzasi va kateti bo‘yicha yechish.

1- masala. To‘g‘ri burchakli uchburchakning gipotenuzasi $c = 13$ cm va kateti $a = 5$ cm berilgan. Uning b kateti, α va β o‘tkir burchaklarini toping.



Yechish. 1) Pifagor teoremasiga ko‘ra:

$$b = \sqrt{c^2 - a^2} = \sqrt{13^2 - 5^2} = \sqrt{169 - 25} = \sqrt{144} = 12 \text{ (cm)}.$$

1- usul. 2) α o‘tkir burchak sinusining ta’rifiga ko‘ra:

$$\sin\alpha = \frac{a}{c} = \frac{5}{13} \approx 0,3846.$$

Bundan $\alpha \approx 23°$.

3) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda

$$\beta = 90° - \alpha \approx 90° - 23° = 67°.$$

Javob: $b = 12$ cm, $\alpha \approx 23°$, $\beta \approx 67°$.

2-usul. 2) β o‘tkir burchak sinusining ta’rifiga ko‘ra:

$$\sin\beta = \frac{b}{c} = \frac{12}{13} \approx 0,9231.$$

Bundan $\beta \approx 67°$.

3) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda

$$\alpha = 90° - \beta \approx 90° - 67° = 23°.$$

Javob: $b = 12$ cm, $\alpha \approx 23°$, $\beta \approx 67°$.

4- hol. Uchburchakni ikkita kateti bo‘yicha yechish.

2- masala. To‘g‘ri burchakli uchburchakning katetlari $a = 8$ cm va $b = 15$ cm berilgan. Uning c gipotenuzasi, α va β o‘tkir burchaklarini toping.

Yechish. 1) Pifagor teoremasiga ko‘ra:

$$c = \sqrt{a^2 + b^2} = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \text{ (cm)}.$$

1- usul. 2) α o‘tkir burchak tangensining ta’rifiga ko‘ra:

$$\operatorname{tg}\alpha = \frac{a}{b} = \frac{8}{15} \approx 0,5333.$$

Bundan $\alpha \approx 28°$.

3) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda

$$\beta = 90° - \alpha \approx 90° - 28° = 62°.$$

Javob: $c = 17$ cm, $\alpha \approx 28°$, $\beta \approx 62°$.

2- usul. 2) β o‘tkir burchak tangensining ta’rifiga ko‘ra:

$$\operatorname{tg}\beta = \frac{b}{a} = \frac{15}{8} = 1{,}875 .$$

Bundan $\beta \approx 62^\circ$.

3) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda

$$\alpha = 90^\circ - \beta \approx 90^\circ - 62^\circ = 28^\circ .$$

Javob: $c = 17$ cm, $\alpha \approx 28^\circ$, $\beta \approx 62^\circ$.

3- usul. 1) α o‘tkir burchak kotangensining ta’rifiga ko‘ra:

$$\operatorname{ctg}\alpha = \frac{b}{a} = \frac{15}{8} = 1{,}875 .$$

Bundan $\alpha \approx 28^\circ$.

2) To‘g‘ri burchakli uchburchakning o‘tkir burchaklari yig‘indisi 90° ga teng. U holda

$$\beta = 90^\circ - \alpha \approx 90^\circ - 28^\circ = 62^\circ .$$

3) Pifagor teoremasiga ko‘ra:

$$c = \sqrt{a^2 + b^2} = \sqrt{8^2 + 15^2} = \sqrt{64 + 225} = \sqrt{289} = 17 \ \text{(cm)}.$$

Javob: $c = 17$ cm, $\alpha \approx 28^\circ$, $\beta \approx 62^\circ$.

Savol, masala va topshiriqlar

1. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90^\circ$, gipotenuza $c = 9\sqrt{2}$ cm, katet $a = 9$ cm. Shu uchburchakning b kateti, α va β o‘tkir burchaklarini toping. Ikki usul bilan yeching.

2. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90^\circ$, katetlari $a = 6\sqrt{3}$ cm va $b = 6$ cm. Shu uchburchakning c gipotenuzasi, α va β o‘tkir burchaklarini toping. Ikki usul bilan yeching.

3. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90^\circ$, katetlari $a = \sqrt{11}$ cm va $b = 5$ cm. Shu uchburchakning c gipotenuzasi, α va β o‘tkir burchaklarini toping. Ikki usul bilan yeching.

4. CD kesma – to‘g‘ri burchakli ABC ($\angle C = 90^\circ$) uchburchakning gipotenuzasiga tushirilgan balandligi. Isbotlang:

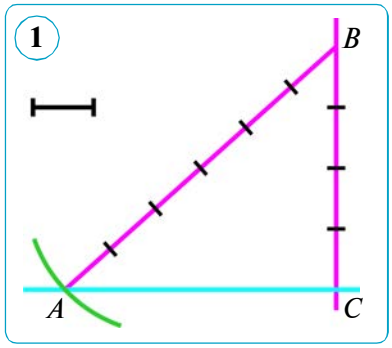
1) $\dfrac{CD}{\sin A} = AB\cos A$; 2) $AD\operatorname{tg}A = BD\operatorname{tg}B$.

5. Hisoblang: $2\sin 60^\circ + 4\cos 60^\circ - \operatorname{ctg}30^\circ - 2\operatorname{tg}45^\circ$.

6. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90^\circ$, gipotenuza $c = 25$ cm, katet $b = 24$ cm. Shu uchburchakning a kateti, α va β o‘tkir burchaklarini toping. Ikki usul bilan yeching.

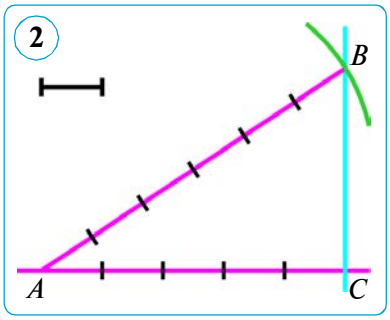
7. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90^\circ$, katetlari $a = 10$ cm va $b = 24$ cm. Shu uchburchakning c gipotenuzasi, α va β o‘tkir burchaklarini toping. Ikki usul bilan yeching.

27. TO‘G‘RI BURCHAKLI UCHBURCHAKLARNI YASASH



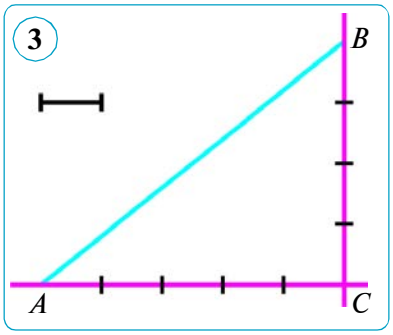
1- masala. Sinusi $\frac{4}{5}$ ga teng bo‘lgan burchakni yasash.

Buning uchun C to‘g‘ri burchak yasaymiz va uning tomonlaridan birida burchak uchidan boshlab 4 ta ixtiyoriy masshtab birligiga teng CB kesmani qo‘yamiz (1- rasm). Markazi B nuqtada va radiusi 5 ta masshtab birligiga teng radiusli yoyni burchakning ikkinchi tomoni bilan kesishguncha chizamiz. Ularning kesishish nuqtasini A bilan belgilaymiz. A va B nuqtalarni birlashtirib, to‘g‘ri burchakli ABC uchburchakni hosil qilamiz. A – izlanayotgan burchak, uning sinusi $\frac{4}{5}$ ga teng bo‘ladi, ya’ni $\sin A = \frac{4}{5}$.



2- masala. Kosinusi $\frac{5}{6}$ ga teng bo‘lgan burchakni yasash.

Buning uchun C to‘g‘ri burchak yasaymiz va uning tomonlaridan birida burchak uchidan boshlab 5 ta ixtiyoriy masshtab birligiga teng AC kesmani qo‘yamiz (2- rasm). Markazi A nuqtada va radiusi 6 ta masshtab birligiga teng radiusli yoyni burchakning ikkinchi tomoni bilan kesishguncha chizamiz. Ularning kesishish nuqtasini B bilan belgilaymiz. A va B nuqtalarni birlashtirib, to‘g‘ri burchakli ABC uchburchakni hosil qilamiz. A – izlanayotgan burchak, uning kosinusi $\frac{5}{6}$ ga teng bo‘ladi, ya’ni $\cos A = \frac{5}{6}$.



3- masala. Tangensi $\frac{4}{5}$ ga teng bo‘lgan burchakni yasash.

Buning uchun C to‘g‘ri burchak yasaymiz va uning tomonlaridan birida burchak uchidan boshlab 5 ta ixtiyoriy masshtab birligiga teng CA kesmani, ikkinchisida esa 4 masshtab birligiga teng CB kesmani qo‘yamiz (3- rasm). A va B nuqtalarni birlashtirib, to‘g‘ri burchakli ABC uchburchakni hosil qilamiz. A – izlanayotgan burchak, uning tangensi $\frac{4}{5}$ ga teng bo‘ladi, ya’ni $\mathrm{tg}A = \frac{4}{5}$.

Berilgan kotangensga ko‘ra burchak yasash talab etilganda ham xuddi shunday yasashga to‘g‘ri keladi, faqat bu holda izlangan burchak uchun AC ga yopishgan katetni olish kerak bo‘ladi.

To‘g‘ri burchakli uchburchakning kateti har doim gipotenuzadan kichik. Shuning uchun o‘tkir burchakning sinusi va kosinusi doimo 1 dan kichikdir.

Katetlar uzunliklarini taqqoslash shuni ko‘rsatadiki, ular o‘zaro teng, biri ikkinchisidan katta yoki kichik bo‘lishi mumkin. Shuning uchun o‘tkir burchak tangensi va kotangenslari istalgan musbat son bo‘lishi mumkin. Demak, ularning har biri katetlarga bog‘liq holda 1 dan kichik, 1 dan katta va 1 ga teng bo‘ladi.

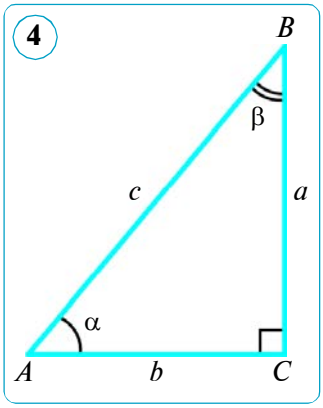
Savol, masala va topshiriqlar

1. 1) $\operatorname{tg}A = \frac{3}{5}$; 2) $\sin A = \frac{2}{3}$ ga teng bo‘lgan, to‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakni yasang.

2. 1) $\sin A = \frac{5}{8}$; 2) $\cos A = \frac{3}{4}$ ga teng bo‘lgan, to‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakni yasang.

3. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90°$, gipotenuza $c = 7\sqrt{2}$ cm, katet $b = 7$ cm. Uchburchakning a kateti, α va β o‘tkir burchaklarini (4- rasm) toping.

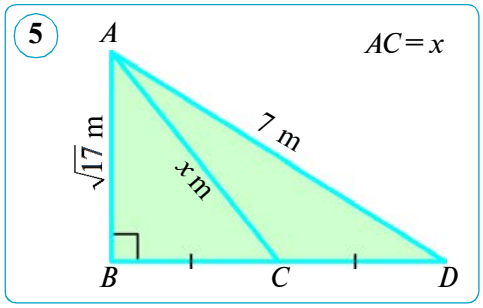
4. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90°$, gipotenuza $c = 12$ cm, $\alpha = 60°$. Uchburchakning a, b katetlari, β o‘tkir burchagini (4- rasm) toping. Masalani ikki usul bilan yeching.

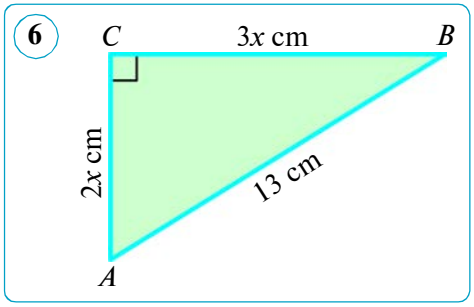


5. Noma’lum uzunliklarni toping (5–6- rasmlar).

6. To‘g‘ri burchakli ABC uchburchakda $\angle C = 90°$, gipotenuza $c = 74$ cm, $\sin\alpha = \frac{12}{37}$. Shu uchburchakning perimetrini (4- rasm) toping.

7. 1) $\sin A = \frac{4}{7}$; 2) $\cos A = \frac{3}{5}$; 3) $\operatorname{tg}A = \frac{2}{5}$ ga teng bo‘lgan to‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakni yasang.





28. AMALIY MASHQ VA TATBIQ

1. Pifagor teoremasining amaliy tatbig‘iga doir masalalar.

1- masala. Suv nilufar gulining ko‘l sathidan ko‘rinadigan qismi 10 cm. Agar gulni boshlang‘ich holatidan bir tomonga 1 m tortilsa, suv sathiga tegadi. Ko‘l-ning shu joydagi chuqurligini toping.

Yechish. Ko‘lning izlangan CD chuqurligini x bilan belgilaymiz (1- rasm). U holda $BD = AD = AC + CD = 0{,}1 + CD = 0{,}1 + x$ (m) ga teng bo‘ladi. Unda to‘g‘ri burchakli BCD uchburchakdan Pifagor teoremasiga ko‘ra quyidagilarga ega bo‘lamiz:

$$BD^2 - CD^2 = BC^2, \quad (0{,}1 + x)^2 - x^2 = 1,$$

bundan:

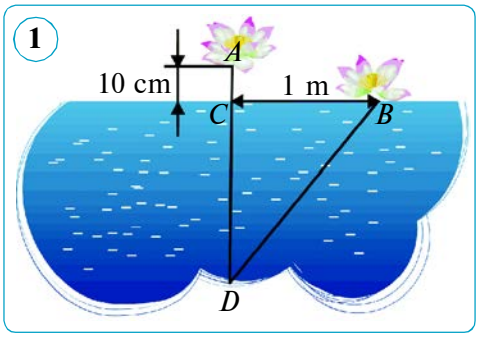
$$0{,}01 + 0{,}2x + x^2 - x^2 = 1;$$

$$0{,}2x = 0{,}99; \quad x = 0{,}99 : 0{,}2;$$

$$x = 9{,}9 : 2; \quad x = 4{,}95 \text{ (m)}.$$

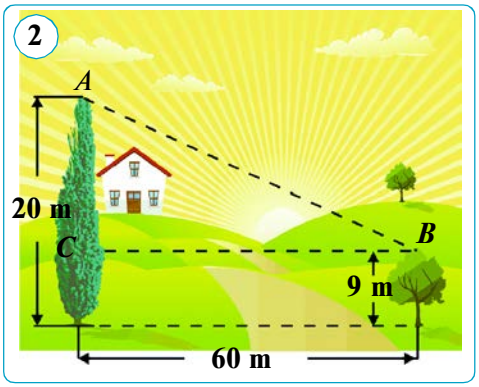
Javob: ko‘lning chuqurligi 4,95 m.

1



2- masala. Bir daraxtning balandligi 20 m, ikkinchisiniki esa 9 m. Bu daraxtlar orasidagi masofa 60 m ni tashkil qiladi. Shu ikki daraxt uchlari orasidagi maso-fani toping (2- rasm). Mustaqil yeching.

2

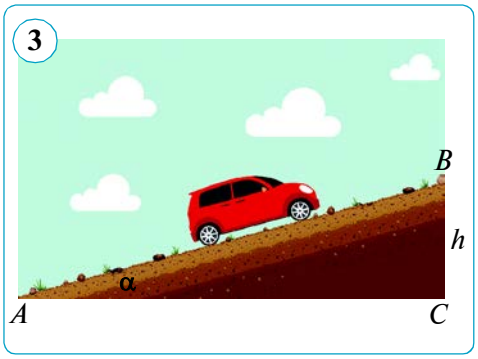


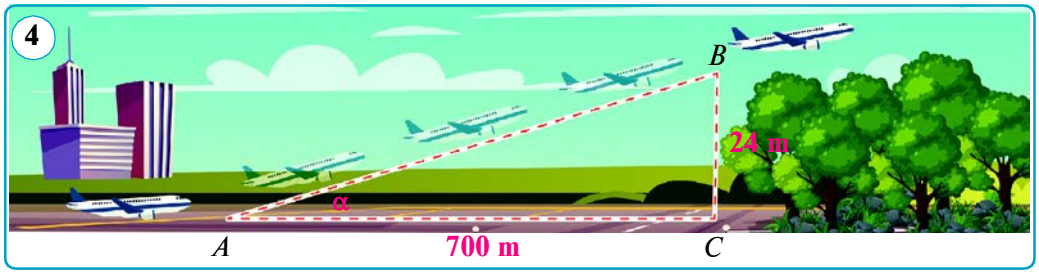
3- masala. Ikkita qarag‘ay daraxti-ning balandliklari mos ravishda 21 m va 28 m, bu daraxtlar orasidagi masofa esa 24 m ni tashkil qiladi. Ikki daraxt uchlari orasidagi masofani toping (2- rasmga qarang). Mustaqil yeching.

2. O‘tkir burchak sinusining amaliy tatbig‘iga doir masala.

Qiya tekis yo‘l ko‘tarilish joyining tikligini gorizontga nisbatan ko‘tarilish burchagi orqali berish mumkin (3- rasm). Ko‘pincha ko‘tarilish joyining tikligini ko‘tarilish burchagidan ko‘ra bosib o‘til-gan yo‘l uzunligining ko‘tarilish baland-ligi orqali berish qulay. Masalan, mashina 100 m masofani bosib o‘tganda 2 m balandlikka ko‘tarilgan bo‘lsin. Bu holda ko‘tarilish joyining tikligi ba-landlikning bosib o‘tilgan yo‘lga nisbati bilan beriladi. Ko‘tarilish balandligi $\frac{2 \text{ m}}{100 \text{ m}} = 0{,}02$ ga teng. Bu nisbat bosib

3





o‘tilgan yo‘lga bog‘liq emas. Qiya tekis yo‘ldan tushishda ham xuddi shunga o‘xshash mulohaza yuritish mumkin.

4- masala. Yengil mashina nishabligi 15° bo‘lgan qiya yo‘l bo‘ylab ko‘tarilmoqda (3- rasmga q.). U qiyalikka ko‘tarilish joyidan 300 m yo‘l bosib o‘tgach gorizontga nisbatan necha metr balandlikda bo‘ladi?

Ko‘rsatma. O‘tkir burchak sinusining ta’rifini qo‘llab, ko‘tarilish balandligini toping.

2. O‘tkir burchak tangensining amaliy tatbig‘iga doir masalalar.

5- masala. Samolyot uchish yo‘lakchasidan havoga ko‘tariladigan nuqtadan 700 m masofada o‘rmonzor joylashgan bo‘lib, daraxtlarning maksimal balandligi 24 m ga teng. Samolyot bu daraxtlarga tegmasligi uchun qanday burchak ostida ko‘tarilishi kerak?

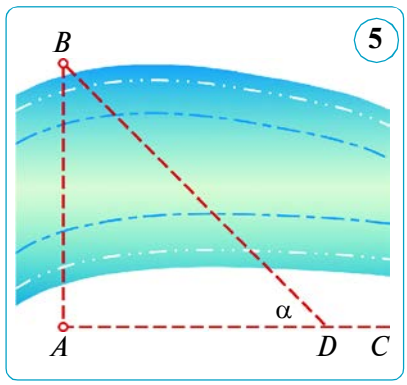
Yechish. To‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakda $AC = 700$ m, $BC = 24$ m (4- rasm). O‘tkir burchak tangensining ta’rifidan topamiz:

$$\operatorname{tg}\alpha = \frac{BC}{AC} = \frac{24}{700} \approx 0,0343 \Rightarrow \alpha \approx 2°.$$

Javob: samolyot daraxtlarga tegmasdan parvoz qilishi uchun uchish nuqtasidan 2° dan kam bo‘lmagan burchak ostida ko‘tarilishi lozim.

6- masala. A punktdan daryo ortidagi borib bo‘lmaydigan B punktgacha bo‘lgan masofani toping (5- rasm).

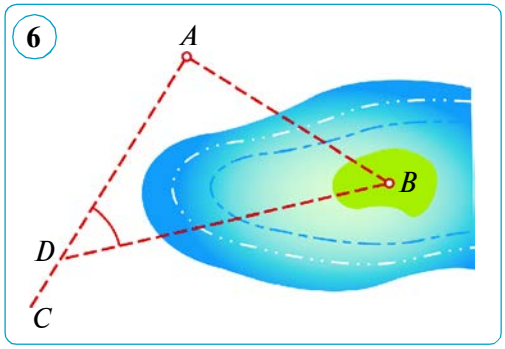
Yechish. Usturlob (astrolyabiya, gorizontal tekislikda joylashgan burchaklarni o‘lchash uchun ishlatiladigan asbob, ya’ni burchak o‘lchagich) yoki ekker yordamida A nuqtada to‘g‘ri BAC burchakni yasaymiz. AC to‘g‘ri chiziqda ixtiyoriy D nuqtani olib, usturlob yordamida ADB burchakni o‘lchaymiz. Deylik, u 44° ga teng bo‘lsin. So‘ngra AD masofani o‘lchaymiz, u 120 m bo‘lsin. AB masofani o‘tkir burchakning tangensidan foydalanib topamiz:

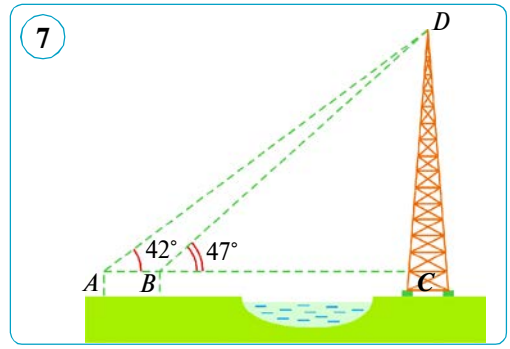


$$\frac{AB}{120} = \operatorname{tg}44° \Rightarrow AB = 120 \cdot \operatorname{tg}44° \approx$$

$$\approx 120 \cdot 0,9657 \approx 116 \text{ (m)}.$$

Javob: ≈ 116 m.





7- masala. A punktdan borib bo‘lmaydigan orolchadagi B punktgacha bo‘lgan masofani toping (6- rasm).

Ko‘rsatma. 5- masalaga o‘xshash muhokama qilinadi. $\angle ADB = 48°$ va $AD = 200$ m deb, masalani yeching.

8- masala. Asosiga borib bo‘lmaydigan obyekt, masalan, elektr uzatgich balandligini o‘lchash talab etilgan bo‘lsin (7- rasm).

Yechish. To‘g‘ri burchakli ACD uchburchakni qaraymiz. Bu uchburchakning A burchagini usturlob yordamida o‘lchashimiz mumkin, deylik, u $42°$ ga teng bo‘lsin.

To‘g‘ri burchakli BCD uchburchakda DBC burchakni o‘lchaymiz, u $47°$ ga teng bo‘lsin.

O‘tkir burchak tangensi ta’rifiga asosan ACD dan topamiz:

$$\frac{CD}{AC} = \operatorname{tg}42° \Rightarrow AC = \frac{CD}{\operatorname{tg}42°}. \qquad (1)$$

O‘tkir burchak tangensi ta’rifiga asosan BCD dan topamiz:

$$\frac{CD}{BC} = \operatorname{tg}47° \Rightarrow BC = \frac{CD}{\operatorname{tg}47°}. \qquad (2)$$

A, B va C nuqtalar bir to‘g‘ri chiziqda yotadi. (1) dan (2) ni ayiramiz:

$$AC - BC = \frac{CD}{\operatorname{tg}42°} - \frac{CD}{\operatorname{tg}47°} \Rightarrow AC - BC = CD\left(\frac{1}{\operatorname{tg}42°} - \frac{1}{\operatorname{tg}47°}\right) \Rightarrow$$

$$\Rightarrow AC - BC = CD\left(\frac{1}{0{,}9004} - \frac{1}{1{,}0724}\right) \Rightarrow AC - BC = CD(1{,}1106 - 0{,}9325) \Rightarrow$$

$$\Rightarrow AC - BC = CD \cdot 0{,}1781 \Rightarrow CD = \frac{AC - BC}{0{,}1781}.$$

$AC - BC$, ya’ni AB masofani bevosita o‘lchashimiz mumkin, deylik, u 12 m ga teng bo‘lsin. U holda

$$CD = \frac{AC - BC}{0{,}1781} = \frac{AB}{0{,}1781} = \frac{12}{0{,}1781} \approx 67{,}4 \text{ (m)}.$$

Javob: $\approx 67{,}4$ m.

Atrofingizdan ko‘rib chiqilgan masalalarga o‘xshash masalalar yetarlicha topiladi. Mustaqil masalalar tuzing va yeching.

29–30. 2- NAZORAT ISHI. XATOLAR USTIDA ISHLASH

1. To‘g‘ri burchakli uchburchakning gipotenuzasi 20 cm ga, o‘tkir burchaklaridan birining sinusi 0,5 ga teng. Uchburchakning katetlarini toping.

2. To‘g‘ri burchakli uchburchakning gipotenuzasi 13 cm ga, o‘tkir burchaklaridan birining kosinusi $\frac{5}{13}$ ga teng. Uchburchakning katetlarini toping.

3. Ifodani soddalashtiring: $(\sin^2\alpha - \cos^2\alpha)^2 + 2\sin\alpha\cos\alpha$.

4. Tomonlari: 1) $a = c = 17$ cm, $b = 16$ cm; 2) $a = 30$ cm, $b = 34$ cm, $c = 16$ cm bo‘lgan uchburchakning balandligini toping.

2- TEST — O‘zingizni sinab ko‘ring!

1. To‘g‘ri burchakli uchburchakning katetlaridan biri 12 cm, gipotenuzasi esa ikkinchi katetdan 6 cm uzun. Gipotenuzaning uzunligini toping.

A) 15 cm; B) 25 cm; D) 26 cm; E) 18 cm.

2. To‘g‘ri burchakli uchburchakning katetlaridan biri 12 cm, ikkinchisi esa gipotenuzadan 8 cm qisqa. Shu uchburchakning gipotenuzasini toping.

A) 15 cm; B) 16 cm; D) 13 cm; E) 25 cm.

3. To‘g‘ri burchakli uchburchakning gipotenuzasi 25 cm, katetlari o‘zaro 3 : 4 nisbatda. Shu uchburchakning kichik katetini toping.

A) 10 cm; B) 15 cm; D) 9 cm; E) 20 cm.

4. Tomonlari 13 cm, 14 cm va 15 cm bo‘lgan uchburchakning eng kichik balandligi necha santimetr?

A) 11,5 cm; B) 11,1 cm; D) 11 cm; E) 11,2 cm.

5. Rombning diagonallari 14 cm va 48 cm ga teng. Shu rombning perimetrini toping.

A) 60 cm; B) 100 cm; D) 80 cm; E) 120 cm.

6. Rombning perimetri 68 cm, diagonallaridan biri 30 cm ga teng. Uning ikkinchi diagonalini toping.

A) 12 cm; B) 8 cm; D) 16 cm; E) 20 cm.

7. To‘g‘ri burchakli uchburchakning katetlaridan biri $5\sqrt{3}$ cm ga, uning qarshisidagi burchak esa 60° ga teng. Uchburchakning gipotenuzasini toping.

A) $5\sqrt{3}$ cm; B) $2\sqrt{15}$ cm; D) 5 cm; E) 10 cm.

8. To‘g‘ri burchakli uchburchakning katetlaridan biri $5\sqrt{3}$ cm, unga yopishgan burchak esa 30° ga teng. Shu uchburchakning ikkinchi katetini toping.

A) $5\sqrt{3}$ cm; B) $2\sqrt{15}$ cm; D) 5 cm; E) 10 cm.

9. To‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakning gipotenuzasi 17 cm ga, katetlari esa 15 cm va 8 cm ga teng. A burchakning sinusini toping.

A) $\frac{8}{15}$; B) $\frac{8}{17}$; D) $\frac{17}{15}$; E) $\frac{15}{17}$.

10. To‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakning gipotenuzasi 37 cm ga, katetlari esa 12 cm va 35 cm ga teng. B burchakning kosinusini toping.

A) $\frac{12}{37}$; B) $\frac{35}{37}$; D) $\frac{12}{35}$; E) $\frac{35}{12}$.



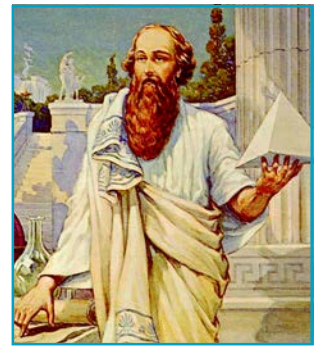
Ingliz tilini o‘rganamiz!

Pifagor teoremasi – Pythagorean theorem
Teskari teorema – inverse function theorem
Trigonometriya – trigonometry
Gipotenuza – hypotenuse

Sinus – sine
Kosinus – cosine
Tangens – tangent
Kotangens – cotangent



Tarixiy ma’lumotlar

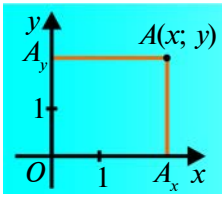


Pifagor
(miloddan oldingi 570–500- y.)

Qadimgi grek faylasufi va matematigi **Pifagor** miloddan oldingi VI asrning ikkinchi yarmida (miloddan oldingi 570–500 yillar) Egey dengizining Samos orolida tug‘ilgan va Tarentda vafot etgan deb taxmin qilinadi. Pifagor Janubiy Italiyaning greklar mustamlakasi bo‘lgan Kroton shahriga (taxminan miloddan oldingi 530-y.) ko‘chib kelib, shu yerda o‘z maktabiga asos solgan. Biz bu maktab olib borgan geometrik tekshirish ishlarining natijalari haqida keyinroq o‘tgan grek matematiklarining asarlaridangina bilamiz. Pifagor olib borgan geometrik ishlarining o‘zi bizgacha yetib kelmagan.

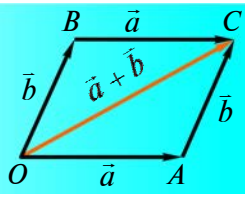
Pifagor birinchi bo‘lib sonlarni juft va toq, tub va murakkab sonlarga ajratgan. Uning maktabida «Pifagor sonlari» deyiladigan natural sonlar uchliklari to‘liq ko‘rib chiqilgan. Pifagor teoremasi juda ko‘p geometrik hisoblashlarning asosini tashkil etadi. Hozirgi kunda Pifagor teoremasining yuzdan ortiq isbotlari mavjud. Ulardan ba’zilari kvadratlarni bo‘laklarga ajratishga asoslangan, bunda katetlarga yasalgan kvadratlar bo‘laklaridan gipotenuzaga yasalgan kvadrat tuzilgan; boshqalari teng shakllarga to‘ldirishga, uchinchilari esa to‘g‘ri burchakning uchidan gipotenuzaga tushirilgan balandlik to‘g‘ri burchakli uchburchakni ikkita o‘xshash uchburchakka ajratishiga asoslangan.

Qadimgi Bobilda teng yonli uchburchakning yon tomoni va asosi uzunligiga ko‘ra uning balandligini topishgan. Ba’zi bir manbalarga ko‘ra, Pifagor maktabida to‘g‘ri chiziqli shakllarni tengdosh shakllarga ajratishning geometrik usullaridan teoremalarni isbotlash va masalalar yechishda foydalanilgan. Chunki to‘g‘ri chiziqli shakllarni geometrik almashtirish masalasi amaliy ishlardan kelib chiqqan.



III BOB

KOORDINATALAR USULI. VEKTORLAR

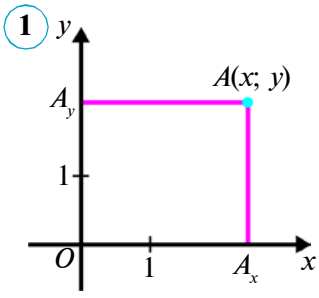


7-§. TEKISLIKDA KOORDINATALAR SISTEMASI

31. TEKISLIKDA NUQTANING KOORDINATALARI. KESMA O‘RTASINING KOORDINATALARI

1. Tekislikda nuqtaning koordinatalari. Tekislikda o‘zaro perpendikular x va y o‘qlarni o‘tkazamiz. Ularning kesishish nuqtasini O harfi bilan belgilaylik. Bu nuqtani har bir o‘q uchun *hisob boshi* deb, har bir o‘qda o‘zaro teng *birlik* kesmani olamiz. Ox o‘qdagi yo‘nalish «*chapdan o‘ngga*», Oy o‘qidagi yo‘nalish esa «*pastdan yuqoriga*» bo‘ladi (1- rasm). Bu holda tekislikda xOy to‘g‘ri burchakli koordinatalar sistemasi aniqlangan, deyiladi. Bu sistemani fanga fransuz olimi **Rene Dekart** kiritgani uchun **Dekart koordinatalar sistemasi** ham deyiladi. Ox o‘qi **abssissalar o‘qi** (yoki x o‘qi), Oy o‘qi esa **ordinatalar o‘qi** (yoki y o‘qi) deyiladi. Abssissalar o‘qi gorizontal, ordinatalar o‘qi vertikal joylashgan.

1

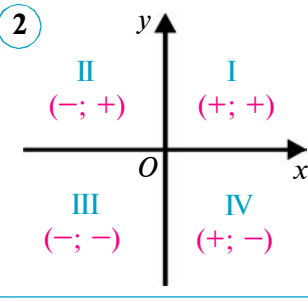


Dekart koordinatalar sistemasi yotgan tekislik **koordinatalar tekisligi** deyiladi.

A – koordinata tekisligida olingan ixtiyoriy nuqta bo‘lsin. A nuqtadan Ox va Oy o‘qlariga parallel to‘g‘ri chiziqlar o‘tkazamiz. Ular Ox va Oy o‘qlari bilan, mos ravishda, A_x va A_y nuqtalarda kesishadi, deylik (1- rasmga q.).

AA_x kesma uzunligi x, AA_y kesma uzunligi y bo‘lsin. x son A nuqtaning **abssissasi**, y son esa A nuqtaning **ordinatasi** deyiladi.

2



x va y sonlar jufti A nuqtaning **koordinatalari** deyiladi va ***A*(*x*; *y*)** kabi belgilanadi. Koordinatalarni ifodalashda birinchi abssissa, keyin ordinata yoziladi.

Shunday qilib: 1) koordinata tekisligida har bir A nuqtaga sonlar jufti $(x\,;\,y)$ mos keladi; 2) ixtiyoriy sonlar jufti $(x\,;\,y)$ni koordinata tekisligidagi biror A nuqtaning koordinatalari deyish mumkin; 3) agar $x \neq y$ bo‘lsa, u holda $(x\,;\,y)$ va $(y\,;\,x)$ juftliklar koordinata tekisligida turli nuqtalarni ifodalaydi.

Koordinata boshi – O nuqtaning koordinatalari $O(0;\ 0)$ dan iborat. Ox o‘qidagi ixtiyoriy B nuqtaning koordinatasi $B(x;\ 0)$, Oy o‘qidagi ixtiyoriy C nuqtaning koordinatasi $C(0;\ y)$ ko‘rinishida bo‘ladi.

Ox va *Oy* o‘qlar tekislikni to‘rtta to‘g‘ri burchakka bo‘ladi, ular *koordinata choraklari* yoki *koordinata burchaklari* deyiladi. Koordinata choraklari rim raqamlari bilan belgilanadi hamda ular soat millariga qarshi yo‘nalish bo‘yicha nomerlanadi. Nuqta koordinatalarining choraklardagi ishoralari belgilanishi 2- rasmda ko‘rsatib o‘tilgan.

Geometrik shakllar va ularning xossalarini koordinatalarda qo‘llab o‘rganishni ko‘rib chiqamiz.

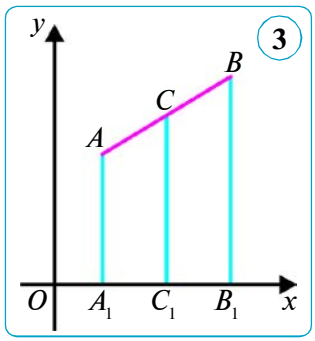
2. Kesma o‘rtasining koordinatalari.

Teorema.

Kesma o‘rtasining koordinatalari quyidagi formulalar bo‘yicha hisoblanadi:

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2},$$

bunda $A(x_1; y_1)$ va $B(x_2; y_2)$ – kesmaning uchlari, $C(x; y)$ – kesmaning o‘rtasi.



Isbot. C nuqtaning x va y kordinatalarini topamiz. AB kesma Ox o‘qini kesmagan bo‘lsin, ya’ni $x_1 < x_2$ holni ko‘rib chiqamiz (3- rasm). Ox o‘qiga AA_1, BB_1 va CC_1 perpendikular to‘g‘ri chiziqlarni o‘tkazamiz. $AA_1 \parallel BB_1 \parallel CC_1$ hamda perpendikularning asoslari $A_1(x_1; 0)$, $B_1(x_2; 0)$ va $C_1(x; 0)$ koordinatalarga ega ekani ravshan. C nuqta AB kesmaning o‘rtasi bo‘lgani uchun, Fales teoremasiga ko‘ra, C_1 nuqta A_1B_1 kesmaning o‘rtasi bo‘ladi va demak, $A_1C_1 = C_1B_1$, ya’ni $x_2 - x = x - x_1$. Bundan ushbu formulani topamiz:

$$x = \frac{x_1 + x_2}{2}.$$

$x_1 = x_2$, ya’ni AB kesma Oy o‘qiga parallel bo‘lsa, uchala nuqta – A_1, B_1 va C_1 bir xil abssissaga ega bo‘ladi. Demak, formula bu holda ham o‘rinli bo‘laveradi.

$x_1 > x_2$ bo‘lgan holda ham yuqoridagi natijaga kelamiz (buni mustaqil tekshirishni o‘zingizga havola qilamiz).

C nuqtaning ordinatasi ham shunga o‘xshash topiladi. A, B va C nuqtalar orqali Oy o‘qiga perpendikular to‘g‘ri chiziqlar o‘tkaziladi. Ushbu formula hosil bo‘ladi:

$$y = \frac{y_1 + y_2}{2}.$$

Masala. Uchlari $A(-2; 1)$, $B(0; 4)$, $C(4; 1)$ va $D(2; -2)$ nuqtalarda bo‘lgan $ABCD$ to‘rtburchakning parallelogramm ekanini isbotlang.

Yechish. Parallelogrammning alomatiga ko‘ra, to‘rtburchakning diagonallari kesishish nuqtasida teng ikkiga bo‘linsa, bu to‘rtburchak parallelogramm bo‘lishi ma’lum. Berilgan $ABCD$ to‘rtburchakning AC va BD diago-

nallari o‘rtasining koordinatalarini topamiz. AC kesmaning o‘rtasi quyidagi koordinataga ega:

$$x = \frac{-2+4}{2} = 1, \quad y = \frac{1+1}{2} = 1.$$

BD kesmaning o‘rtasi quyidagi koordinataga ega:

$$x = \frac{0+2}{2} = 1, \quad y = \frac{4+(-2)}{2} = 1.$$

Shunday qilib, AC va BD diagonallarning kesishish nuqtasi umumiy (1; 1) koordinataga ega ekan. Demak, parallelogramm alomatiga ko‘ra, $ABCD$ to‘rtburchak parallelogrammdir. Shuni isbotlash talab qilingan edi.

Savol, masala va topshiriqlar

1. 1) Koordinata o‘qlari va ularning kesishgan nuqtasi qanday nomlanadi?
2) Koordinatalar tekisligi deb nimaga aytiladi? Tekislikdagi nuqtaning koordinatalari deganda nimani tushunasiz?

2. $A(4; -5)$ nuqtadan koordinatalar o‘qlariga perpendikularlar o‘tkazilgan. Shu perpendikularlar asosining koordinatalarini yozing.

3. Agar: 1) $x = -4$, $y = -6$; 2) $x = -3$, $y = 5$; 3) $x > 0$, $y < 0$; 4) $x > 0$, $y > 0$ bo‘lsa, $A(x; y)$ nuqtaning qaysi chorakda yotishini aniqlang.

4. Agar: 1) $A(-12; -3)$, $B(-8; 1)$; 2) $A(4; -11)$, $B(-4; 0)$ bo‘lsa, AB kesma o‘rtasining koordinatalarini toping.

5. C nuqta – AB kesmaning o‘rtasi. Agar $A(2; -3)$, $C(0{,}5; 1)$ bo‘lsa, B nuqtaning koordinatalarini toping.

6. $A(-4; 0)$, $B(-2; -2)$, $C(0; -6)$ va $D(-2; -4)$ nuqtalar berilgan. $ABCD$ to‘rtburchakning parallelogramm ekanini isbotlang.

7. Agar: 1) $A(-6; 2)$, $B(4; 4)$; 2) $A(-8; -4)$, $B(-1; 3)$ bo‘lsa, AB kesma o‘rtasining koordinatalarini toping.

8. C nuqta – AB kesmaning o‘rtasi, D nuqta esa BC kesmaning o‘rtasi. Agar: 1) 1) $A(-3; 3)$, $B(5; -1)$; 2) $A(-2; -1)$, $C(2; 3)$ bo‘lsa, D nuqtaning koordinatalarini toping.

Bilib qo‘ygan foydali!

Yer sirtidagi nuqtaning geografik uzunligi va kengligi shu nuqtaning ***geografik koordinatalari*** deyiladi. Yer sirtidagi har bir nuqtaga ikkita miqdor – uning geografik uzunligi va kengligi mos qo‘yiladi va aksincha, ikkita miqdor – geografik uzunlik va kenglik bo‘yicha yer sirtidagi muayyan bir nuqta topiladi. Bunda parallel va meridianlar to‘g‘ri burchakli koordinatalar sistemasidagi abssissa va ordinata o‘qlari vazifasini bajaradi.

Masalan, Toshkent shahri 069,20 sharqiy uzunlikda ($\approx 69°$) va 041,26 shimoliy kenglikda ($\approx 41°$), Samarqand shahri esa 066,93 sharqiy uzunlikda ($\approx 67°$) va 039,65 shimoliy kenglikda ($\approx 40°$) joylashgan.



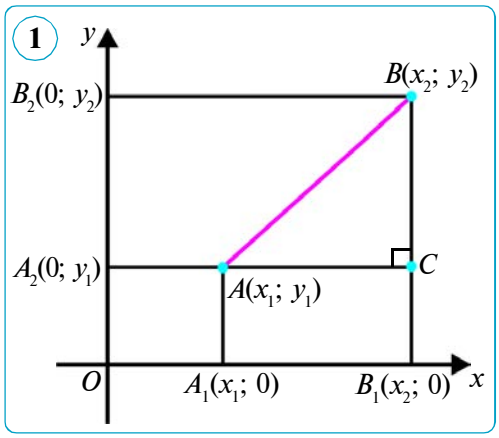
32–33. IKKI NUQTA ORASIDAGI MASOFA. AYLANA TENGLAMASI

1. Ikki nuqta orasidagi masofa.

Teorema.

$A(x_1; y_1)$ va $B(x_2; y_2)$ nuqtalar orasidagi masofa quyidagi formulalar bo‘yicha hisoblanadi:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\ .$$



Isbot. Dastlab $x_1 \neq x_2$ va $y_1 \neq y_2$ holni ko‘rib chiqamiz. Berilgan A va B nuqtalar orqali koordinatalar o‘qlariga perpendikular o‘tkazamiz va ularning kesishish nuqtasini C bilan belgilaymiz (1- rasm). A va C nuqtalar orasidagi masofa $|x_2 - x_1|$ ga, B va C nuqtalar orasidagi masofa esa $|y_2 - y_1|$ ga teng. To‘g‘ri burchakli ABC uchburchakka Pifagor teoremasini qo‘llab topamiz:

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \text{ yoki}$$

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\ . \quad (1)$$

Nuqtalar orasidagi masofa formulasi $x_1 \neq x_2$ va $y_1 \neq y_2$ hol uchun ko‘rib chiqilgan bo‘lsa-da, u boshqa hollar uchun ham o‘z kuchini saqlaydi. Haqiqatan ham, $x_1 = x_2$ va $y_1 \neq y_2$ bo‘lsa, $AB = |y_2 - y_1|$ (1) formula ham shu natijani beradi. $x_1 \neq x_2$ va $y_1 = y_2$ hol ham shunga o‘xshash qaraladi. $x_1 = x_2$ va $y_1 = y_2$ holda A va B nuqtalar ustma-ust tushadi va (1) formula $AB = 0$ nuqtani beradi.

1- masala. Uchlari $A(-2; 1)$, $B(0; 4)$, $C(4; 1)$ va $D(2; -2)$ nuqtalarda bo‘lgan $ABCD$ to‘rtburchakning parallelogramm ekanini isbotlang.

Yechish. Parallelogrammning 2- alomatiga ko‘ra, to‘rtburchakning qarama-qarshi tomonlari o‘zaro teng bo‘lsa, bu to‘rtburchak parallelogramm bo‘lishi ma’lum. Berilgan $ABCD$ to‘rtburchakning tomonlari uzunliklarini topamiz:

$$AB = \sqrt{(0 - (-2))^2 + (4 - 1)^2} = \sqrt{13}\ ; \quad BC = \sqrt{(4 - 0)^2 + (1 - 4)^2} = \sqrt{25} = 5\ ;$$

$$CD = \sqrt{(2 - 4)^2 + (-2 - 1)^2} = \sqrt{13}\ ; \quad AD = \sqrt{(2 - (-2))^2 + (-2 - 1)^2} = \sqrt{25} = 5\ .$$

Shunday qilib, $AB = CD$ va $BC = AD$, ya’ni parallelogramm alomatiga ko‘ra $ABCD$ to‘rtburchak – parallelogramm.

2. Tekislikda shaklning tenglamasi. Tekislikda *shaklning* Dekart koordinatalar sistemasidagi *tenglamasi* deb, shaklga tegishli har qanday nuqtaning koordinatalari qanoatlantiradigan ikkita x, y noma'lumli tenglamaga aytiladi. Aksincha, bu tenglamani qanoatlantiruvchi har qanday ikkita son shaklning biror nuqtasi koordinatalari bo'ladi.

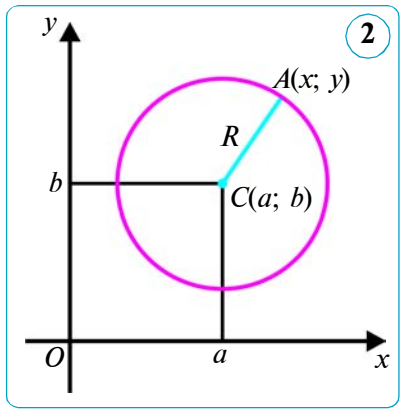
3. Aylana tenglamasi.

Teorema.

To'g'ri burchakli koordinatalar sistemasida markazi $C(a; b)$ nuqtada, radiusi esa R ga teng aylana *tenglamasi* quyidagi ko'rinishga ega:

$$(x-a)^2+(y-b)^2=R^2.$$

Isbot. To'g'ri burchakli koordinatalar sistemasida markazi $C(a; b)$ nuqtada bo'lgan R $(R>0)$ radiusli aylana berilgan bo'lsin (2- rasm). Aylanada ixtiyoriy $A(x; y)$ nuqtani olamiz. Aylana ta'rifiga ko'ra, aylana markazidan aylananing ixtiyoriy nuqtasigacha bo'lgan masofa R ga teng, ya'ni $CA=R$ va demak, $CA^2=R^2$. Bu tenglamani koordinatalar ko'rinishida yozib, quyidagini topamiz:



$$(x-a)^2+(y-b)^2=R^2. \qquad (2)$$

A – aylananing ixtiyoriy nuqtasi. Shuning uchun (2) tenglamani aylanadagi ixtiyoriy nuqtaning koordinatalari qanoatlantiradi.

Aksincha, koordinatalari (2) tenglamani qanoatlantiruvchi har qanday A nuqta aylanaga tegishlidir, chunki undan C nuqtagacha masofa R ga teng. Bundan (2) tenglama haqiqatan ham markazi C nuqtada va radiusi R dan iborat aylananing tenglamasi ekanligi kelib chiqadi. Shunday qilib, shaklning tenglamasi ta'rifidagi har ikkala talab bajariladi. Teorema isbotlandi.

Natija. Markazi koordinatalar boshida, radiusi R bo'lgan aylana tenglamasi ushbu ko'rinishga ega:

$$x^2+y^2=R^2.$$

2- masala. $x^2-4x+y^2+2y-11=0$ tenglama bilan berilgan aylana markazining koordinatalari va radiusini aniqlang.

Yechish. Berilgan tenglamani $(x-a)^2+(y-b)^2=R^2$ ko'rinishga keltiramiz. x^2-4x ni $(x-2)^2-4$ ko'rinishda, y^2+2y ni esa $(y+1)^2-1$ ko'rinishda yozib olamiz. Bu ifodalarni berilgan tenglamaga qo'yib, hosil qilamiz:

$$(x-2)^2-4+(y+1)^2-1-11=0 \quad \text{yoki} \quad (x-2)^2+(y+1)^2=4^2.$$

Bu tenglama markazi $C(2; -1)$ nuqtada va radiusi 4 bo'lgan aylana tenglamasini beradi.

Javob: $(2; -1)$, $R=4$.

Savol, masala va topshiriqlar

1. 1) Nuqtalar orasidagi masofa ularning koordinatalari orqali qanday ifodalanadi?

? 2) Shaklning Dekart koordinatalar sistemasidagi tenglamasi nima? Koordinatalar tekisligida aylana tenglamasi qanday ko‘rinishda beriladi?

2. Agar: 1) $A(-3; 8)$, $B(5; 2)$; 2) $A(8; -1)$, $B(-7; 7)$; 3) $A(5; 0)$, $B(0; -12)$ bo‘lsa, AB kesma uzunligini toping.

3. Agar: 1) $A(2; 1)$ va $B(x; -2)$ nuqtalar orasidagi masofa 5 ga; 2) $A(x; 0)$ va $B(2; -1)$ nuqtalar orasidagi masofa 1 ga teng bo‘lsa, x ni toping.

4. Agar $A(-1; 2)$, $B(2; 6)$ va $C(5; 2)$ bo‘lsa, ABC uchburchakning perimetrini toping.

5. Agar: 1) $C(7; 11)$, $R = 5$; 2) $C(-2; 3)$, $R = 1$ bo‘lsa, markazi C nuqtada, radiusi R bo‘lgan aylana tenglamasini tuzing.

6. Quyidagi tenglama bilan berilgan aylana markazining koordinatalari va radiusini aniqlang: 1) $(x-2)^2 + (y-5)^2 = 7^2$; 2) $(x+1)^2 + (y-5)^2 = 4$.

7. 1) $x^2 - 6x + y^2 + 2y - 6 = 0$; 2) $x^2 + y^2 + 10y + 24 = 0$ tenglama bilan berilgan aylana markazining koordinatalari va radiusini aniqlang.

8. Agar uchburchakning uchlari: 1) $A(0; 0)$, $B(0; 2)$ va $C(2; 0)$; 2) $(1; 0)$, $B(2; \sqrt{3})$ va $C(8; 0)$ bo‘lsa, ABC uchburchakning turini aniqlang.

9. Agar: 1) $C(9; 4)$, $R = 7$; 2) $C(-3; -4)$, $R = 2$ bo‘lsa, markazi C nuqtada, radiusi R bo‘lgan aylana tenglamasini tuzing.

10. Quyidagi tenglama bilan berilgan aylana markazining koordinatalari va radiusini aniqlang:

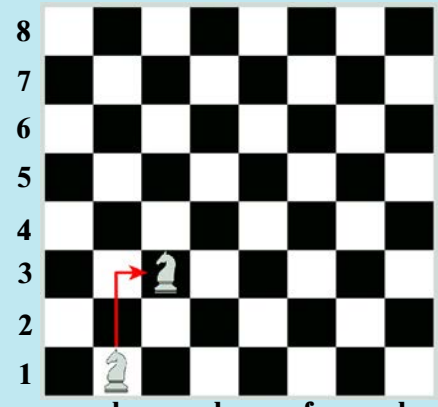
1) $(x-7)^2 + (y+2)^2 = 25$; 2) $(x-4)^2 + y^2 = 1$.

11. $x^2 + y^2 = 100$ tenglama bilan berilgan aylanada: 1) abssissasi 8 ga; 2) ordinatasi -6 ga teng nuqtalarni toping.

Bilib qo‘ygan foydali!

Shaxmat (forscha *shohmat* – shoh yengildi) sport turi bo‘lib, o‘yinning maqsadi raqib shohini mot qilishdan iborat. Oq va qora rangdagi 64 ta katakli taxtada har bir tomon ikki xil rangdagi 16 tadan dona (bittadan shoh va farzin; 2 tadan rux, fil va ot; 8 tadan piyoda) bilan o‘ynaydi.

Shaxmat partiyasining qaydnomasida Siz shaxmatchilarning o‘yin davomida donalar bilan qilgan barcha yurishlarini o‘qiy olishingiz mumkin bo‘ladi. Masalan, ot b1-c3 degan yozuv otning b1 katakdan c3 katakka qilgan harakatini bildiradi. Bularning barchasi shaxmat taxtasidagi koordinatalar sistemasidir.



b1-c3

34. TO‘G‘RI CHIZIQ TENGLAMASI. GEOMETRIK MASALALAR YECHISHNING KOORDINATALAR USULI

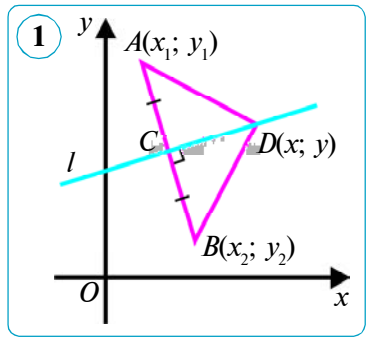
1. To‘g‘ri chiziq tenglamasi.

Teorema.

To‘g‘ri chiziqning to‘g‘ri burchakli koordinatalar sistemasidagi tenglamasi quyidagi ko‘rinishga ega:

$$ax + by + c = 0, \qquad (1)$$

bunda a, b, c – ixtiyoriy sonlar, a va b sonlardan biri nolga teng emas.



Isbot. l to‘g‘ri chiziq to‘g‘ri burchakli koordinatalar sistemasidagi ixtiyoriy to‘g‘ri chiziq bo‘lsin. l ga perpendikulyar biror to‘g‘ri chiziqni o‘tkazamiz va unga l to‘g‘ri chiziq bilan kesishgan nuqtasi C dan boshlab teng CA va CB kesmalarni qo‘yamiz (1- rasm). x_1, y_1 – A nuqtaning koordinatalari, x_2, y_2 – B nuqtaning koordinatalari bo‘lsin. O‘rta perpendikulyar l to‘g‘ri chiziqda yotgan ixtiyoriy $D(x; y)$ nuqta A va B nuqtalardan teng uzoqlashgan bo‘ladi, ya’ni $DA = DB$, bundan $DA^2 = DB^2$. Bu tenglikni koordinatalarda yozib, quyidagini hosil qilamiz:

$$(x - x_1)^2 + (y - y_1)^2 = (x - x_2)^2 + (y - y_2)^2 . \qquad (2)$$

Qavs ichidagi ifodalarni kvadratga oshirib va tenglamadagi o‘xshash hadlarni ixchamlagandan so‘ng, (2) tenglama quyidagi ko‘rinishga keladi:

$$2(x_2 - x_1)x + 2(y_2 - y_1)y + (x_1^2 + y_1^2 - x_2^2 + y_2^2) = 0. \qquad (3)$$

x_1, y_1, x_2, y_2 – ixtiyoriy sonlar, shu sababli $2(x_2 - x_1) = a$, $2(y_2 - y_1) = b$ va $x_1^2 + y_1^2 - x_2^2 + y_2^2 = c$ deb belgilab, ularni (3) tenglamaga qo‘yib:

$$ax + by + c = 0$$

tenglamani hosil qilamiz, bunda a, b va c – biror sonlar.

D – l to‘g‘ri chiziqdagi ixtiyoriy nuqta, shuning uchun (1) tenglamani berilgan to‘g‘ri chiziqdagi ixtiyoriy nuqtaning koordinatasi qanoatlantiradi.

Biror D_0 nuqtaning x_0 va y_0 koordinatalari (1) tenglamani qanoatlantirsin. U holda $D_0A = D_0B$, ya’ni D_0 nuqta A va B nuqtalardan barobar uzoqlashgan bo‘ladi, demak, AB kesmaning o‘rta perpendikulyari l to‘g‘ri chiziqqa tegishli bo‘ladi. A va B – turli ikkita nuqta bo‘lgani uchun $(x_2 - x_1)$ yoki $(y_2 - y_1)$ ayirmalardan biri, ya’ni a va b sonlardan biri nolga teng emasligini aytib o‘tamiz.

1- masala. $A(1; -1)$ va $B(-3; 2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

Yechish. AB to‘g‘ri chiziqning tenglamasi $ax + by + c = 0$ ko‘rinishda ifodalanishini bilamiz. A va B nuqtalar AB to‘g‘ri chiziqda yotadi, demak, ular-

ning koordinatalarini to‘g‘ri chiziq tenglamasiga qo‘yib, ushbu tenglamalarni hosil qilamiz:

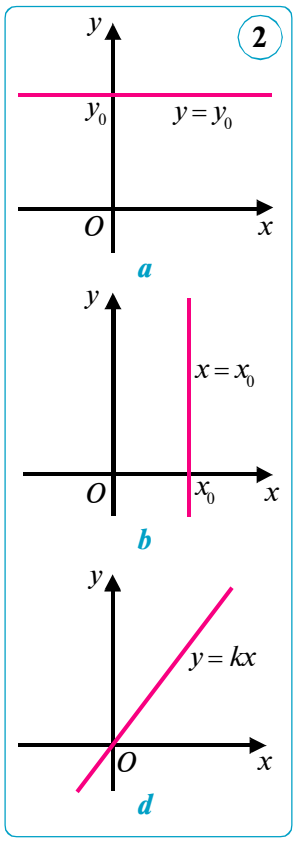
$$a \cdot 1 + b \cdot (-1) + c = 0, \quad a \cdot (-3) + b \cdot 2 + c = 0 \text{ yoki}$$

$$a - b + c = 0, \quad -3a + 2b + c = 0.$$

Bu tenglamalardan a va b koeffitsiyentlarni c orqali ifodalaymiz: $a = 3c$, $b = 4c$. a va b ning bu qiymatlarini to‘g‘ri chiziq tenglamasiga qo‘yib, topamiz: $3cx + 4cy + c = 0$, bunda $c \neq 0$.

Bu tenglama AB to‘g‘ri chiziqning tenglamasi bo‘ladi. Yuqoridagi tenglamani c ga qisqartib, quyidagi ko‘rinishga keltiramiz: $3x + 4y + 1 = 0$.

Bu tenglama izlanayotgan to‘g‘ri chiziq tenglamasidir.



2. To‘g‘ri chiziqning koordinatalar sistemasiga nisbatan joylashishi.

Endi $ax + by + c = 0$ to‘g‘ri chiziq tenglamasining uchta xususiy holini ko‘rib chiqamiz. Har bir hol uchun to‘g‘ri chiziqning koordinatalar o‘qlariga nisbatan qanday joylashganini aniqlaymiz.

1- hol. $a = 0$, $b \neq 0$. Bu holda to‘g‘ri chiziq tenglamasini $by + c = 0$ yoki $y = y_0$ ko‘rinishda yozish mumkin, bunda $y_0 = -\frac{c}{b}$ — biror son. $y = y_0$ to‘g‘ri chiziqning hamma nuqtalari bir xil ordinataga ega, demak, u abssissalar o‘qiga parallel (2- a rasm). Agar $c = 0$ bo‘lsa, u bilan ustma-ust tushadi. $y = 0$ — abssissalar o‘qining tenglamasi.

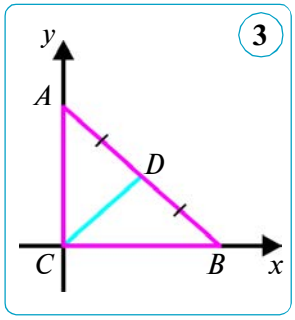
2- hol. $a \neq 0$, $b = 0$. Bu holda to‘g‘ri chiziq tenglamasini $ax + c = 0$ yoki $x = x_0$ ko‘rinishda yozish mumkin, bunda $x_0 = -\frac{c}{a}$ — biror son. $x = x_0$ to‘g‘ri chiziqning hamma nuqtalari bir xil abssissaga ega, demak, u ordinatalar o‘qiga parallel (2- b rasm). Agar $c = 0$ bo‘lsa, u bilan ustma-ust tushadi. $x = 0$ — ordinatalar o‘qining tenglamasi.

3- hol. $a \neq 0$, $b \neq 0$, $c = 0$. Bu holda to‘g‘ri chiziq tenglamasini $ax + by = 0$ yoki $y = kx$ ko‘rinishda yozish mumkin, bunda $k = -\frac{a}{b}$ — biror son.

$y = kx$ to‘g‘ri chiziq koordinatalar boshidan o‘tadi (2- d rasm).

3. Geometrik masalalarni yechishning koordinatalar usuli. Ko‘pgina geometrik masalalarni kesma o‘rtasining koordinatalari va ikki nuqta orasidagi masofani hisoblash formulalaridan foydalanib yechish mumkin. Shu maqsadda to‘g‘ri burchakli koordinatalar sistemasini kiritish va masalaning shartini koordinatalarda yozib olish kerak. Shundan so‘ng masala algebraik hisoblashlar yordamida yechiladi.

2- masala. To‘g‘ri burchakli uchburchakda gipotenuzaning o‘rtasi hamma uchlaridan teng uzoqlashgan. Shuni isbotlang.



3

Yechish. To‘g‘ri burchakli ABC ($\angle C = 90°$) uchburchakni ko‘rib chiqamiz. AB kesmaning o‘rtasini D harfi bilan belgilaymiz. 3- rasmda ko‘rsatilgandek, to‘g‘ri burchakli koordinatalar sistemasini kiritamiz. Agar $BC = a$, $AC = b$ bo‘lsa, u holda uchburchakning uchlari $C(0; 0)$, $B(a; 0)$ va $A(0; b)$ koordinatalarga ega bo‘ladi. Kesma o‘rtasining koordinatalari formulasiga ko‘ra D nuqta koordinatalarini topamiz: $D(0{,}5a; 0{,}5b)$.

Nuqtalar orasidagi masofani topish formulasidan foydalanib, DC va DA kesmalarning uzunliklarini topamiz:

$$DC = \sqrt{(0{,}5a)^2 + (0{,}5b)^2} = \sqrt{0{,}25\left(a^2 + b^2\right)} = 0{,}5\sqrt{a^2 + b^2};$$

$$DA = \sqrt{(0{,}5a)^2 + (0{,}5b - b)^2} = \sqrt{0{,}25a^2 + 0{,}25b^2} = \sqrt{0{,}25\left(a^2 + b^2\right)} = 0{,}5\sqrt{a^2 + b^2}.$$

Shunday qilib, $DA = DB = DC$ ekan. Shuni isbotlash talab qilingan edi.

Savol, masala va topshiriqlar

1. 1) To‘g‘ri chiziqning to‘g‘ri burchakli koordinatalar sistemasida $ax + by + c = 0$ ko‘rinishdagi tenglamaga ega bo‘lishini isbotlang.
2) To‘g‘ri chiziqning $ax + by + c = 0$ tenglamasida $a = 0$ ($b = 0$; $c = 0$) bo‘lsa, to‘g‘ri chiziq qanday joylashadi?

2. $A(3; -1)$, $B(-3; 0)$, $C(12; 5)$, $D(3; 0)$ va $E(-9; -2)$ nuqtalarning qaysilari $x - 3y + 3 = 0$ tenglama bilan berilgan to‘g‘ri chiziqqa tegishli, qaysilari tegishli emas?

3. 1) $A(1; 7)$ va $B(-3; -1)$; 2) $A(2; 5)$ va $B(5; 2)$; 3) $A(0; 1)$ va $B(-4; -5)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

4. $x + y + c = 0$ to‘g‘ri chiziq $(1; 2)$ nuqtadan o‘tsa, uning tenglamasidagi c koeffitsiyent nimaga teng?

5. Agar $ax + by - 1 = 0$ to‘g‘ri chiziqning $(1; 2)$ va $(2; 1)$ nuqtalardan o‘tishi ma’lum bo‘lsa, uning tenglamasidagi a va b koeffitsiyentlar nimaga teng?

6. 1) $x + 2y + 3 = 0$; 2) $3x + 4y = 12$; 3) $4x - 2y - 10 = 0$ tenglama bilan berilgan to‘g‘ri chiziqning koordinatalar o‘qlari bilan kesishish nuqtalarini toping.

7. Agar: 1) $A(3; -1)$, $B(5; 5)$; 2) $A(3; 6)$, $B(-5; -2)$ bo‘lsa, $C(4; 2)$ nuqta AB kesmaning o‘rtasi bo‘lish-bo‘lmasligini tekshiring.

8. $A(0; -2)$, $B(4; 2)$, $C(-4; -5)$ nuqtalarning qaysilari $8x - 4y - 8 = 0$ tenglama bilan berilgan to‘g‘ri chiziqqa tegishli, qaysilari tegishli emas?

9. Agar $A(-1; -1)$, $B(-1; 3)$ va $C(2; 2)$ bo‘lsa, ABC uchburchak tomonlarini o‘z ichiga olgan to‘g‘ri chiziqlar tenglamasini tuzing.

8-§. TEKISLIKDA VEKTORLAR

35. VEKTOR TUSHUNCHASI. VEKTORNING UZUNLIGI VA YO‘NALISHI

1. Vektor kattaliklar. Vektor. Sizga ma’lum bo‘lgan kattaliklar ikki ko‘rinishda bo‘lishi mumkin. Shunday kattaliklar borki, ular o‘zlarining son qiymatlari bilan (berilgan o‘lchov birligida) to‘la aniqlanadi. Masalan, uzunlik, yuza va og‘irlik shular jumlasidandir.

1- ta’rif. *Faqat son qiymati bilan aniqlanadigan kattaliklar* ***skalar kattaliklar*** *deyiladi.*

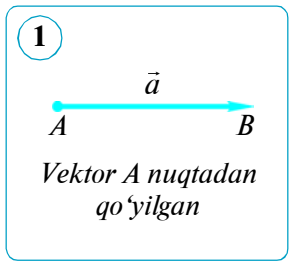
Yana shunday kattaliklar borki, ularni to‘la bilish uchun bu kattaliklarni ifodalovchi son qiymatlaridan tashqari, ularning yo‘nalishlarini ham bilish zarur bo‘ladi. Masalan, tezlik, kuch va bosim shular jumlasidandir.

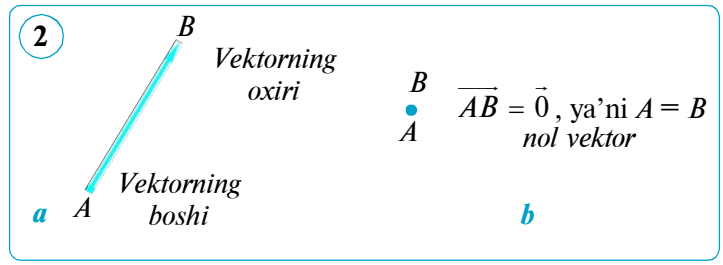
Vektor geometriyaning asosiy tushunchalaridan biri bo‘lib, u son (uzunlik) va yo‘nalishi bilan to‘la aniqlanadi. Ko‘rgazmali bo‘lishi uchun uni yo‘naltirilgan kesma ko‘rinishida tasavvur qilish mumkin. Aslida vektorlar haqida gapirilganda, hammasi o‘zaro parallel bir xil uzunlik va bir xil yo‘nalishga ega bo‘lgan yo‘naltirilgan kesmalarning butun bir sinfini nazarda tutish to‘g‘riroq bo‘ladi.

2- ta’rif. *Son qiymati va yo‘nalishi bilan aniqlanadigan (tavsiflanadigan) kattaliklar* ***vektor kattaliklar*** *yoki* ***vektorlar*** *deb ataladi.*

Fizika, mexanika va matematikaning son bilangina emas, balki yo‘nalishi bilan tavsiflanadigan miqdorlarni tekshiruvchi turli masalalari vektor tushunchasiga olib keladi. Masalan, kuch, tezlik – bular vektorlardir.

Vektor kattaliklarni biz juda ko‘p hollarda uchratamiz. Masalan, transportda ketayotganingizda harakat tezligi, burilish yoki to‘xtash bilan bog‘liq vektor kattaliklarni ko‘rishingiz mumkin. Tabiatni o‘rganuvchi fanlarda ular tezlanish, inersiya kuchi, markazdan qochma kuch va shunga o‘xshash nomlar bilan ataladi. Biz vektor kattaliklarni tabiiy ma’nosini hisobga olmagan holda uning matematik tabiatini o‘rganamiz. Albatta, vektor kattalikning matematik xossalari o‘zining tabiiy ma’nosiga ega bo‘ladi.





Vektor kattalikning son miqdorini kesma orqali ifodalaymiz. Ma'lumki, har qanday kesmaning ikki uchi bor. Ulardan birini vektorning ***boshi*** deb, ikkinchi uchini vektor kattalik yo'nalishiga mos yo'naltiramiz va strelka (yo'nalish) bilan belgilaymiz. Buni vektorning ***uchi*** deymiz.

3- ta'rif. ***Vektor*** *(vektor kattalik) deb, yo'nalishga ega bo'lgan kesmaga aytiladi.*

Vektor kattalik yo'nalishi ko'rsatilgan kesma sifatida tasvirlanadi. Vektorni ifodalovchi kesma uchlari A va B nuqtada bo'lsa, A nuqtadan B nuqtaga yo'nalgan vektor $\overrightarrow{AB}$ kabi belgilanadi. Shuningdek, vektorlar $\vec{a}$, $\vec{b}$ (lotin alifbosining kichik harflari) shaklida ham belgilanishi mumkin (1- rasm).

O'qilishi: $\overrightarrow{AB}$ vektor yoki $\vec{a}$ vektor.

1) Vektorning yo'nalishi uning boshi va oxirini ko'rsatish bilan aniqlanadi. Bunda vektor boshi birinchi o'ringa qo'yiladi (2- *a* rasm).

AB nurni aniqlab bergan yo'nalishi $\overrightarrow{AB}$ *vektorning yo'nalishi* deyiladi. Boshi va oxiri ustma-ust tushgan vektor *nol vektor* deb ataladi. $\overrightarrow{AB} = \vec{0}$ tenglik A va B nuqtalarning ustma-ust tushganini bildiradi (2- *b* rasm).

2) Vektorni ifodalovchi kesmaning uzunligi vektorning *moduli* yoki *absolut qiymati* deb ataladi.

Vektorning moduli $|\overrightarrow{AB}|$ yoki $|\vec{a}|$ kabi belgilanadi (3- rasm).

$\vec{a} = \overrightarrow{AB}$ vektorning moduli AB kesmaning uzunligi hisoblanadi: $|\vec{a}| = |\overrightarrow{AB}|$. Shuning uchun geometriyada vektorning moduli yoki absolut qiymati uning *uzunligi* deb ham ataladi.

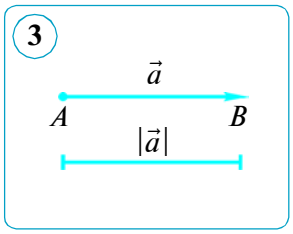
Nol vektorning uzunligi (moduli) nolga teng deb hisoblanadi: $|\vec{0}| = 0$.

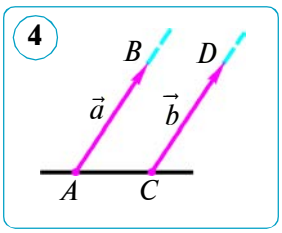
2. Vektorlarning tengligi.

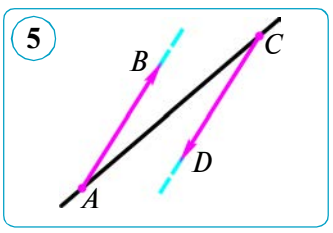
4- ta'rif. *Bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotuvchi vektorlar* ***kollinear vektorlar*** *deyiladi.*

$\vec{a}$ va $\vec{b}$ vektorlarning kollinearligi $\vec{a} \parallel \vec{b}$ kabi belgilanadi.

Agar parallel to'g'ri chiziqlarda yotuvchi ikkita vektor ularning boshi orqali o'tgan to'g'ri chiziqdan bir tomonda yotsa, *yo'nalishdosh vektorlar* (4- rasm); to'g'ri chiziqqa nisbatan turli tomonda yotsa, *qarama-qarshi yo'nalgan vektorlar* deyiladi (5- rasm).







$\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlar: 1) *yo'nalishdosh* bo'lsa, ular $\overrightarrow{AB} \uparrow\uparrow \overrightarrow{CD}$ kabi; 2) *qarama-qarshi yo'nalgan* bo'lsa, $\overrightarrow{AB} \uparrow\downarrow \overrightarrow{CD}$ kabi belgilanadi.

Nol vektor istalgan vektorga kollinear deb hisoblanadi.

5- ta'rif. *Agar* $\vec{a}$ *va* $\vec{b}$ *vektorlarning uzunliklari teng va yo'nalishlari bir xil bo'lsa, bu vektorlar* ***teng vektorlar*** *deb ataladi.*

Shunday qilib, agar $|\vec{a}| = |\vec{b}|$ va $\vec{a} \uparrow\uparrow \vec{b}$ bo'lsa, $\vec{a}$ va $\vec{b}$ vektorlar teng bo'ladi. Vektorlarning tengligi $\vec{a} = \vec{b}$ shaklida yoziladi.

Vektorlarning tengligi uning boshi tekislikning ixtiyoriy nuqtasida bo'la olishini ko'rsatadi (6-rasm), ya'ni vektorning modulini o'zgartirmay, yo'nalishini saqlagan holda uning boshini tekislikning istalgan nuqtasiga ko'chirish mumkin. Bu *vektorni parallel ko'chirish xossasi* deb ataladi.

Savol, masala va topshiriqlar

1. 1) Vektor nima? Vektorlar qanday belgilanadi?
2) Qanday ikki vektor teng vektorlar deb ataladi? Qanday vektorlar bir xil (qarama-qarshi) yo'nalgan vektorlar deyiladi? Vektorning moduli nima?

2. $ABCD$ parallelogrammda (7- rasm): 1) $\overrightarrow{DC}$ vektor bilan yo'nalishdosh; 2) $\overrightarrow{AO}$ vektor bilan yo'nalishdosh; 3) $\overrightarrow{AD}$ vektorga qarama-qarshi yo'nalgan; 4) $\overrightarrow{BD}$ vektorga qarama-qarshi yo'nalgan; 5) $\overrightarrow{AB}$ vektorga teng; 6) $\overrightarrow{OC}$ vektorga teng; 7) $\overrightarrow{OB}$ vektorga teng vektorlarni yozing.

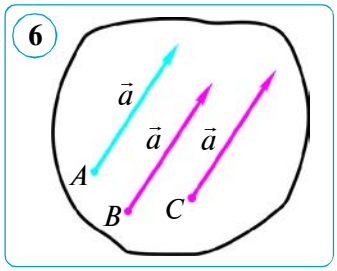
3. $ABCD$ parallelogrammning diagonallari O nuqtada kesishadi. Uning uchlari va diagonallari kesishish nuqtasi bilan belgilangan vektorlarni yozing. Ular ichidan qaysilari: $\overrightarrow{AB}$, $\overrightarrow{BC}$ va $\overrightarrow{BO}$ vektorlarga kollinear?

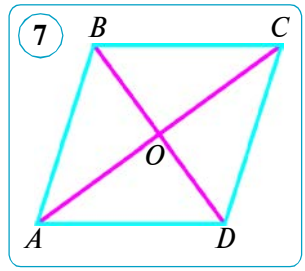
4. Agar: 1) $\overrightarrow{AD} = \overrightarrow{BC}$ va $|\overrightarrow{AD}| = |\overrightarrow{DC}|$; 2) $\overrightarrow{AD} \uparrow\uparrow \overrightarrow{BC}$, $\overrightarrow{AB}$ va $\overrightarrow{DC}$ vektorlar nokollinear bo'lsa, $ABCD$ to'rtburchakning turini aniqlang.

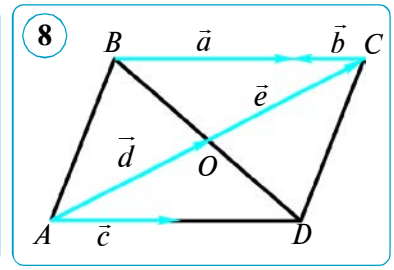
5. $\overrightarrow{AB} = \overrightarrow{CD}$ ekanligi ma'lum. Ushbu tasdiqlar to'g'rimi:
1) $AB \parallel CD$; 2) $|AB| = |CD|$?

6. $ABCD$ – parallelogramm. 8- rasmda tasvirlangan vektorlar ichidan: 1) kollinear; 2) yo'nalishdosh; 3) qarama-qarshi yo'nalgan; 4) teng uzunliklarga ega bo'lgan vektorlar juftlarini ko'rsating.

7. $\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlarning yo'nalishi haqida nima deyish mumkin?







36–37. VEKTORLARNI QO‘SHISH VA AYIRISH

1. Vektorlarni qo‘shish. Bizga $\vec{a}$ va $\vec{b}$ vektorlar berilgan bo‘lsin (1- *a* rasm). Ixtiyoriy A nuqtani belgilaymiz va bu nuqtadan $\vec{a}$ vektorga teng $\overrightarrow{AB}$ vektorni qo‘yamiz. So‘ngra B nuqtadan $\vec{b}$ vektorga teng $\overrightarrow{BC}$ vektorni qo‘yamiz. Endi $\vec{a}$ vektorning boshi A nuqtadan $\vec{b}$ vektor uchi C ga yo‘nalgan vektor o‘tkazamiz (1- *b* rasm). $\overrightarrow{AC}$ *vektor* $\vec{a}$ va $\vec{b}$ *vektorlarning yig‘indisi* deyiladi. Vektorlarni qo‘shishning bu qoidasi «*uchburchak* (*uch nuqta*) *qoidasi*» deyiladi.

$\vec{a}$ va $\vec{b}$ vektorlarning yig‘indisi $\vec{a}+\vec{b}$ kabi belgilanadi.

Uchburchak qoidasini quyidagicha ifodalasak ham bo‘ladi:

agar A, B va C ixtiyoriy nuqtalar bo‘lsa, u holda quyidagi tenglik o‘rinli:

$$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$$

Uchburchak qoidasi istalgan A, B va C nuqtalar uchun, shu bilan bir qatorda ulardan ikkitasi yoki uchtasi ustma-ust tushganda ham o‘rinli bo‘lishi mumkin (1- *d* rasm).

2. Vektorlarni qo‘shish qonunlari. Ma’lumki, parallelogrammning qarama-qarshi tomonlari o‘zaro teng va parallel. Agar yo‘nalishlari bir xil bo‘lsa, parallelogrammning qarama-qarshi tomonlari teng vektorlarni ifodalaydi.

$\vec{a}$ va $\vec{b}$ – nokollinear vektorlar bo‘lsin. Ixtiyoriy A nuqtadan $\overrightarrow{AB}=\vec{a}$ va $\overrightarrow{AD}=\vec{b}$ vektorlarni qo‘yamiz hamda tomonlari shu vektordan tuzilgan $ABCD$ parallelogrammni yasaymiz (2- rasm). Uchburchak qoidasiga ko‘ra:

$$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=\vec{a}+\vec{b} \quad \text{va} \quad \overrightarrow{AC}=\overrightarrow{AD}+\overrightarrow{DC}=\vec{b}+\vec{a}\,.$$

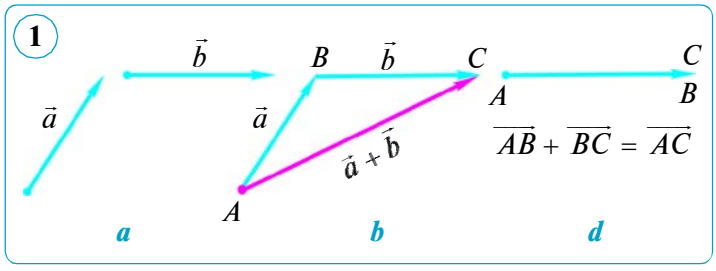
Bulardan $\vec{a}+\vec{b}=\vec{b}+\vec{a}$ kelib chiqadi.

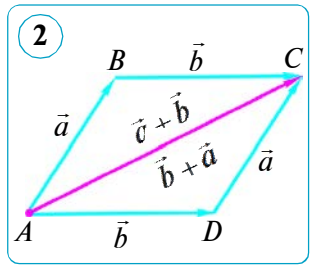
Demak, vektorlar yig‘indisi ularning qanday tartibda ketma-ket joylashishiga bog‘liq emas, ya’ni *istalgan $\vec{a}$ va $\vec{b}$ vektorlar uchun quyidagi tenglik o‘rinli:*

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}\,.$$

Bunga vektorlarni qo‘shishning *o‘rin almashtirish qonuni* deyiladi.

$\vec{a}$ va $\vec{b}$ vektorlardan tuzilgan $ABCD$ parallelogrammda yig‘indi $\overrightarrow{AC}$ vektor qo‘shiluvchi vektorlarning umumiy boshidan chiquvchi diagonaldan iborat.





Odatda, vektorlarni bunday qo‘shish vektorlarni qo‘shishning «*parallelogramm qoidasi* (*usuli*)» deyiladi (2- rasm).

Endi uchta $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar yig‘indisini ko‘raylik. Ixtiyoriy A nuqtadan $\overrightarrow{AB} = \vec{a}$ vektorni, B nuqtadan $\overrightarrow{BC} = \vec{b}$ vektorni, C nuqtadan esa $\overrightarrow{CD} = \vec{c}$ vektorni qo‘yamiz (3- rasm). Uchburchak qoidasini qo‘llab, quyidagiga ega bo‘lamiz:

$$(\vec{a} + \vec{b}) + \vec{c} = (\overrightarrow{AB} + \overrightarrow{BC}) + \overrightarrow{CD} = \overrightarrow{AC} + \overrightarrow{CD} = \overrightarrow{AD};$$

$$\vec{a} + (\vec{b} + \vec{c}) = \overrightarrow{AB} + (\overrightarrow{BC} + \overrightarrow{CD}) = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}.$$

Bundan, *istalgan* $\vec{a}$, $\vec{b}$ va $\vec{c}$ *vektorlar uchun*

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

tenglik o‘rinli ekani kelib chiqadi. Bu vektorlarni qo‘shishning *guruhlash qonuni* (*xossasi*)*dir.*

Vektorlarning har biri noldan farqli bo‘lganda ularning yig‘indisi nol vektor bo‘lishi mumkin.

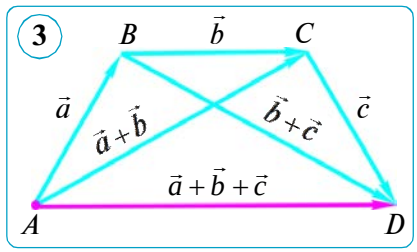
Masalan, ABC uchburchakni qaraylik (4- rasm). Bunda $\overrightarrow{AB}$, $\overrightarrow{BC}$ va $\overrightarrow{CA}$ vektorlar yig‘indisi nol vektor bo‘ladi, ya’ni: $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$, chunki birinchi vektorning boshi bilan uchinchi vektorning uchi ustma-ust tushdi. Demak, yig‘indi vektor nol vektor – nuqta bo‘ldi.

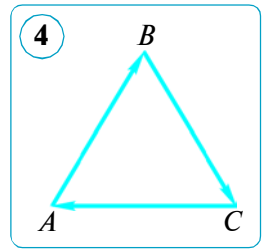
1- ta’rif. *Ikki vektorning yig‘indisi nol vektor bo‘lsa, ular* ***qarama-qarshi vektorlar*** *deb ataladi.*

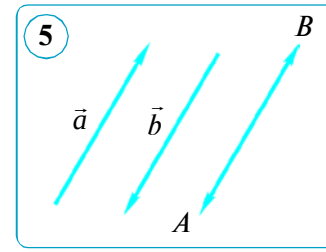
Demak, agar $\vec{a} + \vec{b} = \vec{0}$ bo‘lsa, u holda $\vec{b} = \overrightarrow{BA}$ vektor $\vec{a} = \overrightarrow{AB}$ vektorga (va aksincha) *qarama-qarshi vektor* deyiladi va $\vec{b} = -\vec{a}$, $\vec{a} = -\vec{b}$ kabi yoziladi (5-rasm). Agar qarama-qarshi vektorlarni (uchburchak qoidasi bo‘yicha) qo‘shsak, u holda nol vektor kelib chiqadi. Bunda $|\vec{a}| = |\vec{b}|$, $\vec{a}$ va $\vec{b}$ vektorlar parallel bo‘lib, turli tomonga yo‘nalgan bo‘ladi. Demak, *har bir* $\vec{a}$ *vektor uchun unga qarama-qarshi* $-\vec{a}$ *vektor mavjud* (*ya’ni* $\vec{a} + (-\vec{a}) = \vec{0}$) *bo‘ladi.* Yuqoridagi mulohazalardan quyidagi xulosaga kelamiz.

Agar nol bo‘lmagan ikki vektorning uzunliklari teng va ular qarama-qarshi yo‘nalgan bo‘lsa, ular ***qarama-qarshi vektorlar*** *deyiladi.*

Nol vektor o‘ziga-o‘zi qarama-qarshi vektor hisoblanadi.







3. Vektorlarni ayirish. Vektorlarni ayirish xuddi sonlarni ayirish kabi qo‘shishga teskari amaldir.

2-ta’rif. *$\vec{a}$ va $\vec{b}$ **vektorlarning ayirmasi** deb, shunday $\vec{c}$ vektorga aytiladiki, uning $\vec{b}$ vektor bilan yig‘indisi $\vec{a}$ vektorni beradi: $\vec{c}+\vec{b}=\vec{a}$.*

$\vec{a}$ va $\vec{b}$ vektorlarning ayirmasi xuddi sonlarning ayirmasi kabi belgilanadi: $\vec{a}-\vec{b}$. Ikki vektorning ayirmasi birinchi vektorga ikkinchi vektorga qarama-qarshi vektorni qo‘shish sifatida aniqlanadi va u $\vec{a}+(-\vec{b})$ vektorga teng (6- *b* rasm).

Bizga $\vec{a}$ va $\vec{b}$ vektorlar berilgan bo‘lsin (6- *a* rasm). $\vec{a}$ vektor bilan $\vec{b}$ vektorga qarama-qarshi bo‘lgan $-\vec{b}$ vektorning yig‘indisini ko‘raylik.

Istalgan $\vec{a}$ va $\vec{b}$ vektorlar uchun $\vec{a}-\vec{b}=\vec{a}+(-\vec{b})$ tenglik o‘rinli.

Haqiqatan ham, $(\vec{a}+(-\vec{b}))+\vec{b}=\vec{a}+((-\vec{b})+\vec{b})=\vec{a}+\vec{0}=\vec{a}$.

Agar $\vec{a}$ va $\vec{b}$ vektorlar bitta O nuqtadan qo‘yilgan bo‘lsa, u holda $\vec{a}-\vec{b}$ ayirmani topish uchun quyidagi qoidadan foydalanish qulay (6- *d* rasm):

$$\overrightarrow{OA}-\overrightarrow{OB}=\overrightarrow{BA}.$$

Yuqoridan ko‘rinadiki, *ayriluvchi* vektorning *oxiri ayirma* vektorning *boshi*, *kamayuvchi* vektorning *oxiri* esa *ayirma* vektorning *oxiri* vazifasini o‘tar ekan. Qoidani esda saqlash qulay bo‘lishini ta’minlash maqsadida u sxematik tarzda ko‘rsatildi.

Vektorni qo‘shishda parallelogramm usulidan foydalansak (7- rasm), ayirma vektor parallelogrammning ikkinchi diagonalidan iborat bo‘ladi.

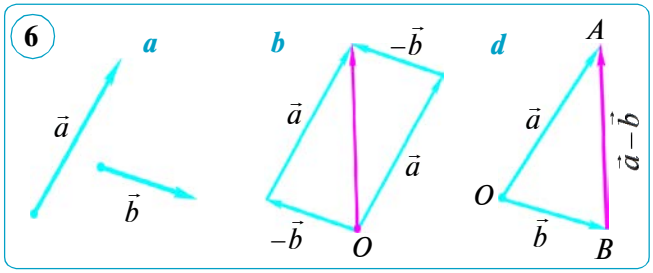
Masala. *ABC* uchburchak berilgan. 1) $\overrightarrow{BA}$; 2) $\overrightarrow{CB}$; 3) $\overrightarrow{CB}+\overrightarrow{BA}$ vektorlarni $\vec{a}=\overrightarrow{AB}$ va $\vec{b}=\overrightarrow{AC}$ vektorlar orqali ifodalang.

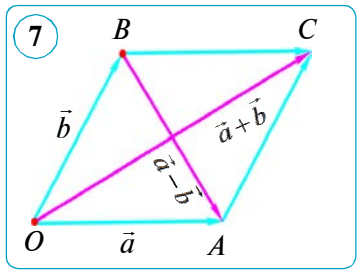
Yechish. 1) $\overrightarrow{BA}$ va $\overrightarrow{AB}$ – qarama-qarshi vektorlar, shuning uchun

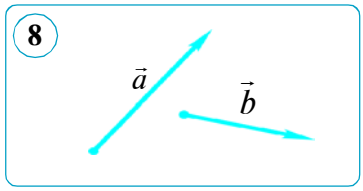
$$\overrightarrow{BA}=-\overrightarrow{AB} \quad \text{yoki} \quad \overrightarrow{BA}=-\vec{a}.$$

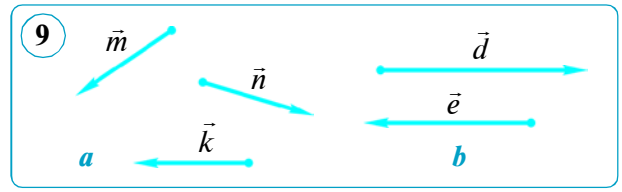
2) Uchburchak qoidasiga ko‘ra: $\overrightarrow{CB}=\overrightarrow{CA}+\overrightarrow{AB}$. Lekin $\overrightarrow{CA}=-\overrightarrow{AC}$, shuning uchun

$$\overrightarrow{CB}=\overrightarrow{AB}+(-\overrightarrow{AC})=\overrightarrow{AB}-\overrightarrow{AC}=\vec{a}-\vec{b}.$$









Savol, masala va topshiriqlar

1. 1) Uchburchak va parallelogramm qoidasiga ko‘ra vektorlar yig‘indisi qanday topiladi? Ikki vektor ayirmasi deb nimaga aytiladi?

2) Berilgan vektorga qarama-qarshi vektor deb nimaga aytiladi?

2. 8- rasmda $\vec{a}$ va $\vec{b}$ vektorlar tasvirlangan. $\vec{a}+\vec{b}$ vektorni ikki usul bilan yasang.

3. 9- rasmda $\vec{m}$, $\vec{n}$ va $\vec{k}$ hamda $\vec{d}$ va $\vec{e}$ vektorlar tasvirlangan. Vektorlarni yasang: 1) $\vec{m}+\vec{n}+\vec{k}$; 2) $\vec{d}+\vec{e}$.

4. 10- rasmda $\vec{a}$, $\vec{b}$ va $\vec{c}$ hamda $\vec{d}$ va $\vec{e}$ vektorlar tasvirlangan. Vektorlarni yasang: 1) $\vec{a}-\vec{b}+\vec{c}$; 2) $\vec{e}-\vec{d}$.

5. $ABCD$ parallelogramm berilgan. $(\overrightarrow{AB}-\overrightarrow{AD})+\overrightarrow{BC}=\overrightarrow{AB}$ tenglik bajarila-dimi? Tekshirib ko‘ring.

6. $ABCD$ rombda: $AD=20$ cm, $BD=24$ cm, O – diagonallarning kesishish nuqtasi. $|\overrightarrow{AD}+\overrightarrow{AB}-\overrightarrow{BC}-\overrightarrow{OB}|$ ni toping.

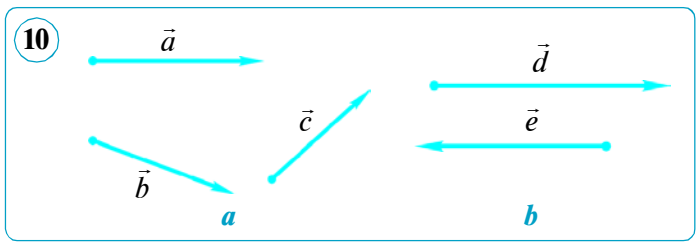
7. $ABCD$ – ixtiyoriy to‘rtburchak. $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AD}+\overrightarrow{DC}$ ekanini isbotlang.

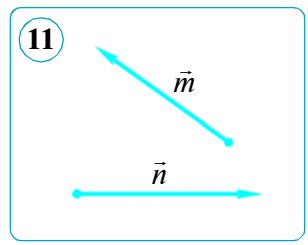
8. $ABCD$ – parallelogramm. $\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}$ vektor tenglikni isbotlang (vektorlarni qo‘shishning «*parallelogramm qoidasi*»).

9. $ABCD$ parallelogrammda: $\overrightarrow{CA}=\vec{a}$, $\overrightarrow{CD}=\vec{b}$. $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{DA}$ vektorlarni $\vec{a}$ va $\vec{b}$ vektorlar orqali ifodalang.

10. E va F – ABC uchburchak AB va AC tomonlarining o‘rtalari. $\overrightarrow{BF}$, $\overrightarrow{EC}$, $\overrightarrow{EF}$ va $\overrightarrow{BC}$ vektorlarni $\vec{a}=\overrightarrow{AE}$ va $\vec{b}=\overrightarrow{AF}$ vektorlar orqali ifodalang.

11. 11- rasmda $\vec{m}$ va $\vec{n}$ vektorlar tasvirlangan. $\vec{m}+\vec{n}$ vektorni ikki usul bilan yasang.





38–39. VEKTORNI SONGA KO‘PAYTIRISH. VEKTORNING KOORDINATALARI

1. Vektorni songa ko‘paytirish. Biror $\vec{a}$ vektorni olamiz va $\vec{a}+\vec{a}+\vec{a}$ yig‘indini topamiz (1- rasm). Bunday yig‘indini $3\cdot\vec{a}$ deb belgilaymiz va bu ifodani $\vec{a}$ vektorning 3 soniga ko‘paytmasi deb atashimiz tabiiydir.

Ta’rif. *Nol bo‘lmagan $\vec{a}$ vektorning k songa ko‘paytmasi deb, shunday $\vec{b}=k\cdot\vec{a}$ vektorga aytiladiki, bunda uning uzunligi $|k|\cdot|\vec{a}|$ songa teng bo‘lib, yo‘nalishi $k>0$ bo‘lganda $\vec{a}$ va $\vec{b}$ vektorlar yo‘nalishi bir xil, $k<0$ bo‘lganda esa qarama-qarshi bo‘ladi.*

Nol vektorning ixtiyoriy songa ko‘paytmasi nol vektor deb hisoblanadi.

$\vec{a}$ vektorning k songa ko‘paytmasi $k\vec{a}$ kabi belgilanadi (son ko‘paytuvchi chap tomonga yoziladi). Ta’rifga ko‘ra:

$$|k\vec{a}|=|k|\cdot|\vec{a}|.$$

Vektorning songa ko‘paytmasi ta’rifidan bevosita quyidagilar kelib chiqadi: 1) *istalgan vektorning nolga ko‘paytmasi nol vektor bo‘ladi;* 2) *istalgan son va ixtiyoriy $\vec{a}$ vektor uchun $\vec{a}$ va $k\vec{a}$ vektorlar kollineardir.*

Endi vektorni songa ko‘paytirishning asosiy xossalarini sanab o‘tamiz.

Istalgan $\vec{a}$, $\vec{b}$ vektorlar va istalgan k, l sonlar uchun quyidagi tengliklar o‘rinli:

1°. $(k\cdot l)\vec{a}=k\cdot(l\vec{a})$ — *guruhlash qonuni.*

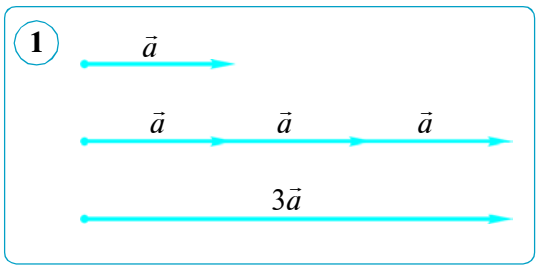
2°. $(k+l)\vec{a}=k\vec{a}+l\vec{a}$ — *birinchi taqsimot qonuni.*

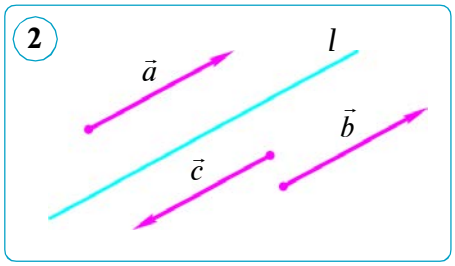
3°. $k(\vec{a}+\vec{b})=k\vec{a}+k\vec{b}$ — *ikkinchi taqsimot qonuni.*

4°. $k\cdot\vec{0}=0\cdot\vec{a}=\vec{0}$.

Parallel to‘g‘ri chiziqlarga yoki bir to‘g‘ri chiziqda yotuvchi ikki vektorning ***kollinear vektorlar*** *deb atalishini yana bir bor eslatib o‘tamiz.*

l to‘g‘ri chiziq va unga parallel bo‘lgan $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar berilgan bo‘lsin (2- rasm). Ta’rifga ko‘ra, $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar kollinear vektorlar bo‘ladi. Bu yerda $\vec{a}$ va $\vec{b}$ vektorlar bir xil yo‘nalgan, $\vec{c}$ vektor esa $\vec{a}$ va $\vec{b}$ vektorlarga nisbatan qarama-qarshi yo‘nalgan.





Ma'lumki, vektorni songa ko'paytirganda ko'paytma vektorning yo'nalishi berilgan vektorga parallel bo'ladi. Bundan quyidagi muhim xulosani hosil qilamiz:

vektorning songa ko'paytmasi shu vektorga kollinear vektordir.

Teorema.

Vektor o'zining moduliga teng songa bo'linsa, shu vektorga kollinear birlik vektor hosil bo'ladi.

Isbot. $\vec{a}$ vektorning moduli $|\vec{a}|$ bo'lsin. $\vec{a}$ vektorning $k = \frac{1}{|\vec{a}|}$ songa ko'paytmasini qaraylik:

$$|k\vec{a}| = |k| \cdot |\vec{a}| = \frac{1}{|\vec{a}|} \cdot |\vec{a}| = 1.$$

Demak, ko'paytma vektor moduli bir birlikka teng.

Moduli birga teng vektorni *birlik vektor* deb ataymiz. Agar $\vec{a}$ vektor bo'yicha yo'nalgan birlik vektorni $\vec{e}$ deb belgilasak, teoremaga ko'ra: $\vec{e} = \frac{\vec{a}}{|\vec{a}|}$ yoki bu tenglikni $|\vec{a}|$ songa ko'paytirsak: $\vec{a} = |\vec{a}| \cdot \vec{e}$.

Natijada biz vektorlarni o'rganishda katta ahamiyatga ega bo'lgan tenglikni hosil qildik, ya'ni *har qanday vektor shu vektor moduli bilan o'ziga kollinear birlik vektorning ko'paytmasiga teng ekan.*

1- masala. k ning qanday qiymatlarida quyidagi mulohazalar to'g'ri:

1) $|k\vec{a}| < |\vec{a}|$; 2) $|k\vec{a}| > \vec{a}$; 3) $|k\vec{a}| = |\vec{a}|$, bu yerda $\vec{a} \neq \vec{0}$?

Yechish. 1) $\vec{a} \neq \vec{0}$ da $|k\vec{a}| < |\vec{a}| \Leftrightarrow |k| \cdot |\vec{a}| < |\vec{a}| \Leftrightarrow |k| < 1 \Leftrightarrow -1 < k < 1$;

2) $\vec{a} \neq \vec{0}$ da $|k\vec{a}| > |\vec{a}| \Leftrightarrow |k| \cdot |\vec{a}| > |\vec{a}| \Leftrightarrow |k| > 1 \Leftrightarrow k < -1$ yoki $k > 1$;

3) $\vec{a} \neq \vec{0}$ da $|k\vec{a}| = |\vec{a}| \Leftrightarrow |k| \cdot |\vec{a}| = |\vec{a}| \Leftrightarrow |k| = 1 \Leftrightarrow k = -1$ yoki $k = 1$.

$\vec{a} \neq \vec{0}$ da $|\vec{a}| > 0$. Bizga ma'lumki, tengsizlik yoki tenglamaning har ikkala qismini musbat songa bo'lsak, munosabat o'zgarmaydi.

Javob: 1) $-1 < k < 1$; 2) $k < -1$ yoki $k > 1$; 3) $k = -1$ yoki $k = 1$ da mulohazalar o'rinli bo'ladi.

2. Vektorning koordinatalari. Tekislikda xOy Dekart koordinatalar sistemasi, ya'ni koordinatalar boshi O nuqta, koordinata o'qlarining yo'nalishi va masshtab birligi – birlik kesma berilgan bo'lsin. Bunda tekislikdagi ixtiyoriy A nuqta o'zining abssissasi x va ordinatasi y ga ega bo'ladi: $A(x; y)$. Moduli bir birlikka ega bo'lgan hamda yo'nalishi Ox o'qi bo'yicha yo'nalgan birlik vektorni $\vec{i}$ bilan, xuddi shuningdek, Oy o'qi bo'yicha yo'nalgan birlik vektorni $\vec{j}$ bilan belgilaymiz (3- *a* rasm).

Tekislikda koordinatalari $(x;\ y)$ bo‘lgan A nuqta berilgan bo‘lsin. OA_xA uchburchakni qaraylik. Bu uchburchakda $\overrightarrow{OA} = \overrightarrow{OA_x} + \overrightarrow{A_xA}$. Ammo $OA_x = x$, $A_xA = OA_y = y$ bo‘lgani uchun $\overrightarrow{OA_x} = x \cdot \vec{i}$, $\overrightarrow{A_xA} = y \cdot \vec{j}$ bo‘ladi. Bundan

$$\vec{a} = \overrightarrow{OA} = x \cdot \vec{i} + y \cdot \vec{j} \qquad (1)$$

tenglikni hosil qilamiz. Bu (1) tenglik vektorning *koordinata ifodasi* deb ataladi.

Demak, boshi koordinatalar boshida, uchi $A\,(x;\ y)$ nuqtada bo‘lgan vektorni koordinata o‘qlari bo‘yicha yo‘nalgan $\vec{i}$ va $\vec{j}$ vektorlar orqali (1) ko‘rinishda yozish mumkin ekan.

Bunda $(\vec{i};\ \vec{j})$ vektorlar juftligi *bazis vektorlar*, x va y sonlar esa $\vec{a}$ vektorning *koordinatalari* deb ataladi.

Agar vektorning (1) koordinata ifodasi ma’lum bo‘lsa, vektor koordinatalari bilan berilgan deyiladi va qisqacha $\vec{a}(x;\ y)$ shaklda yoziladi:

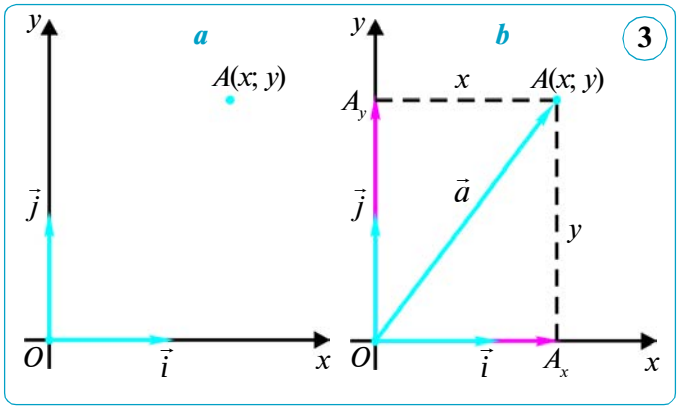
$$\vec{a}(x;y) = x \cdot \vec{i} + y \cdot \vec{j}\ . \qquad (2)$$

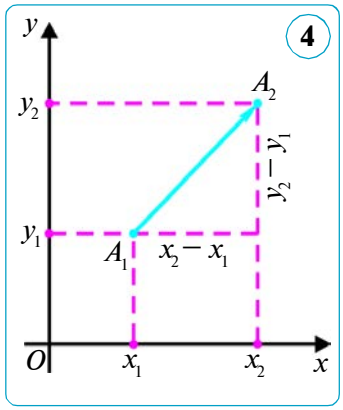
Ta’rif. *Agar* $A_1(x_1;\ y_1)$ *va* $A_2(x_2;\ y_2)$ *bo‘lsa,* $x_2 - x_1$ *va* $y_2 - y_1$ *sonlar* $\overrightarrow{A_1A_2}$ *vektorning koordinatalari deyiladi* (4- rasm).

Vektorning koordinatalari harfiy belgilanishidan keyin qavs ichida yoziladi: $\overrightarrow{A_1A_2}\,(x_2 - x_1;\ y_2 - y_1)$. Ayrim hollarda koordinatalari berilgan vektorlarni belgilashda $\overline{(x_2 - x_1;\, y_2 - y_1)}$ yozuvdan ham foydalaniladi. Nol vektorning koordinatalari nolga teng ekani ravshan: $\vec{0}(0;0)$.

Nuqtalar orasidagi masofani topish formulasiga ko‘ra, $\vec{a}\,(a_1;\ a_2)$ vektorning uzunligi $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$ formula bo‘yicha hisoblanadi.

Qoida. *Vektorning koordinatalarini topish uchun uning oxiri* (*uchi*)*ning koordinatalaridan boshining mos koordinatalarini ayirish kifoya.*





Masalan, $\overrightarrow{OA}$ vektorning koordinatalari vektor oxiri (uchi) A ning koordinatalari bilan to‘la aniqlanadi, ya’ni vektor oxirining koordinatalariga teng bo‘ladi.

Agar $A(x; y)$ bo‘lsa, $\overrightarrow{OA}(x; y) = \overline{(x; y)}$ bo‘ladi.

Koordinatalari teng bo‘lgan vektorlarning *xossasi* va *alomatini* isbotsiz keltiramiz.

Teorema.

Teng vektorlar mos ravishda teng koordinatalarga ega. Va aksincha, agar vektorlarning mos koordinatalari teng bo‘lsa, vektorlar teng bo‘ladi.

1- xulosa. Agar vektor oxirining koordinatalari vektorning koordinatalari bilan teng bo‘lsa, u holda berilgan vektorning boshi koordinatalar boshida bo‘ladi (3- *b* rasm).

2- xulosa. Agar $\vec{a}(a_1; a_2)$ vektor bilan uning oxiri bo‘lgan $B(x_2; y_2)$ nuqtasi koordinatalari berilgan bo‘lsa, u holda vektor boshi $A(x_1; y_1)$ nuqtaning koordinatalarini topish uchun B nuqtaning koordinatalaridan $\vec{a}(a_1; a_2)$ vektorning mos koordinatalarini ayirish kifoya:

$$x_1 = x_2 - a_1; \quad y_1 = y_2 - a_2. \tag{1}$$

3- xulosa. Agar $\vec{a}(a_1; a_2)$ vektor bilan uning boshi bo‘lgan $A(x_1; y_1)$ nuqtasi koordinatalari berilgan bo‘lsa, u holda vektor oxiri $B(x_2; y_2)$ nuqtaning koordinatalarini topish uchun A nuqtaning koordinatalariga $\vec{a}(a_1; a_2)$ vektorning mos koordinatalarini qo‘shish kifoya:

$$x_2 = x_1 + a_1; \quad y_2 = y_1 + a_2. \tag{2}$$

2- masala. Agar $A(-2; 1)$, $B(0; 4)$ va $C(4; 1)$ bo‘lsa, $ABCD$ parallelogrammning to‘rtinchi uchi koordinatasini toping.

Yechish. Agar $ABCD$ to‘rtburchak parallelogramm bo‘lsa, u holda $\overrightarrow{AB} = \overrightarrow{DC}$ bo‘ladi. $(x; y)$ – izlanayotgan D uchining koordinatasi bo‘lsin. $\overrightarrow{AB}$ va $\overrightarrow{DC}$ vektorlarning koordinatalarini topamiz:

$$\overrightarrow{AB} = \overline{(0 - (-2); 4 - 1)} = \overline{(2; 3)}, \quad \overrightarrow{DC} = \overline{(4 - x; 1 - y)}.$$

Shunday qilib, $4 - x = 2$ va $1 - y = 3$, bundan $x = 2$ va $y = -2$.

Javob: $D(2; -2)$.

3- masala. $A(-1; 5)$ nuqta $\vec{a}(2; -3)$ vektorning boshi bo‘lsa, bu vektor oxiri (uchi) B ning koordinatalarini toping.

Yechish. Berilgan ma’lumotlarni so‘nggi (2) munosabatlarga qo‘yib, izlanayotgan koordinatalarni topamiz:

$$x_2 = -1 + 2 = 1, \quad y_2 = 5 + (-3) = 2.$$

Javob: $B(1; 2)$.

4- masala. $A(-3; 0)$ va $B(5; -4)$ nuqtalar berilgan. $\overrightarrow{AB}$ va $\overrightarrow{BA}$ vektorlarning koordinatalarini toping.

Yechish. 1) $\overrightarrow{AB} = \overrightarrow{AB}(5-(-3);-4-0) = \overrightarrow{AB}(8;-4) = \overrightarrow{(8;-4)}$;

2) $\overrightarrow{BA} = -\overrightarrow{AB} = -\overrightarrow{(8;-4)} = \overrightarrow{(-8;-(-4))} = \overrightarrow{(-8;4)}$. *Javob:* (8; –4); (–8; 4).

Eslatma! *Biror vektorning koordinatalari ma'lum bo'lsa, u holda unga qarama-qarshi vektorning koordinatalarini yana qaytadan hisoblamasdan, berilgan vektorning koordinatalari ishorasini qarama-qarshisiga o'zgartirish kifoya.*

Savol, masala va topshiriqlar

1. 1) Berilgan vektorning songa ko'paytmasi deb nimaga aytiladi?
2) Vektorni songa ko'paytirishning xossalarini ayting.
3) Koordinatalar o'qidagi birlik vektorlar qanday belgilanadi?

2. Uzunligi 2 cm ga teng bo'lgan $\vec{a}$ vektorni chizing. $4\vec{a}$, $-2\vec{a}$, $3\vec{a}$, $1{,}5\vec{a}$, $-1{,}5\vec{a}$ vektorlarni yasang.

3. k ning qanday qiymatlarida $\vec{a}$ $(\vec{a} \neq \vec{0})$ va $k\vec{a}$ vektorlar:
1) yo'nalishdosh; 2) qarama-qarshi yo'nalgan; 3) teng bo'ladi?

4. $ABCD$ parallelogrammda O – diagonallarning kesishish nuqtasi, K nuqta – CD tomonning o'rtasi. $\overrightarrow{OA}$ va $\overrightarrow{AK}$ vektorlarni $\overrightarrow{AB} = \vec{a}$ va $\overrightarrow{AD} = \vec{b}$ vektorlar orqali ifodalang.

5. C nuqta AB tomonning o'rtasi. 1) $\overrightarrow{AC}$ vektorni $\overrightarrow{CB}$ vektor orqali; 2) $\overrightarrow{AB}$ vektorni $\overrightarrow{CB}$ vektor orqali; 3) $\overrightarrow{AC}$ vektorni $\overrightarrow{BA}$ vektor orqali ifodalang.

6. Ifodalarni soddalashtiring:
1) $(\overrightarrow{AB} + \overrightarrow{AC}) + (\overrightarrow{BA} + \overrightarrow{CB})$; 2) $\overrightarrow{AB} - \overrightarrow{DB} - \overrightarrow{CA} + \overrightarrow{DA}$.

7. 1) $A(-1;4)$ va $B(3;9)$; 2) $A(2;-5)$ va $B(1;-1)$; 3) $A(3;2)$ va $B(3;2)$ nuqtalar berilgan. $\overrightarrow{AB}$ vektorning koordinatalarini toping.

8. Agar: 1) $\overrightarrow{AB}(7;24)$; 2) $A(0;-1)$ va $B(3;-5)$; 3) $A(2;-4)$ va $B(2;-1)$ bo'lsa, $\overrightarrow{AB}$ vektorning uzunligini toping.

9. Agar: 1) $A(-2;-3)$, $B(-3;-1)$; 2) $A(m;n)$, $B(-m;-n)$ bo'lsa, $\overrightarrow{BA}$ vektorning koordinatalari nimaga teng bo'ladi?

10. $A(-1;-3)$, $B(2;-4)$, $C(-3;-1)$ va $D(5;2)$ nuqtalar berilgan. $\overrightarrow{AC}$ va $\overrightarrow{DB}$ vektorlar tengmi?

11. $\vec{a}(m;24)$ vektorning uzunligi 25 ga teng. m ni toping.

12. $A(5;-3)$ nuqta $\vec{a}(-7;-8)$ vektorning boshi bo'lsa, bu vektor oxiri (B) ning koordinatalarini toping.

13. Agar: 1) $A(-3;1)$ va $B(5;-5)$; 2) $A(12;0)$ va $B(0;-5)$ bo'lsa, $\overrightarrow{AB}$ vektorning uzunligini toping.

40. KOORDINATALARI BILAN BERILGAN VEKTORLAR USTIDA AMALLAR

Koordinatalari bilan berilgan vektorlarni qo‘shish, ayirish va songa ko‘paytirish amallari bilan tanishamiz.

1. Koordinatalari bilan berilgan vektorlarni qo‘shish.

Ta’rif. $\vec{a}\,(a_1;\ a_2)$ va $\vec{b}\,(b_1;\ b_2)$ *vektorlarning yig‘indisi* deb, koordinatalari $c_1 = a_1 + b_1,\ c_2 = a_2 + b_2$ bo‘lgan $\vec{c}\,(c_1;\ c_2)$ vektorga aytiladi.

Shunday qilib,

$$\vec{a}(a_1;\ a_2) + \vec{b}(b_1;\ b_2) = \vec{c}(a_1 + b_1;\ a_2 + b_2) \quad \text{yoki} \quad \overline{(a_1;\ a_2)} + \overline{(b_1;\ b_2)} = \overline{(a_1 + b_1;\ a_2 + b_2)}.$$

Har qanday $\vec{a}\,(x_1;\ y_1)$, $\vec{b}\,(x_2;\ y_2)$ *va* $\vec{c}\,(c_1;\ c_2)$ *vektorlar uchun quyidagi tengliklar o‘rinli:*

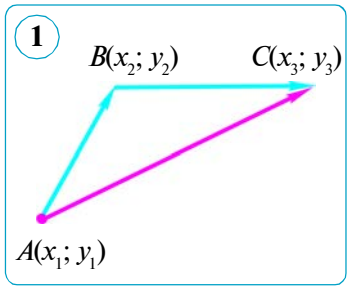
1) $\vec{a} + \vec{b} = \vec{b} + \vec{a}$; 2) $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$; 3) $\vec{a} + \vec{0} = \vec{a}$.

Isbotlash uchun tenglikning o‘ng va chap qismlarida turgan vektorlar koordinatalarining mos koordinatalarini taqqoslash yetarli.

Teorema.

A, B, C nuqtalar qanday bo‘lmasin, quyidagi vektor tenglik o‘rinlidir:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$



Isbot. $A(x_1;\ x_1)$, $B(x_2;\ y_2)$, $C(x_3;\ y_3)$ – berilgan nuqtalar (1- rasm). Qo‘shiluvchi vektorlarni koordinatalar orqali ifodalab, topamiz:

$$\overrightarrow{AB}(x_2 - x_1;\ y_2 - y_1), \quad \overrightarrow{BC}(x_3 - x_2;\ y_3 - y_2).$$

Ta’rifga ko‘ra, yig‘indi vektorning koordinatalarini aniqlash uchun $\overrightarrow{AB}$ va $\overrightarrow{BC}$ vektorlarning mos koordinatalarini qo‘shamiz:

$$x_2 - x_1 + x_3 - x_2 = x_3 - x_1, \quad y_2 - y_1 + y_3 - y_2 = y_3 - y_1.$$

Bu esa $\overrightarrow{AC}$ vektorning koordinatalaridir: $\overrightarrow{AC}(x_3 - x_1;\ y_3 - y_1)$.

Teng vektorlar haqidagi teoremaga ko‘ra: $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$.

Teorema isbotlandi.

2- rasmdan foydalanib, yuqoridagi tenglikning to‘g‘riligini isbotlashni o‘zingizga havola qilamiz.

Shunday qilib, *vektorlarni qo‘shish uchun ularning mos koordinatalarini qo‘shish kifoya ekan.*

2. Koordinatalari bilan berilgan vektorlarni ayirish.

Ta'rif. $\vec{a}\,(a_1;\ a_2)$ va $\vec{b}\,(b_1;\ b_2)$ *vektorlarning ayirmasi* deb, shunday $\vec{c}\,(c_1;\ c_2)$ vektorga aytiladiki, uning $\vec{b}$ vektor bilan yig'indisi $\vec{a}$ vektorni beradi: $\vec{b}+\vec{c}=\vec{a}$.

Bundan $\vec{c}=\vec{a}-\vec{b}$ vektorning koordinatalarini topamiz:

$$c_1=a_1-b_1,\ \ c_2=a_2-b_2.$$

Koordinatalari bilan berilgan vektorlarni ayirish uchun ularning mos koordinatalarini ayirish kifoya, ya'ni:

$$\vec{a}\,(a_1;\ a_2)-\vec{b}\,(b_1;\ b_2)=\vec{c}\,(a_1-b_1;\ \ a_2-b_2)\ \text{ yoki}$$

$$\overline{(a_1;a_2)}-\overline{(b_1;b_2)}=\overline{(a_1-b_1;\ a_2-b_2)}.$$

3. Koordinatalari bilan berilgan vektorlarni songa ko'paytirish.

Ta'rif. $\vec{a}\,(a_1,\ a_2)$ vektorning k songa ko'paytmasi deb, $\overline{(ka_1;\ ka_2)}$ vektorga aytiladi, ya'ni:

$$k\vec{a}=\overline{(ka_1;\ ka_2)}.$$

Ta'rifga ko'ra, $\overline{(a_1;\ a_2)}\cdot k=k\overline{(a_1;\ a_2)}$.

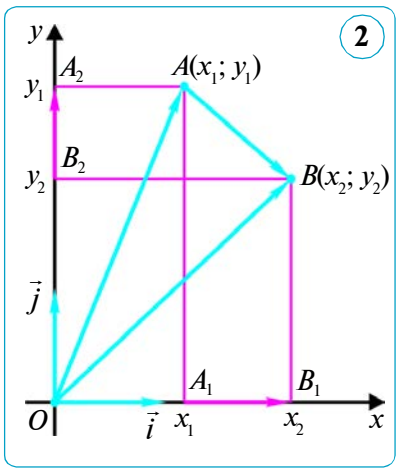
Demak, *vektorni songa (yoki k sonni $\vec{a}$ vektorga ko'paytirish uchun) uning koordinatalarini shu songa ko'paytirish yetarli ekan.*

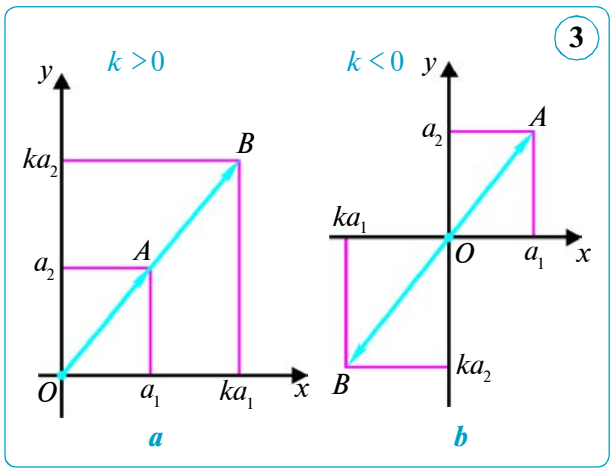
Vektorni songa ko'paytirishning avval keltirilgan ta'rifini 3- rasmdan foydalanib tekshirib ko'ring. Uning xossalari koordinatalarda ham o'rinli bo'ladi. Shu sababli ularni keltirib o'tmadik.

1- masala. $\vec{a}(3;\ 5)$ va $\vec{b}(2;\ 7)$ vektorlar yig'indisini toping.

Yechish. $\vec{a}(3;\ 5)+\vec{b}(2;\ 7)=\overline{(3;\ 5)}+\overline{(2;\ 7)}=\overline{(3+2;\ 5+7)}=\overline{(5;\ 12)}$.

Demak, $\vec{a}+\vec{b}$ vektorning koordinatalari (5; 12) ga teng.





2- masala. $\vec{a}(-3;\ 5)$ va $\vec{b}(3;\ -3)$ vektorlar ayirmasini toping.

Yechish: $\vec{a}(-3;\ 5) - \vec{b}(3;\ -3) = \overline{(-3;\ 5)} - \overline{(3;\ -3)} = \overline{(-3-3;\ 5-(-3))} = \overline{(-6;\ 8)}$.

Javob: $\overline{(-6;\ 8)}$.

3- masala. $\vec{a}(3;\ 5)$ vektorga qarama-qarshi $\vec{b}$ vektorni toping.

Yechish. $\vec{a}$ vektorga qarama-qarshi $\vec{b}$ vektor quyidagiga teng:

$$\vec{b} = -\vec{a} = -1 \cdot \vec{a} = -1 \cdot \overline{(3;\ 5)} = \overline{(-3;\ -5)}.$$

Javob: $\vec{b}(-3;\ -5)$ yoki $\overline{(-3;-5)}$.

4- masala. Agar $\vec{a}(-3;\ 4)$ bo‘lsa, $\vec{b} = 4\vec{a}$ vektorning koordinatalarini toping.

Yechish. $\vec{b} = 4\vec{a} = 4 \cdot \overline{(-3;\ 4)} = \overline{(4 \cdot (-3);\ 4 \cdot 4)} = \overline{(-12;\ 16)}$.

Javob: $\vec{b}\ (-12;\ 16)$ yoki $(-12;\ 16)$.

Savol, masala va topshiriqlar

1. 1) Koordinatalari berilgan ikki vektor qanday qo‘shiladi?
2) Koordinatalari berilgan ikki vektor qanday ayiriladi?
3) Koordinatalari berilgan vektor songa qanday ko‘paytiriladi?

2. Agar $\vec{a}\ (-4;\ 8)$ va $\vec{b}\ (1;\ -4)$ bo‘lsa, shu vektorlar: 1) yig‘indisining; 2) ayirmasinining koordinatalarini toping.

3. $\vec{a}\ (-2;\ 6)$ va $\vec{b}\ (-2;\ 4)$ vektorlar berilgan. 1) $\vec{a} + \vec{b}$; 2) $\vec{a} - \vec{b}$; 3) $\vec{b} - \vec{a}$; 4) $-\vec{a} - \vec{b}$ vektorning koordinatalarini toping.

4. $\vec{a}\ (2;\ 3)$ va $\vec{b}\ (-1;\ 0)$ vektorlar berilgan. Vektorning koordinatalarini toping: 1) $2\vec{a} + \vec{b}$; 2) $\vec{a} - 3\vec{b}$; 3) $2\vec{b} - \vec{a}$.

5. $\vec{a}\ (2;\ -3)$ va $\vec{b}\ (-2;\ -3)$ vektorlar berilgan. Ushbu vektorning koordinatalarini toping: 1) $\vec{c} = \vec{a} - 2\vec{b}$; 2) $\vec{c} = -2\vec{a} + \vec{b}$; 3) $\vec{c} = -3\vec{a} - 2\vec{b}$.

6. $\vec{a} = -2\vec{i} - 3\vec{j}$ va $\vec{b} = -2\vec{j}$ vektorlar berilgan. Ushbu vektorning koordinatalarini toping:

1) $\vec{c} = 2\vec{a} - \vec{b}$; 2) $\vec{c} = -4\vec{a} + 3\vec{b}$; 3) $\vec{c} = -3\vec{a} + 4\vec{b}$.

7. $\vec{a} = -2\vec{i} + 2\vec{j}$ va $\vec{b} = 3\vec{i}$ vektorlar berilgan. 1) $\vec{c} = 3\vec{a} - 2\vec{b}$; 2) $\vec{c} = 4\vec{a} - \vec{b}$ vektorning koordinatalarini toping.

8. $\vec{a} = 2\vec{i} - 3\vec{j}$ va $\vec{b} = 2\vec{j}$ vektorlar berilgan. 1) $\vec{c} = -\vec{a} - 2\vec{b}$; 2) $\vec{c} = \vec{a} - 5\vec{b}$ vektorning koordinatalarini toping.

9. $\vec{a} = -3\vec{i}$ va $\vec{b} = -\vec{i} + 2\vec{j}$ vektorlar berilgan. 1) $\vec{c} = -2\vec{a} + \vec{b}$; 2) $\vec{c} = -\vec{a} - \vec{b}$ vektorning koordinatalarini toping.

41. VEKTORNING FIZIK VA GEOMETRIK TALQINLARI. GEOMETRIK MASALALAR YECHISHNING VEKTOR USULI

1. Vektorning fizik va geometrik talqinlari.

1. Jismga ta'sir etadigan kuch (qo'yilgan kuch)ni yo'nalishi ta'sir etish yo'nalishi bilan bir xil, absolut qiymati esa kuch miqdoriga proporsional vektor bilan tasvirlash qulay. Amaliyot shuni ko'rsatadiki, kuchlarni bunday tasvirlash usulida jismga bir nuqtada ta'sir qiluvchi ikki yoki bir nechta kuchning teng ta'sir etuvchisi shu kuchlarga mos vektorlarning yig'indisi bilan tasvirlanadi. 1- rasmda jismga A nuqtada $\vec{a}$ va $\vec{b}$ vektorlar bilan tasvirlangan ikkita kuch ta'sir etadi. Bu kuchlarning teng ta'sir etuvchisi

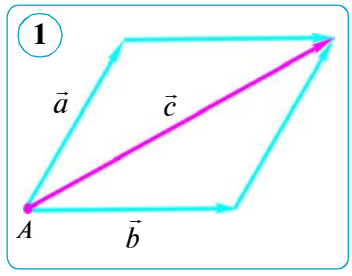
$$\vec{c} = \vec{a} + \vec{b}$$

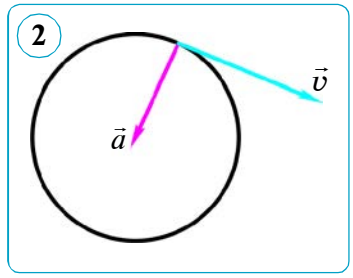
vektor bilan tasvirlanadi.

Kuchni berilgan ikki yo'nalishda ta'sir etuvchi kuchlarning yig'indisi shaklida tasvirlash *kuchni yo'nalishlar bo'yicha yoyish (ajratish)* deyiladi.

2. Fizikada jismning *ilgarilama harakati* deb shunday harakatga aytiladiki, bunda jismning barcha nuqtalari bir xil vaqt oralig'ida, bir xil yo'nalishda bir xil masofaga siljiydi. Shunday qilib, fizikadagi *siljish vektori* darsligimizda qabul qilingan vektor ekan. Farqi shundaki, geometriya darsligida faqat tekislikdagi vektorlar to'g'risidagina gap yuritiladi, fizikada esa boshidanoq fazodagi vektorlar, ularning xossalari to'g'risida ham mulohaza yuritiladi.

3. Fizikada «vektor» so'zi ancha keng ma'noda ishlatiladi. Masalan, tezlik vektor deb yuritiladi. Ammo geometrik vektorning uzunligi metrlarda, tezlikning absolut qiymati esa sekundiga metrlar (m/s)da o'lchanishining o'zidanoq tezlikning geometriyada qabul qilingan ma'nodagi vektor emasligi ko'rinib turibdi. Biz geometriyada tezlikni vektor emas, balki *vektor kattalik* deymiz. Umuman, vektor kattaliklar, o'zlarining modulidan tashqari, yo'nalishi bilan aniqlanadi. Ma'lum masshtab tanlab olinganda vektor kattaliklar geometrik vektorlar bilan tasvirlanadi.





Bunda vektor kattaliklarni qo'shishda ularni tasvirlovchi geometrik vektorlarni qo'shish, vektor kattaliklarni sonlarga ko'paytirishda esa ularni tasvirlovchi geometrik vektorlarni o'sha sonlarga ko'paytirish mos keladi.

Bir misol ko'raylik. 2- rasmda $\vec{v}$ vektor aylanma harakatning tezligini, $\vec{a}$ vektor esa tezlanishni ifodalashi mumkin. Biroq bu vektorlarni fizika nuqtayi nazaridan qo'shish ma'noga ega emas.

Shunday bo‘lsa-da, fizikada tezlik yoki tezlanishlar to‘g‘ridan-to‘g‘ri vektorlar deb aytiladi.

Gap nima to‘g‘risida ketayotganligi aniq tasavvur qilinsa, bunday so‘z erkinligi umumiylikka hech bir ziyon keltirmaydi. Xuddi shunga o‘xshash biz o‘z vaqtida uchburchak tomonining uzunligini, qisqalik uchun, oddiygina qilib uning tomoni deb aytishga kelishib olgan edik va hokazo.

2. Geometrik masalalarni yechishning vektor usuli.

Geometrik masalalarni yechishda va teoremalarni isbotlashda vektorlardan keng foydalaniladi.

1- masala. C nuqta AB kesmaning o‘rtasi, O nuqta esa tekislikning ixtiyoriy nuqtasi. $\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$ ekanini isbot qiling (3- *a* rasm).

Yechish. 1- usul. Uchburchak qoidasiga ko‘ra:

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} \quad \text{va} \quad \overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}.$$

Bu ikki tenglikni qo‘shib, quyidagiga ega bo‘lamiz:

$$2\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} + (\overrightarrow{AC} + \overrightarrow{BC}).$$

C nuqta AB kesmaning o‘rtasi bo‘lganligidan, u holda $\overrightarrow{AC} + \overrightarrow{BC} = \vec{0}$, chunki qarama-qarshi vektorlarning yig‘indisi nol vektorga teng.

Shunday qilib, quyidagiga ega bo‘lamiz:

$$2\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} \quad \text{yoki} \quad \overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}).$$

2- usul. OAB uchburchakni parallelogrammga to‘ldiramiz (3- *b* rasm).

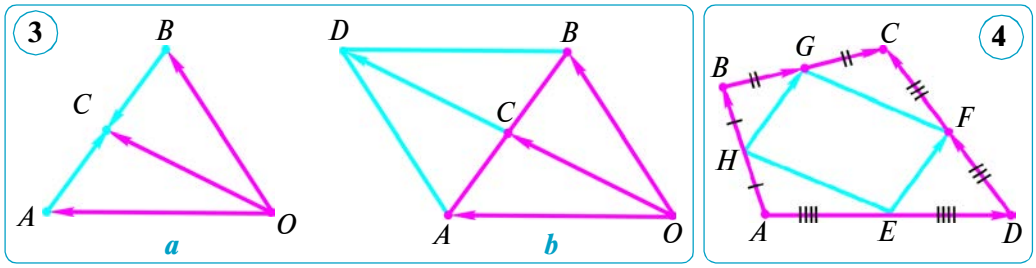
$\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OD}$ (parallelogramm qoidasiga ko‘ra). Parallelogrammning diagonallari kesishish nuqtasida teng ikkiga bo‘linadi, shuning uchun $\overrightarrow{OC} = \overrightarrow{CD}$ va $\overrightarrow{OD} = 2\overrightarrow{OC}$.

Demak, $\overrightarrow{OA} + \overrightarrow{OB} = 2\overrightarrow{OC}$. Bundan:

$$\overrightarrow{OC} = \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}).$$

2- masala. Ixtiyoriy $ABCD$ to‘rtburchak tomonlarining o‘rtalari parallelogrammning uchlari bo‘lishini isbotlang.

Yechish. E, F, G, H – mos ravishda AB, BC, CD va DA tomonlarning o‘rtalari bo‘lsin (4- rasm). Parallelogrammning 3- alomatiga ko‘ra, masalan, EF va HG kesmalarning uzunligi tengligi va parallelligini isbotlash yetarli. Vektor tilida, bu $\overrightarrow{EF}$ va $\overrightarrow{HG}$ vektorlarning tengligini isbotlashdan iboratdir.



Haqiqatan,

$$\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{BC}), \quad \overrightarrow{HG} = \overrightarrow{HD} + \overrightarrow{DG} = \frac{1}{2}(\overrightarrow{AD} + \overrightarrow{DC}).$$

Bundan tashqari, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AD} + \overrightarrow{DC}$ ekani ravshan. Shuning uchun, $\overrightarrow{EF} = \overrightarrow{HG}$. Bundan, EF va HG kesmalarning uzunlik bo‘yicha tengligi va parallel ekani kelib chiqadi. Demak, ixtiyoriy $ABCD$ to‘rtburchak tomonlarining o‘rtalari parallelogrammning uchlari bo‘ladi. Shuni isbotlash talab qilingan edi.

Keltirilgan isbotlardan ko‘rinib turibdiki, masala va teoremalarni vektor usuli bilan yechish algebraik masalalarni yechishga o‘xshaydi. Bu masalani yechishning bir tomonidir va u uch bosqichdan iborat.

Birinchi bosqich. Masala (teorema) shartini vektor ko‘rinishida yozish va qulay vektorlarni kiritish (o‘xshashlik – noma’lumlarni kiritish va algebraik tenglamani tuzish).

Ikkinchi bosqich. Vektor algebrasining vositalari orqali masala sharti shunday almashtiriladiki, masalani vektor ko‘rinishida yechish imkoniyati bo‘lsin (o‘xshashlik – algebraik tenglamani yechish).

Uchinchi bosqich. Olingan vektor munosabat dastlabki atamalarda talqin qilinadi (o‘xshashlik – tenglamani algebraik yechgandan so‘ng javobni yozish).

Savol, masala va topshiriqlar

1. Uchlari $A(3;\ 1)$, $B(1;\ 3)$ va $C(0;\ 2)$ bo‘lgan uchburchak CC_1 medianasining uzunligini toping.

2. K nuqta $ABCD$ parallelogramm AD tomonining o‘rtasi. $\overrightarrow{KC}$ vektorni $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlar orqali ifodalang.

3. $A(2;\ 4)$, $B(3;\ 6)$ va $C(6;\ 14)$ nuqtalar berilgan. $\overrightarrow{AB} + \overrightarrow{AC}$ vektorning koordinatalarini toping.

4. $ABCD$ kvadrat ikki qarama-qarshi uchining koordinatalari berilgan: $A(0;\ 4)$ va $C(6;\ 0)$. Qolgan ikki uchining koordinatalarini toping.

5. $A(-2;\ 3)$ nuqta $\vec{a}(-3;\ 8)$ vektorning boshi bo‘lsa, bu vektor oxiri $(B(x;\ y))$ ning koordinatalarini toping.

6. Trapetsiyaning o‘rta chizig‘i asoslariga parallel va ular uzunligining yarmiga teng ekanini vektor yordamida isbotlang.

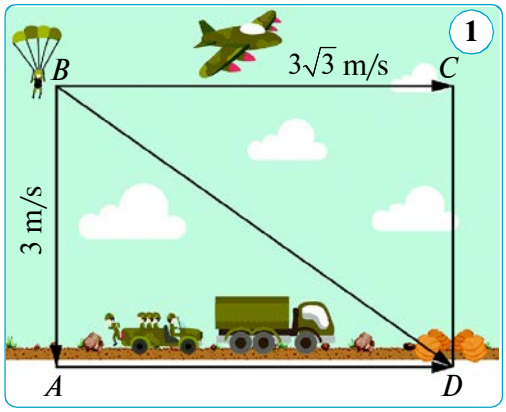
7. $\vec{a}(1;\ 3)$, $\vec{b}(-2;\ 4)$, $\vec{c}(-1;\ -3)$, $\vec{d}(-4;\ 4)$, $\vec{p}(3;\ 9)$, $\vec{q}(-1;\ 2)$ vektorlar berilgan. Shular ichidan: 1) yo‘nalishdosh vektorlarni; 2) bir juft qarama-qarshi yo‘nalgan vektorlarni toping.

8. $ABCD$ rombda N nuqta CD tomonning o‘rtasi. $\overrightarrow{AN}$ vektorni $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorlar orqali ifodalang.

9. $ABCD$ – parallelogramm va shu parallelogrammdan tashqarida yotuvchi ixtiyoriy O nuqta berilgan. $\overrightarrow{OD}$ vektorni $\overrightarrow{OA}$, $\overrightarrow{OB}$ va $\overrightarrow{OC}$ vektorlar orqali ifodalang.

AMALIY KOMPETENSIYANI RIVOJLANTIRUVCHI QO‘SHIMCHA MATERIALLAR

1. Vektorlarning amaliy tatbig‘iga doir masalalar.



1- masala. Parashutchi yerga 3 m/s tezlik bilan tushmoqda, shamol esa uni $3\sqrt{3}$ m/s tezlik bilan surib ketayapti. Bu sharoitda parashutchi yerga qanday burchak ostida tushadi (1- rasm)?

Yechish. Parashutchi B nuqtada bo‘lsin. Og‘irlik kuchi $\overrightarrow{BA}$ va shamol kuchi $\overrightarrow{BC}$ ning teng ta’sir etuvchisi $\overrightarrow{BD}$ bo‘ladi va $ABCD$ – to‘g‘ri to‘rtburchak, AB – vertikal. Demak, $\angle ADB$ burchak qiymatini topish kerak.

$\overrightarrow{BC} = \overrightarrow{AD}$ va $BC = AD$ ($ABCD$ – to‘g‘ri to‘rtburchak, $\angle A = 90°$). Pifagor teoremasiga ko‘ra: $BD^2 = AD^2 + AB^2$, demak:

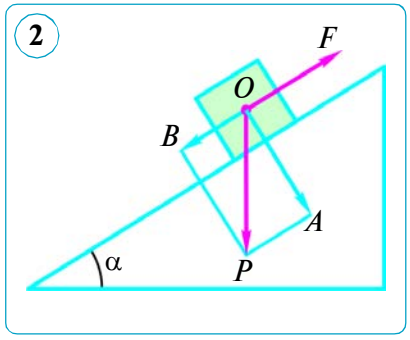
$$BD = \sqrt{AD^2 + AB^2} = \sqrt{(3\sqrt{3})^2 + 3^2} = \sqrt{27+9} = \sqrt{36} = 6 \text{ (cm)}.$$

Shunday qilib, ABD uchburchakda 3 cm li AB katet 6 cm li BD gipotenuzaga qaraganda ikki marta kichik ekan. Demak, $AB = 0{,}5BD$ bo‘lgani uchun to‘g‘ri burchakli uchburchakda 30° li burchak qarshisidagi katetning xossasiga ko‘ra, $\angle ADB = 30°$ yoki $\sin\angle ADB = \frac{AB}{BD} = 0{,}5$, bundan $\angle ADB = 30°$ kelib chiqadi.

Javob: $\angle ADB = 30°$.

2- masala. Parashutchi yerga 4 m/s tezlik bilan tushmoqda, shamol esa uni $4\sqrt{3}$ m/s tezlik bilan surib ketayapti. Bunday sharoitda parashutchi yerga qanday burchak ostida tushadi (1- rasm q.)? Masalani mustaqil yeching.

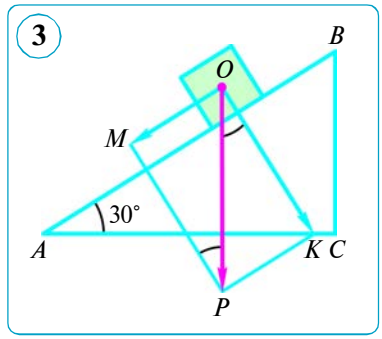
3- masala. Og‘irligi P bo‘lgan yuk qiya tekislikda sirpanib pastga tushib ketmasligi uchun uni qanday F kuch bilan ushlab turish kerak (2- rasm)?



Yechish. Yukning og‘irlik markazi O ga $\vec{P}$ kuch qo‘yilgan bo‘lsin. $\vec{P}$ vektorni o‘zaro perpendikular ikki yo‘nalish bo‘yicha 2- rasmda ko‘rsatilgandek qo‘yamiz. Qiya tekislikka perpendikular bo‘lgan $\overrightarrow{OA}$ kuch yukning siljishiga yo‘l qo‘ymaydi. Yukni ushlab turuvchi $\vec{F}$ kuch unga qarama-qarshi yo‘nalgan $\overrightarrow{OB}$ kuchga miqdor jihatidan teng bo‘lishi kerak. Bundan quyidagi xulosaga kelamiz:

$$F = P\sin\alpha.$$

4- masala. $P = 50$ N yuk og‘ma tekislikda yotibdi. Agar tekislikning gorizontga nisbatan og‘ish burchagi 30° ga teng bo‘lsa, sirpanish kuchi va bosim kuchini toping.



Berilgan: $P = 50$ N, $\angle A = 30°$.

Topish kerak: $F_{\text{og‘ish}}$, $F_{\text{bosim.}}$

Yechish. 1) $\overrightarrow{P}$ kuchni ikkita: sirpanish kuchi yo‘nalishiga parallel hamda bosim kuchi og‘-ma tekislikka perpendikulyar kuchlar bo‘yicha yoyamiz.

2) Parallelogrammni yasaymiz; $\overrightarrow{OP}$ vektor uning diagonali; $OM \parallel AB$, $OK \perp AB$, $PK \parallel AB$, $PM \perp AB$, $\overrightarrow{OM} = \vec{F}_{\text{og‘ish}}$, $\overrightarrow{OK} = \vec{F}_{\text{bosim}}$ ni o‘tkazamiz (3- rasm).

3) $\angle OPM = \angle A = 30°$ ($OP \perp AC$, $PM \perp AB$).

4) To‘g‘ri burchakli OPM uchburchakdan:

$$OM = 0{,}5OP = 0{,}5 \cdot 50 = 25; \quad F_{\text{og‘ish.}} = 25 \text{ N}.$$

5) To‘g‘ri burchakli OPK uchburchakdan Pifagor teoremasiga ko‘ra:

$$OK = \sqrt{OP^2 - PK^2} = \sqrt{OP^2 - OM^2} = \sqrt{50^2 - 25^2} = \sqrt{25^2 \cdot (4-1)} = 25\sqrt{3} \approx 43,$$

ya’ni $F_{\text{bosim.}} \approx 43$ N.

Javob: $F_{\text{og‘ish}} = 25$ N, $F_{\text{bosim}} \approx 43$ N.

5- masala. Tajriba shuni ko‘rsatadiki, agar A jismga ikkita a va b kuch ta’sir ko‘rsatayotgan bo‘lsa, u holda ularning ta’siri bitta c kuch ta’siriga teng bo‘lib, bu c kuch a va b kesmalardan yasalgan parallelogrammning diagonali bilan tasvirlanadi. Teng ta’sir etuvchi kuch *«parallelogramm qoidasi»* bo‘yicha topiladi.

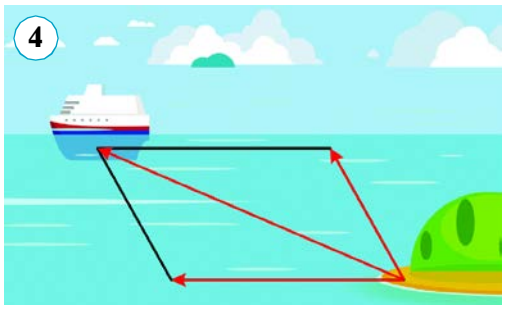
Masalan, suzib ketayotgan kemada (4- rasm) bo‘lgan yoki daryoni qayiqda kesib o‘tayotgan (5- rasm) odamga ko‘ndalang kesim va oqim bo‘lab yo‘nalgan ikkita kuch ta’sir ko‘rsatadi. Shu kuchlarni rasmda belgilang.

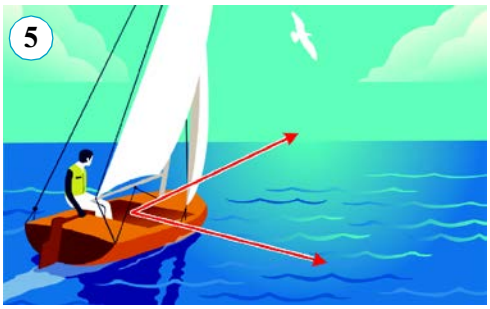
Ushbu masalaga o‘xshash masala tuzing va mos rasmlarda aks ettiring.

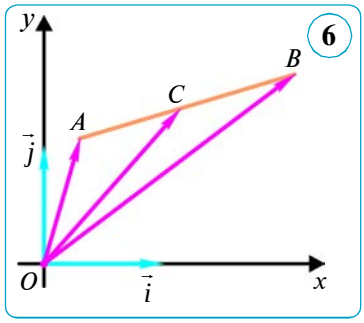
2. Sistema og‘irlik markazining koordinatalarini topish.

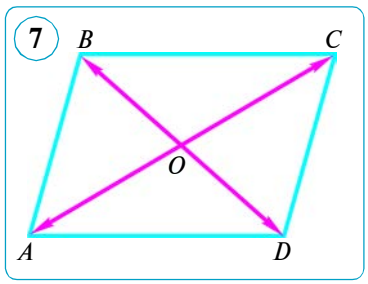
6- masala. Kesmani berilgan nisbatda bo‘lish (koordinata ko‘rinishida).

Agar $\overrightarrow{AC} = \lambda\overrightarrow{CB}$ bo‘lsa, C nuqta AB kesmani λ nisbatda bo‘ladi (6- rasm). Agar kesma oxirlarining koordinatalari $A(x_1; y_1)$, $B(x_2; y_2)$ ma’lum bo‘lsa, C nuq-taning x, y koordinatalarini toping.









Yechish. $\overrightarrow{OA}$, $\overrightarrow{OC}$ va $\overrightarrow{OB}$ vektorlarni yasaymiz. $\overrightarrow{OA}(x_1;\ y_1)$, $\overrightarrow{OC}(x;\ y)$, $\overrightarrow{OB}(x_2;\ y_2)$, $\overrightarrow{AC}(x-x_1;\ y-y_1)$, $\overrightarrow{CB}(x_2-x;\ y_2-y)$ va vektorni λ songa ko‘paytirganda uning koordinatalari λ songa ko‘paytirilishini hisobga olib, quyidagiga ega bo‘lamiz:

$$\overrightarrow{AC}=\lambda\overrightarrow{CB}\Leftrightarrow\overrightarrow{AC}\left(x-x_1;\ y-y_1\right)=$$

$$=\lambda\overrightarrow{CB}\left(x_2-x;\ y_2-y\right)\Leftrightarrow\begin{cases}x-x_1=\lambda(x_2-x);\\ y-y_1=\lambda(y_2-y).\end{cases}$$

Demak, $x=\dfrac{x_1+\lambda x_2}{1+\lambda}$, $y=\dfrac{y_1+\lambda y_2}{1+\lambda}$.

7- masala. $M_1(x_1;\ y_1)$ va $M_2(x_2;\ y_2)$ nuqtalarga mos ravishda m_1 va m_2 ga teng yuklar qo‘yilgan. Bu massalar sistemasining og‘irlik markazi (C nuqta) koordinatalarini toping.

Yechish. Og‘irlik markazi $C-M_1M_2$ kesmada hamda M_1 va M_2 nuqtalarga qo‘yilgan m_1 va m_2 massalardan teskari proporsional masofada yotadi, ya’ni ikkita moddiy nuqtalar sistemasining og‘irlik markazi bo‘lgan C nuqta M_1M_2 kesmani $\lambda=\dfrac{m_2}{m_1}$ nisbatda bo‘ladi. λ ning qiymatini 5- masaladagi formulalarga qo‘yib, shakl almashtirishlardan so‘ng C nuqtaning koordinatalarini topamiz:

$$x_C=\frac{x_1m_1+x_2m_2}{m_1+m_2},\quad y_C=\frac{y_1m_1+y_2m_2}{m_1+m_2}.$$

3. Vektor munosabatni isbotlashga doir masala.

8- masala. $ABCD$ – parallelogramm va uning diagonallari kesishgan O nuqta berilgan. Isbotlang: $\overrightarrow{OA}+\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{OD}$.

Berilgan: $ABCD$ – parallelogramm, $O-AC$ va BD diagonallarning kesishish nuqtasi, $AO=OC$, $BO=OD$ (7- rasm).

Isbot qilish kerak: $\overrightarrow{OA}+\overrightarrow{OC}=\overrightarrow{OB}+\overrightarrow{OD}$.

Isbot. Bu vektor tenglikni isbot qilishning bir necha usulini keltiramiz.

Ayirmaning nol vektorga tengligini ko‘rsatamiz:

1) $(\overrightarrow{OA}+\overrightarrow{OC})-(\overrightarrow{OB}+\overrightarrow{OD})=(\overrightarrow{OA}-\overrightarrow{OB})+(\overrightarrow{OC}-\overrightarrow{OD})=\overrightarrow{BA}+\overrightarrow{DC}=\overrightarrow{BA}+\overrightarrow{AB}=\overrightarrow{BB}=\vec{0}.$

Shakl almashtirish jarayonida yig‘indidan yig‘indini ayirish qoidasi, guruhlash qonuni, uchburchak qoidasi, $\overrightarrow{DC}=\overrightarrow{AB}$ (parallelogrammning qarama-qarshi tomonlari va yo‘nalishdosh vektorlar), nol vektor ta’riflaridan foydalanildi.

2) $(\overrightarrow{OA}+\overrightarrow{OC})-(\overrightarrow{OB}+\overrightarrow{OD})=(\overrightarrow{OA}-\overrightarrow{OD})+(\overrightarrow{OC}-\overrightarrow{OB})=\overrightarrow{DA}+\overrightarrow{BC}=\overrightarrow{DA}+\overrightarrow{AD}=\overrightarrow{DD}=\vec{0}.$

Shakl almashtirishda yig‘indidan yig‘indini ayirish va uchburckak qoidalari, guruhlash qonuni, $\overrightarrow{DC}=\overrightarrow{AB}$ ekani va nol vektor ta’riflaridan foydalanildi.

43–44. 3- NAZORAT ISHI. XATOLAR USTIDA ISHLASH

1. $A(-2; 3)$ va $B(4; 0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq tenglamasini tuzing.

2. Agar $C(4; 9)$ va $R = 5$ bo‘lsa, markazi C nuqtada, radiusi R bo‘lgan aylana tenglamasini tuzing.

3. $\vec{a}(1; 0)$, $\vec{b}(1; 2)$ va $\vec{c}(1; 3)$ vektorlar berilgan. $\vec{a} - \vec{b}$ va $\vec{b} + \vec{c}$ vektorlarning koordinatalarini toping.

4. $\vec{c}(-1; 0)$ va $\vec{d}(1; 2)$ vektorlar berilgan. $2\vec{c} + 3\vec{d}$ vektorning koordinatalarini toping.

3- TEST — O‘zingizni sinab ko‘ring!

1. $A(0; -1)$, $B(1; 0)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq qaysi choraklarda joylashgan?

A) III, IV, I; B) I, II, III; D) II, III, IV; E) II, IV.

2. $A(-2; 0)$, $B(-2; 2)$ nuqtalardan o‘tuvchi to‘g‘ri chiziq qaysi choraklarda yotadi?

A) I, II, III; B) II, III; D) II, IV; E) III, IV, I.

3. Uchlari $A(-4; 0)$, $B(-4; 4)$ nuqtalarda bo‘lgan AB kesma o‘rtasining koordinatalarini toping.

A) $(-2; 0)$; B) $(0; 2)$; D) $(2; -4)$; E) $(-4; 2)$.

4. Uchlari $A(-2; 0)$, $B(0; 2)$, $C(2; 0)$ nuqtalarda bo‘lgan uchburchakning AC tomoni o‘rtasining koordinatalarini toping.

A) $(-1; 1)$; B) $(1; 0)$; D) $(0; 0)$; E) $(0; 1)$.

5. $\vec{a}(-3; 1)$ va $\vec{b}(5; -6)$ vektorlar berilgan. $\vec{c} = \vec{b} - 3\vec{a}$ vektorning koordinatalarini toping.

A) $(14; -9)$; B) $(4; -3)$; D) $(14; -3)$; E) $(9; 3)$.

6. $A(-3; 0)$ va $B(-5; 4)$ nuqtalar berilgan. $\overrightarrow{BA}$ vektorning koordinatalarini toping.

A) $(-8; -4)$; B) $(-8; 4)$; D) $(2; -4)$; E) $(8; -4)$.



Ingliz tilini o‘rganamiz!

Aylana tenglamasi – circle equation
To‘g‘ri chiziq tenglamasi – straight-line equation
Kollinear vektorlar – collinear vectors
Vektor uzunligi – vector length
Teng vektorlar – equal vectors
Skalar – scalar
Qarama-qarshi vektorlar – opposite vectors
Birlik vektor – unit vector
Yo‘nalishdosh – equivalent

Tarixiy ma'lumotlar

1. To‘gri burchakli koordinatalar sistemasini fransuz olimi Rene Dekart fanga kiritgan. To‘g‘ri burchakli koordinatalar sistemasi ba’zan Dekart koordinatalar sistemasi ham deyiladi.



Rene Dekart
(1596–1650)

Rene Dekart (1596–1650) – fransuz faylasufi, matematigi, fizigi, fiziologi. La-Flesh iezuit kollejida ta’lim olgan, grek va lotin tillari, matematika va falsafani o‘rgangan. Dekart falsafasi uning matematikasi, kosmogoniyasi, fizikasi bilan bog‘liq. Matematikada analitik geometriya asoschilaridan biri (to‘g‘ri burchakli koordinatalar sistemasi uning nomi bilan ataladi). Dekart XVII–XVIII asrlar falsafasi va fani taraqqiyotiga salmoqli hissa qo‘shgan.

XVII asrda Dekartning ishlari tufayli butun matematika, xususan geometriyani inqilobiy qayta qurgan koordinatalar metodi vujudga keldi. Algebraik tenglamalarni geometrik grafik orqali talqin qilish va geometrik masalalarning yechimini analitik formulalar, tenglamalar sistemalari yordamida izlash imkoniyati paydo bo‘ldi.

Bizning kunlargacha saqlanib kelgan qulay belgilarning kiritilishida, ya’ni noma’lumlarni belgilash uchun x, y, z; koeffitsiyentlarni belgilash uchun a, b, c lotin harflarini kiritishda, darajalarni x^2, y^2, z^2 ko‘rinishda belgilashda ham Dekartning xizmatlari katta.

2. Vektor tushunchasi XIX asrning o‘rtalarida bir vaqtda bir necha matematikning ishlarida uchraydi. Tekislikda vektorlar bilan ish ko‘rishni ilk bor 1835- yilda italiyalik olim **Bellivitis** (1803–1880) boshlab berdi. Bundan tashqari, **K. Gauss** (1777–1855)ning 1831- yilda «Bikvadratik solishtirmalar nazariyasi» nomli asarida hamda **Y. Argan** (1768–1822) va **K. Vessel** (1745–1818)ning kompleks sonlarni geometrik tasvirlashga doir ishlarida vektor tushunchasi aytib o‘tilgan. Nihoyat, **V. Gamilton** (1805–1865) va **R. Grassman** (1854–1901)ning vektorlar ustida amallar bajarishga doir ishlari vujudga keldi. Gamilton 1845- yilda birinchi bo‘lib vektor va skalar kattaliklarning farqini tushuntirdi. Gamiltonning o‘sha ishida «skalar», «vektor» atamalari yuzaga keldi.

Gamilton «vektor» atamasini lotincha *vehere* – «tashimoq», «sudramoq» so‘zidan hosil qilgan, *vektor* – «tashuvchi», «eltuvchi» demakdir. 1806- yilda Argan yo‘nalgan kesmalarni harf ustiga chiziq qo‘yish bilan belgilagan. Vektorlarning boshi va oxirini ko‘rsatish uchun **A. Myobius** (1790–1868) uni AB ko‘rinishda belgilagan. Grassman vektorlarni «kesmalar» deb atagan, u koordinata o‘qlari bo‘yicha yo‘nalgan e_1, e_2 birlik vektorlarni va vektorlarni $x_1e_1 + x_2e_2$ ko‘rinishida tasvirlashni tavsiya qilgan. Gamilton va **J. Gibss** (1839–1903) vektorlarni grekcha harflar bilan belgilagan. Vektorlarni qora harflar bilan belgilashni 1891- yilda **A. Xevisayd** (1850–1925) taklif etgan. Vektorning uzunligini $|AB|$ ko‘rinishda belgilashni esa 1905- yilda **R. Gans** (1880–1954) kiritgan.



IV BOB
YUZ



9- §. KO‘PBURCHAKNING YUZI

45. YUZ HAQIDA TUSHUNCHA

1. Yuz haqida tushuncha.

Shakllarning yuzlarini aniqlash masalasi juda qadim zamonlarga borib taqaladi. Bu masalaning vujudga kelishini insonlarning amaliy faoliyati taqozo etgan. Har birimiz kundalik turmushimizda yuz haqida birmuncha tasavvurga egamiz. Masalan, Siz to‘g‘ri to‘rtburchak (aytaylik, o‘zingiz yashayotgan xona) va kvadratning yuzini topishni bilasiz. Biz endi shakllarning yuzi to‘g‘risidagi tushunchalarni aniqlash va ularni o‘lchash usullarini topish bilan shug‘ullanamiz.

Agar geometrik shaklni chekli sondagi uchburchaklarga bo‘lish mumkin bo‘lsa, bu shakl *sodda shakl* deyiladi.

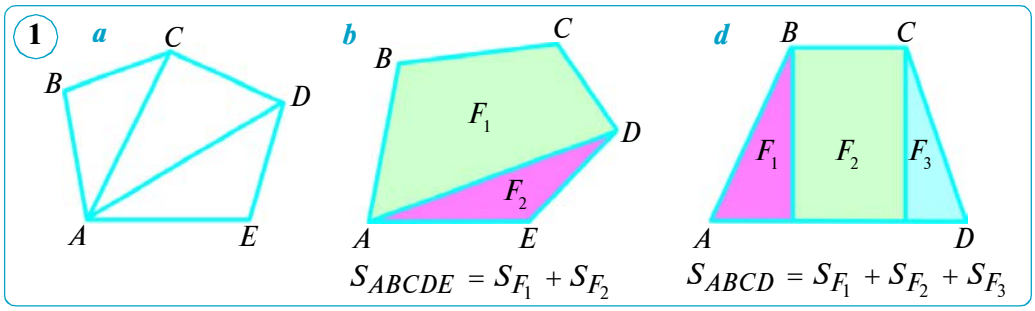
Biz uchburchak deb, tekislikning uchburchak bilan chegaralangan chekli qismiga aytamiz. Qavariq ko‘pburchak o‘zining biror uchidan chiqqan diagonallari bilan uchburchaklarga bo‘linadi (1- *a* rasm).

Yuz musbat miqdor (kattalik) bo‘lib, uning son qiymati quyidagi asosiy xossalarga (aksiomalarga) ega.

1- xossa. *Teng shakllar teng yuzlarga ega.*

2- xossa. *Agar ko‘pburchak bir-birini qoplamaydigan ko‘pburchaklardan tashkil topgan bo‘lsa, u holda uning yuzi bu ko‘pburchaklar yuzlarining yig‘indisiga teng bo‘ladi.*

F ko‘pburchak bir-birini qoplamaydigan ko‘pburchaklardan tashkil topgan degani: 1) *F* bu ko‘pburchaklar yig‘indisidan iboratligi va 2) ko‘pburchaklardan hech qaysi ikkitasi umumiy ichki nuqtalarga ega emasligini



anglatadi. Masalan, 1- *b* va 1- *d* rasmlarda bir-birini qoplamaydigan ko‘pburchaklardan tuzilgan ko‘pburchaklar tasvirlangan.

1- va 2- xossalar yuzlarning *asosiy xossalari* deyiladi.

2. Yuzni o‘lchash. Yuzni o‘lchash kesmalarni o‘lchash kabi o‘lchov birligi uchun qabul qilingan shakl yuzi bilan berilgan shaklni taqqoslashga asoslangan. Natijada berilgan shakl *yuzining sonli qiymatini* olamiz.

Yuz – tekis shakllarni tavsiflovchi asosiy matematik miqdorlardan biri. Sodda hollarda yuz tekis shaklni to‘ldiruvchi birlik kvadratlar – tomoni uzunlik birligiga teng bo‘lgan kvadratlar soni bilan o‘lchanadi.

3- xossa. *Tomoni bir uzunlik o‘lchov birligiga teng bo‘lgan kvadratning yuzi birga teng.*

Shunday qilib, quyidagi teorema o‘rinli bo‘ladi.

Teorema.

Tomonining uzunligi a ga teng bo‘lgan kvadratning yuzi a^2 ga teng.

Odatda, yuz lotincha bosh harf S bilan belgilanadi. Demak, kvadrat uchun

$$S = a^2$$

formula o‘rinli bo‘lib, uzunlik o‘lchovi birligi kvadrati bilan birga aytiladi.

3. Tengdosh shakllar.

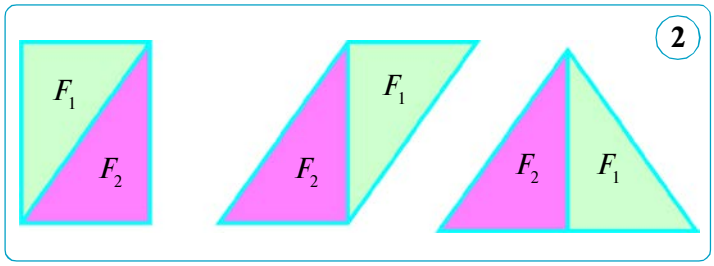
Ta’rif. *Agar ikki ko‘pburchakdan birini bir necha qismga bo‘lib, bu qismlarni boshqacha joylashtirganda ikkinchi ko‘pburchak hosil bo‘lsa, bu ko‘pburchaklar* ***teng tuzilganlar*** *deyiladi.*

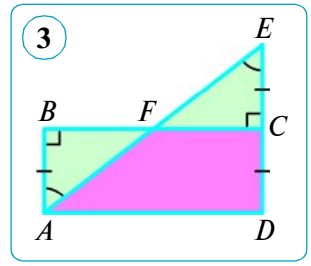
Agar ikkita ko‘pburchakning yuzlari teng bo‘lsa, ular ***tengdosh ko‘pburchaklar*** deb ataladi. 2- rasmdagi ko‘pburchaklar tengdoshdir.

Teng ko‘pburchaklar tengdoshdir (1- xossa), ammo teskari tasdiq, umuman aytganda, to‘g‘ri bo‘lmaydi: agar ikki shakl tengdosh bo‘lsa, bundan ularning tengligi kelib chiqmaydi.

Masala. $ABCD$ to‘g‘ri to‘rtburchak DC tomonining davomida C uchiga nisbatan D nuqtaga simmetrik E nuqta belgilangan (3- rasm). ADE uchburchak yuzining $ABCD$ to‘g‘ri to‘rtburchak yuziga teng ekanini isbotlang.

Isbot. AE va BC tomonlar F nuqtada kesishsin. ABF va ECF uchburchaklar teng (kateti va o‘tkir burchagiga ko‘ra: $AB = EC$, $\angle BAF = \angle E$). Nati-





jada ADE uchburchak $AFCD$ trapetsiya bilan ECF uchburchakdan, $ABCD$ to‘g‘ri to‘rtburchak esa o‘sha $AFCD$ trapetsiya bilan ECF ga teng bo‘lgan ABF uchburchakdan tuzilgan, demak, ADE uchburchak bilan $ABCD$ to‘g‘ri to‘rtburchak teng tuzilgandir (ya’ni tengdoshdir). Shuni isbotlash talab qilingan edi.

Yuz kattalik bo‘lgani uchun kattaliklarning barcha xossalariga ega bo‘ladi. Ularni bir turdagi kattaliklardagi kabi qo‘shish va musbat songa ko‘paytirish mumkin. Ikkita yuzni qo‘shishda va songa ko‘paytirishda yuz hosil bo‘ladi.

Amaliyotda yuzi mavjud bo‘lgan har qanday shaklning yuzini o‘lchash yoki hisoblash mumkin. Ko‘pincha turli yuzlarni aniqlashda formulalardan foydalaniladi. Ayrim shakllarning yuzlarini hisoblash formulalarini chiqarish bilan kelgusi mavzularda shug‘ullanamiz.

Savol, masala va topshiriqlar

1. 1) Sodda shakl deb nimaga aytiladi?
2) Shaklning yuzi deganda nimani tushunasiz?
3) Yuzning asosiy xossalarini ifodalang.
4) Qanday ikki ko‘pburchak teng tuzilgan deyiladi?
5) Tengdosh shakllar nima? Tengdosh shakllar tengmi?

2. Kvadratning tomoni: 1) 1,3 cm; 2) 0,15 dm; 3) 2,5 cm; 4) 18 dm; 5) 2,5 m. Kvadratning yuzini toping.

3. Kvadratning yuzi: 1) 16 dm^2; 2) 144 cm^2; 3) 121 cm^2; 4) 49 mm^2; 5) 196 cm^2; 6) 0,64 dm^2; 7) 6,25 m^2. Kvadratning tomonini toping.

4. Perimetri tomonlari 54 cm va 42 cm ga teng bo‘lgan to‘g‘ri to‘rtburchakning perimetriga teng bo‘lgan kvadratning yuzini toping.

5. 4- rasmdagi Q_1 va Q_2 uchburchaklar tengdosh. Shuni isbotlang.

6. Kvadratning yuzi 36 cm^2. Agar uning hamma tomonini:
1) ikki marta uzaytirilsa; 2) uch marta kamaytirilsa;
3) 2 cm ga uzaytirilsa, uning yuzi qanday o‘zgaradi?

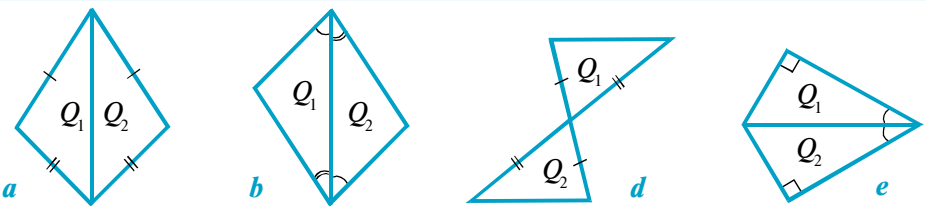
Namuna. Yuzi 81 cm^2 ga teng bo‘lgan kvadratning hamma tomonlari 1 cm ga qisqartirilsa, uning yuzi qanday o‘zgaradi?

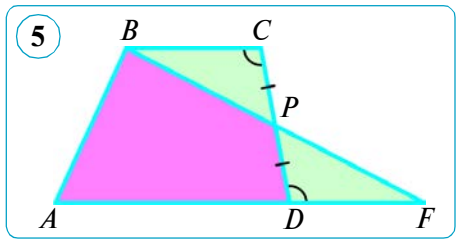
Yechish. Berilgan kvadratning tomoni $a = 9$ cm. Yangi kvadratning tomoni $a_1 = a - 1 = 9 - 1 = 8$ (cm). U holda $S_y = 8^2 = 64$ (cm^2). Berilgan kvadrat tomonlari 1 cm ga kamaytirilsa, uning yuzi

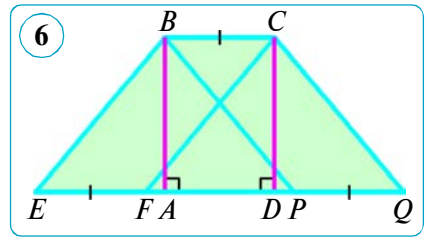
$$S - S_{ya.} = 81 - 64 = 17 \text{ } (cm^2)$$

ga kamayadi. *Javob:* 17 cm^2 ga kamayadi.

4







7. Teng tuzilgan ikkita to‘g‘ri to‘rtburchakdan: 1) bu to‘g‘ri to‘rtburchaklarning tengligi; 2) ularning tengdoshligi kelib chiqadimi?

8. Agar kvadratning hamma tomoni: 1) n marta uzaytirilsa; 2) k marta kamaytirilsa, uning yuzi qanday o‘zgaradi?

9. (*Amaliy ish.*) Biror kvadrat chizing. Tomoni shu kvadratning tomonidan 2 marta katta bo‘lgan ikkinchi kvadratni chizing. Ikkinchi kvadratning yuzi birinchi kvadratning yuzidan necha marta katta?

10. $AD - ABCD$ trapetsiyaning katta asosi bo‘lsin. CD tomonning o‘rtasi P nuqta va B uchi orqali AD nurni F nuqtada kesuvchi to‘g‘ri chiziq o‘tkazilgan (5- rasm). $S_{ABCD} = S_{ABF}$ ekanini isbotlang.

Isbot. 1) $\triangle BCP = \triangle FDP$ – tomoni va unga yopishgan ikki burchagiga ko‘ra ($CP = \ldots - \ldots$, $\angle BCP = \angle\ldots - \ldots$ va ... parallel to‘g‘ri chiziqlarni ... kesuvchi kesganda hosil bo‘lgan ... burchaklar, $\angle BPC = \angle\ldots - \ldots$ bo‘lgani uchun) teng, ya’ni $S_{BCP} = \ldots$.

2) $S_{ABCD} = S_{ABPD} + \ldots$, $S_{ADP} = S_{ABPD} + \ldots$, shuning uchun $S_{ABCD} = \ldots$.

Nuqtalar o‘rniga mos javoblarni yozing.

11. Yuzi: 1) 2,25 cm^2; 2) 0,81 dm^2; 3) 289 mm^2; 4) 5,76 m^2; 5) 400 dm^2 ga teng bo‘lgan kvadratning perimetrini toping.

12. 6- rasmda tasvirlangan ko‘pburchaklar ichidan tengdoshlarini toping.

13. O‘zbekistonning maydoni 448,9 ming km^2. Bu maydonning taqriban 80 % ini tekislik tashkil qiladi. Maydonning tekislik qismi necha ming kvadrat kilometrdan iborat?

Bilib qo‘ygan foydali!



- ***S*** – lotincha «***superfacies***» so‘zidan olingan bo‘lib, bu so‘z «***sirt***» ma’nosini bildiradi.
- Qit’alar, davlatlarning hududlari kvadrat kilometrlarda, katta ekin maydonlarining yuzlari gektarlarda, uncha katta bo‘lmagan yer maydonlari ar (sotix)larda o‘lchanadi.



46–47. TO‘G‘RI TO‘RTBURCHAK VA PARALLELOGRAMMNING YUZI

1. To‘g‘ri to‘rtburchakning yuzi.

Siz to‘g‘ri to‘rtburchakning yuzi qo‘shni tomonlari uzunliklari ko‘paytmasiga tengligini bilasiz va bunga doir masalalar yechgansiz.

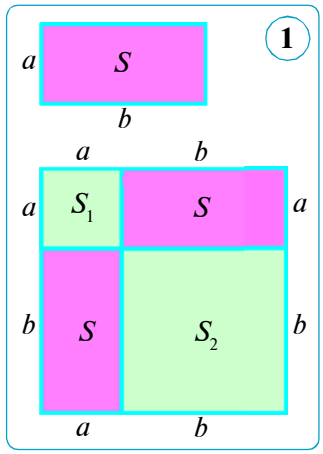
Hozir bu bajarilgan amalning nazariy jihatdan to‘g‘ri ekanligini ko‘rsatamiz.

Teorema.

Tomonlari a va b bo‘lgan to‘g‘ri to‘rtburchakning yuzi

$$S = a \cdot b$$

formula bo‘yicha hisoblanadi.



Isbot. Tomonlari a va b bo‘lgan to‘g‘ri to‘rtburchakni olaylik, bunda a va b – ixtiyoriy musbat sonlar. $S = a \cdot b$ ekanini isbotlaymiz.

Teoremani isbot qilish uchun tomoni $(a + b)$ bo‘lgan kvadrat yasaymiz. Bu kvadratni 1- rasmda ko‘rsatilgan shakldagidek bo‘laklarga ajratamiz. Bunda kvadratning yuzi tomoni a va b ga teng ikki kvadrat hamda tomonlari a va b bo‘lgan ikki to‘g‘ri to‘rtburchakdan tashkil topganini ko‘rish mumkin. Shunday qilib, tomoni $(a + b)$ bo‘lgan kvadrat yuzi $S_1 + 2S + S_2$ ga teng. Ikkinchi tomondan yuzning xossasiga ko‘ra, bu yuza $(a + b)^2$ ga teng, ya’ni

$$S_1 + 2S + S_2 = (a + b)^2, \text{ yoki}$$

$$S_1 + 2S + S_2 = a^2 + 2ab + b^2.$$

Bu tenglikdan $S_1 = a^2$, $S_2 = b^2$ ekanini hisobga olsak,

$$S = a \cdot b$$

ekani kelib chiqadi. Teorema isbotlandi.

1- masala. To‘g‘ri to‘rtburchakning yuzi 150 cm^2 ga teng, tomonlarining nisbati esa 3 : 2 kabi. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

Yechish. To‘g‘ri to‘rtburchakning kichik tomoni $b = 2x$ cm bo‘lsin. U holda katta tomonning uzunligi $a = 3x$ cm ekanligidan foydalanib tenglama tuzamiz va uni yechamiz:

$$S = 3x \cdot 2x, \quad \text{ya’ni} \quad S = 6x^2.$$

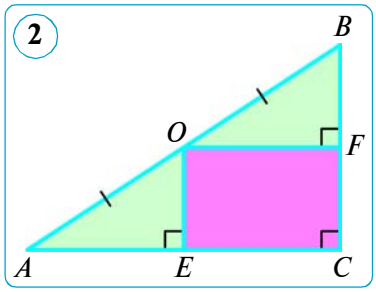
Bundan $x^2 = S : 6$, $x^2 = 150 : 6$, $x^2 = 25$, $x = 5$ (cm).

Demak, to‘g‘ri to‘rtburchakning kichik tomoni: $b = 2 \cdot 5 = 10$ (cm) ga, katta tomoni esa $a = 3 \cdot 5 = 15$ (cm) ga teng.

Endi uning perimetrini hisoblaymiz:

$$P = 2 \cdot (a + b) = 2 \cdot (15 + 10) = 2 \cdot 25 = 50 \text{ (cm)}.$$

Javob: $P = 50$ cm.



2- masala. To‘g‘ri burchakli uchburchakning katetlari 12 cm va 24 cm ga teng. Gipotenuzaning o‘rtasidan uchburchakning katetlariga perpendikulyarlar o‘tkazilgan. Hosil bo‘lgan to‘g‘ri to‘rtburchakning yuzini toping.

Berilgan: to‘g‘ri burchakli $\triangle ABC$ da: $AO = OB$, $OE \perp AC$, $OF \perp CB$, $AC = 24$ cm, $BC = 12$ cm (2- rasm).

Topish kerak: S_{CEOF}.

Yechish. Bizga ma’lumki, bir to‘g‘ri chiziqqa o‘tkazilgan ikki perpendikular o‘zaro parallel bo‘ladi. Fales teoremasiga ko‘ra:

$$AE = EC = 0{,}5AC = 0{,}5 \cdot 24 = 12 \text{ (cm)},$$

$$CF = FB = 0{,}5BC = 0{,}5 \cdot 12 = 6 \text{ (cm)}.$$

Demak, $S_{CEOF} = CE \cdot CF = 12 \cdot 6 = 72$ (cm^2). *Javob:* 72 cm^2.

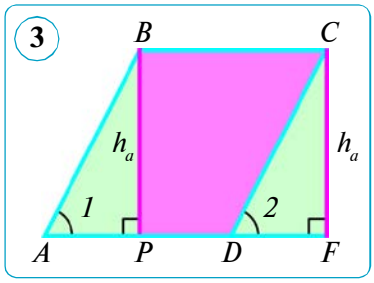
2. Parallelogrammning yuzi.

Parallelogrammning istalgan tomonini uning *asosi* deb olish mumkin, u holda qarama-qarshi tomonning ixtiyoriy nuqtasidan asosni o‘z ichiga olgan to‘g‘ri chiziqqa tushirilgan perpendikulyar parallelogrammning *balandligi* deyiladi. Balandlik tomonga yoki tomonning davomiga tushishi mumkin. 3- rasmda BP va CF – $ABCD$ parallelogrammning balandliklaridir.

Teorema.

Parallelogrammning yuzi asosi bilan balandligining ko‘paytmasiga teng:

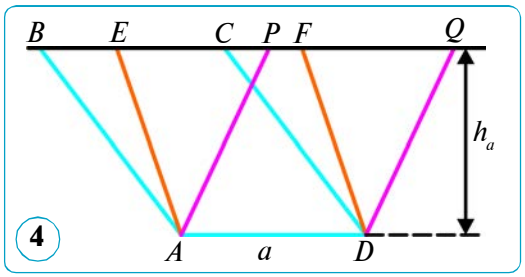
$$\boldsymbol{S = a \cdot h_a}.$$

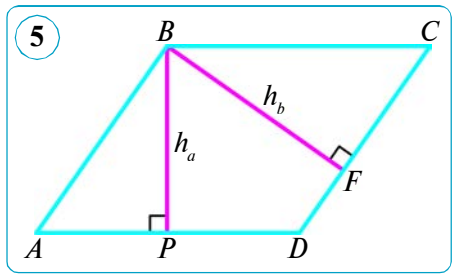


$ABCD$ parallelogrammning asosi uchun $AD = a$ tomon olingan, balandligi esa h_a ga teng bo‘lsin (3- rasm).

$S = a \cdot h_a$ ekanini isbot qilish talab etiladi.

Isbot. Asosi parallelogrammning BC tomoniga teng, balandligi esa h_a dan iborat bo‘lgan $PBCF$ to‘g‘ri to‘rtburchak yasaymiz. ABP va DCF uchburchaklar teng (gipotenuzasi va o‘tkir burchagiga ko‘ra: $AB = DC$ – gipotenuzalar, $\angle 1 = \angle 2$ – mos burchaklar). $ABCD$ parallelogramm $PBCD$ trapetsiya bilan ABP uchburchakdan, $PBCF$ to‘g‘ri to‘rtburchak esa o‘sha $PBCD$ trapetsiya bilan ABP ga teng bo‘lgan DCF uchburchakdan tuzilgan. Demak, $ABCD$ parallelogramm bilan yasalgan $PBCF$ to‘g‘ri to‘rtburchak teng tuzilgandir (ya’ni, tengdosh). Bundan, $ABCD$ parallelogrammning yuzi $PBCF$ to‘g‘ri to‘rtburchakning yuziga, ya’ni ah_a teng, degan natija chiqadi.





Shunday qilib, asosi a va unga tushirilgan balandligi h_a bo‘lgan parallelogrammning S yuzi quyidagi formula bo‘yicha hisoblanadi:

$$S = a \cdot h_a.$$

Shu formulani isbotlash talab qilingan edi.

1- natija. Agar ikkita parallelogramm bitta asosga ega va balandliklari teng bo‘lsa, ular teng tuzilgandir.

Berilgan: $ABCD$, $AEFD$ va $APQD$ parallelogrammlar bitta $AD = a$ asosga ega hamda balandliklari teng (h_a) (4- rasm).

Isbot qilish kerak: $ABCD$, $AEFD$ va $APQD$ parallelogrammlar teng tuzilgan.

Isbot. Masalan, $ABCD$ va $AEFD$ parallelogrammlarning teng tuzilganini isbotlaymiz. BAE va CDF uchburchaklar teng (uchburchaklar tengligining birinchi alomatiga ko‘ra), chunki $BA = CD$ va $AE = DF$ hamda $\angle BAE = \angle CDF$ (mos tomonlari parallel burchaklar bo‘lgani uchun). Demak, $ABCD$ parallelogramm $AECD$ trapetsiya bilan BAE uchburchakdan, $AEFD$ parallelogramm esa $AECD$ trapetsiya bilan BAE uchburchakka teng bo‘lgan CDF uchburchakdan tuzilgan. Demak, $ABCD$ va $AEFD$ parallelogrammlar teng tuzilgan.

Shunga o‘xshash, qolgan parallelogrammlarning teng tuzilgani isbotlanadi.

3- masala. Parallelogrammning tomonlari 25 cm va 20 cm, birinchi tomoniga tushirilgan balandlik 8 cm. Shu parallelogrammning ikkinchi tomoniga tushirilgan balandligini toping.

Berilgan: $ABCD$ parallelogrammda:

$$AD = a = 25 \text{ cm}, \quad DC = b = 20 \text{ cm}, \quad h_a = 8 \text{ cm (5- rasm)}.$$

Topish kerak: h_b.

Yechish. 1) $S = ah_a = 25 \cdot 8 = 200$ (cm^2).

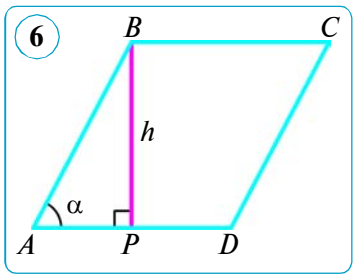
2) $S = bh_b$, ya’ni $200 = 20 \cdot h_b$. Bundan $h_b = 200 : 20 = 10$ (cm).

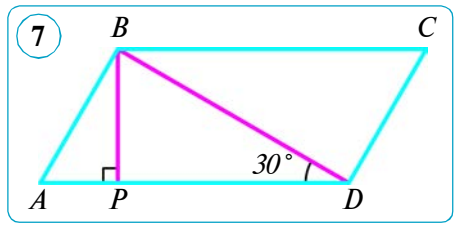
Javob: 10 cm.

2- natija. Parallelogrammning yuzi uning ikki tomoni bilan ular orasidagi burchak sinusi ko‘paytmasiga teng. Shuni isbotlang.

Yechish. $ABCD$ parallelogrammda $AD = a$, $AB = b$ va $\angle BAD = \alpha$ bo‘lsin. U holda parallelogarammning yuzi $\boldsymbol{S = ab \sin\alpha}$ formula bo‘yicha hisoblanadi. Shuni isbotlaymiz.

$ABCD$ parallellorammning BP balandligini o‘tkazamiz va uni $BP = h_a = h$ bilan belgilaymiz (6- rasm). U holda h balandlik to‘g‘ri burchakli ABP uch-





burchakning α o‘tkir burchagi qarshisida yotgan katet bo‘ladi. h ni b tomon va α burchakning sinusi ko‘paytmasi bilan ifodalaymiz:

$$h = b\sin\alpha.$$

Parallelogrammning yuzini hisoblash $S = ah$ formulasiga h ning bu ifodasini qo‘yib, ushbu formulani hosil qilamiz:

$$S = ab\sin\alpha.$$

4- masala. *Berilgan:* $ABCD$ – parallelogramm, $AD = 20$ cm, $BD = 16$ cm, $\angle BDA = 30°$.

Topish kerak: S_{ABCD}.

Yechish. 1- usul. 1) Berilgan parallelogrammning BP balandligini o‘tkazamiz va BDP uchburchakni ko‘rib chiqamiz (7- rasm). U to‘g‘ri burchakli, chunki $BP \perp AD$. BP balandlikni topamiz. 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng, shuning uchun

$$BP = 0{,}5BD = 0{,}5 \cdot 16 = 8 \text{ (cm)}.$$

2) Shunday qilib, $ABCD$ parallelogrammning yuzi quyidagiga teng bo‘ladi:

$$S = AD \cdot BP = 20 \cdot 8 = 160 \text{ (cm}^2\text{)}$$

2- usul. To‘g‘ri burchakli BDP uchburchakdan BP ni BD tomon (gipotenuza) va $\angle BDP = 30°$ burchakning sinusi bilan ifodalaymiz va parallelogrammning yuzi formulasiga qo‘yib, izlanayotgan yuzani topamiz:

$$S = AD \cdot BP = AD \cdot BD \cdot \sin\angle BDP = 20 \cdot 16 \cdot \sin 30° = 20 \cdot 16 \cdot 0{,}5 = 160 \text{ (cm}^2\text{)}.$$

Javob: $S = 160$ cm^2.

Savol, masala va topshiriqlar

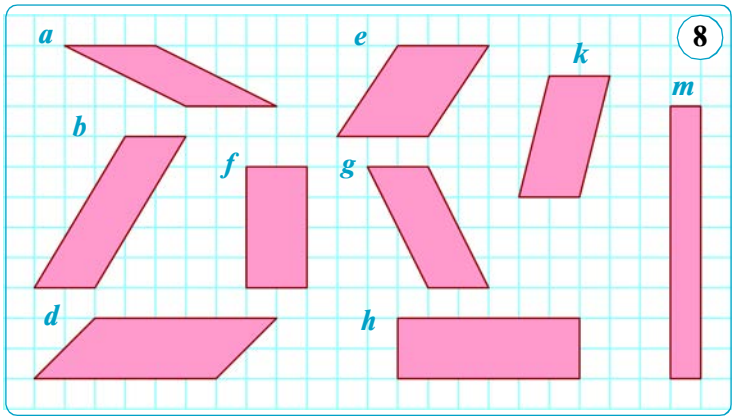
1. 1) To‘g‘ri to‘rtburchakning yuzi nimaga teng?

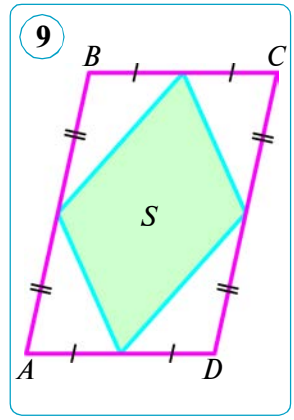
2) Parallelogrammning asosi va balandligi deganda nimani tushunasiz?

3) Parallelogrammning yuzi uning ikki qo‘shni tomoni va ular orasidagi burchak bo‘yicha qanday topiladi?

2. To‘g‘ri to‘rtburchakning ikki tomoni: 1) 30 cm va 2,9 cm; 2) 34 dm va 0,6 dm; 3) 2,5 dm va 12 cm. Shu to‘g‘ri to‘rtburchakning perimetri va yuzini toping.

3. To‘g‘ri to‘rtburchakning bir tomoni 15 dm, ikkinchi tomoni esa undan 5 marta ortiq. Shu to‘g‘ri to‘rtburchakning perimetri va yuzini toping.





4. Yuzi 240 m^2 bo‘lgan basketbol maydoni sport maydonining 15 % ini tashkil etadi. Sport maydoni yuzi butun maktab maydonining 32 % ini tashkil etadi. Maktab maydonining yuzini toping.

5. To‘g‘ri to‘rtburchakning bir tomoni 23 cm, ikkinchi tomoni esa undan 17 cm uzun. To‘g‘ri to‘rtburchakning perimetri va yuzini toping.

6. Agar to‘g‘ri to‘rtburchakning yuzi 20 cm^2 va 1) uzunligi 5 cm ga; 2) uzunligi enining 125 % iga; 3) tomonlaridan biri x ga teng bo‘lsa, perimetri nimaga teng bo‘ladi?

7. Agar $ABCD$ to‘g‘ri to‘rtburchakda: 1) $AB = 9$ cm, $BC = 4$ cm; 2) $AB : BC = 5 : 7$, $P_{ABCD} = 48$ cm bo‘lsa, uning yuzini toping.

8. Parallelogrammning tomoni 16 cm ga, unga tushirilgan balandlik esa 9 cm ga teng. Shu parallelogrammga tengdosh kvadratning tomonini toping.

9. a – parallelogrammning asosi, h – balandligi, S – yuzi. Agar:

1) $a = 10$ cm, $h_a = 0{,}5$ m bo‘lsa, S ni; 2) $h_a = 4$ cm, $S = 48$ cm^2 bo‘lsa, a ni; 3) $a = 24$ cm, $S = 120$ cm^2 bo‘lsa, h_a ni toping.

10. 8- rasmdagi tengdosh parallelogrammlarni toping.

11. Agar to‘g‘ri to‘rtburchakning: 1) asosi 5 marta kamaytirilib, balandligi 8 marta uzaytirilsa; 2) asosi ham, balandligi ham 2,5 marta kamaytirilsa, uning yuzi qanday o‘zgaradi?

12. 9- rasmdagi S shaklning yuzi parallelogramm yuzining qanday qismini tashkil etadi?

13. To‘g‘ri to‘rtburchakning ikki tomoni: 1) 24 cm va 20 cm; 2) 3,5 dm va 8 cm; 3) 8 m va 4,5 m; 4) 3,2 dm va 1,5 dm. Uning yuzini toping.

14. Parallelogrammning yuzi 36 cm^2, balandliklari 3 cm va 4 cm. Shu parallelogrammning perimetrini toping.

15. Parallelogrammning tomonlari 20 cm va 28 cm, ular orasidagi burchak 30° ga teng. Shu parallelogrammning yuzini ikki usul bilan toping.

48. UCHBURCHAKNING YUZI

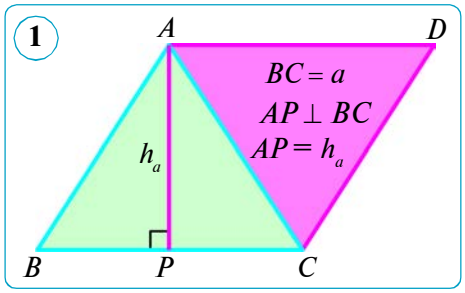
Uchburchak yuzini hisoblash formulasini topish uchun parallelogramm shakliga keltirish usulidan foydalanamiz.

Teorema.

Uchburchakning yuzi uning asosi bilan balandligi ko‘paytmasining yarmiga teng:

$$S = \frac{1}{2} a \cdot h_a ,$$

bunda a – uchburchakning asosi, h_a – asosga tushirilgan balandlik.



Isbot. ABC – berilgan uchburchak bo‘lsin (1- rasm). $\triangle ABC$ ni rasmda ko‘rsatilgandek $ABCD$ (asosi BC) parallelogrammga to‘ldiramiz. $\triangle BAC$ va $\triangle DCA$ lar teng, chunki parallelogrammning diagonali uni ikki teng uchburchakka ajratadi. Shuning uchun bu uchburchaklarning yuzlari teng. Demak, $ABCD$ parallelogrammning yuzi $\triangle ABC$ yuzining ikkilanganiga teng: $2S = a \cdot h_a$.

Bundan, $S = \frac{ah_a}{2}$. Teorema isbotlandi.

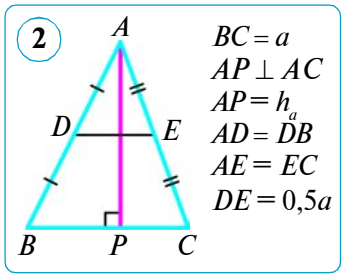
Uchburchakning yuzini hisoblash formulasini boshqacha ham o‘qish mumkin:

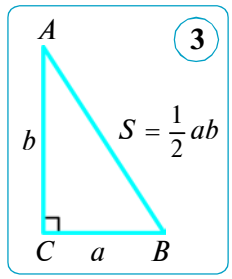
uchburchakning yuzi uning o‘rta chizig‘i bilan balandligining ko‘paytmasiga teng (2- rasm):

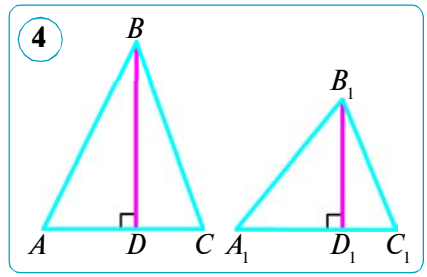
$$S = \frac{a}{2} \cdot h_a .$$

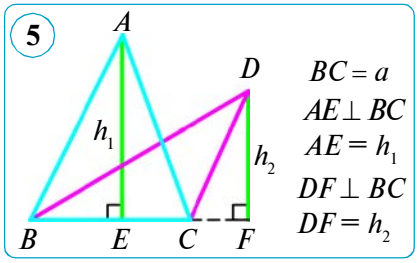
1- natija. *To‘g‘ri burchakli uchburchakning yuzi katetlari ko‘paytmasining yarmiga teng,* chunki bir katetni asos va ikkinchisini balandlik qilib olish mumkin (3- rasm).

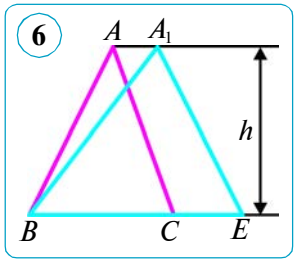
2- natija. *Ikkita uchburchak yuzlarining nisbati asoslari bilan balandliklari ko‘paytmasining nisbati kabidir* (4- rasm).

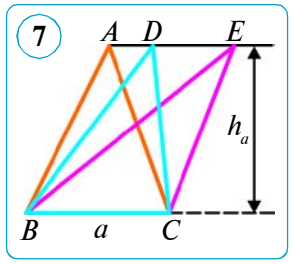












Isbot. $\frac{S_{ABC}}{S_{A_1B_1C_1}} = \frac{0{,}5AC \cdot BD}{0{,}5A_1C_1 \cdot B_1D_1} = \frac{AC \cdot BD}{A_1C_1 \cdot B_1D_1}$.

3- natija. *Asoslari teng bo‘lgan ikki uchburchak yuzlarining nisbati balandliklarining nisbati kabidir* (5- rasm).

Isbot. $\frac{S_{ABC}}{S_{DBC}} = \frac{0{,}5a \cdot h_1}{0{,}5a \cdot h_2} = \frac{h_1}{h_2}$.

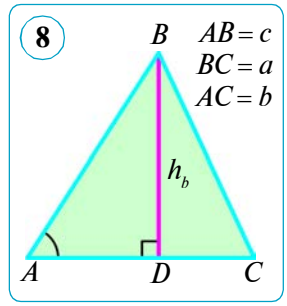
4- natija. *Balandliklari teng bo‘lgan ikki uchburchak yuzlarining nisbati asoslarining nisbati kabidir* (6- rasm).

Isbot. $\frac{S_{ABC}}{S_{A_1BE}} = \frac{0{,}5 \cdot BC \cdot h}{0{,}5 \cdot BE \cdot h} = \frac{BC}{BE} = \frac{a}{a_1}$, bunda $BC = a$, $BE = a_1$.

5- natija. *Asoslari va balandliklari teng bo‘lgan uchburchaklar tengdoshdir* (7- rasm).

Isbot. $S_{BAC} = S_{BDC} = S_{BEC} = 0{,}5ah_a$.

6- natija. *Uchburchakning yuzi uning ikki tomoni va ular orasidagi burchak sinusi ko‘paytmasining yarmiga teng* (8- rasm).



Isbot. ABC uchburchakning tomonlari $BC = a$, $AC = b$, $AB = c$ bo‘lsin. U holda $S = \frac{1}{2}bc\sin A$ ekanini isbotlaymiz. Buning uchun ABC uchburchakning $BD = h_b$ balandligini o‘tkazamiz (8- rasm). h_b ni c tomon va A burchakning sinusi bilan ifodalaymiz: $h_b = c\sin A$. Uchburchakning yuzini hisoblash formulasi $S = \frac{1}{2}bh_b$ ga h_b ning shu ifodasini qo‘yib, ushbu formulani hosil qilamiz:

$$S = \frac{1}{2}bc\sin A.$$

Uchburchakning yuzini a, b tomonlari va C burchak sinusi, a, c tomonlari va B burchak sinusi orqali hisoblash formulalari shunga o‘xshash keltirib chiqariladi.

Sunday qilib, uchburchakning yuzi ikki tomoni va ular orasidagi burchak sinusiga ko‘ra ushbu formulalar bo‘yicha hisoblanadi:

$$S = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B = \frac{1}{2}bc\sin A.$$

Uchburchak yuzini tomonlari orqali hisoblash formulasi I asrda yashagan qadimgi grek olimi **Geron** tomonidan topilgan bo‘lib, u *Geron formulasi* deb ataladi. Geron formulasi uchburchakning uchala tomoni uzunligi ma’lum bo‘lganda uning yuzini hisoblash uchun ishlatiladi.

Geron formulasining isbotini keltirib chiqaramiz.

Ma’lumki, uchburchakning yuzi uning asosi bilan balandligi ko‘paytmasining yarmiga teng:

$$S = \frac{1}{2}a \cdot h_a = \frac{1}{2}b \cdot h_b = \frac{1}{2}c \cdot h_c .$$

Balandlik o‘rniga uning uchburchak tomonlari orqali ifodasi

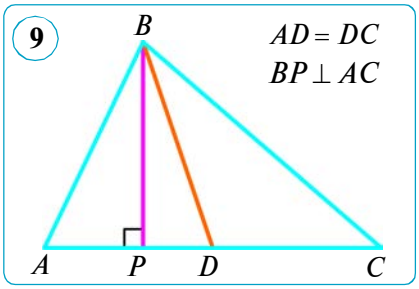
$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)}\ \beta = 90°,\ h_b = \frac{2}{b}\sqrt{p(p-a)(p-b)(p-c)}\ \text{va}$$

$$h_c = \frac{2}{c}\sqrt{p(p-a)(p-b)(p-c)}$$

larni qo‘yib, uni soddalashtirib ushbu Geron formulasini hosil qilamiz:

$$S = \sqrt{p(p-a)(p-b)(p-c)}\ , \text{ bunda } p = \frac{a+b+c}{2}.$$

9

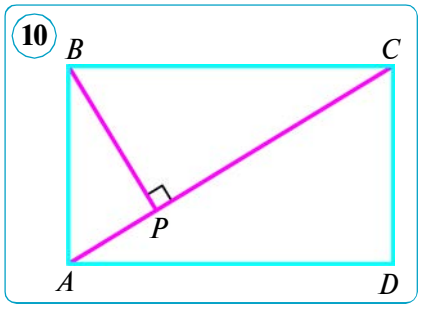


1- masala. Uchburchakning medianasi uni ikkita tengdosh uchburchakka bo‘lishini isbot qiling.

Isbot. BD — ABC uchburchakning medianasi bo‘lsin (9- rasm). ABD va CBD uchburchaklar teng AD va DC tomonlarga hamda umumiy BP balandlikka ega, ya’ni uchburchaklar 5- natijaga ko‘ra tengdoshdir:

$$S_{ABD} = S_{CBD}.$$

10



2- masala. Berilgan: $ABCD$ — to‘g‘ri to‘rtburchak, $AC = 20$ cm, $BP = 12$ cm, $BP \perp AC$ (10- rasm).

Topish kerak: S_{ABCD}.

Yechish. 1) $S_{ABC} = 0{,}5AC \cdot BP =$
$= 0{,}5 \cdot 20 \cdot 12 = 120$ (cm^2).

2) $S_{ABCD} = 2 \cdot S_{ABC} = 2 \cdot 120 = 240$ (cm^2).

Javob: $S_{ABCD} = 240$ cm^2.

Savol, masala va topshiriqlar

1. 1) Uchburchakning yuzi nimaga teng?
2) To‘g‘ri burchakli uchburchakning yuzi qanday hisoblanadi?
3) Tomonlariga ko‘ra uchburchakning yuzi qanday hisoblanadi?

2. To‘g‘ri burchakli uchburchakning katetlari: 1) 4 cm va 7 cm; 2) 1,2 dm va 25 cm. Shu to‘g‘ri burchakli uchburchakning yuzini toping.

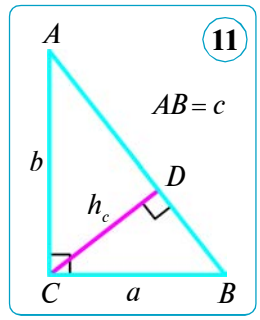
3. Bir uchburchakning asosi 20 cm, balandligi 8 cm. Ikkinchi uchburchakning asosi 40 cm. Uchburchaklar tengdosh bo‘lishi uchun ikkinchi uchburchakning balandligi qanday bo‘lishi kerak?

4. ABC uchburchakda $AB = 5AC$. Uchburchakning B va C uchlaridan o‘tkazilgan balandliklarining nisbati nimaga teng?

5. Noma’lum miqdorlarni toping. a – uchburchakning asosi, h – asosiga o‘tkazilgan balandlik, S – uchburchakning yuzi.

	1	2	3	4	5	6
a	69 cm	0,8 dm	?	0,25 m	?	0,9 m
h_a	0,5 m	?	20 dm	100 cm	4,8 cm	?
S	?	4 cm^2	2000 cm^2	?	9,6 mm^2	36 dm^2

6. Katetlarning (a va b) ko‘paytmasi gipotenuza (c) bilan to‘g‘ri burchak uchidan gipotenuzaga tushirilgan balandlikning (h_c) ko‘paytmasiga teng (11- rasm).



Yechish. Agar katetlardan birini asos uchun qabul qilsak, u holda ikkinchisi balandlik bo‘ladi. Shuning uchun, to‘g‘i burchakli uchburchakning yuzi katetlar ko‘paytmasining yarmiga teng bo‘ladi:

$S = \frac{1}{2}ab$, bundan $ab = 2S$; $S = \frac{1}{2}ch_c$, bundan $ch_c = 2S$.

Demak, $ab = ch_c$ ekan. Shuni isbotlash talab qilingan edi.

Uchburchakning katetlari: 1) 12 cm va 16 cm; 2) 5 cm va 12 cm ga teng. c (Pifagor teoremasiga ko‘ra) va h_c ($ab = ch_c$ ga ko‘ra)ni toping.

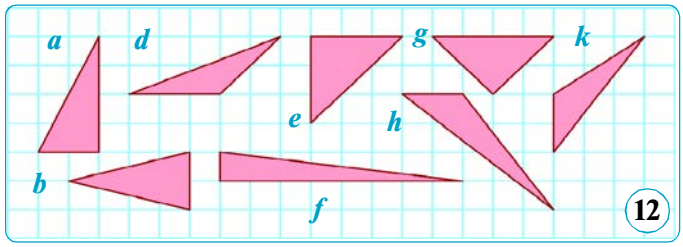
7. 12- rasmdagi tengdosh uchburchaklarni ko‘rsating. Javobingizni asoslang.

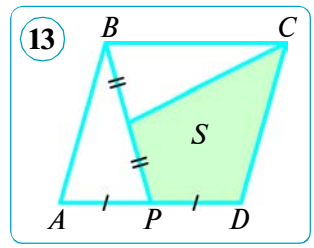
8. Tomonlari: 1) 39 cm, 42 cm, 45 cm; 2) 35 cm, 29 cm, 8 cm; 3) 20 cm, 20 cm, 32 cm ga teng bo‘lgan uchburchakning yuzini toping.

9. 13- rasmdagi S shaklning yuzi parallelogramm yuzining qanday qismini tashkil etadi?

10. Uchburchakning yuzi 150 cm^2 ga teng. Uchburchakning balandliklari 15 cm, 12 cm va 20 cm ga teng bo‘lsa, uning perimetrini toping.

11. Uchburchakning ikki tomoni 5 dm va 6 dm, ular orasidagi burchak 30°. Uchburchakning yuzini toping. Masalani ikki usul bilan yeching.





49–50. ROMB VA TRAPETSIYANING YUZI

1. Rombning yuzi. **Romb** – tomonlari teng bo‘lgan parallelogrammdir. Tomoni a va balandligi h_a bo‘lgan rombning yuzi

$$S = ah_a$$

formula bo‘yicha hisoblanadi.

Bizga ma‘lumki, rombning hamma balandliklari o‘zaro teng.

Bundan tashqari, rombning yuzini diagonallari orqali ham hisoblash mumkin.

Teorema.

Rombning yuzi uning diagonallari ko‘paytmasining yarmiga teng:

$$S = \frac{1}{2} d_1 \cdot d_2 ,$$

bunda d_1 va d_2 – rombning diagonallari.

Isbot. Ma’lumki, rombning AC diagonali uni ikkita o‘zaro teng yonli uchburchakka ajratadi (1- rasm). Ikkinchi diagonal esa birinchisiga perpendikular bo‘lib, hosil bo‘lgan uchburchaklar balandliklari yig‘indisiga teng bo‘ladi. Shuning uchun rombning yuzi:

$$S = S_{ABCD} = S_{ABC} + S_{ADC} = \frac{1}{2} AC \cdot BO + \frac{1}{2} AC \cdot DO = \frac{1}{2} AC \cdot (BO + DO) =$$

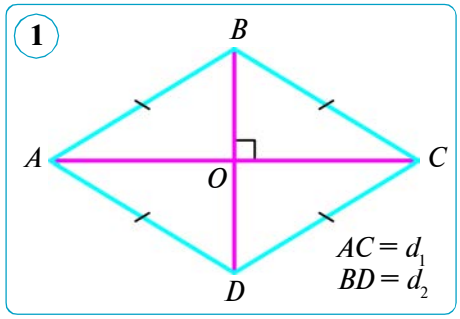
$$= \frac{1}{2} \underline{AC} \cdot \underline{\underline{BD}} = \frac{1}{2} \underline{d_1} \cdot \underline{\underline{d_2}} .$$

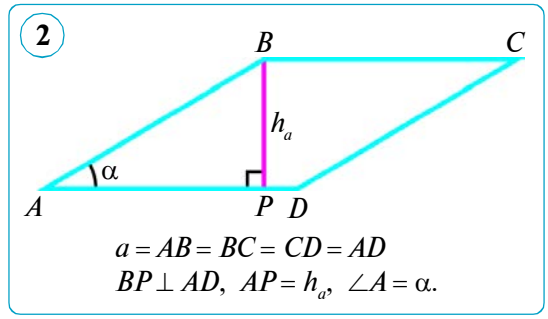
Demak, $S = \frac{1}{2} d_1 \cdot d_2$. Teorema isbotlandi.

1- masala. $ABCD$ rombning tomoni a ga, o‘tkir burchagi esa α ga teng. Shu rombning yuzini toping. $\alpha = 30^\circ$ da uning yuzini toping.

Yechish. 1) $ABCD$ rombda $AB = BC = CD = AD = a$, $\angle A = \alpha$ bo‘lsin. $BP \perp AD$ ni o‘tkazamiz (2- rasm). U holda h_a balandlik to‘g‘ri burchakli ABP uchburchakning α o‘tkir burchagi qarshisida yotgan katet bo‘ladi. h_a ni α burchakning sinusi bilan ifodalaymiz: $h_a = a \sin\alpha$. Rombning yuzini hisoblash formulasi $S = ah_a$ ga h_a ning bu ifodasini qo‘yib, ushbu formulani hosil qilamiz:

$$S = a^2 \sin\alpha .$$





2) Rombning yuzini $S = a^2 \sin\alpha$ formuladan foydalanib topamiz:

$$S = a^2 \sin 30^\circ = a^2 \cdot 0{,}5 = 0{,}5a^2 \text{ (kv. birl.)}.$$

Javob: $S = 0{,}5a^2$ kv. birl.

2- masala. Rombning diagonallaridan biri ikkinchisidan 1,5 marta katta, yuzi esa 27 cm^2 ga teng. Shu rombning diagonallarini toping.

Berilgan: $ABCD$ – romb; $S_{ABCD} = 27$ cm^2; $AC = 1{,}5BD$ (1- rasmga q.)

Topish kerak: AC, BD.

Yechish. $BD = x$ cm bo‘lsin, u holda $AC = 1{,}5x$ cm bo‘ladi.

$S_{ABCD} = \frac{1}{2} AC \cdot BD$, bunga belgilashlarni qo‘yamiz: $27 = \frac{1}{2} \cdot 1{,}5x \cdot x$.

U holda $x^2 = 36$ bo‘ladi, bundan $x = 6$ (cm). Shunday qilib,

$$BD = 6 \text{ cm}, \quad AC = 1{,}5 \cdot 6 = 9 \text{ (cm)}.$$

Javob: 9 cm, 6 cm.

2. Trapetsiyaning yuzi. Har qanday ko‘pburchakni diagonallar o‘tkazish yo‘li bilan uchburchaklarga ajratish mumkin. Ixtiyoriy ko‘pburchakning yuzini hisoblash uchun u avval uchburchaklarga ajratiladi, so‘ngra uchburchaklar yuzi hisoblanadi. Ko‘pburchak yuzi esa uni tashkil qilgan bir-birini qoplamaydigan uchburchaklar yuzlari yig‘indisiga teng bo‘ladi. Trapetsiya yuzlarini hisoblashda shu usuldan foydalanamiz.

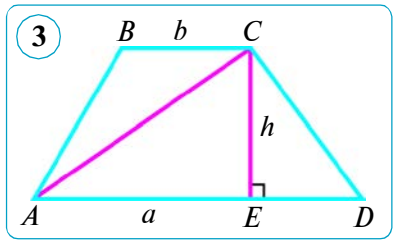
Teorema.

Trapetsiyaning yuzi uning asoslari yig‘indisining yarmi bilan balandligi ko‘paytmasiga teng:

$$S = \frac{a+b}{2} \cdot h,$$

bunda a va b – trapetsiyaning asoslari, h – trapetsiyaning balandligi.

Isbot. Asoslari $AD = a$, $BC = b$ va balandligi $CE = h$ ($CE \perp AD$) bo‘lgan $ABCD$ trapetsiyani qaraylik (3- rasm).



Trapetsiyada AC diagonalni o‘tkazamiz. Bunda $ABCD$ trapetsiya ABC va ACD uchburchaklarga ajraladi. Trapetsiya yuzi esa bu uchburchaklar yuzlari yig‘indisiga teng bo‘ladi.

Parallel to‘g‘ri chiziqlar orasidagi masofa o‘zgarmas bo‘lgani uchun ABC va ACD uchburchaklarning balandliklari o‘zaro teng.

Bundan, $S_{ABC} = \frac{1}{2} BC \cdot CE = \frac{1}{2} b \cdot h$ va $S_{ACD} = \frac{1}{2} AD \cdot CE = \frac{1}{2} a \cdot h$.

Trapetsiyaning yuzi $S = S_{ABC} + S_{ACD}$, ya’ni:

$$S = \frac{1}{2} a \cdot h + \frac{1}{2} b \cdot h \quad \text{yoki} \quad S = \frac{a+b}{2} \cdot h.$$

Teorema isbotlandi.

Natija. *Trapetsiyaning yuzi o‘rta chizig‘i bilan balandligining ko‘paytmasiga teng.*

Ushbu natija trapetsiyaning o‘rta chizig‘i asoslari yig‘indisining yarmiga tengligidan kelib chiqadi.

3- masala. Trapetsiyaning asoslari 15 cm va 30 cm ga, yuzi 225 cm^2 ga teng. Shu trapetsiyaning balandligini toping.

Yechish. Trapetsiyaning o‘rta chizig‘i:

$$\frac{a+b}{2} = \frac{15+30}{2} = \frac{45}{2} = 22,5 \ \text{(cm)}.$$

Demak, trapetsiyaning balandligi:

$$h = S_{\text{tr.}} : \frac{a+b}{2} = 225 : 22,5 = 10 \ \text{(cm)}.$$

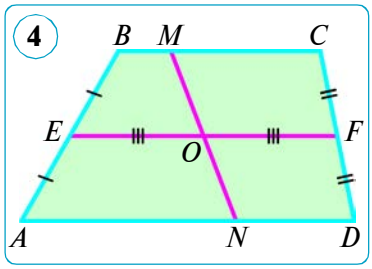
Javob: $h = 10$ cm.

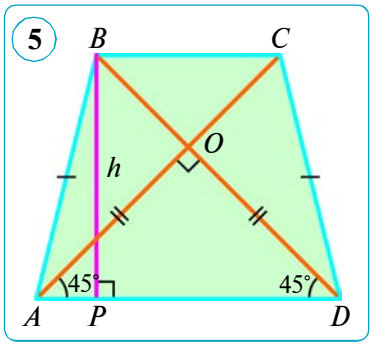
4- masala. Trapetsiyaning o‘rta chizig‘i o‘rtasidan o‘tib, asoslarini kesuvchi to‘g‘ri chiziq bu trapetsiyani ikkita tengdosh bo‘lakka bo‘lishini isbot qiling.

Yechish. $ABCD$ – berilgan trapetsiya ($AD \| BC$), EF – uning o‘rta chizig‘i, MN esa o‘rta chiziqning o‘rtasi O orqali o‘tuvchi hamda asoslarini M va N nuqtalarda kesuvchi to‘g‘ri chiziq bo‘lsin (4- rasm). $ABMN$ va $MNDC$ trapetsiyalar mos ravishda teng EO va OF o‘rta chiziq hamda berilgan trapetsiyaning balandligiga teng balandlikka ega. Demak, bu trapetsiyalarning yuzlari teng, ya’ni ular tengdoshdir:

$$S_{ABMN} = S_{MNDC}.$$

Shuni isbotlash talab qilingan edi.





5- masala. Teng yonli trapetsiyaning diagonallari o‘zaro perpendikular bo‘lsa, u holda trapetsiyaning balandligi uning o‘rta chizig‘iga, yuzi esa balandligining kvadratiga teng bo‘ladi. Shuni isbot qiling.

Berilgan: $ABCD$ trapetsiya – teng yonli ($AB = DC$), $AC \perp BD$, $AD = a$, $BC = b$ bo‘lsin (5- rasm).

Isbot qilish kerak:

$$1)\ h = \frac{a+b}{2}; \qquad 2)\ S_{\text{tr.}} = h^2 = \left(\frac{a+b}{2}\right)^2.$$

Yechish. 1) $\triangle AOD$ – teng yonli va to‘g‘ri burchakli, shuning uchun $\angle ADO = 45°$.

2) B uchidan $BP \perp AD$ ni o‘tkazamiz. Hosil bo‘lgan BPD uchburchak ham teng yonli va to‘g‘ri burchakli, chunki $\angle ADO = 45°$ va demak, $\angle PBD = 45°$. Bundan: $DP = BP$. Bizga

ma'lumki, teng yonli trapetsiyaning kichik asosi uchidan o'tkazilgan balandlikning xossasiga ko'ra:

$$BP = DP = \frac{a+b}{2}.$$

3) $S_{\text{tr.}} = \frac{a+b}{2} \cdot h = h \cdot h = h^2$ yoki $S_{\text{tr.}} = \frac{a+b}{2} \cdot h = \frac{a+b}{2} \cdot \frac{a+b}{2} = \left(\frac{a+b}{2}\right)^2$.

Shunday qilib, teng yonli trapetsiyaning diagonallari o'zaro perpendikular bo'lganda uning balandligi o'rta chizig'iga, yuzi esa balandligining kvadratiga tengligi to'la isbotlandi.

Savol, masala va topshiriqlar

1. 1) Rombning yuzi tomoni va balandligi bo'yicha qanday topiladi?
2) Rombning yuzi diagonallari orqali qanday topiladi? Uni ifodalang.
3) Trapetsiyaning yuzi nimaga teng?

2. Rombning yuzi 40 cm², balandligi esa 5 cm ga teng. Shu rombning perimetrini toping.

3. Agar rombning: 1) balandligi 16 cm, o'tkir burchagi 30° ga; 2) tomoni 1,8 dm, o'tkir burchagi 30° ga teng bo'lsa, uning yuzini toping.

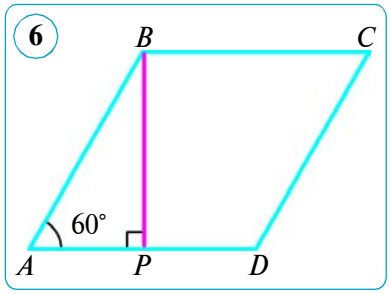
4. Rombning yuzi 60 cm², diagonallaridan biri 10 cm ga teng. Shu rombning ikkinchi diagonalini toping.

5. Rombning yuzi 30 cm², perimetri esa 24 cm ga teng. Shu rombning balandligini toping.

6. *Berilgan:* $ABCD$ — romb. $\angle BAD = 60°$, $BP \perp AD$, $BP = 12$ cm (6- rasm).

Topish kerak: S.

Yechish. To'g'ri burchakli BPA uchburchakni ko'rib chiqamiz. O'tkir burchak sinusi ta'rifiga ko'ra: $\sin A = \frac{BP}{AB}$. Bunga berilganlarni qo'yib, AB ni topamiz:



$$\sin A = \frac{BP}{AB} \Rightarrow AB = \frac{BP}{\sin A} = \frac{12}{\sin 60°} = 12 : \frac{\sqrt{3}}{2} = 12 \cdot \frac{2}{\sqrt{3}} = \frac{24}{\sqrt{3}} \text{ (cm)}.$$

Tomoni va o'tkir burchagiga ko'ra rombning yuzini topish formulasiga

$$AB = a = \frac{24}{\sqrt{3}}, \quad \sin 60° = \frac{\sqrt{3}}{2}$$

qiymatlarni qo'yib, quyidagini topamiz:

$$S = a^2 \cdot \sin 60° = \left(\frac{24}{\sqrt{3}}\right)^2 \cdot \frac{\sqrt{3}}{2} = \frac{576}{3} \cdot \frac{\sqrt{3}}{2} = 96\sqrt{3} \text{ (cm}^2\text{)}.$$

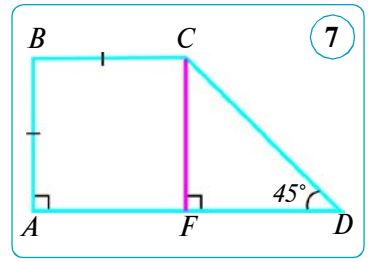
Javob: $96\sqrt{3}$ cm².

7. Diagonallari: 1) 1,5 dm va 1,8 dm; 2) 24 cm va 15 cm; 3) 3,2 cm va 0,5 dm bo‘lgan rombning yuzini toping.

8. 1) Trapetsiyaning asoslari 11 cm va 18 cm ga, balandligi esa 6 cm ga teng. Shu trapetsiyaning yuzini toping.

2) Trapetsiyaning asosi 26 cm, balandligi 10 cm, yuzi esa 200 cm^2. Shu trapetsiyaning ikkinchi asosini toping.

9. $ABCD$ to‘g‘ri burchakli trapetsiyada $AB = BC = 18$ cm, $\angle D = 45^\circ$ (7- rasm). Trapetsiyaning yuzini toping. Bo‘sh joylarga mos javoblarni yozing.



Yechish. $CF \perp AD$ ni o‘tkazamiz. 1) $ABCF$ – kvadrat, chunki $ABCF$ to‘rtburchakning qo‘shni tomonlari AB va ..., shuning uchun $AF = CF = ...$ (cm).

2) $\triangle CFD$ – to‘g‘ri burchakli, yasashga ko‘ra $\angle F = 90^\circ$ va shartga ko‘ra $\angle D = 45^\circ$, shuning uchun $\angle DCF = ...^\circ$ va demak, $\triangle CFD$ – ... va $DF = ... = ...$ (cm).

3) $AD = AF + ... = ... + ... = ...$ (cm) va $S_{ABCD} = ... \cdot ... = ... \cdot ... = ...$ (cm^2).

Javob: ... cm^2.

10. Romb burchaklarining nisbati 1 : 5 ga, tomoni esa a ga teng. Shu rombning yuzini toping.

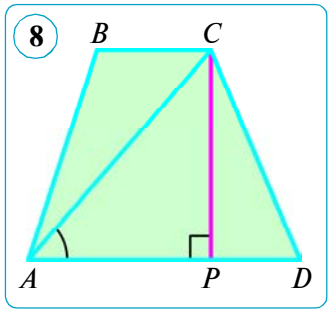
11. $ABCD$ trapetsiyada: $AD = 20\sqrt{2}$ cm, $BC = 10\sqrt{2}$ cm, $AC = 24$ cm, $\angle CAD = 45^\circ$ (8- rasm). Trapetsiyaning yuzini toping.

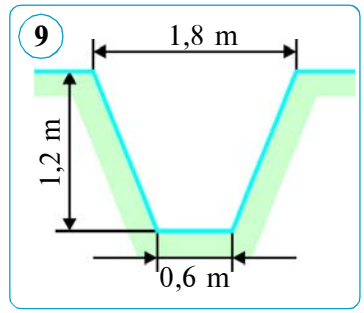
12. Diagonallari: 1) 3,5 dm va 1,4 dm; 2) 28 cm va 17 cm; 3) 4,2 cm va 1,5 dm bo‘lgan rombning yuzini toping.

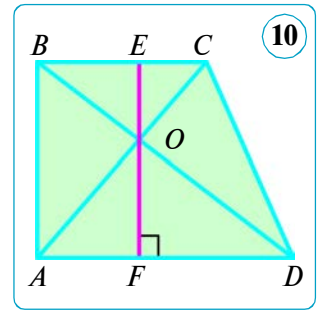
13. Teng yonli trapetsiyaning diagonallari o‘zaro perpendikular va balandligi 5 cm ga teng. Shu trapetsiyaning yuzini toping.

14. Teng yonli trapetsiya shaklidagi chuqurlik ko‘ndalang kesimining yuzini toping (9- rasm).

15. Trapetsiyaning asoslari 16 cm va 12 cm. Diagonallarining kesishish nuqtasidan asoslarigacha bo‘lgan masofalar 6 cm va 4 cm ga teng (10- rasm). Shu trapetsiyaning yuzini toping.



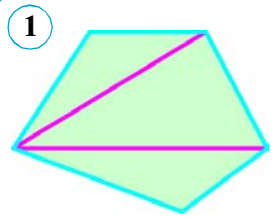




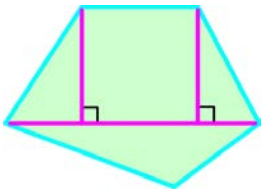
51. KO‘PBURCHAKNING YUZI

Ko‘pburchakning yuzini hisoblash uchun uni o‘zaro kesishmaydigan, ya’ni umumiy ichki nuqtalari bo‘lmagan uchburchaklarga ajratish va ularning yuzlari yig‘indisini topish mumkin. Qavariq ko‘pburchakni uchburchaklarga ajratish uchun, masalan, uning bir uchidan diagonallar o‘tkazish yetarli (1- *a* rasm). Ba’zan boshqacha ajratishlardan foydalanish qulay (1- *b* rasm).

1



a



b

1- masala. $ABCDE$ ko‘pburchakda $BD \parallel AE$, $CP \perp AE$ ekani ma’lum (2-rasm)

$S_{ABCDE} = 0{,}5(BD \cdot CP + AE \cdot OP)$ ekanini isbotlang.

Isbot. Berilgan shaklning trapetsiya va uchburchakdan tashkil topganini ko‘rish qiyin emas. Shu sababli yuzning xossasiga ko‘ra:

$$S_{ABCDE} = S_{BCD} + S_{ABDE} = 0{,}5BD \cdot CO + 0{,}5(AE + BD) \cdot OP =$$
$$= 0{,}5(\underline{BD} \cdot \underline{CO} + AE \cdot OP + \underline{BD} \cdot \underline{OP}) = 0{,}5(BD \cdot (CO + OP) +$$
$$+ AE \cdot OP) = 0{,}5(BD \cdot CP + AE \cdot OP).$$

Demak, $S_{ABCDE} = 0{,}5(BD \cdot CP + AE \cdot OP)$.

2- masala. AC va BD – $ABCD$ to‘rtburchakning diagonallari, O – diagonallarining kesishish nuqtasi (3- rasm). Agar $S_{AOB} = S_1$, $S_{BOC} = S_2$, $S_{COD} = S_3$ va $S_{AOD} = S_4$ bo‘lsa, $S_1 \cdot S_3 = S_2 \cdot S_4$ ekanini isbotlang.

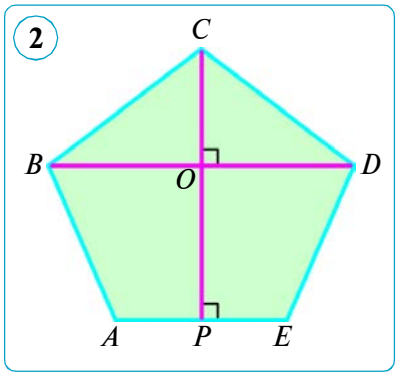
Isbot. 1) $AE \perp BD$ va $CF \perp BD$ larni o‘tkazamiz.

2) $\dfrac{S_1}{S_4} = \dfrac{0{,}5OB \cdot AE}{0{,}5OD \cdot AE} = \dfrac{OB}{OD}$ (1) va $\dfrac{S_2}{S_3} = \dfrac{0{,}5OB \cdot CF}{0{,}5OD \cdot CF} = \dfrac{OB}{OD}$ (2).

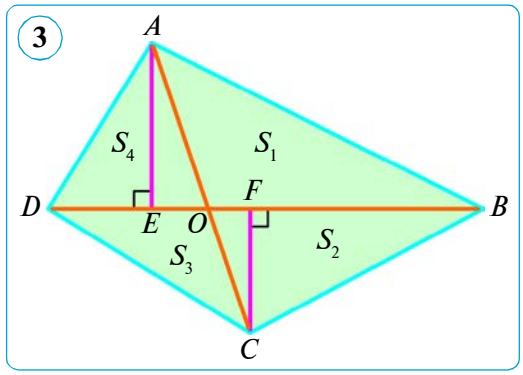
3) (1) va (2) dan topamiz:

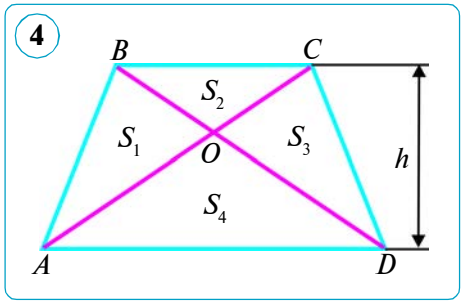
$$\frac{S_1}{S_4} = \frac{S_2}{S_3} \Rightarrow S_1 \cdot S_3 = S_2 \cdot S_4 .$$

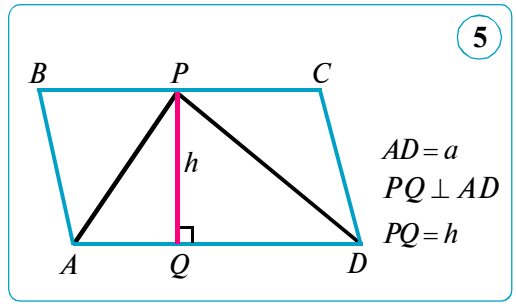
2



3







3- masala. BC va AD – $ABCD$ trapetsiyaning asoslari, O – AC va BD diagonallarining kesishish nuqtasi (4- rasm). $AD = a$, $BC = b$.

$S_{AOB} = S_1$, $S_{BOC} = S_2$, $S_{COD} = S_3$ va $S_{AOD} = S_4$ bo‘lsa, quyidagini isbot qiling:

1) $S_1 = S_3 = \sqrt{S_2 \cdot S_4}$; 2) $S_{\text{tr.}} = \left(\sqrt{S_2} + \sqrt{S_4}\right)^2$.

Isbot. 1) $S_{ABC} = S_{DBC} = \frac{1}{2}bh \Rightarrow S_1 + \underline{S_2} = S_3 + \underline{S_2} \Rightarrow S_1 = S_3$.

2) Bizga $S_1 \cdot S_3 = S_2 \cdot S_4$ ekani ma’lum. $S_1 = S_3$ ni nazarga olsak, $S_1 = S_3 = \sqrt{S_2 \cdot S_4}$ kelib chiqadi. Masalaning birinchi qismi isbotlandi.

3) Trapetsiyaning yuzi to‘rtta uchburchak yuzlarining yig‘indisiga teng ekanligi va yuqoridagi natijalarni e’tiborga olib, quyidagiga ega bo‘lamiz:

$$S_{\text{tr.}} = \underline{S_1} + S_2 + \underline{S_3} + S_4 = S_2 + 2S_1 + S_4 =$$
$$= \left(\sqrt{S_2}\right)^2 + 2\sqrt{S_2 \cdot S_4} + \left(\sqrt{S_4}\right)^2 = \left(\sqrt{S_2} + \sqrt{S_4}\right)^2.$$

Demak, $S_{\text{tr.}} = \left(\sqrt{S_2} + \sqrt{S_4}\right)^2$. Masalaning ikkinchi qismi isbotlandi.

4- masala. Parallelogramm bilan umumiy asos va umumiy balandlikka ega bo‘lgan uchburchakning yuzi parallelogramm yuzining yarmiga teng.

Isbot. AD asos va h balandlik $ABCD$ parallelogramm va APD uchburchak uchun umumiy (5- rasm). $S_{APD} = 0{,}5S_{ABCD}$ ekanini isbotlaymiz.

$S_{ABCD} = ah$ (1) va $S_{APD} = 0{,}5ah$ (2) ekani ma’lum. (2) tenglikdagi ah o‘rniga S_{ABCD} ni qo‘yib, topamiz:

$$S_{APD} = 0{,}5ah = 0{,}5S_{ABCD}.$$

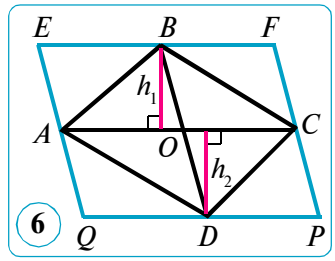
Eslatma! Yuqorida keltirilgan masalani quyidagicha ham o‘qish mumkin:

uchburchak bilan umumiy asos va umumiy balandlikka ega bo‘lgan parallelogrammning yuzi uchburchak yuzidan ikki marta katta.

5- masala. Qavariq to‘rtburchakning uchlari orqali uning diagonallariga parallel to‘g‘ri chiziqlar o‘tkazilsa, u holda hosil bo‘lgan parallelogrammning yuzi berilgan to‘rtburchak yuzidan ikki marta katta bo‘lishini isbotlang.

Isbot. $ABCD$ – berilgan qavariq to‘rtburchak, O – AC va BD diagonallarning kesishish nuqtasi, h_1 va h_2 – to‘rtburchakning B va D uchlaridan AC diagonalga tushirilgan balandliklar; $EFPQ$ – to‘rtburchakning uchlari orqali

uning diagonallariga parallel o‘tkazilgan to‘g‘ri chiziqlar kesishishidan hosil bo‘lgan parallelogramm (6- rasm).



6

$S_{EFPQ} = 2S_{ABCD}$ ekanini isbotlaymiz.

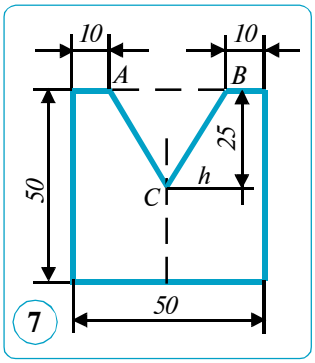
Yasashga ko‘ra, parallelogrammning EF va QP tomonlari AC diagonalga parallel hamda teng. Shuning uchun, AC diagonal hosil bo‘lgan $EFPQ$ parallelogrammni ikkita – $AEFC$ va $ACPQ$ parallelogrammlarga ajratadi.

Yuqorida keltirilgan eslatmadagi xulosani qo‘llab, $S_{EFPQ} = 2S_{ABCD}$ tenglikni isbotlaymiz: $S_{EFPQ} = S_{AEFC} + S_{ACPQ} = 2S_{ABC} + 2S_{ADC} = 2(S_{ABC} + S_{ADC}) = 2S_{ABCD}$.

Demak, $S_{EFPQ} = 2S_{ABCD}$.

Savol, masala va topshiriqlar

1. 7- rasmdagi shaklning yuzini toping.



7

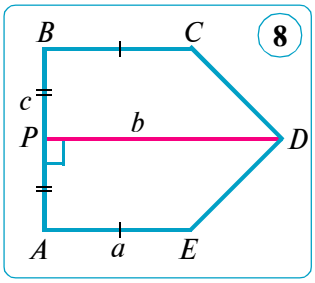
Yechish. Rasmda tasvirlangan shaklning yuzini A va B nuqtalarni tutashtirib, uni kvadratga to‘ldirish orqali topish qulaydir. Berilgan shaklning yuzi hosil bo‘lgan kvadrat yuzi bilan ABC uchburchak yuzining ayirmasiga teng:

$S = S_{\text{kv.}} - S_{ABC} = \ldots^2 - 0{,}5 \cdot (50 - 2 \cdot 10) \cdot \ldots =$
$= \ldots - 375 = \ldots$ (kv. birl.).

Nuqtalar o‘rniga mos sonlarni qo‘ying.

Javob: ... kv. birl.

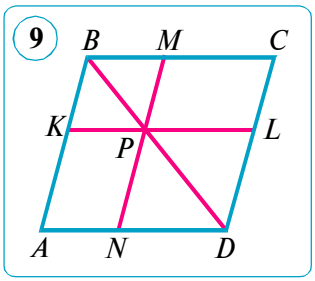
2. 8- rasmdagi shakl yuzini hisoblash uchun formula keltirib chiqaring. Bunda $AE \parallel BC \parallel PD$, $AE = BC$, $AP = PB$, $PD \perp AB$.



3. *Berilgan:* $ABCD$ – to‘g‘ri to‘rtburchak, $AB = 12$ cm, $AD = 16$ cm; E, F, P va Q nuqtalar – mos tomonlarning o‘rtalari.

Topish kerak: $S_{EFCPQA.}$.

4. *Berilgan:* $ABCD$ – parallelogramm, $P \in BD$, $KL \parallel BC$, $MN \parallel AB$ (9- rasm).



Isbot qilish kerak: $S_{AKPN} = S_{PMCL}$.

5. AC va BD – $ABCD$ to‘rtburchakning diagonallari, O – ularning kesishish nuqtasi. $S_{AOD} = 12$, $S_{BOC} = 8$, $S_{AOB} = 6$. S_{COD} ni toping.

6. To‘g‘ri to‘rtburchak shaklidagi yer maydonining yuzi 400 ha. Agar: 1) maydonning bo‘yi 10 km bo‘lsa; 2) maydon kvadrat shaklida bo‘lsa, uning perimetri qanday bo‘ladi?

52. AMALIY MASHQ VA TATBIQ

I. Tadqiqot uchun masalalar.

1- masala. To‘g‘ri to‘rtburchakning tomonlari natural son va perimetri 4 ga karrali bo‘lgan masalani ko‘rib chiqamiz.

Perimetri 72 cm ga teng va tomonlari natural son bo‘lgan barcha to‘g‘ri to‘rtburchaklar ichidan eng katta yuzaga ega bo‘lganini toping. U qanday shakl bo‘ladi? Xulosa chiqaring.

Yechish. To‘g‘ri to‘rtburchakda: $P = 2 \cdot (a + b) = 72$ cm – perimetr, $p = a + b = 36$ cm – yarim perimetr, ya’ni qo‘shni tomonlar yig‘indisi. a va b ning qiymatlari ma’lum bo‘lgandagina $S = a \cdot b$ ni hisoblay olamiz. Masalada qo‘yilgan savolga javob berish uchun to‘g‘ri to‘rtburchakning qo‘shni tomonlarini topishga harakat qilamiz.

Buning uchun 36 ni ikkita natural sonning yig‘indisi ko‘rinishida ifodalaymiz:

$$a + b = 36 = 1 + 35 = 2 + 34 = 3 + 33 = \ldots = 33 + 3 = 34 + 2 = 35 + 1.$$

Bundan ko‘rinadiki, qo‘shni tomonlari yig‘indisi 36 sm ga teng bo‘lgan 35 ta turli to‘g‘ri to‘rtburchak mavjud. Ma’lumotlarni jadvalga kiritib, ularni tahlil qilamiz va xulosa chiqaramiz:

a cm	**1**	2	...	**17**	**18**	**19**	20	...	**34**	**35**
b cm	**35**	34	...	**19**	**18**	**17**	16	...	**2**	**1**
$(a + b)$ cm	**36**	**36**		**36**	**36**	**36**	**36**	...	**36**	**36**
$S = a \cdot b$ cm^2	**35**	68	...	**323**	**324**	**323**	320	...	**68**	**35**

Jadvaldan ko‘rinadiki, eng kichik yuzga $a = 1$ cm va $b = 35$ cm yoki $a = 35$ cm va $b = 1$ cm bo‘lganda, eng katta yuzga esa $a = b = 18$ cm – tomoni 18 cm ga teng kvadrat bo‘lgandagina erishiladi. Qolgan to‘g‘ri to‘rtburchaklarning perimetrlari 72 cm bo‘lsa-da, ammo yuzlari

$$18 \cdot 18 = 324 \text{ (cm}^2\text{)}$$

dan kichik bo‘ladi.

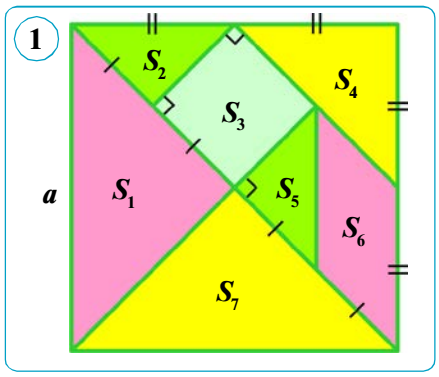
Jadvalni tahlil qilish natijasida quyidagi xulosalarga kelamiz.

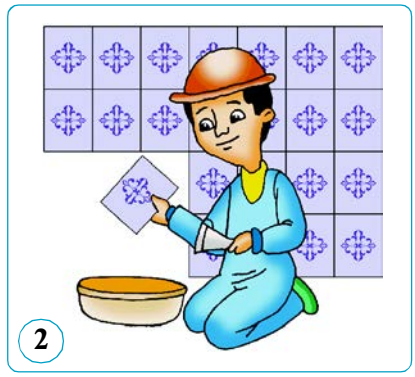
1- xulosa. Agar to‘g‘ri to‘rtburchakning tomonlari natural son va perimetri 4 ga karrali bo‘lsa, eng katta yuza quyidagi formula bo‘yicha topiladi:

$$S_{\max} = \left(\frac{P}{4}\right)^2 \text{ kv. birl.}$$

2- xulosa. Agar to‘g‘ri to‘rtburchakning tomonlari natural son va perimetri 2 ga karrali bo‘lsa, u holda perimetrlari teng bo‘lgan barcha to‘g‘ri to‘rtburchaklar ichidan tomonlaridan biri 1 ga va ikkinchi tomoni esa 1 ni yarim perimetrga to‘ldiruvchi son bo‘lgandagina eng kichik yuzga ega bo‘ladi.

3- xulosa. To‘g‘ri to‘rtburchakning qo‘shni tomonlari uzunliklari bir-biriga yaqinlashgan sari yuza ortib boradi.





2- masala. Xitoycha «tangram» o‘yinida kvadrat 1- rasmda ko‘rsatilgandek uchburchaklar va to‘rtburchaklarga ajratilgan. Bulardan turlicha shakllar yasash mumkin. Agar kvadratning tomoni 8 cm ga teng bo‘lsa, bo‘lingan shakllarning yuzlarini toping.

Yechish. $a = 8$ cm – kvadratning tomoni. $S = a^2 = 8^2 = 64$ (cm^2) – berilgan kvadratning yuzi. Endi shakldagi bo‘lakchalarning yuzlarini topamiz.

1) S_1 va S_7 – kvadrat yuzining to‘rtdan biriga teng. Demak,

$$S_1 = S_7 = S : 4 = 64 : 4 = 16 \ \ (\text{cm}^2).$$

2) Teng yonli to‘g‘ri burchakli uchburchakning yuzi gipotenuza kvadratining to‘rtdan biriga teng. Demak,

$$S_2 = S_5 = 0{,}25 \cdot (a : 2)^2 = 0{,}25 \cdot 4^2 = 0{,}25 \cdot 16 = 4 \ \ (\text{cm}^2).$$

3) S_3 kvadratning yuzi ikkita S_2 uchburchak yuzlari yig‘indisiga teng. Demak, $S_3 = 2S_2 = 2 \cdot 4 = 8$ (cm^2).

4) S_4 uchburchakning katetlari berilgan kvadrat tomonining yarmiga teng, ya’ni $a : 2 = 8 : 2 = 4$ (cm). Teng yonli uchburchakning yuzi kateti kvadratining yarmiga teng, ya’ni $S_4 = 0{,}5 \cdot 4^2 = 0{,}5 \cdot 16 = 8$ (cm^2).

5) Asoslari va balandliklari teng bo‘lgan kvadrat bilan parallelogramm tengdosh, shuning uchun $S_6 = S_3 = 2 \cdot 4 = 8$ (cm^2).

Javob: $S_1 = S_7 = 16$ cm^2; $S_2 = S_5 = 4$ cm^2; $S_3 = S_4 = S_6 = 8$ cm^2.

3- masala. Usta bo‘yi 2,25 m va eni 1,8 m bo‘lgan to‘g‘ri to‘rtburchak shaklidagi devor qismini kafel bilan qoplamoqchi. Buning uchun unga tomoni 15 cm li kvadrat shaklidagi kafeldan nechta kerak bo‘ladi (2- rasm)?

Yechish. 1) Qoplanmoqchi bo‘lgan devorning yuzini topamiz va uni kvadrat santimetrda ifodalaymiz:

$$2{,}25 \cdot 1{,}8 = 4{,}05 \ (\text{m}^2) = 4{,}05 \cdot 10000 \ \text{cm}^2 = 40500 \ \text{cm}^2.$$

2) Bir dona kafelning yuzini topamiz: $a^2 = 15^2 = 225$ (cm^2).

3) To‘g‘ri to‘rtburchak shaklidagi devorni qoplash uchun nechta kafel kerak bo‘lishini topamiz:

$$40500 : 225 = 180 \ \text{(ta)}.$$

Javob: 180 ta kafel.

Quyidagi masalani yechishni o‘zingizga havola qilamiz.

4- masala. Tomoni 4 m ga teng bo‘lgan kvadrat shaklidagi yo‘lakchani qoplash uchun tomoni 20 cm li kafeldan nechta kerak bo‘ladi?

AMALIY KOMPETENSIYANI RIVOJLANTIRUVCHI QO‘SHIMCHA MATERIALLAR

KATAKLI QOG‘OZDA YUZLARNI HISOBLASH

Katakli qog‘ozda berilgan qavariq va qavariq bo‘lmagan ko‘pburchaklarning yuzini hisoblash uchun «**Pik formulasi**» deb ataluvchi formulani keltiramiz. Har bir katak tomoni uzunligi 1 cm bo‘lsin. Katakli qog‘ozdagi to‘g‘ri chiziqlar kesishish nuqtalarini – birlik kvadrat uchlarini **tugun nuqtalar** deb ataymiz. U holda ko‘pburchakning yuzi quyidagi formula bo‘yicha hisoblanadi:

$$S = \frac{M}{2} + N - 1.$$

Bu formulada M – ko‘pburchak chegarasida yotgan tugun nuqtalar soni, N – ko‘pburchak ichida yotgan tugun nuqtalar soni.

Bu formulani ko‘pburchakning uchlari tugun nuqtalarda bo‘lgan har qanday ko‘pburchak uchun qo‘llasa bo‘ladi.

1- masala. 1- rasmdagi shakl yuzini hisoblang.

Yechish. 1- usul. 1) Barcha to‘liq kvadratlar soni 59 ta bo‘lib, ularning yuzi 59 cm^2; kvadratning yarmiga teng bo‘lgan uchburchaklar soni 16 ta bo‘lib, ularning yuzi $16 : 2 = 8$ (cm^2); bitta asosi 2 cm, balandligi 3 cm ga teng uchburchak bor, uning yuzi 3 cm^2 ga teng.

Shunday qilib, berilgan ko‘pburchakning yuzi:

$$S = 59 + 8 + 3 = 71 \ (cm^2).$$

2- usul. Shu javobning Pik formulasi yordamida qanday topilishini ko‘rib chiqamiz. Tugun nuqtalarni belgilab olamiz.

1) Shakl ichida yotgan tugun nuqtalarni (qora rangda belgilangan) sanaymiz: ular 50 ta, ya’ni $N = 50$.

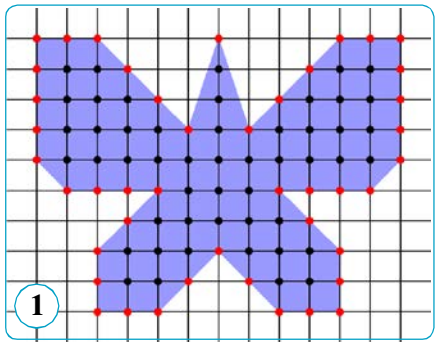
2) Shakl tomonlarida yotgan tugun nuqtalarni (qizil rangda belgilangan) sanaymiz: ular 44 ta, ya’ni $M = 44$. Pik formulasini qo‘llaymiz:

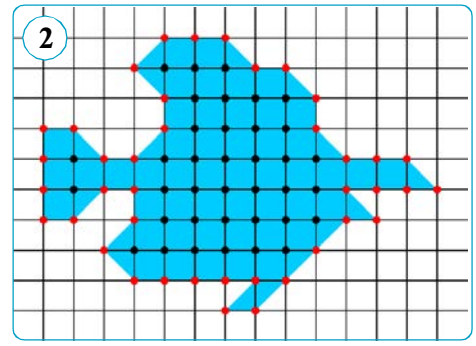
$$S = \frac{44}{2} + 50 - 1 = 22 + 49 = 71 \ (cm^2).$$

Demak, ikkala usulda ham bir xil natija kelib chiqdi. *Javob:* 71 cm^2.

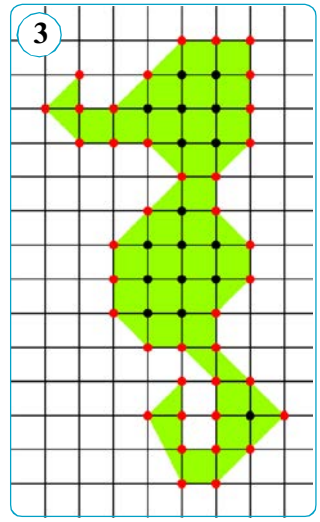
2- masala. 2- rasmdagi ko‘pburchak yuzini hisoblang.

Yechish. 1) Ko‘pburchak tomonlarida yotgan tugun nuqtalarni (qizil rangda belgilangan) sanaymiz: ular 40 ta, ya’ni $M = 40$.





2) Ko‘pburchak ichida yotgan tugun nuqtalarni (qora rangda belgilangan) sanaymiz: ular 37 ta, ya’ni $N = 37$.



Pik formulasiga ko‘ra:

$$S = \frac{40}{2} + 37 - 1 = 20 + 36 = 56 \ (\text{cm}^2).$$

Javob: 56 cm^2.

3- masala. 3- rasmdagi ko‘pburchak yuzini hisoblang.

Yechish. 1- usul. 1) Ko‘pburchak tomonlarida yotgan tugun nuqtalarni (qizil rangda belgilangan) sanaymiz: ular 39 ta, ya’ni $M = 39$.

2) Ko‘pburchak ichida yotgan tugun nuqtalarni (qora rangda belgilangan) sanaymiz: ular 17 ta, ya’ni $N = 17$.

Pik formulasiga ko‘ra:

$$S = \frac{39}{2} + 17 - 1 = 19{,}5 + 16 = 35{,}5 \ (\text{cm}^2).$$

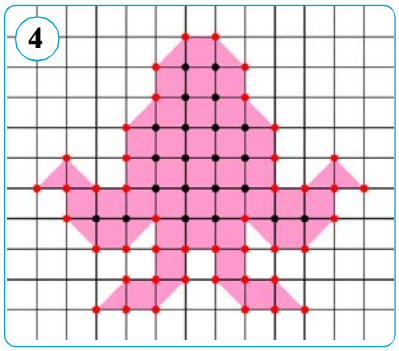
2- *usul.* Olingan javobning to‘g‘ri ekaniga yana bir bor ishonch hosil qilmoqchi bo‘lsangiz, dastlab berilgan ko‘pburchakni turli usullar bilan o‘rganilgan qavariq ko‘pburchaklarga ajrating. So‘ngra hosil bo‘lgan shakllar yuzlarini tegishli formulalar yordamida hisoblang. Olingan natijalarni qo‘shib, 1- usulda chiqqan natija bilan solishtiring. Agar hisoblashlarni to‘g‘ri bajarsangiz, albatta har ikki natija bir xil bo‘ladi. Berilgan ko‘pburchak chizmada turli shakllarga ajratib ko‘rsatilmasa ham bo‘ladi. Hisoblash usulini tanlash o‘zingizga bog‘liq. Hisoblashlarni og‘zaki bajarsa ham bo‘ladi.

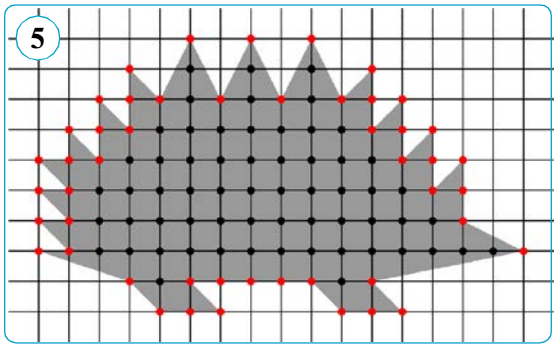
Barcha to‘liq kvadratlar soni 26 ta, ularning yuzi 26 cm^2; kvadratning yarmiga teng bo‘lgan uchburchaklar soni 17 ta, ularning yuzi $17 : 2 = 8{,}5$ (cm^2); bitta asosi 2 cm, balandligi 1 cm ga teng uchburchak bor, uning yuzi 1 cm^2 ga teng. Shunday qilib, berilgan ko‘pburchakning yuzi: $26 + 8{,}5 + 1 = 35{,}5$ (cm^2).

Demak, har ikkala natija bir xil.

Javob: 35,5 cm^2.

4- masala. 4- va 5- rasmdagi ko‘pburchaklar yuzini Pik formulasini qo‘llab hisoblang.



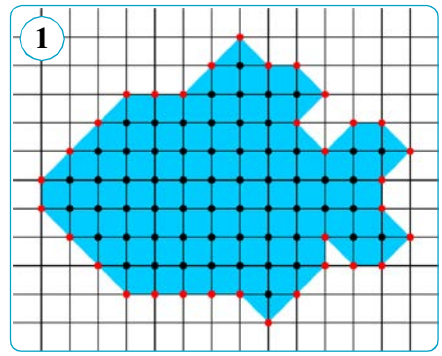


53–54. 4- NAZORAT ISHI. XATOLAR USTIDA ISHLASH

1. Tomonlari 27 cm va 21 cm ga teng to‘g‘ri to‘rtburchakning perimetriga teng bo‘lgan kvadratning yuzini toping.

2. To‘g‘ri to‘rtburchakning yuzi 540 cm^2, ikki tomonining nisbati 3 : 5 kabi. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

3. Parallelogrammning yuzi 24 cm^2. Agar uning balandliklari 3 cm va 4 cm ga teng bo‘lsa, uning perimetrini toping.



4. 1- rasmda tasvirlangan shaklning yuzini bo‘laklarga bo‘lib hamda Pik formulasini qo‘llab toping.

4- TEST — O‘zingizni sinab ko‘ring!

1. Agar to‘g‘ri to‘rtburchakning tomonlari 4 marta orttirilsa, uning yuzi necha marta ortadi?

A) 4; B) 8; D) 16; E) 32.

2. To‘g‘ri to‘rtburchakning yuzi 400 ha, tomonlarining nisbati 4 : 1 ga teng. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

A) 10 km; B) 5 km; D) 2 km; E) 8 km.

3. To‘g‘ri to‘rtburchakning uzunligi 25 % ga orttirildi. Uning yuzi o‘zgarmasligi uchun enini necha foizga kamaytirish kerak?

A) 20 %; B) 16 %; D) 25 %; E) 18 %.

4. Kvadratning tomonini necha marta kamaytirganda yuzi 4 marta kichrayadi?

A) 1,5 marta; B) 2 marta; D) 3 marta; E) 3,5 marta.

5. Yuzi 144 cm^2, balandliklari 8 cm va 12 cm bo‘lgan parallelogrammning perimetrini toping.

A) 40 cm; B) 30 cm; D) 80 cm; E) 60 cm.

6. $ABCD$ parallelogrammning AC diagonaliga BO perpendikular tushirilgan. $AO = 8$ cm, $OC = 6$ cm va $BO = 4$ cm bo‘lsa, parallelogrammning yuzini toping.

A) 50 cm^2; B) 28 cm^2; D) 52 cm^2; E) 56 cm^2.

7. Rombning yuzi 40 cm^2 ga, uning perimetri 20 cm ga teng. Shu rombning balandligini toping.

A) 2 cm; B) 8 cm; D) 4 cm; E) 16 cm.

8. Asoslari 5 cm va 9 cm ga teng bo‘lgan trapetsiyaning yuzi 35 cm^2 ga teng. Shu trapetsiyaning balandligini toping.

A) 9 cm; B) 8 cm; D) 5 cm; E) 10 cm.

9. Asoslari 8 va 12 ga teng bo‘lgan teng yonli trapetsiyaning diagonallari o‘zaro perpendikular. Trapetsiyaning yuzini toping.

A) 100; B) 64; D) 144; E) 76.

10. Trapetsiyaning yuzi 30 cm^2 ga, balandligi 6 cm ga teng bo‘lsa, uning o‘rta chizig‘i qanchaga teng bo‘ladi?

A) 2,5 cm; B) 5 cm; D) 7,5 cm; E) 4,5 cm.



Ingliz tilini o‘rganamiz!

Kvadrat ildiz – square root
Uchburchak – triangle
O‘rta chiziq – midline
Geron formulasi – formula of Heron
Yuza – area



Tarixiy ma’lumotlar

Ibn Sinoning «Donishnoma» asarining beshinchi bobi «To‘rtburchaklar, ularda joylashgan uchburchaklar va ularning munosabatlariga doir asosiy geometrik masalalar» mavzusiga bag‘ishlangandir. Asarda parallel chiziqlar haqida quyidagicha fikrlar aytib o‘tilgan.

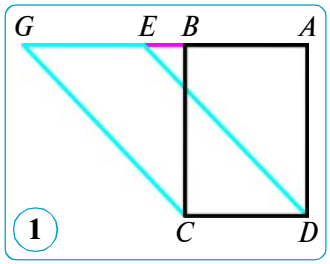


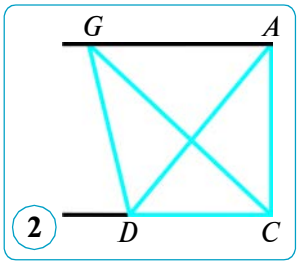
Abu Ali ibn Sino
(980–1037)

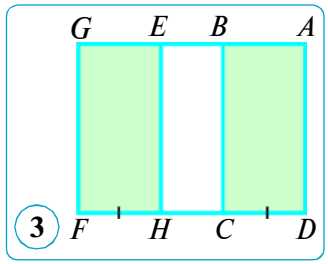
1- teorema. O‘zaro parallel ikki chiziq orasiga joylashgan, umumiy asosga ega va qarama-qarshi tomonlari parallel shakllar tengdosh bo‘ladi (ya’ni ularning yuzlari teng). Masalan, asoslari CD bo‘lgan $ABCD$ va $EGCD$ tekis shakllar o‘zaro tengdosh bo‘ladi (1- rasm).

2- teorema. O‘zaro parallel chiziqlar orasiga joylashgan va umumiy asosga ega bo‘lgan uchburchaklar tengdosh bo‘ladi. Masalan, CD asosga ega bo‘lgan ACD va GCD uchburchaklar tengdosh bo‘ladi (2- rasm).

3- teorema. O‘zaro parallel chiziqlar orasiga joylashgan va asoslari teng bo‘lgan to‘rtburchaklar tengdosh bo‘ladi. Masalan, $ABCD$ va $GEHF$ to‘rtburchaklar tengdoshdir (3- rasm).









V BOB
AYLANA



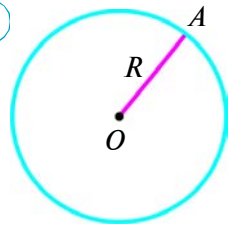
10-§. AYLANADAGI BURCHAKLAR

55. TO‘G‘RI CHIZIQ VA AYLANANING O‘ZARO JOYLASHUVI. AYLANAGA URINMA VA UNING XOSSALARI

1. Aylana haqida boshlang‘ich ma’lumotlar.

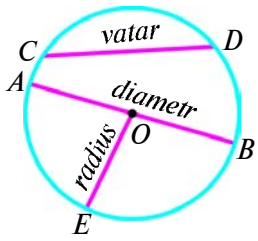
Ta’rif. *Tekislikning berilgan nuqtasidan bir xil uzoqlikda yotgan barcha nuqtalaridan iborat shakl* ***aylana*** *deyiladi.*

1



O markazli, *R* radiusli aylana, ya’ni (*O*, *R*)

a



CD – vatar, *OE* – radius, *AB* – diametr

b

Berilgan *O* nuqta ***aylananing markazi*** deyiladi. Aylananing ixtiyoriy nuqtalaridan uning markazigacha bo‘lgan masofa ***aylananing radiusi*** deyiladi. Shuningdek, aylana nuqtasini uning markazi bilan tutashtiruvchi har qanday kesma ham *radius* deyiladi. Shunday qilib, markazi *O* nuqta va radiusi *R* bo‘lgan aylana – berilgan *O* nuqtadan *R* ga teng masofada joylashgan tekislikning hamma nuqtalaridan tuzilgan geometrik shakldir.

Odatda, *O* markazli va *R* radiusli aylana quyidagicha belgilanadi: (*O*, *R*) (1-*a* rasm).

Aylananing ixtiyoriy ikki nuqtasini tutashtiruvchi kesma ***vatar*** deyiladi. Aylananing markazidan o‘tuvchi vatar uning ***diametri*** deyiladi.

1-*b* rasmda aylananing radiusi va ikki vatari tasvirlangan bo‘lib, vatardan biri aylananing diametridir: *OE* – radius, *CD* – vatar, *AB* – diametr.

Odatda, diametr *d* harfi bilan belgilanadi. Bizga ma’lumki, diametr radiusdan ikki marta katta, ya’ni $d = 2R$ ga teng.

2. To‘g‘ri chiziq va aylananing o‘zaro joylashuvi.

Bu mavzuda tekislikda to‘g‘ri chiziq bilan aylananing o‘zaro joylashishini ko‘rib chiqamiz. Agar to‘g‘ri chiziq aylana markazidan o‘tsa, u holda u aylanani ikki nuqtada, ya’ni bu to‘g‘ri chiziqda yotuvchi diametr uchlarida kesib o‘tishi ko‘rinib turibdi.

Berilgan *l* to‘g‘ri chiziq bilan (*O*, *R*) aylana nechta umumiy nuqtaga ega, degan savolga javob berish uchun aylananing markazi *O* dan *l* to‘g‘ri chiziqqacha bo‘lgan *d* masofani shu aylananing *R* radiusi bilan taqqoslash kerak.

Aylananing markazidan to‘g‘ri chiziqqa tushirilgan perpendikular *aylana markazidan to‘g‘ri chiziqqacha masofa* deb ataladi.

Uch hol bo‘lishi mumkin: 1) $d > R$; 2) $d = R$; 3) $d < R$. Endi bu hollarni ko‘rib chiqamiz.

1- hol. *Agar aylananing markazidan to‘g‘ri chiziqqacha bo‘lgan masofa aylananing radiusidan katta bo‘lsa, to‘g‘ri chiziq bilan aylana umumiy nuqtaga ega bo‘lmaydi, ya’ni kesishmaydi.*

Haqiqatan ham, agar $d > R$ bo‘lsa (2- *a* rasm), l to‘g‘ri chiziqning O markazga eng yaqin nuqtasi (bu to‘g‘ri chiziqning istalgan nuqtasi ham) (O, R) aylanaga tegishli bo‘lmaydi, chunki u markazdan aylana radiusidan katta masofada bo‘ladi.

2- hol. *Agar aylananing markazidan to‘g‘ri chiziqqacha masofa aylananing radiusiga teng bo‘lsa, u holda to‘g‘ri chiziq bilan aylana bitta va faqat bitta umumiy nuqtaga ega bo‘ladi.*

Haqiqatan ham, agar $d = R$ bo‘lsa (2- *b* rasm), l to‘g‘ri chiziqning O markazga eng yaqin nuqtasi aylananing radiusiga teng masofada bo‘ladi va demak, u nuqta aylanaga ham tegishli bo‘ladi. l to‘g‘ri chiziqning qolgan hamma nuqtalari O markazdan aylananing radiusidan katta masofada bo‘ladi, demak, aylanaga tegishli bo‘lmaydi.

3- hol. *Aylananing markazidan to‘g‘ri chiziqqacha bo‘lgan masofa aylananing radiusidan kichik bo‘lsa ($d < R$), u holda to‘g‘ri chiziq bilan aylana ikkita umumiy nuqtaga ega bo‘ladi.*

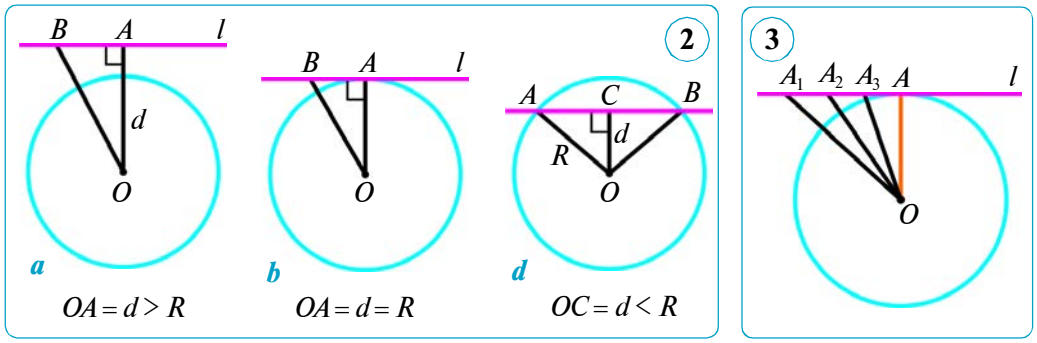
To‘g‘ri chiziqning aylana ichidagi qismi vatar bo‘ladi (2- *d* rasm). Bu holda to‘g‘ri chiziq aylanaga nisbatan *kesuvchi* deyiladi.

Vatarning uzunligi AB ni aylananing radiusi va markazidan to‘g‘ri chiziqqacha masofa d orqali ifodalash mumkin:

$$AB = 2\sqrt{R^2 - d^2}\ .$$

Bu tenglikni o‘zingiz isbot qiling.

Xulosa. *To‘g‘ri chiziq bilan aylana umumiy nuqtalarga ega bo‘lmasligi, bir yoki ikki umumiy nuqtaga ega bo‘lishi mumkin.*



2. Aylanaga urinma.

Ta'rif. *Aylana bilan faqat bitta umumiy nuqtaga ega bo'lgan to'g'ri chiziq shu aylanaga* ***urinma****, ularning umumiy nuqtasi esa* ***urinish nuqtasi*** *deyiladi.*

2- *b* rasmda l to'g'ri chiziq O markazli aylanaga urinma, A – urinish nuqtasi. Aylana l to'g'ri chiziqqa urinadi, deyish ham mumkin.

Urinmaning xossasi haqidagi teoremani isbotlaymiz.

1-teorema.

Aylanaga urinma shu aylananing urinish nuqtasiga o'tkazilgan radiusga perpendikulardir.

Isbot. l to'g'ri chiziq aylanaga A nuqtada o'tkazilgan urinma bo'lsin (3- rasmga q.). $R = OA$ ning l ga perpendikular bo'lishini isbot qilamiz. Shartga ko'ra, l to'g'ri chiziqning A nuqtasidan boshqa hamma nuqtalari aylanadan tashqarida yotadi. Shuning uchun bu to'g'ri chiziqning A dan boshqa har qanday A_1 nuqtasi uchun $OA_1 > OA$. Demak, OA masofa O nuqtadan l to'g'ri chiziqning nuqtalarigacha bo'lgan masofalarning eng qisqasidir. Nuqtadan to'g'ri chiziqqacha eng qisqa masofa esa shu to'g'ri chiziqqa tushirilgan perpendikular bo'ladi. Bundan, $OA \perp l$ ekani kelib chiqadi.

Teorema isbotlandi.

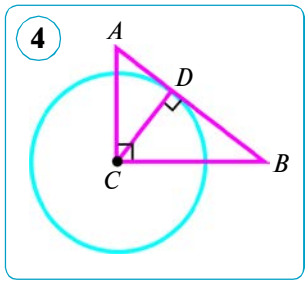
Endi urinmaning xossasiga teskari teoremani isbotlaymiz.

2-teorema.

Radiusga perpendikular va uning aylanada yotgan uchidan o'tuvchi to'g'ri chiziq shu aylanaga urinmadir.

Isbot. Agar aylana markazidan to'g'ri chiziqqacha bo'lgan masofa aylana radiusiga teng ($d = R$) bo'lsa (2- *b* rasmga q.), l to'g'ri chiziqning O markazga eng yaqin nuqtasi aylananing radiusiga teng bo'ladi, demak, u nuqta aylanaga ham tegishli bo'ladi. l to'g'ri chiziqning qolgan hamma nuqtalari O markazdan aylananing radiusidan katta masofada bo'ladi, demak, aylanaga tegishli bo'lmaydi. Ta'rifga ko'ra, l to'g'ri chiziq shu aylanaga urinma bo'ladi. Teorema isbot bo'ldi.

Masala. To'g'ri burchakli ACB ($\angle C = 90°$) uchburchakning katetlari $AC = 3$ cm va $BC = 4$ cm. Markazi C nuqtada bo'lgan radiusi 2,4 cm ga teng aylana o'tkazilgan. Bu aylana bilan AB to'g'ri chiziq o'zaro qanday holatda bo'ladi?



Yechish. $\triangle ACB$ ($\angle C = 90°$) da: $AC = 3$ cm, $BC = 4$ cm. Pifagor teoremasiga ko'ra:

$$AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ (cm)}.$$

$CD \perp AB$ ni o'tkazamiz (4- rasm). Uchburchakning yuzini ikki xil hisoblash mumkin, ya'ni

$CA \cdot CB = AB \cdot CD$ tenglik o‘rinli. Bundan $CD = CA \cdot CB : AB = 3 \cdot 4 : 5 = 2{,}4$ (cm). Demak, C nuqtadan AB to‘g‘ri chiziqqacha bo‘lgan masofa radius uzunligiga teng bo‘lgani uchun AB to‘g‘ri chiziq aylanaga urinadi.

Javob: AB – urinma.

Savol, masala va topshiriqlar

1. 1) Aylana nima? Aylana: markazi, radiusi, vatari va diametri deb nimaga aytiladi? Qanday to‘g‘ri chiziq aylanaga urinma deyiladi?

2) Urinmaning qanday xossasi va alomatini bilasiz?

2. $d - R$ radiusli aylananing markazidan l to‘g‘ri chiziqqacha bo‘lgan masofa. Agar: 1) $R = 8$ cm, $d = 6$ cm; 2) $R = 10$ cm, $d = 8{,}4$ cm; 3) $R = 14{,}4$ dm, $d = 7{,}4$ dm; 4) $R = 1{,}6$ dm, $d = 24$ cm; 5) $R = 4$ cm, $d = 40$ mm bo‘lsa, l to‘g‘ri chiziq bilan aylana o‘zaro qanday joylashgan bo‘ladi?

3. $ABCD$ kvadratning tomoni 8 cm ga va markazi A nuqtada bo‘lgan aylananing radiusi 7 cm ga teng. AB, BC, CD va BD to‘g‘ri chiziqlardan qaysi biri shu aylanaga nisbatan kesuvchi bo‘ladi?

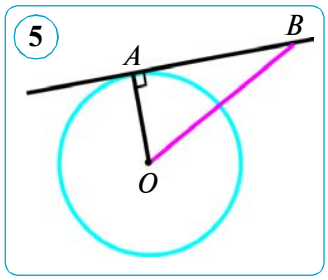
4. AB to‘g‘ri chiziq O markazli aylananing A nuqtasiga o‘tkazilgan urinma. Agar $AB = 24$ cm, aylananing radiusi 7 cm ga teng bo‘lsa, OB kesmaning uzunligini toping (5- rasm).

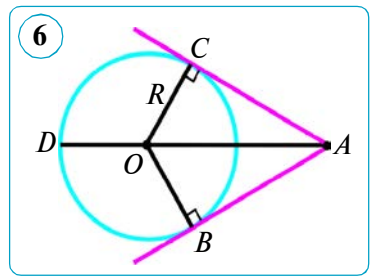
5. To‘g‘ri burchakli ACB ($\angle C = 90°$) uchburchakda $AB = 10$ cm, $\angle ABC = 30°$. Markazi A nuqtada bo‘lgan aylana o‘tkazilgan. Bu aylananing radiusi qanday bo‘lganda: 1) aylana BC to‘g‘ri chiziqqa urinadi; 2) BC to‘g‘ri chiziq bilan umumiy nuqtaga ega bo‘lmaydi; 3) BC to‘g‘ri chiziq bilan ikkita umumiy nuqtaga ega bo‘ladi?

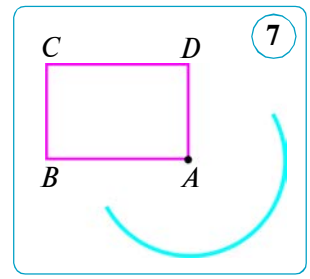
6. Aylana tashqarisidagi bir nuqtadan unga ikkita urinma o‘tkazilsa, o‘sha nuqtadan urinish nuqtalarigacha bo‘lgan masofalar teng. Shuni isbotlang (6- rasm).

7. Agar aylana radiusi 5 cm ga teng, aylana markazidan to‘g‘ri chiziqqacha bo‘lgan masofa: 1) 6 cm; 2) 5 cm; 3) 4 cm bo‘lsa, to‘g‘ri chiziq bilan aylana o‘zaro qanday joylashgan bo‘ladi?

8. $ABCD$ to‘g‘ri to‘rtburchak berilgan, unda $AB = 16$ cm, $AD = 12$ cm (7- rasm). AC, BC, CD va BD to‘g‘ri chiziqlardan qaysi biri radiusi 12 cm li A markazli aylanaga urinma bo‘ladi?





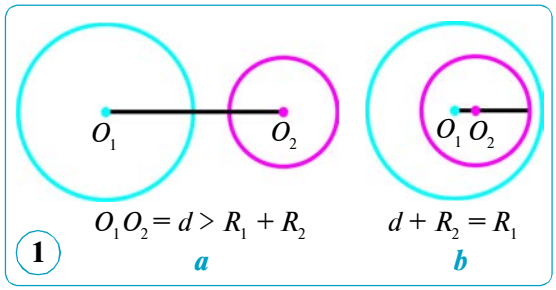


56. IKKI AYLANANING O‘ZARO JOYLASHUVI. MARKAZIY BURCHAK VA YOYNING GRADUS O‘LCHOVI

1. Ikki aylananing o‘zaro joylashuvi.

Ikki aylana o‘zaro joylashadigan hollarni ko‘rib chiqamiz.

1) Ikki aylana umumiy nuqtaga ega bo‘lmaydi. Bu holda ular aylanadan tashqarida (1- *a* rasm) yoki biri ikkinchisining ichida bo‘ladi (1- *b* rasm).

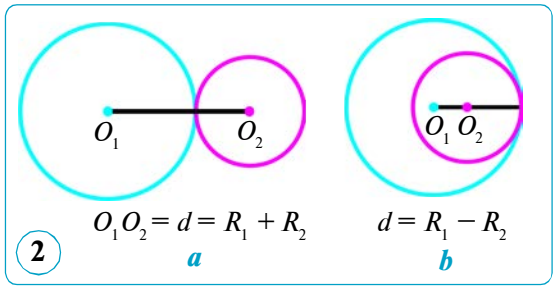


2) Ikki aylana bitta umumiy nuqtaga ega bo‘ladi (2- rasm). Bu holda, aylanalar bir-biriga *urinadi,* deyiladi. Ammo bu holda aylanalar *tashqi* tomondan (2- *a* rasm) yoki *ichki* tomondan urinishi mumkin (2- *b* rasm).

3) Ikki aylana ikkita umumiy nuqtaga ega bo‘lishi mumkin (3- rasm). Bu holda aylanalar bir-biri bilan *kesishadi*, deyiladi.

Umumiy markazga ega bo‘lgan aylanalar *konsentrik aylanalar* deyiladi (4- rasm).

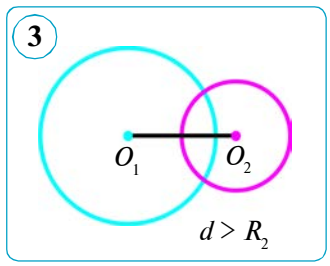
Ikki aylananing o‘zaro joylashishi ularning radiusi va markazlar orasidagi masofaga bog‘liq bo‘ladi.

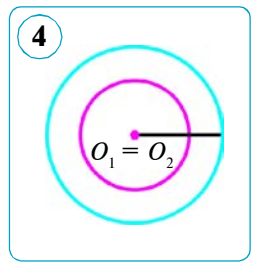


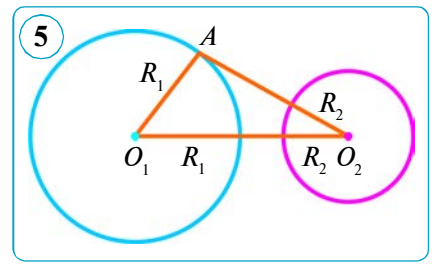
Teorema.

Agar ikki aylananing markazlari orasidagi masofa ularning radiuslari yig‘indisidan katta yoki ayirmasidan kichik bo‘lsa, bu aylanalar umumiy nuqtaga ega bo‘lmaydi.

Isbot. O_1, O_2 markazli va radiuslari mos ravishda R_1, R_2 ($d = R_1 + R_2 < O_1O_2$) bo‘lgan ikkita aylana berilgan bo‘lsin (5- rasm). Aylanadagi A nuqtani ko‘rib chiqamiz: $O_1A = R_1$. U holda $O_2A \geq O_1O_2 - O_1A > R_1 + R_2 - R_1 = R_2$ va demak, A nuqta ikkinchi aylanaga tegishli emas. Demak, bu aylanalar umumiy nuqtaga ega bo‘lmaydi.







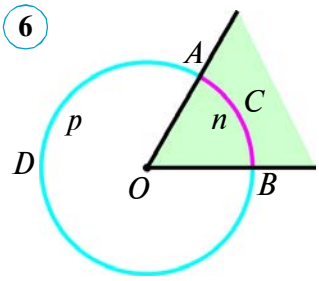
Ikkita aylana bitta umumiy nuqtaga ega bo‘lgan holni, shuningdek, ikkita aylana ikkita umumiy nuqtaga ega bo‘lgan hollarni mustaqil ko‘rib chiqing.

2. Markaziy burchak.

Ta’rif. *Uchi aylananing markazida bo‘lgan burchak* ***markaziy burchak*** *deb ataladi.*

Umumiy uchi aylananing O markazida bo‘lgan ikki nur OA va OB ikkita markaziy burchakni belgilaydi, ulardan biri qavariq soha bilan chegaralangan bo‘ladi. Aylananing ikki A va B nuqtasi uni ikki yoyga ajratadi. Bu yoylar bir-biridan farqlanishi uchun ularning har biriga bittadan oraliq nuqta (yoyning uchlaridan farqli) yoki lotincha kichik harf qo‘yilib belgilanadi hamda ACB (yoki AnB) va ADB (yoki ApB) *yoylar* deyiladi (6- rasm). Yoylarni quyidagicha belgilash qabul qilingan: $\cup ACB$ (yoki $\cup AnB$) va $\cup ADB$ (yoki $\cup ApB$). Ayrim hollarda yoy oraliq nuqtasiz belgilanadi: $\cup AB$ (ikki yoydan qaysi biri haqida gap ketayotgani tushunarli bo‘lganda). Agar yoyning uchlarini tutashtiruvchi kesma aylana diametri bo‘lsa, yoy *yarim aylana* deyiladi. 7- *b* rasmda ikkita yarim aylana tasvirlangan, ulardan biri alohida ajratib ko‘rsatilgan.

6



$\angle AOB$ – markaziy burchak

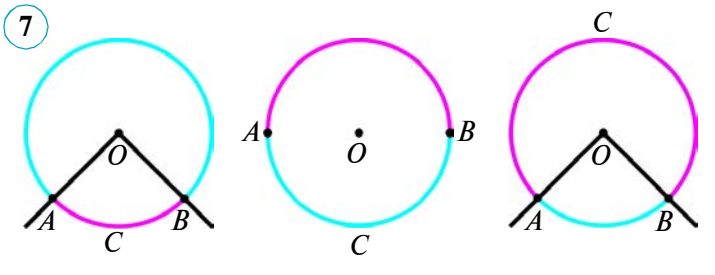
3. Yoyning gradus o‘lchovi.

Ta’rif. ***Aylana yoyining burchak kattaligi*** *deb, aylananing shu yoyga mos markaziy burchagining kattaligiga aytiladi.*

Aylana yoyini graduslarda o‘lchash mumkin. *Agar O markazli aylananing ACB yoyi yarim aylanadan kichik yoki yarim aylanaga teng bo‘lsa, u holda uning gradus o‘lchovi AOB markaziy burchak gradus o‘lchoviga teng hisoblanadi* (7- *a, b* rasm). *Agar ACB yoy yarim aylanadan katta bo‘lsa, u holda uning gradus o‘lchovi* $360^\circ - \angle AOB$ *ga teng hisoblanadi* (7- *d* rasm).

Bundan, *oxirlari umumiy bo‘lgan aylana ikki yoyining gradus o‘lchovlari yig‘indisi* 360° *ga tengligi* kelib chiqadi.

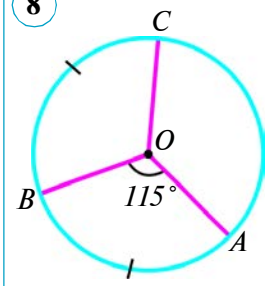
7



$\cup ACB = \cup AOB$ $\cup ACB = 180^\circ$ $\cup ACB = 360^\circ - \angle AOB$

a *b* *d*

8



Aylana ikki yoyining burchak kattaliklari (ya'ni ularga mos markaziy burchaklar) teng bo'lganda va faqat shundagina bu yoylar teng bo'ladi.

Masala. O nuqta – aylana markazi, $\angle AOB = 115°$, $\cup BC = \cup AB$ (8- rasm). AOC burchakni toping.

Yechish. AOB burchak aylananing markaziy burchagi, AB yoy esa yarim aylanadan kichik, shuning uchun $\cup AB = \angle AOB = 115°$. Masala shartiga ko'ra, $\cup BC = \cup AB$ va demak, BC yoy 115° ga teng. $\cup ABC = \cup AB + \cup BC = 230° > 180°$, ya'ni ABC yoy yarim aylanadan katta, shuning uchun $\angle AOC = 360° - ABC = 360° - 230° = 130°$. *Javob:* $\angle AOC = 130°$.

Savol, masala va topshiriqlar

1. 1) Aylana berilgan nuqtada urinadi deganda, nimani tushunasiz?
2) Konsentrik aylanalar deb nimaga aytiladi?
3) Markaziy burchak nima? Aylana yoyi qanday belgilanadi?
4) Aylana yoyining burchak kattaligi nima?

2. Agar ikki aylananing markazlari orasidagi masofa 2 cm, radiuslari mos ravishda: 1) 3 cm va 5 cm; 2) 2 cm va 5 cm bo'lsa, ular bir-biriga nisbatan o'zaro qanday joylashgan bo'ladi?

3. Agar radiuslari 4 cm va 6 cm ga teng aylanalar: 1) tashqi tomondan urinsa; 2) ichki tomondan urinsa, ularning markazlari orasidagi masofa nimaga teng?

4. Aylana markazidan o'tuvchi ikki to'g'ri chiziq bu aylanada nechta yoy va nechta markaziy burchakni aniqlaydi?

5. Berilgan aylananing nuqtasidan radiusiga teng ikkita vatar o'tkazilgan. Ular orasidagi burchakni toping.

6. Markaziy burchakka mos yoy aylananing: 1) $\frac{2}{5}$; 2) $\frac{4}{15}$; 3) $\frac{7}{12}$; 4) $\frac{5}{9}$; 5) $\frac{13}{18}$; 6) $\frac{17}{20}$; 7) $\frac{23}{30}$ qismiga teng. Shu markaziy burchakni toping.

7. Aylana ikki nuqta bilan ikki yoyga bo'linadi. Agar: 1) ulardan birining burchak kattaligi ikkinchisining burchak kattaligidan 40° ortiq bo'lsa; 2) bu yoylarning burchak kattaliklari 2 : 7 nisbatda bo'lsa, har qaysi burchak kattaligini toping.

8. A, B, C nuqtalar markazi O nuqtada bo'lgan aylanada yotadi. Agar $\cup ABC = 70°$ bo'lsa, AOC burchakni toping.

9. Aylananing: 1) $\frac{1}{5}$; 2) $\frac{1}{6}$; 3) $\frac{1}{9}$; 4) $\frac{1}{10}$; 5) $\frac{1}{12}$ qismini tashkil qiluvchi AB yoyga mos keluvchi markaziy burchaklarni toping. Bu hollarning har birida AB yoyning burchak kattaligini belgilar yordamida yozing.

10. Aylananing radiusi: 1) 7,8 cm; 2) 10,5 cm; 3) 0,8 dm. Aylananing diametrini toping.

57. AYLANAGA ICHKI CHIZILGAN BURCHAK

Ta'rif. *Uchi aylanada yotuvchi, tomonlari esa shu aylanani kesib o'tuvchi burchak* ***aylanaga ichki chizilgan burchak*** *deyiladi.*

1- rasmda ABC burchak aylanaga ichki chizilgan, AnC yoy shu burchakning ichiga joylashgan. Bunday holda, ichki chizilgan ABC burchak AnC yoyga tiralgan deb ham aytiladi.

Teorema.

Aylanaga ichki chizilgan burchak o'zi tiralgan yoyning yarmi bilan o'lchanadi:

$$\angle ABC = \frac{1}{2} \cup AC.$$

Isbot. $\angle ABC$ — O markazli aylananing AC yoyiga tiralgan ichki chizilgan burchagi bo'lsin (2- rasm). Aylana markazining shu ichki chizilgan burchakka nisbatan joylashishining uch holini ko'rib chiqamiz.

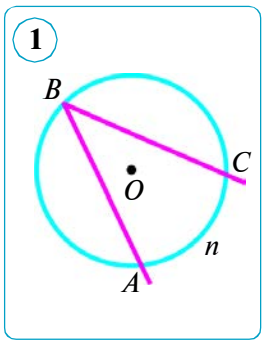
1- hol. Aylana markazi ichki chizilgan burchakning tomonlaridan biri, masalan, BC tomonda yotadi (2- *a* rasm). OA radiusni o'tkazamiz va AOC markaziy burchakni qaraymiz. U BOA uchburchakning tashqi burchagidir. Uchburchak tashqi burchagining xossasiga ko'ra: $\angle AOC = \angle OBA + \angle OAB$. Ammo $\angle OBA = \angle OAB$, chunki AOB uchburchak teng yonli ($OA = OB = R$). OBA va OAB burchaklar esa teng yonli uchburchakning asosidagi burchaklardir. Demak, $\angle AOC = 2\angle ABC$ (1). Markaziy burchakning kattaligi shu burchakka mos yoyning burchak kattaligiga teng bo'lishini bilasiz (56- mavzu). Bu holda AC yoy yarim aylanadan kichik, shuning uchun markaziy burchak xossasiga ko'ra:

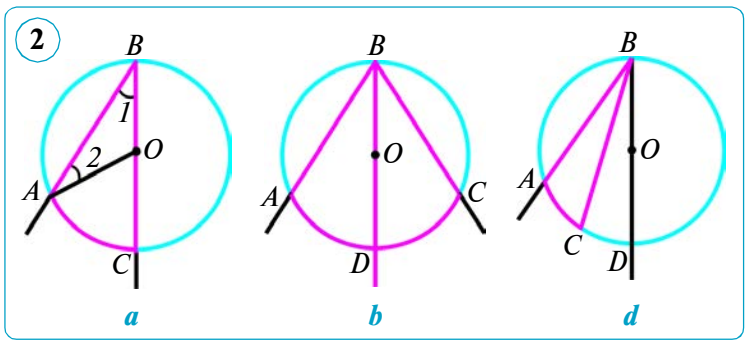
$$\angle AOC = \cup AC. \qquad (2).$$

(1) va (2) tengliklardan: $2\angle ABC = \cup AC$, ya'ni $\angle ABC = \frac{1}{2} \cup AC$.

Teorema 1- hol uchun isbotlandi.

2- hol. Aylananing markazi O ichki chizilgan burchak tomonlari orasida yotadi. BO nurni o'tkazamiz, u AC yoyni biror D nuqtada kesadi (2- *b* rasm).





D nuqta AC yoyni ikkita $\cup AD$ va $\cup DC$ yoyga bo‘ladi. Demak, isbotga ko‘ra (1- hol): $\angle ABD = \frac{1}{2}\cup AD$ va $\angle DBC = \frac{1}{2}\cup DC$. Bu tengliklarni hadma-had qo‘shib, quyidagilarni hosil qilamiz:

$$\angle ABC = \angle ABD + \angle DBC = \frac{1}{2}\cup AD + \frac{1}{2}\cup DC = \frac{1}{2}(\cup AD + \cup DC) = \frac{1}{2}\cup AC.$$

3- hol. Aylananing markazi O ichki chizilgan burchakdan tashqarida yotadi. Bu holning isbotini 2- *d* rasmdan foydalanib, o‘zingiz mustaqil bajaring.

1- natija. *Bir yoyga tiralgan hamma ichki chizilgan burchaklar o‘zaro tengdir* (3- *a* rasm):

$$\angle B = \angle B_1 = \angle B_2 = \ldots = \frac{1}{2}\cup AC.$$

2- natija. *Diametrga* (*yarim aylanaga*) *tiralgan hamma ichki chizilgan burchaklar to‘g‘ri burchakdir* (3- *b* rasm):

$$\angle B = \angle B_1 = \angle B_2 = \ldots = 90^\circ.$$

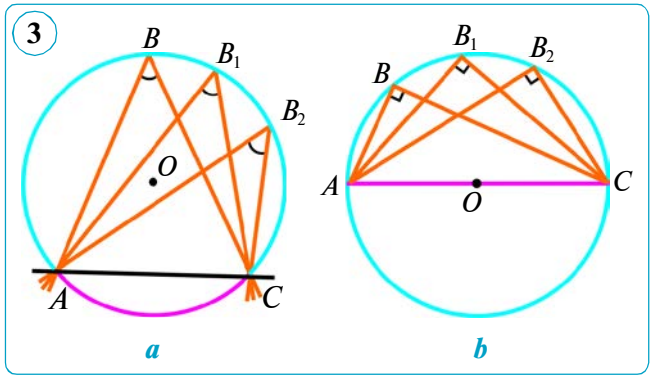
Masala. Aylananing radiusiga teng vatar o‘tkazilgan. Shu vatar: 1) aylana markazidan; 2) berilgan vatar uchlaridan farqli aylananing ixtiyoriy nuqtasidan qanday burchak ostida ko‘rinadi?

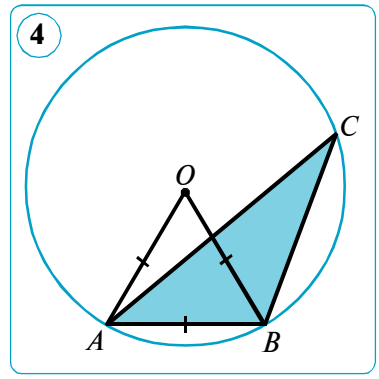
Yechish. $AB - O$ markazli aylananing radiusiga teng vatar bo‘lsin (4- rasm). U holda AOB uchburchak teng tomonli va demak, markaziy burchak (aylana markazidan AB vatar ko‘rinadigan burchak) 60° ga teng. A va B nuqtalardan farqli aylananing ixtiyoriy C nuqtasidan ichki chizilgan ACB burchak (C nuqtadan AB vatar ko‘rinadigan burchak) markaziy burchakning yarmiga teng, ya’ni 30° ga teng.

Javob: 1) 60°; 2) 30°.

Savol, masala va topshiriqlar

1. 1) Qanday burchak aylanaga ichki chizilgan burchak deyiladi?
2) Ichki chizilgan burchak qanday o‘lchanadi?
3) Yarim aylanaga tiralgan ichki chizilgan burchak nimaga teng?





2. (*Og'zaki.*) Ichki chizilgan burchak 25° ga teng. Shu ichki burchakka tiralgan yoyning kattaligini toping.

3. AB va BC – markazi O nuqtada bo'lgan aylananing vatarlari, $\angle ABC = 30°$. Agar aylana radiusi 10 sm ga teng bo'lsa, AC vatarning uzunligini toping.

4. 1) 5- rasmda O nuqta – aylana markazi, $\angle AOB = 88°$. $\angle ACB$ ni toping.

Yechish. AOB burchak berilgan aylananing ... burchagi bo'ladi va ...° ga teng. Demak, $\cup ADB = ...°$. ACB burchak ... chizilgan burchak bo'ladi va ... yoyga tiraladi, shuning uchun $\angle ACB = \frac{1}{2} \cup ... = ...°$.

Javob: $\angle ACB = ...°$.

2) 6- rasmda $\cup CAB = 130°$. $\angle CAB$ ni toping.

Yechish. CAB burchak aylanaga ichki chizilgan burchak bo'ladi va $\cup CDB$ yoyga tiralgan. Bundan:

$$\cup CDB = 360° - \cup CAB = 360° - 130° = 230°,$$

$$\angle CAB = \frac{1}{2} \cup CDB = \frac{1}{2} \cdot 230° = 115°.$$

Javob: $\angle CAB = 115°$.

3) 7- rasmda $\angle APE = 46°$, $\angle BCE = 34°$. $\angle AEP$ ni toping.

Yechish. PAB va BCP ichki chizilgan burchaklar bitta BP ..., demak, $\angle PAB = \angle ... = ...$. AEP uchburchakdan quyidagiga ega bo'lamiz:

$$\angle AEP = 180° - (\angle ... + \angle ...) = 180° - (... + ...) = ...$$

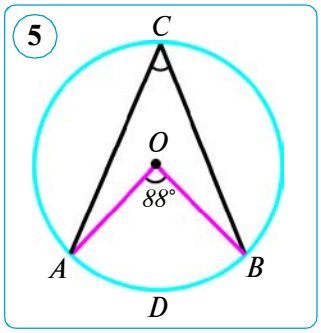
Javob: $\angle AEP = ...$.

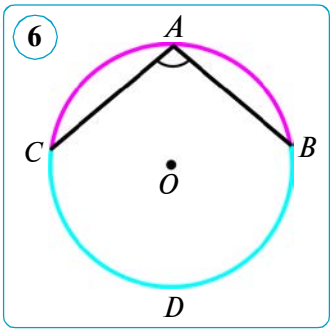
5. Aylanada yotuvchi A, B, C nuqtalar bu aylanani uchta yoyga bo'ladi. Bu yoylarning gradus o'lchovlari nisbati 3 : 5 : 7 kabi. ABC uchburchakning burchaklarini toping.

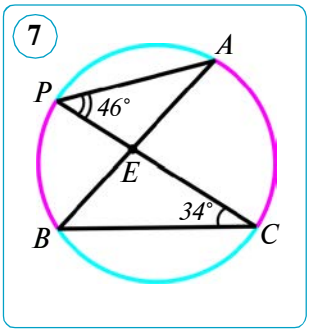
6. Vatar aylanani ikki yoyga bo'ladi. Agar bu yoylar burchak kattaliklarining nisbati: 1) 5 : 4; 2) 7 : 3 kabi bo'lsa, vatar aylana nuqtalaridan qanday burchak ostida ko'rinadi?

7. Aylanaga AB diametr va AC vatar o'tkazilgan. Agar AC va CB yoylarning gradus o'lchovi 7 : 2 nisbatda bo'lsa, BAC burchakni toping.

8. AB va AC – aylana vatarlari, $\angle BAC = 70°$, $\cup AB = 120°$. AC yoyning gradus miqdorini toping.







58. AYLANANING KESUVCHILARI HOSIL QILGAN BURCHAKLAR

1. Urinma bilan vatardan tuzilgan burchak.

1-teorema.

Urinma bilan vatardan tuzilgan burchak o‘z ichiga olgan yoyning yarmi bilan o‘lchanadi.

Isbot. AB urinma va BC vatar bo‘lsin. $\angle ABC = \frac{1}{2} \cup BmC$ ekanini isbot qilamiz (1-rasm). Buning uchun C uchidan $CD \parallel AB$ ni o‘tkazsak, $\angle ABC = \angle BCD$, chunki ular ichki almashinuvchi burchaklar.

Ammo $\angle C = \frac{1}{2} \cup BnD$ va $CD \parallel AB$ bo‘lgani uchun $\cup BnD = \cup BmC$ va $\angle B = \angle C = \frac{1}{2} \cup BnD = \frac{1}{2} \cup BmC$.

Teorema isbotlandi.

1-masala. AB vatar 56° li yoyni tortib turadi. Shu vatarning uchlaridan aylanaga o‘tkazilgan urinmalar bilan vatardan hosil bo‘lgan burchaklarni toping.

Berilgan: $(O,\ R)$, AB – vatar, $\angle AOB = 56^\circ - AB$ vatarni tortib turgan markaziy burchak, $AC \perp OA$, $BC \perp OB$ (2-rasm).

Topish kerak: $\angle CAB$, $\angle CAB$, $\angle BAD$, $\angle ABE$.

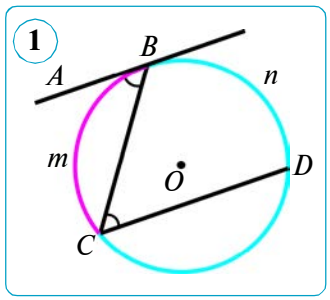
Yechish. Urinma bilan vatar orasidagi yoy $\cup AB = 56^\circ$ (1-hol) yoki $\cup AnB = 360^\circ - 56^\circ = 304^\circ$ (2-hol) bo‘ladi.

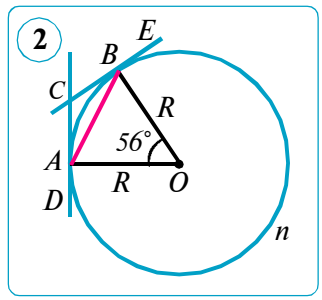
Shunday qilib, 1-holda $\angle CAD = \frac{1}{2} \cup AB = \frac{1}{2} \cdot 56^\circ = 28^\circ$, 2-holda esa $\angle BAD = \frac{1}{2} \cup AnB = \frac{1}{2} \cdot 304^\circ = 152^\circ$ ga ega bo‘lamiz.

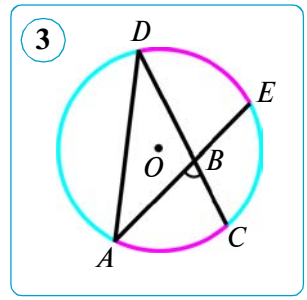
Bizga ma’lumki, aylana tashqarisidagi bir nuqtadan aylanaga o‘tkazilgan urinmalarning urinish nuqtalarigacha bo‘lgan kesmalari teng bo‘ladi. Shuning uchun $\triangle ACB$ – teng yonli.

Demak, $\angle CAB = \angle CBA = 28^\circ$ va $\angle BAD = \angle ABE = 152^\circ$.

Javob: $\angle CAB = \angle CBA = 28^\circ$, $\angle BAD = \angle ABE = 152^\circ$.







2. Ikkita vatarning kesishishidan hosil bo‘lgan burchaklar.

2-teorema.

Ixtiyoriy ikkita vatarning kesishishidan hosil bo‘lgan har qaysi vertikal burchak o‘z tomonlari tiralgan yoylar yig‘indisining yarmiga teng.

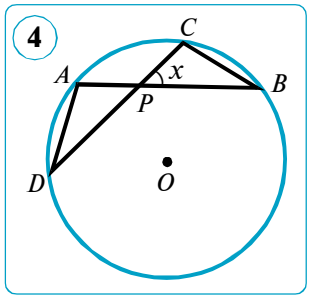
Isbot. $\angle ABC - CD$ va AE vatarlarning kesishishidan hosil bo‘lgan burchaklardan bittasi bo‘lsin (3- rasm).

$\angle ABC = \frac{1}{2}(\cup AC + \cup DE)$ ekanini isbotlaymiz. Buning uchun A va D nuqtalarni birlashtiramiz, u holda $\angle ABC$ $\triangle ABD$ ga nisbatan tashqi burchak bo‘ladi. Demak, $\angle ABC = \angle ADC + \angle DAE$. Ammo $\angle ADC = \frac{1}{2} \cup AC$ va $\angle DAE = \frac{1}{2} \cup DE$. Shuning uchun

$$\angle ABC = \frac{1}{2} \cup AC + \frac{1}{2} \cup DE = \frac{1}{2}(\cup AC + \cup DE).$$

$\angle ABD = \angle CBE = \frac{1}{2}(\cup AD + \cup EC)$ ekanligi xuddi yuqoridagidek isbotlanadi. Buni mustaqil isbotlang.

2- masala. AB va CD – bir aylananing vatarlari, P – ularning kesishish nuqtasi. Agar BPD burchak BPC burchakdan 4 marta katta, CDA burchak esa BPC dan 26° ga katta bo‘lsa, CBP burchakni toping.



Berilgan: $\angle BPD = 4\angle BPC$, $\angle CDA = \angle BPC + 26°$ (4- rasm).

Topish kerak: $\angle CBP$.

Yechish. $\angle BPD + \angle BPC = 180°$,

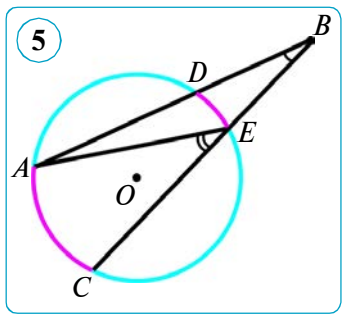
$4\angle BPC + \angle BPC = 180°$, bundan $5\angle BPC = 180°$ va nihoyat, $\angle BPC = 36°$. $\angle CDA = \angle BPC + 26° = 36° + 26° = 62°$. $\angle CBA = \angle CDA = 62°$, chunki ular bitta $\cup AC$ ga tiralgan ichki chizilgan burchaklar. Bundan $\angle CBP = \angle CBA = 62°$.

Javob: $\angle CBP = 62°$.

3. Aylana tashqarisidagi bir nuqtadan unga o‘tkazilgan ikki kesuvchi orasidagi burchak.

3-teorema.

Aylananing tashqarisidagi bir nuqtadan unga o‘tkazilgan ikki kesuvchi orasidagi burchak (ABC) kesuvchilar orasidagi yoylar (AC va DE) ayirmasining yarmiga teng.



Isbot. B – aylana tashqarisidagi nuqta, BA va BC kesuvchilar bo‘lsin. $\angle B = \frac{1}{2}(\cup AC - \cup DE)$ bo‘lishini isbotlaymiz. Buning uchun A va E nuqtalarni birlashtiramiz (5- rasm).

$\angle AEC$ $\triangle AEB$ ga tashqi burchak bo‘ladi. Demak, $\angle AEC = \angle B + \angle DAE$, bundan $\angle B = \angle AEC - DAE$. Ammo $\angle AEC = 0{,}5 \cup AC$ va $\angle DAE = 0{,}5 \cup DE$. Bularni o‘z o‘rinlariga qo‘ysak:

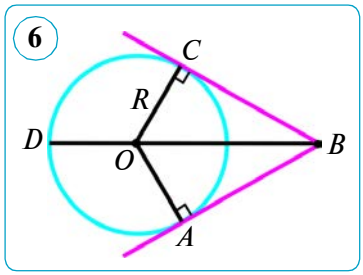
$$\angle B = \frac{1}{2} \cup AC - \frac{1}{2} \cup DE = \frac{1}{2}(\cup AC - \cup DE).$$

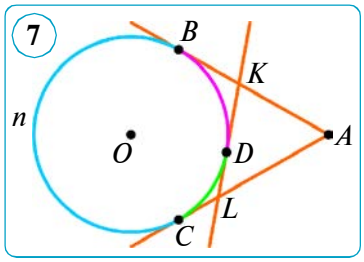
Demak, $\angle B = \frac{1}{2}(\cup AC - \cup DE)$. Teorema isbotlandi.

4. Aylana tashqarisidagi bir nuqtadan unga o‘tkazilgan ikki urinmaning xossasi.

4-teorema.

Aylana tashqarisidagi bir nuqtadan unga ikkita urinma o‘tkazilsa, ularning o‘sha nuqtadan urinish nuqtalarigacha bo‘lgan kesmalari teng va aylananing markazi ular orasidagi burchak bissektrisasida yotadi, bu burchak 180° bilan urinmalar tiralgan yoy ayirmasiga teng.





Isbot. BC va BA to‘gri chiziqlar aylanaga C va A nuqtalardan o‘tuvchi urinmalar, BD esa ABC burchak bissektrisasi bo‘lsin. $AB = CB$ va O markazning BD da yotishi hamda $\angle B = 180° - \cup AC$ ekanini ko‘rsatamiz (6- rasm).

OA va OC radiuslar o‘tkazilsa, $OA \perp BA$ va $OC \perp BC$ bo‘lgani uchun: $\triangle AOB$ va $\triangle COB$ – to‘g‘ri burchakli. $\triangle AOB = \triangle COB$, chunki BO gipotenuza umumiy, $OA = OC = R$. Uchburchaklarning tengligidan: $AB = BC$. Endi $OC = OA = R$ va $OA \perp BA$, $AB = BC$ va $OC \perp BC$ bo‘lgani uchun O markaz doimo BD bissektrisada yotadi. Aylana tashqarisidagi bir nuqtadan o‘tkazilgan ikki kesuvchi orasidagi burchakni o‘lchash haqidagi teoremaga asosan:

$$\angle B = 0{,}5(\cup ADC - \cup AC) =$$
$$= 0{,}5(360° - \cup AC - \cup CA) = 180° - \cup AC.$$

Demak, $\angle B = 180° - \cup AC$ bo‘ladi. Teorema isbotlandi.

3- masala. Aylananing A, B va C nuqtalari uni $11 : 3 : 4$ nisbatdagi yoylarga bo‘ladi. A, B va C nuqtalardan urinmalar o‘tkazilib, bir-biri bilan kesishguncha davom ettirilgan. Hosil bo‘lgan uchburchakning burchaklarini toping.

Yechish. 1) $\cup BnC : \cup CD : \cup DB = 11 : 3 : 4$, urinish nuqtalariga urinmalar o‘tkazishdan hosil bo‘lgan uchburchak AKL bo‘lsin (7- rasm). A, AKL va ALK burchaklarni topamiz:

$$\cup BnC = \frac{360°}{11+3+4} \cdot 11 = 220°; \qquad \cup CD = \frac{360°}{11+3+4} \cdot 3 = 60°;$$

$\cup DB = \frac{360^\circ}{11+3+4} \cdot 4 = 80^\circ$;

$\cup CDB = \cup CD + \cup DB = 60^\circ + 80^\circ = 140^\circ$;

$\angle A = 180^\circ - \cup CDB = 180^\circ - 140^\circ = 40^\circ$,

$\angle BKD = 180^\circ - \cup DB = 180^\circ - 80^\circ = 100^\circ$,

$\angle AKL = 180^\circ - \angle BKD = 180^\circ - 100^\circ = 80^\circ$,

$\angle ALK = 180^\circ - (\angle A + \angle AKL) = 180^\circ - (40^\circ + 80^\circ) = 180^\circ - 120^\circ = 60^\circ$.

Javob: $\angle A = 40^\circ$, $\angle AKL = 80^\circ$, $\angle ALK = 60^\circ$.

Savol, masala va topshiriqlar

1. 1) Urinma bilan vatardan tuzilgan; ikki vatarning kesishishidan hosil bo‘lgan; ikki kesishuvchi vatar orasidagi burchak qanday o‘lchanadi?

2) Bir nuqtadan o‘tkazilgan ikki urinma qanday xossaga ega?

2. Aylana radiusiga teng AB vatar A nuqtada o‘tkazilgan urinma bilan qanday burchaklar hosil qiladi?

3. Aylanani kesuvchi ikki vatari orasidagi burchaklardan biri 70° ga teng. Shu burchakka qo‘shni bo‘lgan burchaklarning yig‘indisini toping.

4. 8- rasmda tasvirlangan x noma’lum miqdorni toping.

5. Ikki radius orasidagi burchak 150° ga teng. Bu radiuslarning oxirlaridan aylanaga o‘tkazilgan urinmalar orasidagi burchakni toping.

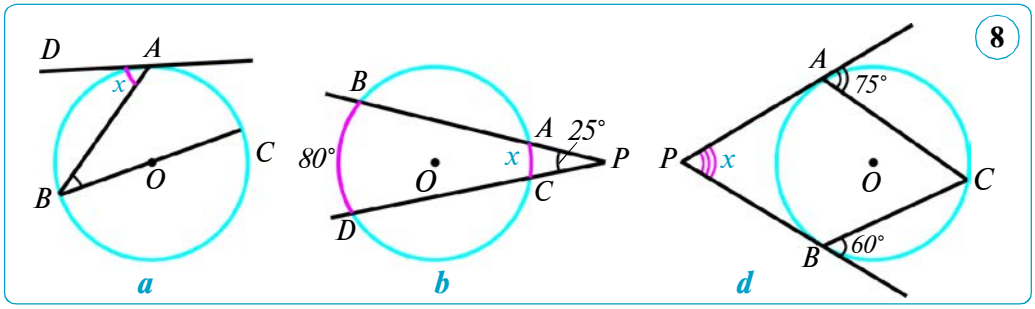
6. B nuqtadan aylanaga o‘tkazilgan BA va BC urinmalar aylanani urinish nuqtalarida: 1) 5 : 4; 2) 12 : 6; 3) 9 : 6; 4) 13 : 7; 5) 2 : 3 nisbatda ikki yoyga bo‘ladi. ABC burchakning miqdorini toping.

7. Aylanani: 1) 2 : 7; 2) 4 : 5 nisbatda bo‘luvchi vatarning uchlaridan ikkita urinma o‘tkazilgan. Hosil bo‘lgan uchburchakning burchaklarini toping.

8. Aylanadan tashqaridagi nuqtadan o‘tkazilgan ikki urinmaning urinish nuqtalari aylanani: 1) 1 : 9; 2) 3 : 15; 3) 7 : 11; 4) 3 : 7 nisbatdagi ikkita yoyga ajratadi. Urinmalar orasidagi burchakni toping.

9. 1) 52°; 2) 74°; 3) 104° li markaziy burchak tashkil etgan ikki radiusning uchlariga o‘tkazilgan urinmalar orasidagi burchakni toping.

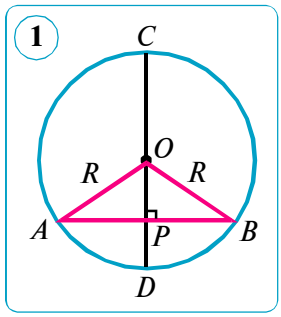
10. Aylananing radiusi diametridan 40 mm qisqa. Aylananing diametrini toping.



59. AYLANA VATARI VA DIAMETRINING XOSSALARI

1-teorema.

Vatarga perpendikular diametr shu vatarni va unga tiralgan yoyni teng ikkiga bo‘ladi.



Isbot. Markazi O nuqtada va radiusi R bo‘lgan aylana, AB vatarga perpendikular CD diametr, CD va AB larning kesishish nuqtasi P berilgan bo‘lsin (1- rasm). $AP = PB$ va $\cup AD = \cup DB$ ekanini isbotlaymiz. Agar AB vatar diametr bo‘lsa, P nuqta O nuqta bilan ustma-ust tushadi va shu nuqtada AB vatar hamda uni tortib turgan yarim aylananing ADB yoyi teng ikkiga bo‘linadi, ya’ni tasdiq o‘rinli bo‘ladi. AB vatar diametr bo‘lmasin. OA va OB radiuslarni o‘tkazamiz. Hosil bo‘lgan AOB uchburchak – teng yonli, chunki $OA = OB = R$. OP – teng yonli uchburchakning AB tomoniga tushirilgan balandlik teng yonli uchburchakning xossasiga ko‘ra, uchburchakning asosiga o‘tkazilgan mediana va O uchidagi burchagining bissektrisasi bo‘ladi. Vatarning o‘rtasi orqali o‘tgan diametr esa AB vatarni teng ikkiga bo‘ladi, ya’ni $AP = PB$. $OP - AOB$ burchakning bissektrisasi ekanidan $\angle AOP = \angle BOP$ ni hosil qilamiz. Bu burchaklar tiralgan yoylar bo‘lgani uchun $\cup AD = \cup DB$. Teorema isbotlandi.

2-teorema.

Aylana vatari uning diametridan katta bo‘lmaydi.

Isbot. OPB uchburchak – to‘g‘ri burchakli (1- rasmga q.). U holda, bu uchburchakda OB – gipotenuza, PB – katet. Ma’lumki, katet gipotenuzadan katta emas, ya’ni $PB \le OB$. Bundan, $2PB \le 2 \cdot OB$ hamda $2PB = AB$ va $2OB = 2R = d$. Demak, $AB \le d$ ekan.

1-natija. *Vatarning o‘rtasidan o‘tuvchi diametr shu vatarga perpendikulardir.*

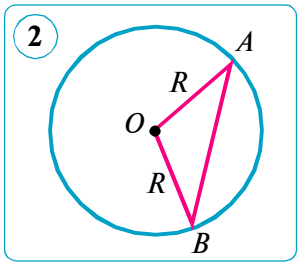
2-natija. *Vatarning o‘rta perpendikulari aylananing diametri bo‘ladi.*

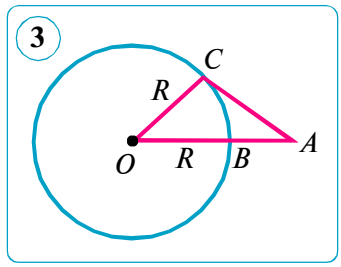
Bu natijalarni isbot qilish o‘zingizga havola qilinadi.

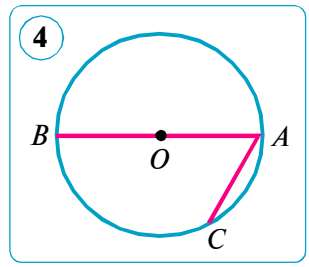
1-masala. Diametr eng katta vatar ekanini isbotlang.

Yechish. O markazli va R radiusli aylana hamda diametrdan farqli ixtiyoriy AB vatar berilgan bo‘lsin (2- rasm). OA va OB kesmalarni o‘tkazamiz. AOB uchburchakda AB tomon qolgan ikki tomon yig‘indisidan kichik, ya’ni $AB < OA + OB = R + R = 2R$. Demak, AB vatar diametrdan kichik bo‘ladi.

2-masala. A nuqta R radiusli aylanadan tashqarida va bu aylananing O markazidan d masofada joylashgan. A nuqtadan shu aylanadagi nuqtagacha bo‘lgan eng qisqa masofa qanchaga teng?







Yechish. B – aylananing OA kesma bilan kesishgan nuqtasi bo‘lsin (3- rasm). AB masofa A nuqtadan aylanadagi nuqtalargacha mumkin bo‘lgan masofalar ichida eng kichigi ekanini ko‘rsatamiz. Haqiqatan ham, aylananing istalgan C nuqtasi uchun $AB + BO < AC + CO$ tengsizlik bajariladi. $BO = CO = R$ ni e’tiborga olib, oxirgi tengsizlikdan $AB < AC$ tengsizlikni hosil qilamiz. $AO = d$ va $BO = R$ ni hisobga olsak, izlanayotgan eng qisqa masofa AB kesmaning uzunligiga, ya’ni $d - R$ ga teng ekani kelib chiqadi.

Savol, masala va topshiriqlar

1. 1) Vatarga perpendikulyar diametr qanday xossaga ega?
2) Aylana vatari uning diametridan katta bo‘lishi mumkinmi?
3) Vatarning o‘rta perpendikulyari diametr bo‘lmasligi mumkinmi?

2. Aylana chizing hamda uning bir-biriga perpendikulyar ikkita AB va CD diametrlarini o‘tkazing. A, B, C va D nuqtalar ajratgan aylana yoylarining gradus o‘lchovini toping.

3. 8 cm li vatar aylanadan 90° li yoy ajratadi. Aylana markazidan vatargacha bo‘lgan masofani toping.

4. Berilgan aylananing nuqtasidan radiusiga teng ikkita vatar o‘tkazilgan. Ular orasidagi burchakni toping.

5. Aylananing berilgan nuqtasidan diametr va radiusga teng vatar o‘tkazilgan. Diametr bilan vatar orasidagi burchakni toping (4- rasm).

6. Aylanada undan 90° li yoy ajratuvchi ikkita parallel vatar o‘tkazilgan. Ulardan birining uzunligi 8 sm. Vatarlar orasidagi masofani toping.

7. Aylananing markazidan boshqa nuqtada kesishuvchi ikki vatari kesishish nuqtasida teng ikkiga bo‘linmasligini isbotlang.

8. Aylanadagi A nuqtadan aylana radiusiga teng ikki vatar AB va AC o‘tkazilgan. B va C nuqtalar to‘g‘ri chiziq bilan tutashtirilgan. Aylananing radiusi 12 cm. Aylana markazidan BC vatargacha bo‘lgan masofani toping.

9. Aylanada undan 90° li yoy ajratuvchi ikkita parallel vatar o‘tkazilgan. Ulardan birining uzunligi 10 cm. Vatarlar orasidagi masofani toping.

10. Aylananing radiusi 13 cm ga teng. Shu aylanada 10 cm ga teng vatar o‘tkazilgan. Aylana markazidan vatargacha bo‘lgan masofani toping.

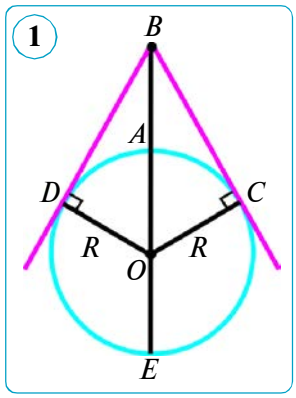
11. AB kesma – markazi O nuqtada bo‘lgan aylananing diametri, AC va CB – shu aylananing teng vatarlari. COB burchakni toping.

AMALIY KOMPETENSIYANI RIVOJLANTIRUVCHI QO‘SHIMCHA MATERIALLAR

GORIZONTNING UZOQLIGI

1- masala. (*Tayanch masala.*) Kesuvchi bilan uning tashqi qismi ko‘paytmasi urinma kvadratiga teng. Shuni isbotlang.

Yechish. O markazli aylana tashqarisida olingan B nuqtadan BE kesuvchi, BC va BD urinmalar o‘tkazilgan bo‘lsin (1- rasm).



$BC^2 = BE \cdot BA$ ekanini isbotlaymiz. Buning uchun to‘g‘ri burchakli BOC ($\angle C = 90°$) uchburchakni ko‘rib chiqamiz. Bundan Pifagor teoremasiga ko‘ra:

$$BC^2 = BO^2 - OC^2.$$

Bu tenglikka $BO = BA + AO = BA + R$ va $OC = R$ belgilashlarni qo‘yib, hosil bo‘lgan tenglikni shakl almashtiramiz:

$$BC^2 = (BA + R)^2 - R^2 \Rightarrow BC^2 = BA^2 + 2BA \cdot R + R^2 - R^2 \Rightarrow$$
$$\Rightarrow BC^2 = BA^2 + 2BA \cdot R \Rightarrow BC^2 = BA \cdot (BA + 2R) \Rightarrow$$
$$\Rightarrow BC^2 = BA \cdot BE$$

Shuni isbotlash talab qilingan edi.

1. Gorizont haqida tushuncha.

Uzoqni ko‘rish uchun hech narsa xalaqit qilmaydigan ochiq joyda turib olisga qaraganingizda siz o‘zingizni yer sirti (dengiz sirti) go‘yoki osmon bilan tutashib ketgandek va undan keyin hech narsa yo‘qdek ko‘rinadigan aylana markazida turgandek his qilasiz. Bu – gorizontdir. Gorizont chizig‘i tutqich bermaydi: siz unga yaqinlashgan sari, u sizdan uzoqlashaveradi. Unga borib bo‘lmaydi, ammo shunga qaramasdan u haqiqatda mavjud. Har bir kuzatish nuqtasi uchun shu yerdan turib qaraganda yer sirtini ko‘rish mumkin bo‘lgan ma’lum chegarasi bo‘ladi va bu chegaraning uzoqligini hisoblash qiyin emas. Gorizontga bog‘liq bo‘lgan geometrik nisbatlarni tushunish uchun Yer sharining ma’lum qismini tasvirlaydigan 1- rasmga (yoki 2- rasmga) murojaat qilamiz. Yerdan BA balandlikdagi B nuqtada kuzatuvchining ko‘zi joylashadi. Shu kuzatuvchi bu joyda o‘zining tevarak-atrofini qanday uzoqlikkacha ko‘ra oladi? Qarash nuri Yer sirtiga urinadigan C va D (1- rasm) yoki C (2- rasm) nuqtalargacha ekani ravshan: bundan narida Yer qarash nuridan pastda bo‘ladi. Bu nuqtalar (va DAC yoyda yotgan boshqa nuqtalar ham) yer sirti ko‘rinadigan qismining chegarasini tasvirlaydi, ya’ni gorizont chizig‘ini hosil qiladi. Kuzatuvchiga mana shu joyda osmon yerga tutashgandek bo‘lib ko‘rinadi, chunki kuzatuvchi bu nuqtalarda bir vaqtning o‘zida ham osmonni, ham yerdagi narsalarni ko‘radi.

2. Gorizontning uzoqligi.

Gorizont chizig‘i kuzatuvchidan qanday uzoqlikda bo‘ladi? Boshqacha aytganda, tekis joyda biz markazida o‘zimizni ko‘rgan doira radiusining kattaligi

qancha? Kuzatuvchining yer sirtidan ko‘tarilgan balandligi ma’lum bo‘lsa, gorizontning uzoqligi qanday hisoblanadi?

Masala kuzatuvchining ko‘zidan yer sirtiga o‘tkazilgan urinma (2- rasm) BC kesmasining uzunligini hisoblashga keltiriladi. 1- masaladan ma’lumki, urinmaning kvadrati kesuvchining tashqi kesmasi $BA = h$ bilan kesuvchining hamma uzunligi, ya’ni $BE = h + 2R$ ning ko‘paytmasiga teng: $d^2 = (h + 2R) \cdot h$, bunda R – Yerning radiusi, $BC = d$ – kuzatuvchidan ko‘rinadigan eng uzoq masofa. Kuzatuvchi ko‘zining yerdan ko‘tarilishi Yer sharining diametriga ($2R$ ga) nisbatan juda kichik, masalan, samolyotning eng baland ko‘tarilishi Yer shari diametrining taxminan 0,001 ulushinigina tashkil qiladi, u holda $2R + h \approx 2R$ deb olish mumkin, unda formula yanada soddalashadi:

$$d^2 \approx 2Rh.$$

Demak, gorizontning uzoqligini juda oddiy formula bo‘yicha hisoblash mumkin:

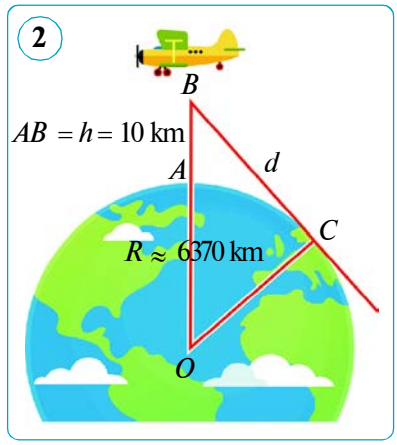
$$d \approx \sqrt{2Rh},$$

bunda: R – Yer sharining radiusi (taxminan 6400 km yoki aniqrog‘i 6371 km), h – yer sirtidan kuzatuvchi ko‘tarilgan balandlik, $\sqrt{6400} = 80$, unda formula quyidagicha ko‘rinishni olishi mumkin:

$$d \approx 80\sqrt{2h} \approx 113\sqrt{h},$$

bunda h albatta kilometrning bo‘laklarida ifodalanishi lozim.

2- masala. Yerdan 10 km balandlikda uchayotgan samolyotdan qancha uzoqlikdagi masofani ko‘rish mumkin? (Yerning radiusi taxminan 6370 km.)



Yechish. $OA = R \approx 6370$ km, $AB = h = 10$ km. $BC = d$ ni topamiz (2- rasm). Kesuvchi bilan uning tashqi qismi ko‘paytmasi urinmaning kvadratiga teng ekanini bilasiz, ya’ni

$$d^2 = (h + 2R) \cdot h \quad \text{yoki}$$

$$d^2 = (10 + 2 \cdot 6370) \cdot 10 = 127\,500,$$

bundan:

$$d = \sqrt{127500} = \sqrt{51 \cdot 2500} = 50\sqrt{51} \approx$$

$$\approx 50 \cdot 7{,}141 = 357{,}05 \approx 360 \text{ (km)}.$$

Javob: ≈ 360 km.

3- masala. Yerdan 4 km balandlikka ko‘tarilgan havo sharidan qancha uzoqlikdagi masofa ko‘rinadi? Yerning radiusi taxminan 6370 km. *Javob:* $\approx 225{,}8$ km.

4- masala. Kavkazdagi Elburs cho‘qqisi dengiz sathidan ≈ 5600 m (anig‘i 5642 m) balandlikda joylashgan. Shu cho‘qqidan qanday uzoqlikni ko‘rish mumkin? Yerning radiusi taxminan 6370 km. *Javob:* ≈ 270 km.

Eslatma! *Yuqorida yechilgan masalalarda gorizontning uzoqligiga ta’sir qiladigan fizik omillarni hisobga olmadik. Gorizontning uzoqligi ko‘pgina omillarga bog‘liq holda birmuncha ortishi yoki kamayishi mumkin.*

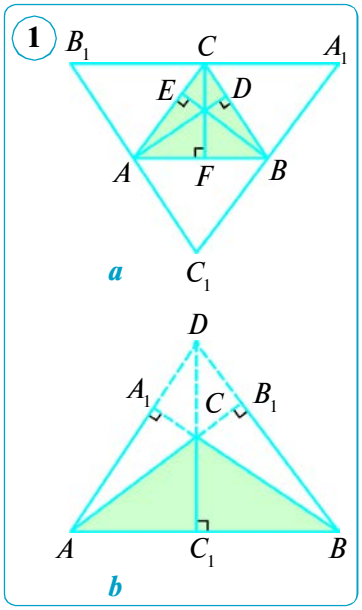
UCHBURCHAKNING AJOYIB NUQTALARI

Uchburchakning to‘rtta ajoyib nuqtasini ko‘rib chiqamiz.

1. Uchburchak balandliklarining kesishish nuqtasi.

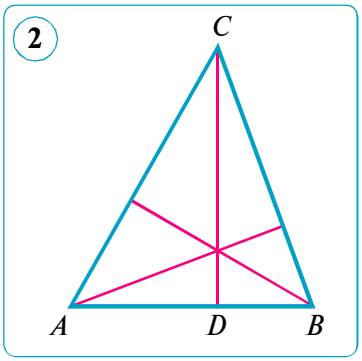
1-teorema.

Uchburchakning balandliklari (yoki ularning davomi) bir nuqtada kesishadi.



Isbot. AD, BF va CE – ABC uchburchakning balandliklari (1- *a* rasm). Uchburchakning uchlari orqali qarama-qarshi yotgan tomonlariga parallel qilib to‘g‘ri chiziqlar o‘tkazib, natijada tomonlari ABC uchburchakning balandliklariga perpendikular bo‘lgan yangi $A_1B_1C_1$ uchburchakni hosil qilamiz. Yasashga ko‘ra, C_1BCA va B_1ABC to‘rtburchaklar – parallelogramm, bundan $C_1A = BC$ va $BC = AB_1$ ekani kelib chiqadi. Demak, A nuqta – B_1C_1 kesmaning o‘rtasi. Xuddi shuningdek, B nuqta – A_1C_1 ning o‘rtasi, C esa A_1B_1 ning o‘rtasi ekani isbotlanadi.

Shunday qilib, AD, BF va CE balandliklar $A_1B_1C_1$ uchburchakning o‘rta perpendikularida yotadi. Demak, ular bir nuqtada kesishadi. Uchburchakning balandliklari kesishmasligi ham mumkinligini qayd qilib o‘tamiz. O‘tmas burchakli uchburchak balandliklari ularning davomida bitta nuqtada kesishadi, ammo balandliklarning o‘zi kesishmaydi (1- *b* rasm).



Uchburchak balandliklari (yoki ularning davomi)ning kesishish nuqtasi uning *ortomarkazi* ham deyiladi.

Masala. Uchburchak tomonlaridan qaysi biri ortomarkazga yaqin joylashgan?

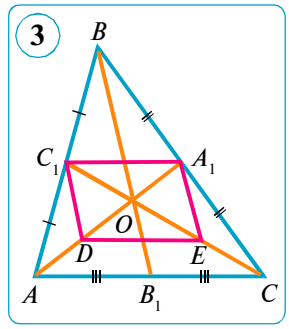
Yechish. ABC uchburchakda $AC > BC$ bo‘lsin (2- rasm). Uchburchakning CD balandligi uchun $AD > BD$ tengsizlik va demak, $\angle ACD > \angle BCD$ tengsizlik bajarilishidan foydalanamiz. Bu balandlik nuqtalari shu uchdan chiquvchi tomonlardan eng kichigiga yaqin joylashganini bildiradi. Demak, uchburchakning ortomarkazi kichik tomonga yaqin joylashadi.

2. Uchburchak medianalarining kesishish nuqtasi.

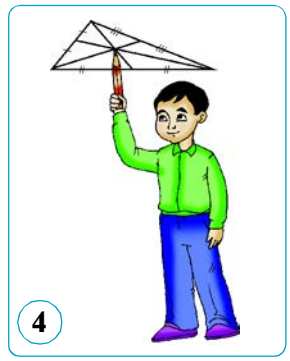
2-teorema.

Uchburchak medianalari bir nuqtada kesishadi va bu nuqtada uchidan boshlab hisoblaganda 2 : 1 nisbatda bo‘linadi.

Isbot. ABC uchburchakda AA_1, BB_1 va CC_1 medianalar o‘tkazilgan bo‘lsin (3- rasm). Ular biror O nuqtda kesishishini hamda $AO : OA_1 = BO : OB_1 = CO : OC_1 = 2 : 1$ bo‘lishini isbotlaymiz.



$O - AA_1$ va CC_1 medianalarning kesishish nuqtasi, D va E mos ravishda AO va CO kesmalarning o‘rtasi bo‘lsin. C_1A_1 kesma ABC uchburchakning o‘rta chizig‘i va uchburchak o‘rta chizig‘ining xossasiga ko‘ra: $C_1A_1 \parallel AC$, $C_1A_1 = 0{,}5AC$. Bundan tashqari, $DE - AOC$ uchburchakning o‘rta chizig‘i va o‘sha xossaga ko‘ra: $DE \parallel AC$, $DE = 0{,}5AC$. Demak, DC_1A_1E to‘rtburchakning ikki tomoni parallel va teng. Shunday qilib, DC_1A_1E – parallelogramm, uning DA_1 va C_1E diagonallari kesishish nuqtasida teng ikkiga bo‘linadi. Demak, $AD = DO = OA_1$, $CE = EO = OC_1$, ya’ni AA_1 va CC_1 medianalar O nuqtada $2 : 1$ nisbatda bo‘linadi.



Xuddi shuningdek, uchinchi BB_1 mediana – AA_1 va CC_1 medianalarning har biri bilan kesishish nuqtasida $2 : 1$ nisbatda bo‘linishi isbotlanadi. Har bir mediana uchun bunday bo‘linish yagona va demak, uchala mediana bir nuqtada kesishar ekan.

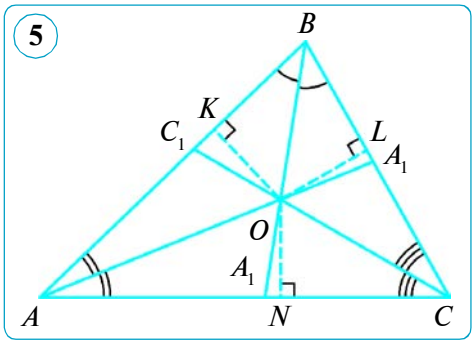
Uchburchak medianalarining kesishish nuqtasi *sentroid* yoki *og‘irlik markazi* ham deyiladi. Bunday nomlanishini quyidagi tajribada tekshirib ko‘ring: karton qog‘ozdan ixtiyoriy uchburchak qirqib oling va uning medianalarini o‘tkazing, so‘ngra igna yoki uchi o‘tkir ochilgan qalam uchini medianalarning kesishish nuqtasiga qo‘yib, muvozanatda ushlashga harakat qiling (4- rasm).

3. Uchburchak bissektrisalarining kesishish nuqtasi.

3- t e o r e m a .

Uchburchakning uchala bissektrisasi bir nuqtada kesishadi.

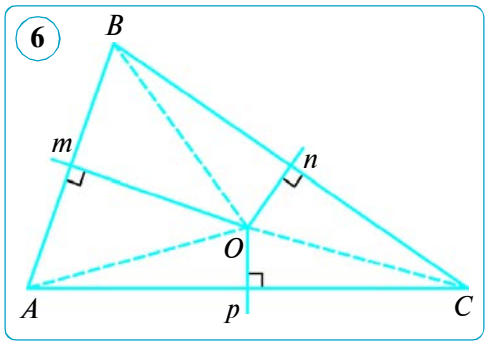
Isbot. ABC uchburchakning AA_1 va BB_1 bissektrisalari kesishgan nuqtasini O bilan belgilaymiz. U nuqtadan mos ravishda AB, BC va CA to‘g‘ri chiziqlarga OK, OL va OM perpendikularni o‘tkazamiz (5- rasm). Bizga ma’lumki, burchak bissektrisasining ixtiyoriy nuqtasidan burchak tomonlarigacha bo‘lgan masofalar teng. Shunga asosan, $OK = OK$ va $OK = OL$. Shuning uchun $ON = OL$, ya’ni O nuqta ACB burchakning tomonlaridan teng uzoqlashgan bo‘ladi va demak, shu burchakning CC_1 bissektrisasida yotadi. Bundan, ABC uchburchakning uchala bissektrisasi O nuqtada kesishishi kelib chiqadi. Teorema isbotlandi.



4. Uchburchak o‘rta perpendikularlarining kesishish nuqtasi.

4-teorema.

Uchburchak tomonlarining o‘rta perpendikularlari bir nuqtada kesishadi.



Isbot. $\triangle ABC$ berilgan (6- rasm). Uning AB va BC tomonlariga m va n o‘rta perpendikularlar o‘tkazamiz. Ular biror O nuqtada kesishadi (kesishuvchi to‘g‘ri chiziqlarga perpendikular to‘g‘ri chiziqlar kesishadi). Bizga ma’lumki, kesma o‘rta perpendikularining ixtiyoriy nuqtasidan kesma uchlarigacha bo‘lgan masofalar teng. Shunga ko‘ra, $OA = OB$ (1) va $OB = OC$ (2) bo‘ladi. (1) va (2) tengliklardan topamiz: $OA = OC$. Demak, AC tomonning o‘rta perpendikulari p ham O nuqtadan o‘tadi. Shunday qilib, O nuqta $\triangle ABC$ ning uchala uchidan teng uzoqlashgan bo‘ladi: $OA = OB = OC$. Bundan, $\triangle ABC$ ning tomonlariga o‘tkazilgan uchala m, n va p o‘rta perpendikulari O nuqtada kesishishi kelib chiqadi. Teorema isbotlandi

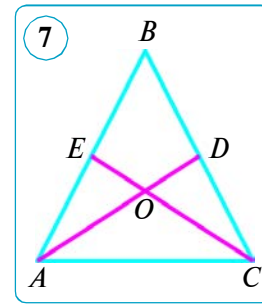
Savol, masala va topshiriqlar

1. 1) Har doim ham uchburchakning balandliklari kesishadimi?

? 2) Uchburchakning nechta ajoyib nuqtasini bilasiz? Ularni ayting.

2. Teng tomonli uchburchakning ajoyib nuqtalari qanday joylashgan bo‘ladi?

3. Agar uchburchakda ikkita mediana teng bo‘lsa, u holda u teng yonli bo‘ladi. Shuni isbotlang.



Yechish. ABC uchburchakda AD va CE medianalar teng hamda O nuqtada kesishsin (7- rasm). AOE va COD uchburchaklarni ko‘rib chiqamiz. O nuqta teng AD va CE medianalarning har birini $2:1$ nisbatda bo‘ladi. Shuning uchun, $AO = CO$, $EO = DO$ bo‘ladi. Bundan tashqari, vertikal burchaklar bo‘lgani uchun: $\angle AOE = \angle COD$. Demak, uchburchaklar tengligining birinchi alomatiga ko‘ra: $\triangle AOE = \triangle COD$. Bundan, $AE = CD$ ekani kelib chiqadi.

Bu kesmalar mediananing ta’rifiga ko‘ra, AB va CB tomonlarning yarmiga teng. Demak, $AB = CB$, ya’ni ABC uchburchak teng yonli ekan. Shuni isbotlash talab qilingan edi.

4. Teng yonli uchburchakning to‘rttala ajoyib nuqtasi bir to‘g‘ri chiziqda yotishini isbotlang. U qaysi to‘g‘ri chiziq bo‘ladi?

5. Uchburchak medianalarining kesishish nuqtasi ortomarkaz bilan ustma-ust tushadi. Berilgan uchburchak teng tomonli ekanini isbotlang.

6. Uchburchakning uchi balandliklari kesishgan nuqtasi bo‘lishi mumkinmi?

7. Uchburchak medianalarining kesishish nuqtasi medianalardan birini ayirmasi 3 cm ga teng bo‘laklarga bo‘ladi. Bu mediana uzunligini toping.

61–62. 5- NAZORAT ISHI. XATOLAR USTIDA ISHLASH

1. AB – O markazli aylananing diametri. Agar $OA = OC = AC$ bo‘lsa, BCO burchakni toping.

2. 1) Aylana tashqarisida berilgan nuqtadan aylana nuqtalarigacha bo‘lgan eng katta va eng kichik masofalar mos ravishda 50 cm va 20 cm ga teng. Berilgan aylananing radiusini toping.
2) Aylana markazidan B nuqtagacha masofa 3 cm ga, radius 10 cm ga teng. B nuqtadan aylanagacha bo‘lgan eng kichik va eng katta masofani toping.

3. AB va AC to‘g‘ri chiziqlar O markazli aylanaga B va C nuqtalarda urinadi. Agar $\angle OAB = 30^\circ$ va $AB = 5$ cm bo‘lsa, BC ni toping.

4. Aylana 11 : 16 : 9 nisbatda uch yoyga bo‘lingan va bo‘linish nuqtalari tutashtirilgan. Hosil bo‘lgan uchburchak burchaklarining kattaliklarini toping.

5- TEST — O‘zingizni sinab ko‘ring!

1. Aylana markazidan B nuqtagacha masofa 5 cm ga, radius 12 cm ga teng. B nuqtadan aylanagacha bo‘lgan eng kichik va eng katta masofani toping.
A) 7 cm, 17 cm; B) 7 cm, 12 cm; D) 5 cm, 7 cm; E) 7 cm, 24 cm.

2. Aylana tashqarisida berilgan nuqtadan aylana nuqtalarigacha bo‘lgan eng katta va eng kichik masofalar mos ravishda 30 cm va 10 cm ga teng. Berilgan aylananing radiusini toping.
A) 20 cm; B) 10 cm; D) 15 cm; E) 5 cm.

3. AB – O markazli aylananing diametri. Agar $OA = OC = BC$ bo‘lsa, CAO burchakni toping.
A) 60°; B) 30°; D) 90°; E) 120°.

4. Radiusi R ga teng bo‘lgan aylanadagi nuqtadan uzunliklari R ga teng bo‘lgan ikkita vatar o‘tkazildi. Vatarlar orasidagi burchakni toping.
A) 120°; B) 110°; D) 135°; E) 40°.

5. Aylanani kesuvchi ikki vatari orasidagi burchaklardan biri 80° ga teng. Shu burchakka qo‘shni bo‘lgan burchaklarning yig‘indisini toping.
A) 200°; B) 90°; D) 100°; E) 160°.

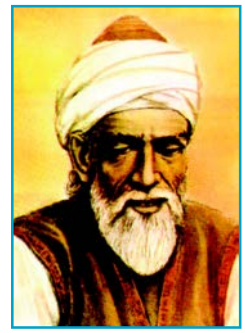
6. Aylana tashqarisidagi nuqtadan aylanaga ikkita urinma o‘tkazilgan. Agar urinmalar orasidagi burchak 72° bo‘lsa, aylananing urinish nuqtalari orasidagi katta yoyini toping.
A) 248°; B) 240°; D) 252°; E) 236°.



Ingliz tilini o‘rganamiz!

Aylana – circle	**Diametr** – diameter
Vatar – chord	**Markaziy burchak** – central angle
Radius – radius	**Aylanaga urinma** – tangent to the circle
Yoy – arc	**Perpendikular** – perpendicular

Tarixiy ma'lumotlar



Abul Vafo Buzjoniy
(940–998)

Abul Vafo Buzjoniy 940- yil Xuroson viloyatining Hirot va Nishopur shaharlari orasidagi Buzjon shahrida (hozirda Turkmanistonning Sarxatobod shahri atrofida) tug'ilgan. U Bag'dodda o'qigan va ijod qilgan.

Abul Vafo Buzjoniyning «Hunardmandlar geometrik yasashlardan nimalarni bilishlari zarur» nomli kitobining birinchi va ikkinchi boblari chizg'ich va sirkul yordamida yasashlarga bag'ishlangan. Biz sizga Abul Vafoning aylananing markazini topish masalasini keltiramiz.

«Agar «Aylananing markazi qanday topiladi?» deb so'ralsa, uning aylanasida A va B nuqtalarni belgilab, AB masofa bilan A va B nuqtalarni markaz qilib ikkita teng aylana yasaymiz, ular C va D nuqtalarda kesishadi (1- *rasm). CD chiziqni o'tkazamiz va uni aylana bilan E va F nuqtalarda kesishguncha davom ettiramiz, so'ngra EF chiziqni O nuqtada teng ikkiga bo'lamiz. U holda O nuqta aylananing markazi bo'ladi».*

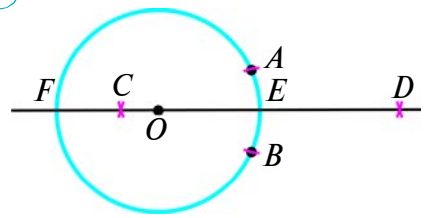
Abul Vafoning bu usuli A va B nuqtalarni markaz qilib yoy chizilganda ularning kesishgan nuqtalarini tutashtiruvchi CD to'g'ri chiziq berilgan aylananing markazidan o'tib, uning AB vatariga perpendikular bo'lishiga asoslangan.

Hozir bu masala quyidagicha yechiladi: faraz qilaylik, bizga markazi belgilanmagan aylana berilgan va uning markazini aniqlash talab qilingan (2- rasm).

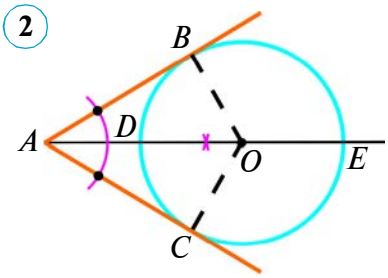
A nuqtadan bu aylanaga AB va AC urinmalarni o'tkazamiz hamda BAC burchakning bissektrisasini yasaymiz. Bissektrisa aylanani D va E nuqtalarda kesadi. DE ni teng ikkiga bo'lsak, bo'linish nuqtasi O aylananing markazi bo'ladi. Nega? Yoki B nuqtada AB urinmaga perpendikular o'tkazsak, u bissektrisani O nuqtada kesadi. O nuqta aylana markazi bo'ladi. Nega?

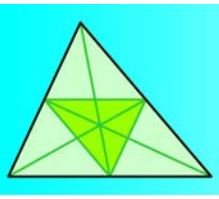
Shu bilan bir qatorda Abul Vafo o'z asarida yoyiq yoyni to'liq aylanaga to'ldirish, aylanaga uning tashqarisidagi nuqtadan urinma o'tkazish, aylanaga unda yotgan nuqtadan urinma o'tkazish kabi yasash usullarini bergan.

1



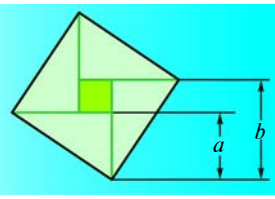
2





VI BOB

TAKRORLASH



8- SINFDA O‘TILGAN MAVZULARNI TAKRORLASH UCHUN MASHQLAR

1. To‘rtburchakning uchta tashqi burchagi mos ravishda 142°, 22° va 136° ga teng. Shu to‘rtburchakning burchaklarini toping.

2. To‘rtburchakning eng kichik tomoni 7 cm ga teng, qolgan tomonlarining har biri oldingisidan mos ravishda 4 cm ga katta. Shu to‘rtburchakning perimetrini toping.

3. To‘g‘ri burchakli trapetsiyaning o‘tkir burchagi 45° ga teng. Kichik yon tomoni hamda kichik asosi 24 cm ga teng. Shu trapetsiyaning katta asosini toping.

4. Teng yonli uchburchakning tomonlari: 1) 6 cm, 5 cm va 5 cm; 2) 24 cm, 15 cm va 15 cm; 3) 3,2 dm, 20 cm va 20 cm; 4) 22 cm, 60 cm va 60 cm. Shu uchburchakning yuzi va yon tomoniga o‘tkazilgan balandlikni toping.

5. $ABCD$ to‘rtburchakda: $AB = CD$, $AD = BC$, A burchak B burchakdan uch marta katta. Shu to‘rtburchakning burchaklarini toping.

6. $ABCD$ teng yonli trapetsiyada $BC = 20$ cm, $AB = 24$ cm va $\angle D = 60°$ bo‘lsa, uning AD asosini toping.

7. $\triangle ABC$ da AE va BD – balandliklar. $AC = 20$ cm, $BD = 16$ cm va $BC = 32$ cm. AE ni toping.

8. To‘g‘ri burchakli uchburchakning yuzi 168 cm^2 ga teng. Agar katetlaridan biri ikkinchisining $\frac{7}{12}$ qismiga teng bo‘lsa, uchburchakning katetlarini toping.

9. Uchburchakning yuzi 24 cm^2. Uchburchakning 16 cm ga teng tomoniga o‘tkazilgan balandligini toping.

10. $ABCD$ romb berilgan. AC va BD diagonallar mos ravishda 30 cm va 12 cm ga teng. Rombning yuzini toping.

11. Uchta tomoniga ko‘ra uchburchakning yuzini toping:
1) 15, 15, 18; 2) 39, 42, 45; 3) 4, 13, 15; 4) 29, 25, 6.

12. ABC uchburchakda $BC = 34$ cm. BC kesmaning o‘rtasidan AC to‘g‘ri chiziqqa o‘tkazilgan EF perpendikular AC tomonni $AF = 25$ cm va $FC = 15$ cm li kesmalarga ajratadi. ABC uchburchakning yuzini toping.

13. Rombning diagonallari 18 dm va 24 dm. Shu rombning perimetri va parallel tomonlari orasidagi masofani toping.

14. Teng yonli trapetsiyaning balandligi yon tomonidan ikki marta kichik. Trapetsiyaning burchaklarini toping.

15. Teng tomonli uchburchakning ixtiyoriy nuqtasidan tomonlarigacha bo‘lgan masofalar yig‘indisi o‘zgarmas (bir xil) va shu uchburchakning balandligiga teng. Shuni isbotlang.

16. Aylananing A, B va C nuqtalari uni: 1) $14:6:4$; 2) $13:12:5$; 3) $17:10:9$ nisbatdagi yoylarga bo‘ladi. A, B va C nuqtalardan urinmalar o‘tkazilib, bir-biri bilan kesishguncha davom ettirilgan. Hosil bo‘lgan uchburchakning burchaklarini toping.

17. To‘g‘ri to‘rtburchakning bo‘yi 30 % ga orttirilsa va eni 30 % ga kamaytirilsa, uning yuzi qanday o‘zgaradi?

18. Agar uchburchakning asosi 20 % uzaytirilib, balandligi 20 % ga qisqartirilsa, uning yuzi qanday o‘zgaradi?

19. To‘g‘ri to‘rtburchakning yuzi 540 cm^2, ikki tomonining nisbati $3:5$ kabi. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

20. Parallelogrammning yuzi 24 cm^2 ga teng. Agar balandliklari 3 cm va 4 cm ga teng bo‘lsa, uning perimetrini toping.

21. Biror $ABCD$ parallelogrammni chizing. Vektorlarni yasang:
1) $\overrightarrow{AB}+\overrightarrow{BC}$; 2) $\overrightarrow{AD}+\overrightarrow{DC}$; 3) $\overrightarrow{AB}-\overrightarrow{AD}$; 4) $\overrightarrow{DB}-\overrightarrow{DA}$.

22. Agar: 1) $A(0;\ 1)$, $B(1;\ 0)$; 2) $A(-2;\ 1)$, $B(-4;\ 3)$ bo‘lsa, $\overrightarrow{AB}$ vektorning koordinatalari nimaga teng bo‘ladi?

23. ABC uchburchakda AA_1 – mediana, O – AA_1 ning o‘rtasi. $\overrightarrow{BO}$ vektorni $\vec{a}=\overrightarrow{BA}$ va $\vec{b}=\overrightarrow{BC}$ vektorlar orqali ifodalang.

24. $ABCD$ parallelogramm diagonallari O nuqtada kesishadi, P nuqta OB ning o‘rtasi. $\overrightarrow{AP}$ vektorni $\overrightarrow{AB}=\vec{a}$ va $\overrightarrow{AC}=\vec{b}$ vektorlar orqali ifodalang.

25. 240° li yoyning uchlaridan o‘tkazilgan urinmalar kesishguncha davom ettirilgan. Ular orasidagi burchakni toping.

26. Parallelogrammning burchaklaridan biri ikkinchisidan 4 marta katta. Shu parallelogrammning katta burchagini toping.

27. To‘g‘ri to‘rtburchakning yuzi 288 cm^2, ikki tomonining nisbati $1:2$ ga teng. Shu to‘g‘ri to‘rtburchakning perimetrini toping.

28. Parallelogrammning tomonlaridan biriga o‘tkazilgan balandligi shu tomondan uch marta kichik. Parallelogrammning yuzi 48 cm^2. Shu tomon va balandligini toping.

29. Kvadratning yuzi 16 cm^2. Agar: 1) uning hamma tomonini ikki marta qisqartirsak; 2) uning hamma tomonini uch marta uzaytirsak, kvadratning yuzi qanday o‘zgaradi?

30. Agar: 1) 1) $A(7;\ -5)$, $B(-9;\ -3)$; 2) $A(-8;\ 2)$, $B(-12;\ -4)$; 2) $A(8;\ -1)$, $B(-16;\ -11)$ bo‘lsa, AB kesma o‘rtasi – C nuqta kooordinatalarini toping.

YAKUNIY NAZORAT ISHI. XATOLAR USTIDA ISHLASH

1. To‘g‘ri to‘rtburchakning kichik tomoni 10 cm ga teng, diagonallari esa 60° li burchak ostida kesishadi. Shu to‘g‘ri to‘rtburchakning diagonallarini toping.

2. Uchburchakning tomonlari 11 cm, 7 cm va 10 cm ga teng. Berilgan uchburchak o‘rta chiziqlaridan hosil bo‘lgan uchburchakning perimetrini toping.

3. Uchburchakning tomonlari 21 cm, 72 cm va 75 cm ga teng. Shu uchburchakning yuzini toping.

4. Aylanaga tashqaridagi nuqtadan o‘tkazilgan ikki urinma orasidagi burchak 75° ga teng. Shu urinma tomonlarini o‘z ichiga olgan yoylarni toping.

5. $\vec{a}(2;-3)$ va $\vec{b}(-2;-3)$ vektorlar berilgan. $\vec{m}=\vec{a}-2\vec{b}$ vektorning koordinatalarini toping.

6- TEST O‘zingizni sinab ko‘ring!

1. To‘rtburchakning burchaklari o‘zaro $3:5:4:6$ nisbatda. To‘rtburchakning kichik burchagini toping.

A) 80°; B) 30°; D) 60°; E) 40°.

2. Qavariq to‘rtburchakning diagonallari uni nechta uchburchakka ajratadi?

A) 4; B) 5; D) 6; E) 8.

3. To‘g‘ri to‘rtburchakning eni 5 cm ga teng, bo‘yi undan 7 cm ga ortiq. To‘g‘ri to‘rtburchakning perimetrini hisoblang.

A) 32 cm; B) 34 cm; D) 24 cm; E) 26 cm.

4. Har bir ichki burchagi 162° bo‘lgan qavariq ko‘pburchakning nechta tomoni bor?

A) 18 ta; B) 20 ta; D) 15 ta; E) 12 ta.

5. Parallelogrammning ikki tomoni nisbati $3:7$ ga, uning perimetri esa 18 cm ga teng. Shu parallelogrammning kichik tomonini toping.

A) 2,7 cm; b) 3,4 cm; d) 5,4 cm; E) 4,5 cm.

6. To‘g‘ri to‘rtburchak shaklidagi maydonning eni 32 m. Agar maydonning yuzi 2 gektar bo‘lsa, uning bo‘yi necha metr bo‘ladi?

A) 610 m; B) 615 m; D) 625 m; E) 630 m.

7. Rombning balandligi 5 cm ga, diagonallarining ko‘paytmasi 80 cm^2 ga teng. Uning perimetrini toping.

A) 32 cm; B) 16 cm; D) 24 cm; E) 28 cm.

8. $\vec{a}(2;\ -3)$ va $\vec{b}(-2;\ -3)$ vektor berilgan. $\vec{m}=-\vec{a}+2\vec{b}$ vektorning koordinatalarini toping.

A) (–6; –3); B) (–3; 6); D) (–2; −9); E) (2; –3).

9. $\vec{a}(3;\ 2)$ va $\vec{b}(0;\ -1)$ vektor berilgan. $2\vec{a}-4\vec{b}$ vektorning modulini toping.

A) 10; B) 6; C) 8; D) 3.

Ilova. O‘tkir burchakli trigonometrik funksiyalarning qiymatlari jadvali

Graduslar	sinα 1° ≤ α ≤ 45°	tgα 1° ≤ α ≤ 45°	ctgα 1° ≤ α ≤ 45°	cosα 1° ≤ α ≤ 45°	Graduslar
1	≈ 0,0175	≈ 0,0175	≈ 57,290	≈ 0,9998	89
2	≈ 0,0349	≈ 0,0349	≈ 28,636	≈ 0,9994	88
3	≈ 0,0523	≈ 0,0524	≈ 19,081	≈ 0,9986	87
4	≈ 0,0698	≈ 0,0699	≈ 14,301	≈ 0,9976	86
5	≈ 0,0872	≈ 0,0875	≈ 11,430	≈ 0,9962	85
6	≈ 0,1045	≈ 0,1051	≈ 9,514	≈ 0,9945	84
7	≈ 0,1219	≈ 0,1228	≈ 8,144	≈ 0,9925	83
8	≈ 0,1392	≈ 0,1405	≈ 7,115	≈ 0,9903	82
9	≈ 0,1564	≈ 0,1584	≈ 6,314	≈ 0,9877	81
10	≈ 0,1736	≈ 0,1763	≈ 5,671	≈ 0,9848	80
11	≈ 0,1908	≈ 0,1944	≈ 5,145	≈ 0,9816	79
12	≈ 0,2079	≈ 0,2126	≈ 4,705	≈ 0,9781	78
13	≈ 0,2250	≈ 0,2309	≈ 4,331	≈ 0,9744	77
14	≈ 0,2419	≈ 0,2493	≈ 4,011	≈ 0,9703	76
15	≈ 0,2588	≈ 0,2679	≈ 3,732	≈ 0,9659	75
16	≈ 0,2756	≈ 0,2867	≈ 3,487	≈ 0,9613	74
17	≈ 0,2924	≈ 0,3057	≈ 3,271	≈ 0,9563	73
18	≈ 0,3090	≈ 0,3249	≈ 3,078	≈ 0,9511	72
19	≈ 0,3256	≈ 0,3443	≈ 2,904	≈ 0,9455	71
20	≈ 0,3420	≈ 0,3640	≈ 2,747	≈ 0,9397	70
21	≈ 0,3584	≈ 0,3839	≈ 2,605	≈ 0,9336	69
22	≈ 0,3746	≈ 0,4040	≈ 2,475	≈ 0,9272	68
23	≈ 0,3907	≈ 0,4245	≈ 2,356	≈ 0,9205	67
24	≈ 0,4067	≈ 0,4452	≈ 2,246	≈ 0,9135	66
25	≈ 0,4226	≈ 0,4663	≈ 2,145	≈ 0,9063	65
26	≈ 0,4384	≈ 0,4877	≈ 2,050	≈ 0,8988	64
27	≈ 0,4540	≈ 0,5095	≈ 1,963	≈ 0,8910	63
28	≈ 0,4695	≈ 0,5317	≈ 1,881	≈ 0,8829	62
29	≈ 0,4848	≈ 0,5543	≈ 1,804	≈ 0,8746	61
30	0,5000	≈ 0,5774	≈ 1,732	≈ 0,8660	60
31	≈ 0,5150	≈ 0,6009	≈ 1,664	≈ 0,8572	59
32	≈ 0,5299	≈ 0,6249	≈ 1,600	≈ 0,8480	58
33	≈ 0,5446	≈ 0,6494	≈ 1,540	≈ 0,8387	57
34	≈ 0,5592	≈ 0,6745	≈ 1,483	≈ 0,8290	56
35	≈ 0,5736	≈ 0,7002	≈ 1,428	≈ 0,8192	55
36	≈ 0,5878	≈ 0,7265	≈ 1,376	≈ 0,8090	54
37	≈ 0,6018	≈ 0,7536	≈ 1,327	≈ 0,7986	53
38	≈ 0,6157	≈ 0,7813	≈ 1,280	≈ 0,7880	52
39	≈ 0,6293	≈ 0,8098	≈ 1,235	≈ 0,7771	51
40	≈ 0,6428	≈ 0,8391	≈ 1,192	≈ 0,7660	50
41	≈ 0,6561	≈ 0,8693	≈ 1,150	≈ 0,7547	49
42	≈ 0,6691	≈ 0,9004	≈ 1,111	≈ 0,7431	48
43	≈ 0,6820	≈ 0,9325	≈ 1,072	≈ 0,7314	47
44	≈ 0,6947	≈ 0,9657	≈ 1,036	≈ 0,7193	46
45	≈ 0,7071	1,0000	1,000	≈ 0,7071	45
Graduslar	**cosα** **45° ≤ α ≤ 89°**	**ctgα** **45° ≤ α ≤ 89°**	**tgα** **45° ≤ α ≤ 89°**	**sinα** **45° ≤ α ≤ 89°**	**Graduslar**

JAVOBLAR

7- sinfda o‘tilganlarni takrorlash. 5. 9 dm. **7.** 3 cm. **9.** Ha, teng. **10.** 52°, 63°, 65°. **11.** 60°. **13.** 24°, 72°, 84°. **14.** Yo‘q, kelib chiqmaydi. **18.** 58°.

I bob. 1- mavzu. 2. 1) $n=8$; 2) $n=11$; 3) $n=24$. **4.** 80°. **5.** 1) $n=12$; 2) $n=36$; 3) $n=40$. **6.** $n=8$ ta. **7.** 1) $n=20$ ta; 2) $n=15$ ta; 3) $n=6$ ta. **9.** 1) $n=24$ ta; 2) $n=8$ ta; 3) $n=5$ ta. **10.** 36°, 72°, 108°, 144°. **2- mavzu. 2.** 25,5 cm, 50,5 cm. **3.** 1) 35°, 145°, 35°, 145°; 3) 85°, 105°, 85°, 105°. **4.** $P_{ABO}=20$ cm; $P_{BOC}=24$ cm. **5.** $AB=DC=16$ cm, $AD=BC=4$ cm. **3- mavzu. 2.** 1) Ha, to‘g‘ri. **3.** 32 cm. **7.** 26 cm yoki 28 cm. **8.** 45°, 135°, 135°, 45°. **9.** 26 cm. **4- mavzu. 2.** 1) 9 cm; 2) 7 cm. **3.** 12 cm. **4.** $AB=DC=4$ cm, $BC=AD=8$ cm. **6.** 1) $4+7<12$ – uchburchak tengsizligi bajarilmadi; yo‘q, bo‘lishi mumkin emas. **7.** 7 cm, 14 cm, 7 cm, 14 cm. **5–6- mavzular. 2.** 10 cm. **3.** $BP=12$ cm. **5.** 7 cm. **6.** 40°, 140°, 40°, 140°. **9.** 12 cm, 24 cm, 30 cm, 42 cm. **10.** 64 cm. **12.** 30 cm. **13.** 32 cm. **7–8- mavzular. 3.** 150°. **4.** 23 cm. **6.** 27 cm, 11 cm. **7.** 20 cm, 14 cm. **10.** 90°, 90°, 100°, 80°. **12.** 70 cm. **9- mavzu. 3.** $AC=5$ cm. **4.** $OB_1=3{,}2$ cm, $OB_2=4{,}8$ cm, $OB_3=6{,}4$ cm. **6.** 2) 19 cm. **8.** $x=4$. **9.** $OB_1=9$ cm, $OB_2=13{,}5$ cm, $OB_3=18$ cm. **10–11- mavzular. 2.** 2,5 cm, 3,5 cm, 5,5 cm. **4.** 22 cm, 10 cm. **6.** 2) 15 cm. **9.** 24 cm, 12 cm. **10.** 3 cm. **11.** 30 cm, 10 cm. **12.** 12 cm.

II bob. 15- mavzu. 2. a) $\cos\alpha$; b) $\operatorname{tg}\alpha$; d) $\sin\alpha$; e) $\operatorname{ctg}\alpha$. **4.** a) Ha, chunki $0{,}98<1$; b) yo‘q, chunki $\sqrt{2}>1$; d) ha, chunki $\sqrt{5}-2<1$. **5.** $ML=24$, $MN=25$. **6.** $\sin M=\frac{5}{13}$, $\cos M=\frac{12}{13}$, $\operatorname{tg}M=\frac{5}{12}$, $\operatorname{ctg}M=\frac{12}{5}$. **16- mavzu. 2.** a) To‘g‘ri, chunki $a=c\sin\alpha$; d) noto‘g‘ri, chunki $c=\frac{a}{\sin\alpha}$. **3.** Ha, chunki tangensning qiymati istalgan musbat son bo‘ladi. **4.** 1) 16 cm; 2) 50 cm. **6.** 16 cm. **7.** 5 cm. **8.** 50 cm. **17- mavzu. 2.** 1) 13; 2) 9; 3) 2,5. **3.** 1) 40 cm; 2) 100 cm. **4.** $x=\sqrt{3}$; $y=\sqrt{2}$. **5.** 1) 0,5; 2) $4\sqrt{2}$; 3) 0,8; 4) 1,5. **18- mavzu. 2.** 1) Yo‘q, chunki $121+49\neq 289$; 2) ha, chunki $3^2+1{,}6^2=3{,}4^2$, $11{,}56=11{,}56$. **5.** Ikkita yechimga ega. **6.** 1) Ha, chunki $12^2+35^2=37^2$; 2) yo‘q, chunki $11^2+20^2\neq 25^2$. **19- mavzu. 1.** 1) 9,6 cm, 9,6 cm, 8 cm. **2.** $\frac{2\sqrt{3}}{3}h$. **3.** 1) $h_b=\frac{12}{7}\sqrt{6}$ cm; 2) $h_c=11{,}2$ dm; 3) $h_b=6{,}72$ cm. **4.** 1) $h=6\sqrt{3}$ cm. **5.** $h_a=\frac{15}{4}\sqrt{7}$ cm; $h_c=\frac{5}{2}\sqrt{7}$ cm. **7.** $h_a=\frac{3}{2}\sqrt{15}$ cm. **20–21- mavzular. 2.** 1) $\frac{5}{13}$; 2,4; $\frac{5}{12}$. **4.** 1) 2; 2) 1; 3) 1. **5.** 1) $\operatorname{ctg}^2\alpha$; 2) $\operatorname{tg}\alpha$. **7.** 1) $\frac{2}{\sqrt{5}}$; $\frac{1}{\sqrt{5}}$; $\frac{1}{2}$. **9.** 1) $\cos\alpha=\frac{15}{17}$; $\operatorname{tg}\alpha=\frac{8}{15}$; $\operatorname{ctg}\alpha=\frac{15}{8}$. **12.** 1) $\sin^2\alpha$; 2) $\cos^3\alpha$. **14.** 1) $\sin^2\alpha$; 2) $\sin^3\alpha$. **22- mavzu. 2.** 1) $x\approx 50°$; 2) $x\approx 14°$; 3) $x\approx 34°$; 4) $x\approx 74°$. **3.** 1) $\sin B=0{,}6$; $\cos B=0{,}8$. **5.** $\cos A=0{,}5$; $\operatorname{tg}A=\sqrt{3}$. **7.** 1) $\sin\alpha=0{,}6$; $\cos\alpha=0{,}8$. **8.** 1) $\sin\alpha$; 2) $\cos^2\alpha$. **23- mavzu. 1.** 1) 1,5; 3) 0,5. **3.** $\frac{32\sqrt{3}}{3}$; $\frac{16\sqrt{3}}{3}$. **4.** 12; 6. **5.** 1) $\sin^2\alpha$; 2) $\sin^2\alpha$. **7.** 2. **8.** 1) 0,5; 2) 0,5; 3) 1. **24- mavzu. 1.** a) 1) $\approx 0{,}0523$; 2) $\approx 0{,}3584$; 3) $\approx 0{,}7660$; 4) $\approx 0{,}6428$; e) 1) $\approx 5{,}671$; 2) $\approx 1{,}732$; 3) $\approx 0{,}2679$; 4) $\approx 11{,}430$. **2.** b) 1) $\approx 42°$; 2) $\approx 50°$; 3) $\approx 87°$; d) 1) $\approx 25°$; 2) $\approx 85°$; 3) $\approx 10°$. **4.** 1. **6.** 1) 1; 2) 0. **7.** 1) $\approx 0{,}9397$; 4) $\approx 23{,}078$. **8.** $x\approx 8°$. **25- mavzu. 1.** 14 cm. **2.** 45°, 45°. **3.** $a\approx 6{,}691$; $b\approx 7{,}431$; $\beta\approx 48°$. **5.** $\cos^2\alpha$. **7.** $a=4$ cm; $b=4\sqrt{3}$ cm, $\beta=60°$. **26- mavzu. 1.** $b=9$ cm, $\alpha=\beta=45°$. **2.** $c=12$ cm, $\alpha=60°$, $\beta=30°$. **5.** 0. **7.** $c=26$ cm. **27- mavzu. 3.** $a=7$ cm, $\alpha=\beta=45°$. **4.** $a=6\sqrt{3}$ cm, $b=6$ cm, $\beta=30°$. **5.** $a=5$ cm (5- rasm); $AC=2\sqrt{13}$ cm, $BC=3\sqrt{13}$ cm (6- rasm). **6.** 168 cm.

III bob. 31- mavzu. 3. 1) III chorak; 2) II chorak; 3) IV chorak; 4) I chorak. **4.** 1) (–10; –1); 2) (0; –5,5); 3) (–2; 1). **5.** 1) $B(-1;5)$. **8.** 1) $D(3;0)$; 2) $D(4;5)$. **32–33- mavzular. 2.** 1) 10; 2) 17;

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3) 13. **3.** 1) $x_1 = -2$; $x_2 = 6$. **4.** $P = 16$. **5.** 1) $(x-7)^2 + (y-11)^2 = 25$; 2) $(x+2)^2 + (y-3)^2 = 1$. **6.** 1) (2; 5), $R = 7$; 2) (−1; 5), $R = 2$. **7.** 1) $C(3; -1)$, $R = 4$; 2) $C(0; -5)$, $R = 1$. **8.** 1) Teng yonli. **9.** 1) $(x-9)^2 + (y-4)^2 = 49$; 2) $(x+3)^2 + (y+4)^2 = 4$. **10.** 1) $C(7; -2)$, $R = 5$; 2) $C(4; 0)$, $R = 1$. **11.** 1) (5; −12) va (5; 12); 2) (−5; −12) va (5; −12). **34- mavzu. 3.** 1) $2x - y + 5 = 0$; 2) $x + y - 7 = 0$; 3) $3x - 2y + 2 = 0$. **4.** $c = -3$. **5.** $a = b = \frac{1}{3}$. **6.** 1) (0; −1,5) va (−3; 0); 2) (0; 3) va (4; 0); 3) (0; −5) va (2,5; 0). **9.** $x + 1 = 0$, $x - 3y - 8 = 0$, $x - y = 0$. **35- mavzu. 2.** 1) $\overrightarrow{DC} \uparrow\uparrow \overrightarrow{AB}$; 2) $\overrightarrow{AO} \uparrow\uparrow \overrightarrow{OC}$; 3) $\overrightarrow{CB} \uparrow\downarrow \overrightarrow{AD}$ va $\overrightarrow{DA} \uparrow\downarrow \overrightarrow{AD}$; 5) $\overrightarrow{DC} = \overrightarrow{AB}$; 6) $\overrightarrow{AO} = \overrightarrow{OC}$; 7) $\overrightarrow{DO} = \overrightarrow{OB}$. **36−37- mavzular. 5.** Ha, bajariladi. **6.** $|\overrightarrow{AO}| = 16$ cm. **7.** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AD} + \overrightarrow{DC} \Rightarrow \overrightarrow{AC} = \overrightarrow{AC}$. **9.** $\overrightarrow{AB} = -\vec{b}$; $\overrightarrow{BC} = -\vec{a} + \vec{b}$; $\overrightarrow{DA} = \vec{a} - \vec{b}$. **10.** $\overrightarrow{BF} = -2\vec{a} + \vec{b}$; $\overrightarrow{EC} = -\vec{a} + 2\vec{b}$; $\overrightarrow{EF} = -\vec{a} + \vec{b}$; $\overrightarrow{BC} = -2\vec{a} + 2\vec{b}$. **38−39- mavzular. 4.** $\overrightarrow{OA} = -\frac{1}{2}\vec{a} - \frac{1}{2}\vec{b}$; $\overrightarrow{AK} = \frac{1}{2}\vec{a} + \vec{b}$. **5.** 1) $\overrightarrow{AC} = \overrightarrow{CB}$; 2) $\overrightarrow{AB} = 2\overrightarrow{CB}$; 3) $\overrightarrow{AC} = -\frac{1}{2}\overrightarrow{BA}$. **7.** 1) (4; 5); 2) (−1; 4); 3) (0; 0). **8.** 1) 25; 2) 5; 3) 3. **9.** 1) (1; −2); 2) $(2m; 2n)$. **11.** $m = 7$. **12.** $B(-2; -11)$. **40- mavzu. 2.** 1) (−3; 4); 2) (−5; 12). **3.** 1) (−4; 10); 2) (0; 2); 4) (4; −10). **4.** 1) (3; 6); 2) (5; 3); 3) (−4; −3). **5.** 1) (6; 3); 2) (−6; 3); 3) (−2; 15). **6.** 1) $\vec{c}(-4; -4)$; 2) $\vec{c}(8; 6)$. **7.** 1) $\vec{c}(-12; 6)$; 2) $\vec{c}(-11; 8)$. **8.** 1) $\vec{c}(-2; -1)$; 2) $\vec{c}(2; -13)$. **41- mavzu. 1.** $CC_1 = 2$. **2.** $\overrightarrow{KC} = \overrightarrow{AB} + \frac{1}{2}\overrightarrow{AD}$. **3.** (5; 12). **4.** $B(5; 5)$, $D(1; -1)$. **5.** $B(-5; 11)$. **8.** $\overrightarrow{AN} = \frac{1}{2}\overrightarrow{AB} + \overrightarrow{AD}$.

IV bob. 45- mavzu. 2. 2) 0,0225 dm²; 5) 6,25 m². **6.** 1) 4 marta ortadi; 2) 9 marta kamayadi; 3) 28 cm² ga ortadi. **11.** 2) 3,6 dm; 3) 68 mm; 5) 80 dm. **13.** 359,12 ming km². **46−47- mavzular. 2.** 1) $P = 65{,}8$ cm, $S = 87$ cm²; 3) $P = 7{,}4$ dm, $S = 3$ dm². **4.** $S = 5000$ m². **5.** 1) $P = 126$ cm, $S = 920$ cm². **8.** 12 cm. **9.** 1) $S = 500$ cm²; 2) $a = 12$ cm; 3) $h_a = 5$ cm. **11.** 1) 1,6 marta ortadi; 2) 6,25 marta kamayadi. **13.** 2) 280 cm²; 4) 4,8 dm². **14.** $P = 42$ cm. **15.** $S = 280$ cm². **48- mavzu. 2.** 1) 14 cm²; 2) 150 cm². **3.** 4 cm. **4.** 5 : 1. **8.** 1) 756 cm²; 2) 84 cm²; 3) 192 cm². **10.** 60 cm. **11.** 7,5 dm². **49−50- mavzular. 2.** 1) 32 cm. **3.** 1) 512 cm²; 2) 1,62 dm². **4.** 12 cm. **5.** 5 cm. **7.** 1) 1,35 dm²; 2) 180 cm²; 3) 8 cm². **8.** 1) 87 cm²; 2) 14 cm. **10.** 1) $0{,}5a^2$ kv. birl. **11.** 360 cm². **12.** 1) 2,45 dm²; 2) 238 cm²; 3) 31,5 cm². **14.** 1) 1,44 m². **15.** 1) 140 cm². **51-mavzu. 1.** 2125 kv. birl. **2.** $(a + b) \cdot c$. **3.** 144 cm². **5.** 16 kv. birl. **6.** 1) 20,8 km; 2) 8 km.

V bob. 55- mavzu. 3. AB va BD kesuvchi. **4.** 25 cm. **5.** 1) $R = 5$ cm; 2) $R < 5$ cm; 3) $R > 5$ cm. **8.** CD. **56- mavzu. 2.** 1) Aylanalar ichki tomondan bir-biriga urinadi; 2) umumiy nuqtaga ega emas, biri ikkinchisining ichida yotadi. **3.** 1) 10 cm; 2) 2 cm. **6.** 1) 144°; 2) 96°; 3) 210°; 4) 200°; 5) 260°; 6) 306°; 7) 276°. **7.** 1) 160°, 200°; 2) 80°, 280°. **8.** 70°. **9.** 1) 72°; 2) 60°; 3) 40°; 4) 36°; 5) 30°. **10.** 1) 15,6 cm; 2) 21 cm; 3) 1,6 dm. **57- mavzu. 3.** $AC = 10$ cm. **4.** 1) $\angle ACB = 44°$; 3) $\angle AEP = 100°$. **5.** 36°, 60°, 84°. **6.** 1) 100° yoki 80°; 2) 126° yoki 54°. **7.** $\angle BAC = 20°$. **8.** 100°. **58- mavzu. 3.** 220°. **4.** *a*) $x = 45°$; *b*) $x = 30°$; *d*) $x = 90°$. **5.** 30°. **6.** 1) $\angle ABC = 20°$; 2) $\angle ABC = 60°$; 3) $\angle ABC = 36°$; 4) $\angle ABC = 54°$; 5) $\angle ABC = 36°$. **7.** 1) 100°, 40°, 40°. **8.** 1) 144°; 2) 120°; 3) 40°; 4) 72°. **9.** 1) 128°; 3) 76°. **59- mavzu. 3.** 4 cm. **6.** 8 cm. **9.** 10 cm. **11.** 90°.

VI bob. 1. 38°, 158°, 44°, 120°. **2.** 52 cm. **3.** 48 cm. **4.** 1) 12 cm²; 4,8 cm; 2) 108 cm²; 14,4 cm. **6.** 44 cm. **7.** 10 cm. **9.** 3 cm. **10.** 180 cm². **13.** 60 dm, 14,4 dm. **14.** 30°, 150°, 150°, 30°. **17.** 9 % ga kamayadi. **19.** 96 cm. **20.** 28 cm. **22.** 1) (1; −1); 2) (−2; 2). **23.** $\overrightarrow{BO} = \frac{1}{2}\vec{a} + \frac{1}{4}\vec{b}$ **25.** 60°. **26.** 144°. **27.** 72 cm. **28.** 12 cm, 4 cm. **29.** 1) 4 cm² ga kamayadi; 2) 128 cm² ga ortadi.

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TOXTAXODJAYEVA MUYASSAR ABDUVAHOBOVNA

GEOMETRIYA

Umumiy o‘rta ta’lim maktablarining 8- sinfi uchun darslik

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Ijaraga berilgan darslik holatini ko‘rsatuvchi jadval

T/r	O‘quvchining ismi, familiyasi	O‘quv yili	Darslikning olingandagi holati	Sinf rahbarining imzosi	Darslikning topshirilgandagi holati	Sinf rahbarining imzosi
1						
2						
3						
4						
5						
6						

Darslik ijaraga berilib, o‘quv yili yakunida qaytarib olinganda yuqoridagi jadval sinf rahbari tomonidan quyidagi baholash mezonlariga asosan to‘ldiriladi:

Yangi	Darslikning birinchi marotaba foydalanishga berilgandagi holati.
Yaxshi	Muqova butun, darslikning asosiy qismidan ajralmagan. Barcha varaqlari mavjud, yirtilmagan, ko‘chmagan, betlarida yozuv va chiziqlar yo‘q.
Qoniqarli	Muqova ezilgan, birmuncha chizilib, chetlari yedirilgan, darslikning asosiy qismidan ajralish holati bor, foydalanuvchi tomonidan qoniqarli ta’mirlangan. Ko‘chgan varaqlari qayta ta’mirlangan, ayrim betlariga chizilgan.
Qoniqarsiz	Muqova chizilgan, yirtilgan, asosiy qismidan ajralgan yoki butunlay yo‘q, qoniqarsiz ta’mirlangan. Betlari yirtilgan, varaqlari yetishmaydi, chizib, bo‘yab tashlangan. Darslikni tiklab bo‘lmaydi.